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A Universal Low-Complexity Demapping Algorithm for Non-Uniform Constellations

Hao Wang ^{1,2} , Mingqi Li ^{2,*}  and Chao Wang ^{3,*}

¹ School of Communication and Information Engineering, Shanghai University, Shanghai 200444, China; wanghao01@sari.ac.cn

² Shanghai Advanced Research Institute, Chinese Academy of Sciences, Shanghai 201210, China

³ Key Laboratory of Specialty Fiber Optics and Optical Access Networks, Joint International Research Laboratory of Specialty Fiber Optics and Advanced Communication, Shanghai University, Shanghai 200444, China

* Correspondence: limq@sari.ac.cn (M.L.); wangchao@staff.shu.edu.cn (C.W.)

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Abstract: A non-uniform constellation (NUC) can effectively reduce the gap between bit-interleaved coded modulation (BICM) capacity and Shannon capacity, which has been utilized in recent wireless broadcasting systems. However, the soft demapping algorithm needs a lot of Euclidean distance (ED) calculations and comparisons, which brings great demapping complexity to NUC. A universal low-complexity NUC demapping algorithm is proposed in this paper, which creates subsets based on the quadrant of the two-dimensional NUC (2D-NUC) received symbol or the sign of the in-phase (I)/quadrature (Q) component of the one-dimensional NUC (1D-NUC) received symbol. ED calculations and comparisons are only carried out on the constellation points contained in subsets. To further reduce the number of constellation points contained in subsets, the proposed algorithm takes advantage of the condensation property of NUC and regards a constellation cluster containing several constellation points as a virtual point. Analysis and simulation results show that, compared with the Max-Log-MAP algorithm, the proposed algorithm can greatly reduce the demapping complexity of NUC with negligible performance loss.

Keywords: broadcasting; non-uniform constellation; soft demapping algorithm; demapping complexity

1. Introduction

Shannon defined the channel capacity limitation in his landmark paper [1]. Bit-interleaved coded modulation (BICM) is commonly utilized to approach Shannon capacity in modern communication systems. The BICM transmitter consists of a cascaded forward error correction (FEC) encoder, bit interleaver, and constellation mapper, and the capacity of the BICM system is called BICM capacity [2]. A gap between the BICM capacity and Shannon capacity appears when gray labeled uniform quadrature amplitude modulation (QAM) constellation is applied, which becomes larger as the constellation order increases. Signal shaping is an efficient way to reduce this gap, which can be classified into probabilistic shaping and geometrical shaping [3]. Probabilistic shaping sends symbols with unequal probability. Since a shaping decoder is needed in the receiver, there are difficulties in its application. Geometrical shaping changes the position of constellation points, and the receiver only needs to store new constellations. Non-uniform constellation (NUC) belongs to the latter.

NUC can be classified into one-dimensional NUC (1D-NUC) and two-dimensional NUC (2D-NUC). The former restricts the shape of NUC to a square, while the latter has no constraint on the shape of NUC. Compared with 1D-NUC, 2D-NUC can provide higher shaping gain, but the optimization and demapping complexity is also greatly increased. NUC has been applied in Digital Video

Broadcasting-Next Generation Handheld (DVB-NGH) [4] and Advanced Television Systems Committee 3rd Generation (ATSC 3.0) [5], and its application in 5G New Radio (NR) is considered in [6].

A soft demapping algorithm is usually utilized to calculate the log-likelihood ratio (LLR) of each bit to obtain optimal performance in the decoding phase. The Log-MAP algorithm is a commonly used algorithm, which can be simplified to the Max-Log-MAP algorithm by applying the Max-Log approximation. For the Max-Log-MAP demapping algorithm, the Euclidean distances (EDs) between the received symbol and all constellation points need to be calculated. Then, these EDs are compared to determine the required EDs to calculate the LLR of each bit. 1D-NUC can be decomposed into two independent non-uniform pulse amplitude modulation (NU-PAM) constellations, so only one-dimensional EDs need to be calculated and compared. 2D-NUC cannot be decomposed, so two-dimensional EDs of all constellation points need to be calculated and compared. A large number of ED calculations and comparisons in the Max-Log-MAP algorithm lead to the high demapping complexity of 2D-NUC and high-order 1D-NUC.

The condensed symbols reduction (CSR) algorithm proposed in [7] and the quadrant search reduction (QSR) algorithm proposed in [8] can efficiently reduce the number of EDs that need to be calculated and compared. The CSR algorithm is based on the condensation characteristic of NUC. For a constellation cluster containing several constellation points, the ED only needs to be calculated and compared once. CSR algorithm is suitable for NUCs designed for low and medium code rates (CRs) FEC codes. As CR increases, the constellation points are barely condensed, and the complexity of the CSR algorithm is still high. QSR algorithm is based on the quadrant symmetry characteristic of NUC. When symbols are received in a particular quadrant, they are more likely to be sent from certain constellation points. QSR algorithm selects these constellation points and creates a subset, ED calculation and comparison only need to be performed on the constellation points contained in the subset. Otherwise, the QSR algorithm is sensitive to noise, the number of constellation points in the subset is greatly increased when facing the lower signal to noise ratio (SNR). Therefore, the QSR algorithm is suitable for NUCs designed for medium and high CRs. Moreover, there is a problem when the QSR algorithm creates the subset. It considers which constellