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Time Reversal and Fractional Fourier Transform-Based Method for LFM Signal Detection in Underwater Multi-Path Channel

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Abstract: Fractional Fourier transform (FrFT) is a useful tool to detect linear frequency modulated (LFM) signal. However, the detection performance of the FrFT-based method will deteriorate drastically in underwater multi-path environment. This paper proposes a novel method based on time-reversal and fractional Fourier transform (TR-FrFT) to solve this problem. We make use of the focusing ability of time-reversal to mitigate the influence of multi-path, and then improve the detection performance of FrFT. Simulated results show that, compared to FrFT, the difference between peak value and maximum pseudo-peak value of the signal processed by TR-FrFT is improved by 8.75 dB. Lake experiments results indicate that, the difference between peak value and maximum pseudo-peak value of the signal processed by TR-FrFT is improved by 7.6 dB. The detection performance curves of FrFT and TR-FrFT detectors with simulated data and lake experiments data verify the effectiveness of proposed method.

Keywords: linear frequency modulation; time-reversal; fractional Fourier transform; detection method; underwater multi-path environment



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1. Introduction

The linear frequency modulated (LFM) signal is a broadband signal and its bandwidth utilization is high. This characteristic makes the LFM signal receive more attention in the field of underwater acoustic communication. For example, in the orthogonal chirp division multiplexing (OCDM) communication system [1], a bank of orthogonal LFM signals acts as the carrier to modulate transmission signal. In addition, LFM signal has the lower side-lobe after pulse compression, and its Doppler frequency is not sensitive. Thus, the LFM signal is often applied in some detection or ranging systems [2,3] by using matched filtering to compress the received signal. It is easy to detect an LFM signal in a high signal-to-noise environment. However, the detection of the LFM signal in the presence of high level noise is a tough task.

Fractional Fourier transform (FrFT) is a generalization of Fourier transform (FT). In recent years, it has been attracting more attention in the signal processing field [4–7]. In order to practically realize the FrFT-based engineering application, research on discrete fractional Fourier transform (DFrFT) is particularly needed. There are three main methods for calculating the DFrFT, namely, linear combination-type DFrFT [8], eigenvector decomposition-type DFrFT [9] and sampling-type FrFT [10,11]. The simplest definition of DFrFT is linear combination-type DFrFT. The computational complexity of linear combination-type DFrFT approach is $O(N \log N)$, which is same as the fast Fourier transform (FFT). However, this approach produces an error deviation in terms of continuous FrFT. Although the eigenvector decomposition-type DFrFT has a smaller deviation error with respect to the continuous

FrFT, the computational complexity of this method reaches $O(N^2)$. The sampling-type FrFT provides the best approximation and has a fast computing speed. For example, the Reference [11] proposed the Pei sampling-type approach. It requires two chirp operations and one FFT operation. The computational complexity of the Pei sampling-type approach is slightly larger than FFT. However, compared with other DFrFT approaches, the sampling-type FrFT cannot perform the inverse operation, which will limit its application scope. Designers require one to select the proper method to numerically calculate DFrFT according to demands. The spectrum of infinite-length LFM signal in the optimal order fractional Fourier domain is a pulse function [12]. While the spectrum of finite-length LFM signal is a sinc function [6], it means that LFM signal can be easily detected in fractional Fourier domain. Several methods based on FrFT have been proposed to detect LFM signal with low signal-to-noise ratio. The authors in [13] compared the performance of FrFT detector and Fourier transform (FT) detector. In [14], a lower computational complexity LFM signal detector based on the integration of the 4th-power modulus of the fractional Fourier transform is proposed. An adaptive FrFT detection algorithm is shown in [15]. It combined statistic-based and FrFT-based method to detect moving target with a low speed in heavy sea clutter. The signal to be detected at the receiving end is simply modeled as a LFM signal corrupted by noise. However, in practice, the received signal is propagated in the channel. Hence, the impacts of the channel, especially wireless channel, cannot be ignored. For example, in the underwater acoustic channel [16], due to multiple reflections from boundaries or scatters, the received signal can be viewed as the superposition of a number of amplitude-weighted and delayed replicas of the original emitted signal. In this scenario, the FrFT of LFM signal has multiple peaks [17] in the optimal order fractional Fourier domain. The presence of spurious peaks indicates that the received signal has been expanded. For FrFT detector, it is hard to decide whether the peak value is produced by noise or multi-path when the signal-to-noise ratio is low. The performance of these LFM detection methods drastically declines in this condition.

As discussed above, channel multi-path negatively affects the performance of traditional detector. Time-reversal (TR) provides an opportunity to utilize multi-path. TR can make the extended signal focusing and lower the effects of multi-path. In ultrasound and acoustic domain, Fink et al. have demonstrated that TR has the ability of super-resolution focusing. In their work [18,19], they showed that the resolution is only limited by the correlation of channel passed by transmitted signal, while it is no longer dependent on the sensor aperture size. They also validated this theory by acoustic and seismic imaging experiments. Furthermore, they proposed TR cavity [20], iterative TR [21], TR operator decomposition (DORT) [22] and other methods [23,24] for target detection. Kuperman et al. verified the focusing ability of TR in ocean experiments [25–27]. It is worth noting that TR focusing ability is based on the reciprocity of the channel, while long time delays will produce large mismatches that lead to TR focusing degradation [28]. In the electromagnetic field, TR focusing has been confirmed in [29]. Moura and Jin derived TR detectors, analyzed the performance and verified their theory by real electromagnetic data [30,31]. In addition, TR focusing has also acquired considerable attention and been applied in other fields, such as communication [32,33], positioning [34,35] and imaging [36].

In this paper, we propose a TR-FrFT-based method for LFM signal detection in the underwater multi-path environment. In our work, TR can be regarded as a pre-filter to mitigate the effect of multi-path, and then the signal is transformed into the optimal order fractional Fourier domain to be detected. The main contribution of this paper is summarized as:

- (1) We propose a novel method for LFM signal detection in underwater multi-path environment. The new method can achieve energy focusing on the optimal order fractional Fourier domain and realize low signal to noise ratio detection.
- (2) Simulations and lake experiments were conducted and verified the effectiveness of the proposed method for LFM signal detection.

(3) Compared with the FrFT and matched filter, the proposed method has superior detection performance in the underwater multi-path environment.

The rest of this paper is organized as follows. In Section 2, after providing preliminaries of FrFT and the related detection method, problem statement is introduced. In Section 3, firstly, the theory about time-reversal is briefly presented. Secondly, we devise the proposed detection method. Simulations and lake experiments compared with other methods are provided in Section 4. Section 5 concludes the work.

2. LFM Signal Detection Based on FrFT

2.1. Definition and Properties of FrFT

The FrFT of signal $x(t)$ with angle α is defined by

$$X_p(u) = \{F^p[x(t)]\}(u) = \int_{-\infty}^{+\infty} x(t)K_p(t, u)dt, \quad (1)$$

where $K_p(t, u)$ is the

$$K_p(t, u) = \begin{cases} \sqrt{(1 - j \cot \alpha)} \exp j\pi [(t^2 + u^2) \cot \alpha - 2tu \csc \alpha], & \alpha \neq n\pi, \\ \delta(t - u), & \alpha = 2n\pi, \\ \delta(t + u), & \alpha = (2n + 1)\pi, \end{cases} \quad (2)$$

where α is the rotation angle and represents the α -th power of the ordinary Fourier transform operator, p is the fractional order, $\alpha = p\pi/2$, $F^p[\cdot]$ denotes the fractional Fourier transform operator, t represents the integral variable and the variable u has different physical meanings according to different fractional order p . When the rotation angle $\alpha = \pi/2$ (the order $p = 1$), the transformation kernel is

$$K_1(t, u) = e^{-j2\pi ut} \quad (3)$$

The FrFT of $x(t)$ with $\alpha = \pi/2$ is

$$X_1(t, u) = \int_{-\infty}^{\infty} e^{-j2\pi ut} x(t)dt. \quad (4)$$

$X_1(t)$ is the Fourier transform of $x(t)$. Thus, FrFT is a generalization of FT. The inverse FrFT is

$$x(t) = F^{-p}[X_p(u)] = \int_{-\infty}^{\infty} X_p(u)K_{-p}(t, u)du. \quad (5)$$

Equation (5) indicates that signal $x(t)$ can be characterized by a set of orthogonal basis functions $K_{-p}(t, u)$ with weight coefficient $X_p(u)$. These basis functions are complex exponential functions of LFM.

In the following, we list some useful properties of FrFT, which will be applied to the mathematical derivation of proposed detection method.

(1) Linear property:

$$\{F^p[ax(t) + by(t)]\}(u) = aX_p(u) + bY_p(u) \quad (6)$$

(2) Time shift property:

$$\{F^p[x(t - \tau)]\}(u) = X_p(u - \tau \cos \alpha) \exp(j\pi \tau^2 \sin \alpha \cos \alpha - j2\pi u \tau \sin \alpha) \quad (7)$$

In (6), a and b are arbitrary constants. Moreover, τ is the time shift value. More properties of FrFT refer to [6].

2.2. FrFT Detection Method for LFM Signal

The LFM signal [37] $s(t)$ can be expressed as

$$s(t) = A \exp(j\pi\mu t^2 + j2\pi f_0 t + \varphi), -T/2 \leq t \leq T/2, \quad (8)$$

where A is the signal amplitude, μ is the chirp rate, f_0 is the starting frequency, φ is the initial phase, T is the time width of LFM signal.

Without loss of generalization, both the starting frequency f_0 and the initial phase φ in (8) are set to 0, and (8) is reduced to

$$s(t) = A \exp(j\pi\mu t^2). \quad (9)$$

Substituting (9) into (1), we obtain the FrFT of the signal $s(t)$:

$$\begin{aligned} S_p(u) &= \{F^p[s(t)]\}(u) \\ &= A\sqrt{1-j\cot\alpha} \exp(j\pi u^2 \cot\alpha) \int_{-\infty}^{+\infty} \exp(j\pi\mu t^2) \exp(j\pi t^2 \cot\alpha - j2\pi tu \csc\alpha) dt. \end{aligned} \quad (10)$$

For an infinite time width LFM signal, when $\alpha = \operatorname{arccot}(-\mu)$, Equation (10) can be simplified as [38]

$$\begin{aligned} S_p(u) &= \{F^p[s(t)]\}(u) \\ &= A\sqrt{1-j\cot\alpha} \exp(j\pi u^2 \cot\alpha) \delta(u \csc\alpha), \end{aligned} \quad (11)$$

where δ is Kronecker delta function. Equation (10) is a pulse function of u .

For a determined time width $[-T/2, T/2]$ signal, when $\alpha = \operatorname{arccot}(-\mu)$, Equation (10) can be simplified as [6]

$$\begin{aligned} S_p(u) &= \{F^p[x(t)]\}(u) \\ &= A\sqrt{1-j\cot\alpha} \exp(j\pi u^2 \cot\alpha) T \operatorname{sinc}[\pi T \csc(\alpha)u]. \end{aligned} \quad (12)$$

When $T \rightarrow +\infty$, $T \operatorname{sinc}[\pi T \csc(\alpha)u] = \delta(u \csc\alpha)$, it means that (12) is equivalent to (11). The result shows that the finite-length LFM signal in the FrFT domain follows the sinc function distribution under the condition that the rotation angle is $\alpha_{opt} = \operatorname{arccot}(-\mu)$, which is called the optimal rotation angle. This indicates that the LFM signal has its energy concentrated in the optimal order fractional Fourier domain, where the optimal order $p_{opt} = 2\alpha_{opt}/\pi$. We can use this property to detect LFM signal in the optimal order domain. To this end, the objective is to find the optimal order p_{opt} , i.e.,

$$\begin{aligned} p_{opt} &= \arg \max_p |S_p(u)|^2 \quad \forall u, \\ &\text{subject to } 0 \leq p \leq 2 \end{aligned} \quad (13)$$

which is a non-convex problem, we can employ the brutal searching method to find the optimal solution. However, it will cost heavy computations. In this paper, we adopt an existing coarse-to-fine scanning method [39], as shown in Algorithm 1.

Algorithm 1 The optimal transform order corresponding to the maximum value in FrFT domain

- 1: Initialization: setting the transform order range $\mathcal{P} \in [0, 2]$; setting the scanning space $s_s = 0.1$; setting the initial iteration number $Num = 0$;
- 2: **while** $I > Num$ **do**
- 3: Compute $p' = \max_{p' \in \mathcal{P}} \left\{ \max |S_p(u)|^2 \right\}$;
- 4: Update: $\mathcal{P} = [\max(0, p' - s_s/2), \min(p' + s_s/2, 2)]$; $s_s = s_s/10$; $Num = Num + 1$;
- 5: **return** result

Output: The optimal transform order $p_{opt} = p'$.

2.3. Problem Statement

In this section, we consider the FrFT detection problem for LFM signal in the underwater multi-path channel. The schematic of FrFT detection is shown in Figure 1. Considering the typical ray model [40], the underwater acoustic channel associated with the target can be expressed as

$$h(t) = \sum_{i=1}^M a_i \delta(t - \tau_i), \quad (14)$$

where a_i and τ_i are the amplitude and the delay of the i th path, respectively. M is the total number of paths.

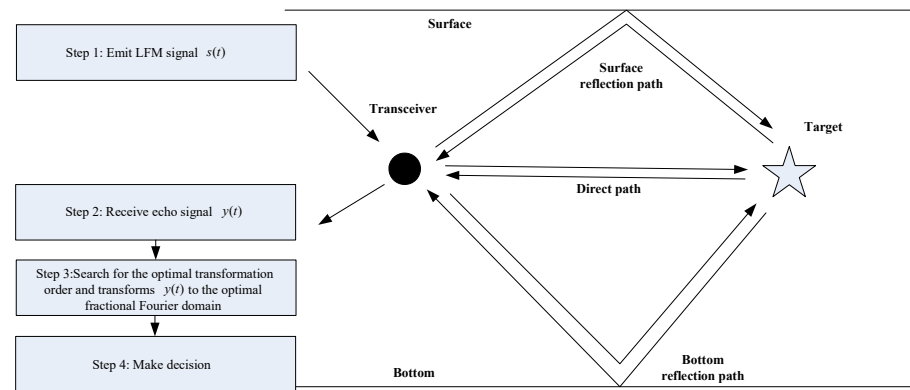


Figure 1. The schematic of Fractional Fourier transform (FrFT) detection.

The traditional FrFT detection method has four steps.

Step 1: the transceiver emits LFM signal $s(t)$ to illuminate the target.

Step 2: the transceiver receives the echo signal $y(t)$.

$$\begin{aligned} y(t) &= s(t) * h(t) + n_1(t) \\ &= \sum_{i=1}^M a_i s(t - \tau_i) + n_1(t), \end{aligned} \quad (15)$$

where the symbol $*$ denotes convolution and $n_1(t)$ is the additive noise.

Step 3: the processor searches for the optimal transformation order and transforms $y(t)$ to the optimal fractional Fourier domain.

Step 4: the processor decides whether there is a target.

Substituting (15) into (1) with $x(t)$ set to $y(t)$, it can be verified that the FrFT of the received signal $Y_p(u)$ is given by

$$Y_p(u) = \sum_{i=1}^M A_i S_p(u - u_i) + N_{1p}(u), \quad (16)$$

with

$$A_i = a_i e^{j\pi\tau_i^2 \sin\alpha \cos\alpha - j2\pi u \tau_i \sin\alpha}, i = 1, 2, \dots, M, \\ u_i = \tau_i \cos\alpha, i = 1, 2, \dots, M, \quad (17)$$

and $S_p(u)$ is the FrFT form of the emitted signal $s(t)$ with transform order p . $N_{1p}(u)$ is the FrFT form of the additive noise $n_1(t)$ with transform order p .

The echo signal $y(t)$ is extended in the fractional Fourier domain under the influence of multipath. When the signal-to-noise ratio (SNR) is high, the main lobe (corresponding to the direct path) and side lobes (corresponding to other paths) are clearly distinguishable, it is then easy to be detected. When the SNR is low, in the fractional Fourier domain, the noise will be disturbed and coupled with the spectrum of echo signal $y(t)$, and then the detection performance of the FrFT detector will be seriously deteriorated.

3. TR-FrFT Method for LFM Signal Detection in Underwater Multi-Path Environment

3.1. The Basic TR Processing Method

In this section, we briefly review the basic of TR processing. The method is composed of three phases. During phase 1, the source emits a LFM signal $s(t)$ passing through the underwater acoustic channel $h_1(t)$ and received by the receiver. The expression of received signal $y(t)$ is referred to (15). In the second phase, the sonar system time-reverses $y(t)$ in time domain and obtains $y(-t)$. Next, the signal $y(-t)$ is normalized to the power of the original signal $s(t)$ by a compensation factor a

$$a = \sqrt{\frac{P\{s(t)\}}{P\{y(t)\}}}, \quad (18)$$

where $P\{s(t)\} = \frac{1}{T_s} \int_{T_s} |s(t)|^2 dt$ is the power of $s(t)$ during the observation time T_s . Moreover, $P\{y(t)\} = \frac{1}{T_y} \int_{T_y} |y(t)|^2 dt$ is the power of $y(t)$ during the observation time T_y .

After time reversal and power compensation, the re-emitted signal $y_{tr}(t)$ is

$$y_{tr}(t) = ay(-t) \\ = as(-t) * h_1(-t) + an_1(-t). \quad (19)$$

During phase 3, $y_{tr}(t)$ is transmitted back to the underwater acoustic channel $h_2(t)$ again. The resulting signal $z(t)$ is

$$z(t) = y(-t) * h_2(t) + n_2(t) \\ = as(-t) * h_1(-t) * h_2(t) + an_1(-t) * h_2(t) + n_2(t). \quad (20)$$

where $n_2(t)$ is the additive noise in phase 3.

If the underwater acoustic channel is time invariant in the TR processing interval, we have $h_1(t) = h_2(t) = h(t)$. Equation (20) can be simplified as

$$z(t) = s(-t) * a \int |h(t)|^2 dt + an_1(-t) * h(t) + n_2(t). \quad (21)$$

Since the term $a \int |h(t)|^2 dt$ is a constant, we can acquire $z(t) \propto s(-t)$ from (21). Clearly, it means that the time extended signal can be compressed and focusing after TR processing.

3.2. TR-FrFT Method

As described in Section 2.3, in an underwater multi-path environment, the result of received signal is extended in FrFT domain. The false peaks will confuse the detector. Considering time-reversal processing has the characteristics of energy focusing and compression, we propose a novel method based on TR-FrFT to detect LFM signal in this complex environment. A schematic illustration of the novel method is shown in Figure 2.

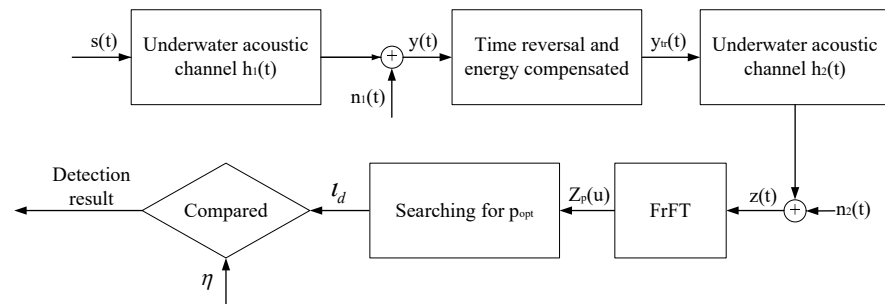


Figure 2. Schematic illustration of the novel method based on TR-FrFT used to detect LFM signal in an underwater multi-path environment.

The TR method is presented in Section 3.1. Substituting (14) into (20), $z(t)$ can be re-written as

$$z(t) = a \sum_{i=1}^M a_i^2 s(-t) + a \sum_{i=1}^M \sum_{\substack{l=1 \\ i \neq l}}^M a_i a_l s(-t - \tau_{li}) + a \sum_{i=1}^M a_i^2 n_1(-t - \tau_i) + n_2(t), \quad (22)$$

where $\tau_{li} = \tau_l - \tau_i$ is the delay difference between the l -th path and i -th path.

Substituting (22) into (1) with $x(t)$ set to $z(t)$, the FrFT of $z(t)$ is

$$\begin{aligned} Z_p(u) &= \{F^p[z(t)]\}(u) \\ &= a \sum_{i=1}^M a_i^2 S_p(-u) + a \sum_{i=1}^M \sum_{\substack{l=1 \\ i \neq l}}^M a_i a_l B_{\tau_{li}} S_p(-u + u_{\tau_{li}}) + a \sum_{i=1}^M a_i^2 C_{\tau_i} N_{1p}(u + u_{\tau_i}) + N_{2p}(u), \end{aligned} \quad (23)$$

with

$$\begin{aligned} B_{\tau_{li}} &= e^{j\pi\tau_{li}^2 \sin \alpha \cos \alpha - j2\pi u \tau_{li} \sin \alpha}, & i = 1, 2, \dots, M, & l = 1, 2, \dots, M, \\ C_{\tau_i} &= e^{j\pi\tau_i^2 \sin \alpha \cos \alpha - j2\pi u \tau_i \sin \alpha}, & i = 1, 2, \dots, M, & l = 1, 2, \dots, M, \\ u_{\tau_{li}} &= \tau_{li} \cos \alpha, & i = 1, 2, \dots, M, & l = 1, 2, \dots, M, \\ u_{\tau_i} &= \tau_i \cos \alpha, & i = 1, 2, \dots, M, & l = 1, 2, \dots, M, \end{aligned} \quad (24)$$

and $Z_p(u)$ is the FrFT form of the emitted signal $z(t)$ with transform order p . $N_{2p}(u)$ is the FrFT form of the additive noise $n_2(t)$ with transform order p .

There are four terms on the right hand side of (23). The first term is the focusing signal with the coherent superposition of each paths. In the second term, the delay of each paths are different in the FrFT domain resulting in non-coherent superposition. The last two are about noise in the FrFT domain. Thus, processed by TR-FrFT, the optimal order FrFT of $z(t)$ is focusing. We can use the method shown in Algorithm 1 to search the optimal transform order p_{opt} . Once the optimal transform order p_{opt} is found, the largest peak will appear in the optimal order fractional Fourier domain.

Detecting the LFM signal is a binary detection problem. There are two situations, i.e., H_0 : LFM signal is absent. H_1 : LFM signal is present.

$$H_0 : Z_{p_{opt}}(u) = a \sum_{i=1}^M a_i^2 C_{\tau_i} N_{1p}(u + u_{\tau_i}) + N_{2p}(u), \quad (25)$$

$$H_1 : Z_{p_{opt}}(u) = a \sum_{i=1}^M a_i^2 S_p(-u) + a \sum_{i=1}^M \sum_{\substack{l=1 \\ i \neq l}}^M a_i a_l B_{\tau_{li}} S_p(-u + u_{\tau_{li}}) + a \sum_{i=1}^M a_i^2 C_{\tau_i} N_{1p}(u + u_{\tau_i}) + N_{2p}(u). \quad (26)$$

We choose the largest peak in FrFT domain as test statistic

$$\ell_d = |Z_{p_{opt}}(u)|. \quad (27)$$

The resulting detection decision rule can be expressed as

$$\ell_d \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \quad (28)$$

where η denotes a threshold. The LFM signal is assumed to be presented when the test statistic $\ell_d > \eta$. Otherwise, there is no LFM signal.

4. Experiments

4.1. Simulation Experiments

In this section, we conduct simulation experiments to show the FrFT detection problem described in Section 2.3 and to verify the proposed method for LFM signal detection in underwater multi-path channel. The frequency range of the transmitted LFM signal is 20–22 kHz, and the pulse length is 200 ms. The simulated underwater acoustic channel is generated by the acoustic toolbox Bellhop [41]. The simulation environment is as follows: shallow water wave-guide, the sound speed profile is shown in Figure 3, the bottom is to be modeled as an acoustic elastic half-space, the density is 2 g/cm³ and other parameters are presented in Table 1. The impulse response of the simulated channel is shown in Figure 4. The main parameters of acoustic channel generated by Bellhop are reported in Table 2. The simulated channel contains 6 paths. The first path has two surface reflections and one bottom reflection. The second path and the sixth path have one surface reflection and one bottom reflection. The third path has one surface reflection. The fifth path has one bottom reflection. The fourth path is the direct path.

Table 1. Parameters of simulation environment.

Parameter Name	Numerical Value
Attenuation	1.5 (dB/m) kHz
Water depth	100.431 m
Source depth	30 m
Receiver depth	50 m
Range	1000 m

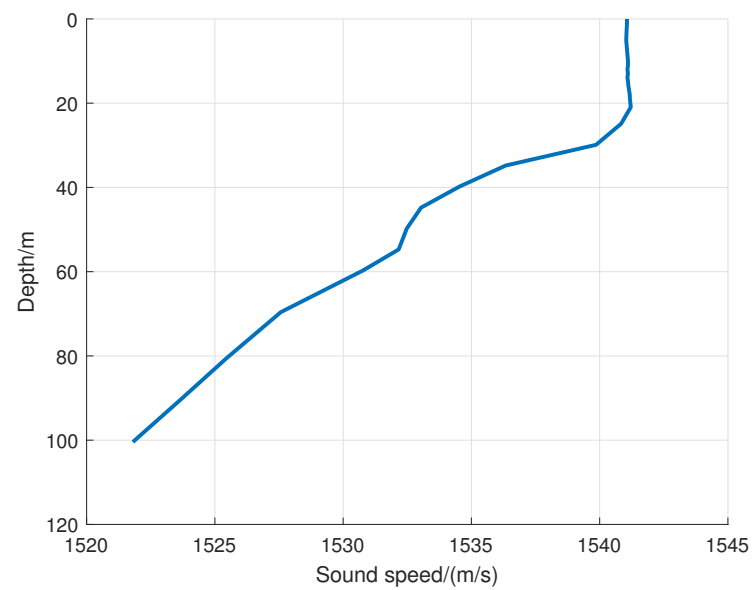


Figure 3. Sound speed profile.

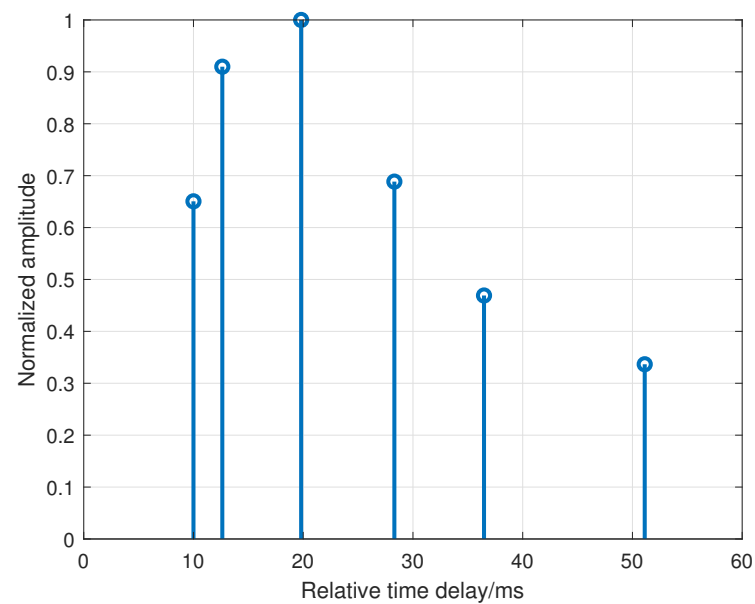


Figure 4. The simulated channel generated by Bellhop.

Table 2. Acoustic channel parameters.

Number	Amplitude	Relative Time Delay (ms)	Number of Sea Surface Reflections	Number of Seafloor Reflections
1	5.403×10^{-4}	51.759	2	1
2	1.105×10^{-3}	28.946	1	1
3	1.460×10^{-3}	13.267	1	0
4	1.044×10^{-3}	10.639	0	0
5	1.605×10^{-3}	20.451	0	1
6	7.529×10^{-4}	37.115	1	1

Calculated by Algorithm 1, the optimal order of $y(t)$ is 1.0141. When the SNR_s defined in Section 4.3 is 0 dB, the waveform of the received signal is illustrated in Figure 5. The result processed by the optimal order FrFT is shown in Figure 6. We can find that, due to the influence of multi-path effect, the result in time domain is expended, and the corresponding result processed by the optimal order FrFT has multiple peaks. The difference between the maximum spurious peak and the main peak is 1.07 dB. The presence of spurious peaks will affect the decision of the detector.

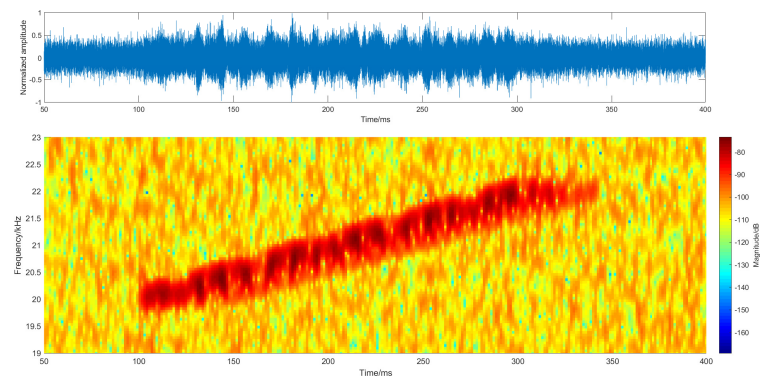


Figure 5. The waveform of received signal.

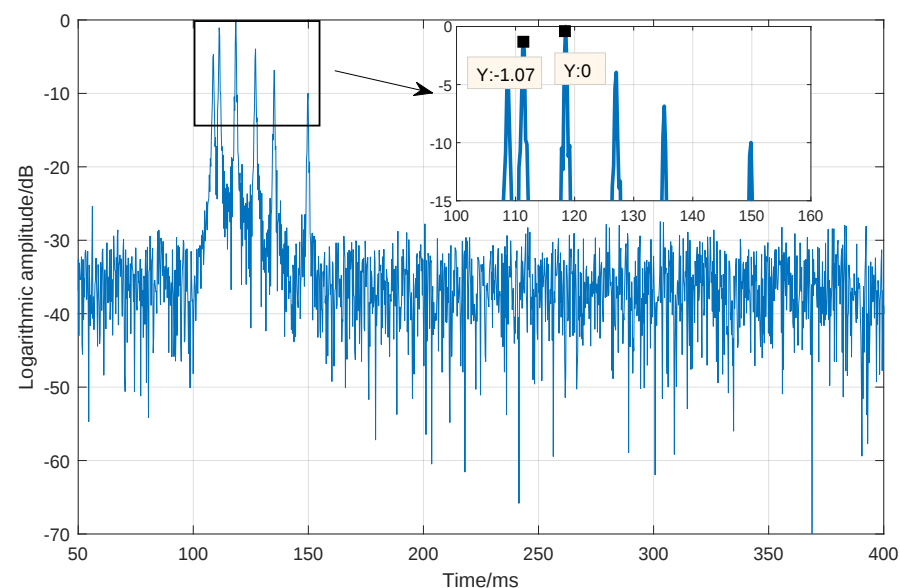


Figure 6. The result of received signal processed by the optimal order FrFT.

We now use the TR-FrFT method processing the received data. First, let the received signal pass through a TR system and acquire Figure 7. Second, calculating the optimal order of $z(t)$, we can obtain 0.9782 and we plot the result of TR-FrFT method in the optimal order fractional Fourier domain in Figure 8. The difference shown in Table 3 between the maximum spurious peak and the main peak is 9.82 dB. Compared to FrFT, the difference is improved 9.01 dB. Observe that the received signal processed by TR-FrFT achieves focusing in the optimal order fractional Fourier domain. Compared to the FrFT method, TR-FrFT has the ability to resist multi-path effects and a higher SNR.

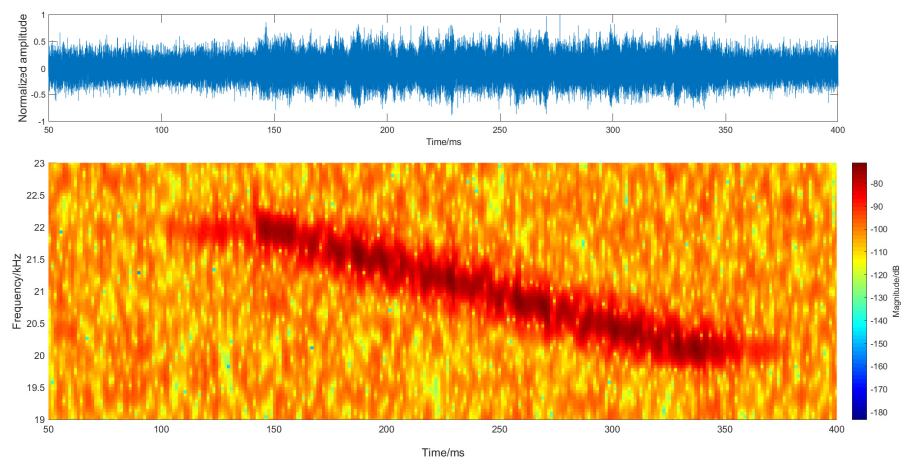


Figure 7. The waveform of received signal processed by TR.

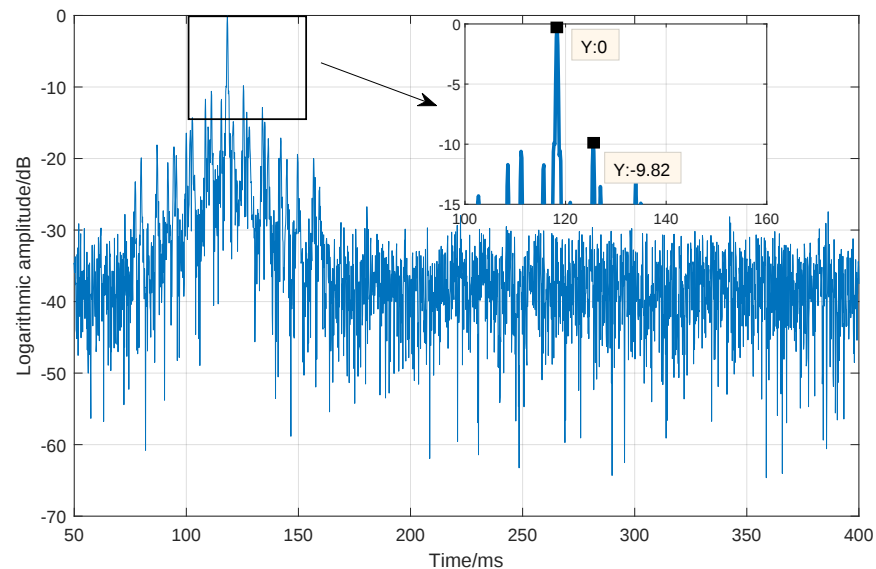


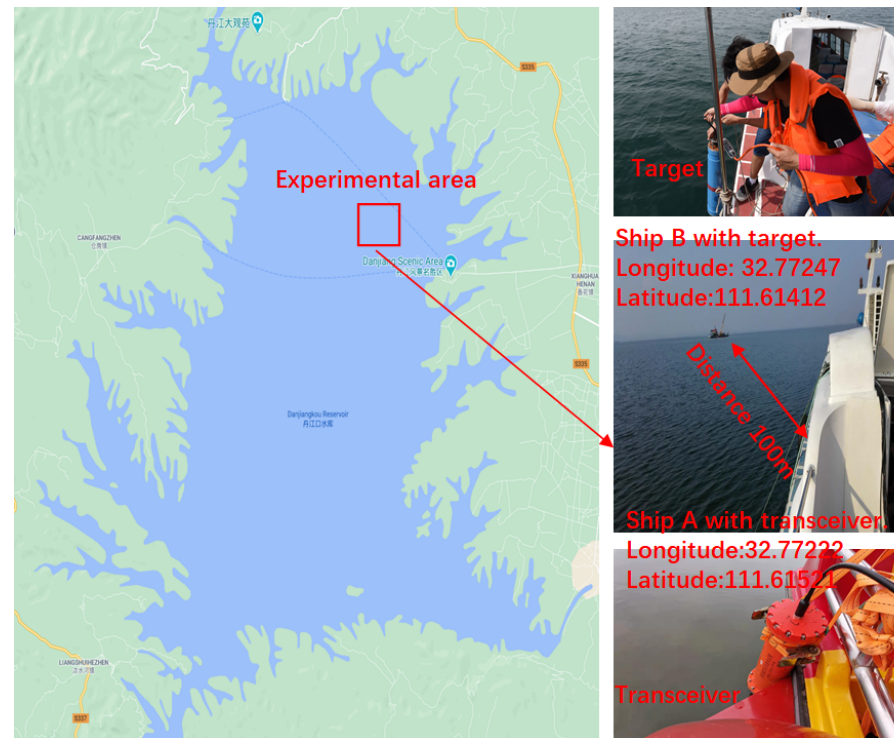
Figure 8. The result of received signal processed by TR-FrFT.

4.2. Lake Experiments

The LFM signal detection experiments were carried out in Danjiangkou Reservoir, Xichuan County, Nanyang City, Henan Province on 15 July 2019. The original picture of experimental work performed is shown in Figure 9. The depth of the experimental water is about 40 m. In the lake experiments, we used two ships, named Ship A and Ship B respectively, which are 100 m apart. The longitude and latitude of these ships are shown in Figure 9. Ship A carries a transceiver with the functions of transmitting and receiving signals, and the depth of the transceiver is 4 m. Ship B carries a target simulator, which is used to simulate the underwater target echo signal. The depth of the target simulator is 4 m. The parameters of transceiver and target simulator are shown in Table 4 and 5 respectively. The transceiver emits the LFM signal, having the same parameters as described in Section 4.1. The target simulator records the LFM signal and sends it back. The transceiver receives the simulated echo signal. The waveform of the received echo signal is shown in Figure 10 in the latest revised version. FrFT is performed on the received signal, and the result is shown in Figure 11.

Table 3. The comparison results of FrFT and TR-FrFT with simulated data.

Method	The Difference Between Main and Side Lobes
FrFT	1.07 dB
TR-FrFT	9.82 dB

**Figure 9.** The original picture of experimental work performed.**Table 4.** Parameters of transceiver.

Parameter Name	Value
Length, width, and height	15 cm × 15 cm × 60 cm
Source level	192 dB ± 1.5 dB (18 kHz~24 kHz) dB
Receiving sensitivity	−185 dB
Passband	19 kHz~23 kHz (±1 dB)
Stopband	15 kHz~19 Hz (≥20 dB), 23 kHz~27 kHz (≥20 dB)

Table 5. Parameters of target simulator.

Parameter Name	Value
Length, width, and height	15 cm × 15 cm × 120 cm
Target strength	5 dB~25 dB
Receiving and transmitting switching time	5 ms
The duration of recording signal	10 ms~5000 ms

Figure 11 shows that, in the underwater multi-path environment, the received signal is expanded, both in the time domain and the fractional Fourier domain. It is consistent to the simulation results in Figure 6. The difference shown in Table 6 between the maximum spurious peak and the main peak is 2.6 dB of FrFT from Figure 11.

We use TR to process the echo signal. The transceiver emits time reversed signal. The target simulator records the TR signal and sends it back. The transceiver receives the new simulated echo signal. The waveform of the received new echo signal is shown in

Figure 12. The TR-FrFT results are shown in Figure 13. Comparing Figure 12 with Figure 10, it can be seen that, the expanded received signal in the time domain is focused after being processed by TR, while in the time frequency domain, the signal has little change before and after processed by TR. The reason for this phenomenon is that the received signal has a high signal-to-noise ratio, which makes the difference not prominent. However, we can find a significant change in the optimal order fractional Fourier domain, which can be seen in Figure 13. The results of the received signal processed by TR-FrFT indicate that the main peak is enhanced and the false peaks are suppressed. The difference between the maximum spurious peak and the main peak is improved 10.2 dB. The difference is improved 7.6 dB comparing to FrFT. The TR-FrFT method has the ability to resist multi-path effects and achieve signal focusing in the fractional Fourier domain.

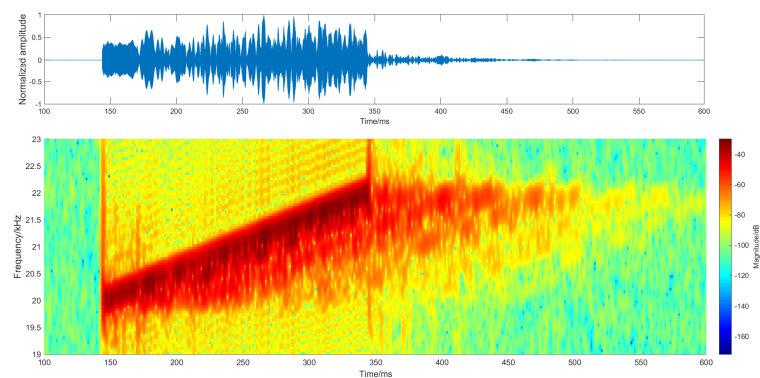


Figure 10. The waveform of received signal with lake experiments data.

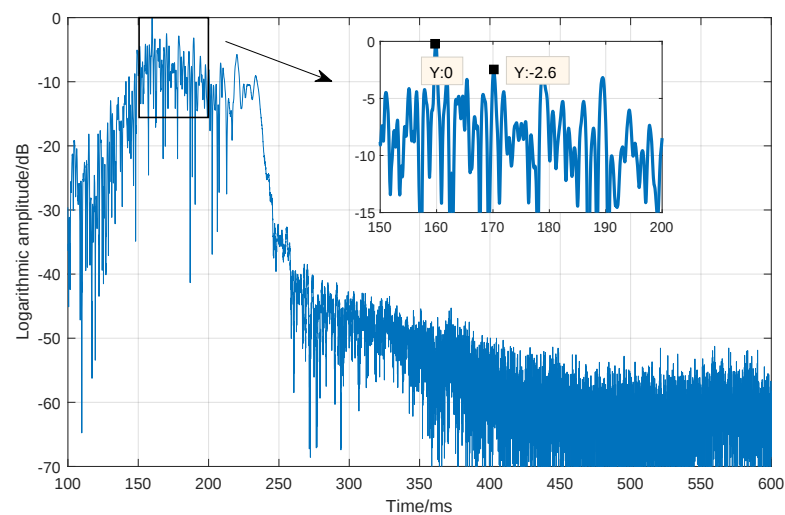


Figure 11. The result of received signal processed by FrFT with lake experiments data.

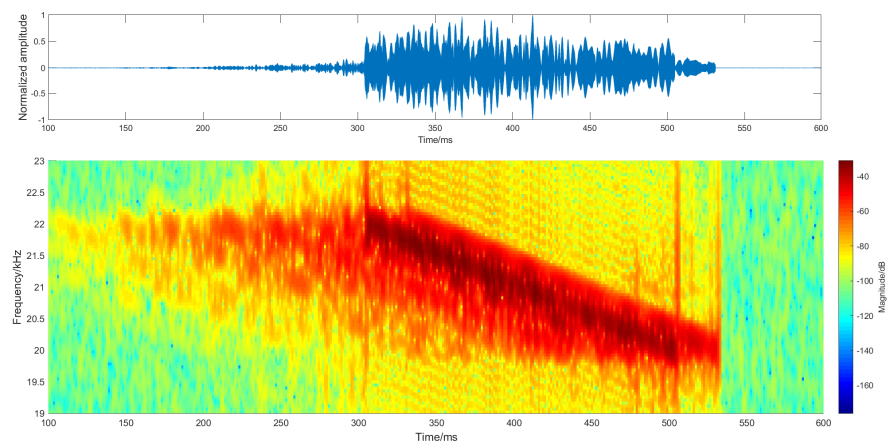


Figure 12. The waveform of received signal processed by TR with lake experiments data.

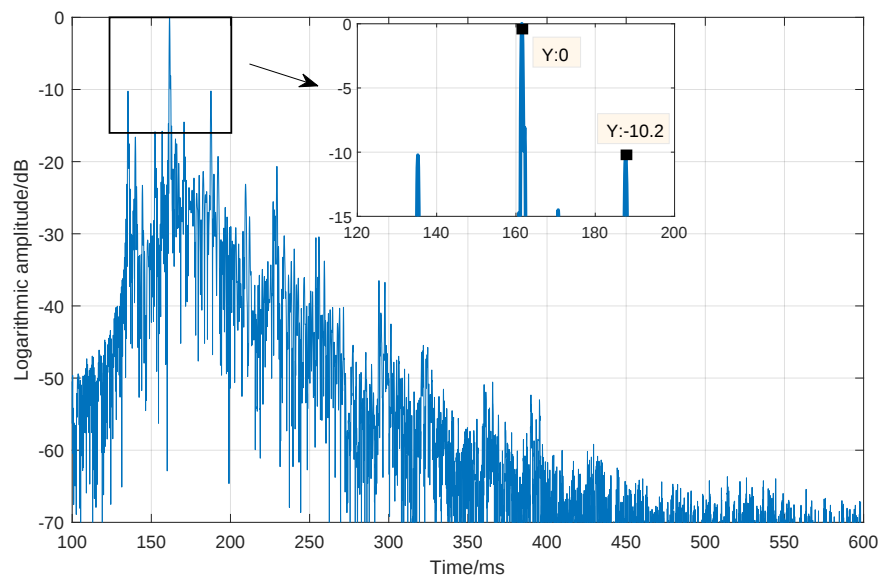


Figure 13. The result of received signal processed by TR-FrFT with lake experiments data.

Table 6. The comparison results of FrFT and TR-FrFT with lake experiments data.

Method	The Difference Between Main and Aide Lobes
FrFT	2.6 dB
TR-FrFT	10.2 dB

4.3. Detection Performance Comparison

In this subsection, we compare the detection performance of TR-FrFT detector and FrFT detector in both simulated data and lake experiments data. To generate the performance curves as a function of signal to noise ratio (SNR). We add numerically generated noise to the simulated data and real data. The SNR_s corresponding to simulated data is defined by

$$\begin{aligned} \text{SNR}_s &= 10 \log \left(\frac{P\{s(t) * h_1(t)\}}{P\{n_1(t)\}} \right) \\ &= 10 \log \left(\frac{\frac{1}{T} \int_T |s(t) * h_1(t)|^2 dt}{\sigma_n^2} \right), \end{aligned} \quad (29)$$

where T is the observation time of received signal; σ_n^2 is the power of $n_1(t)$.

For real data, (29) is no longer suitable for calculating SNR, because it is hard to separate the detected signal and mixed noise. The SNR_r , which is an approximate calculation of SNR_s , corresponding to real lake experiments data, is defined by

$$\text{SNR}_r = 10 \log \left(\frac{P_s}{\sigma_n^2} \right), \quad (30)$$

where P_s is the power of the experimental measured LFM signal.

In order to obtain the performance curve of TR-FrFT detector for a given probability of false alarm P_F , we need to select the threshold η . We generate 6000 independent Monte Carlo trials and compute the test statistic given by (27). Then, we sort the results of test statistics in ascend order. The threshold is selected to result in a given P_F . After that, we compute the P_D by the chosen threshold. To do so, We generate 6000 new independent Monte Carlo trials containing both signal and noise. Compare each test statistics with the chosen threshold and count the numbers that the test statistic exceeds the threshold. The detection probability P_D is calculated as a ratio of A over B. A is the number of test statistics that exceed the threshold. B is the number of new experiments.

With the same arguments, we can obtain the performance curves of FrFT and TR-FrFT detectors. In order to make the simulation environment close to the real environment, we add the Doppler frequency shift parameter to calculate the detection performance curve. Figures 14–16 depict, for $P_F = 0.1$, the detection performance of simulated data for FrFT detector and TR-FrFT detector. The Doppler frequency shifts are set to 0 Hz, 40 Hz and 80 Hz in these three simulated experiments. Figure 17 shows the detection performance of lake experiment data. Observe that the proposed TR-FrFT detector has the best performance compared to the FrFT detector in both simulated and lake experiments data.

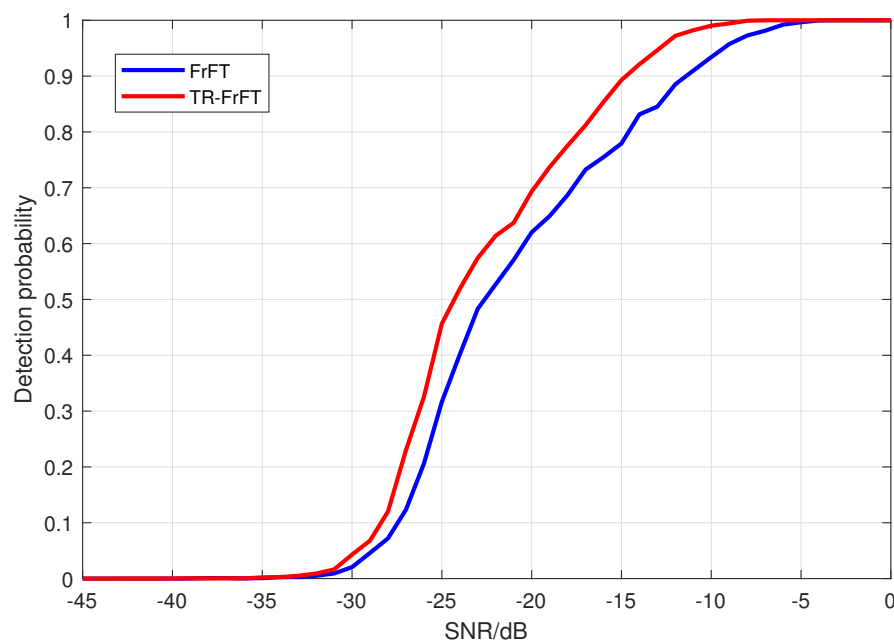


Figure 14. Detection probability vs. SNR for FrFT and TR-FrFT with the simulated data. The false alarm rate is $P_F = 0.1$. The Doppler frequency shift is 0 Hz.

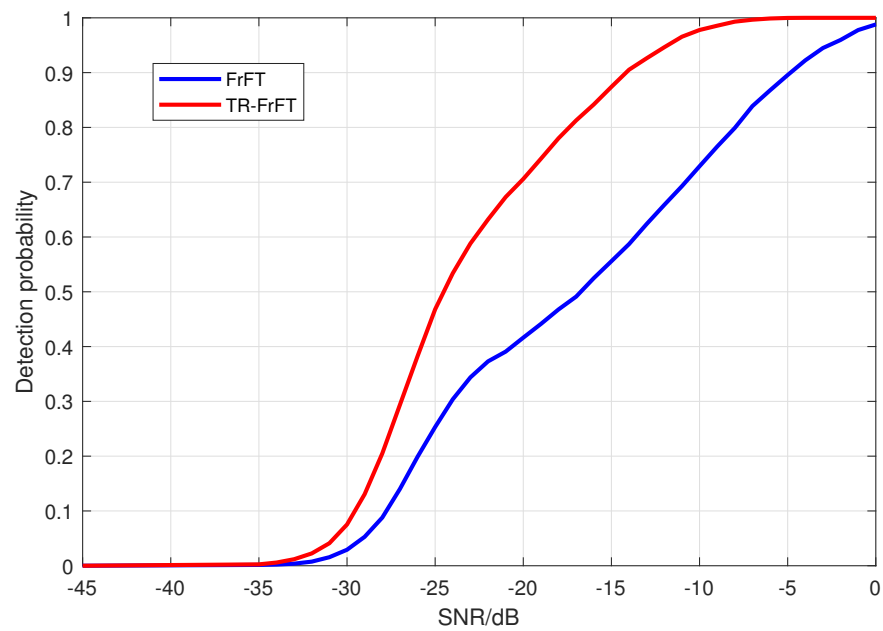


Figure 15. Detection probability vs. SNR for FrFT and TR-FrFT with the simulated data. The false alarm rate is $P_F = 0.1$. The Doppler frequency shift is 40 Hz.

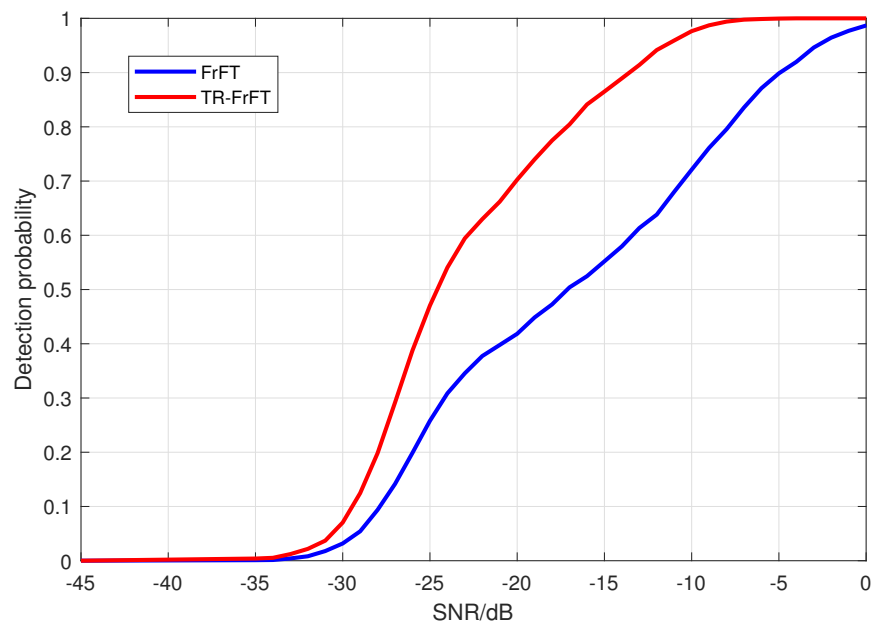


Figure 16. Detection probability vs. SNR for FrFT and TR-FrFT with the simulated data. The false alarm rate is $P_F = 0.1$. The Doppler frequency shift is 80 Hz.

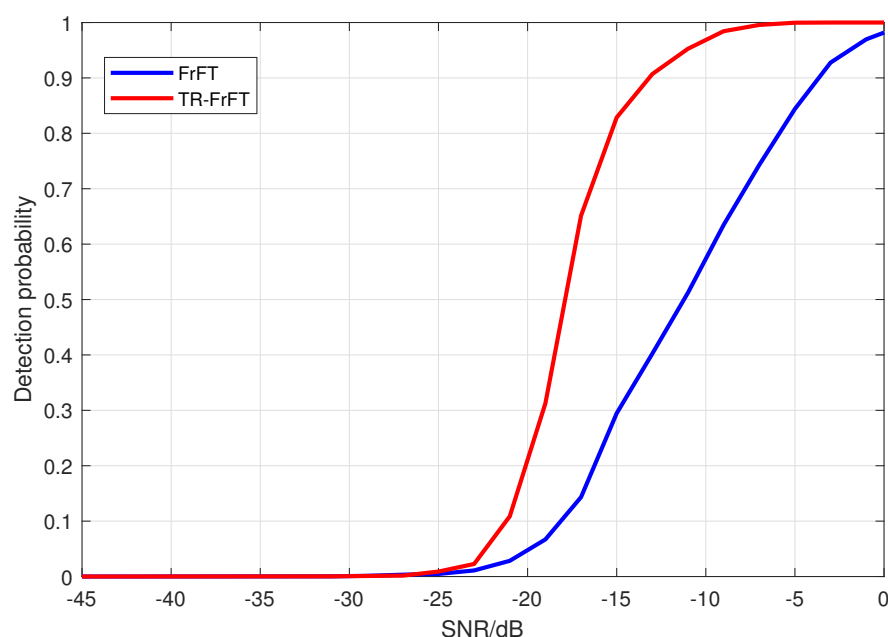


Figure 17. Detection probability vs. SNR. The false alarm rate is $P_F = 0.1$.

5. Conclusions

This paper proposes a novel method named TR-FrFT to detect LFM signal in underwater multi-path environment. We show that the LFM signal in multi-path channel is extended in fractional Fourier domain. TR method is employed to mitigate the multi-path effect. The proposed detection method is devised and verified by simulation data and lake experiments data, respectively. In addition, we compare the detection performance of proposed method with FrFT detector. The results show that TR-FrFT detector is superior to FrFT detector in the underwater multi-path environment.

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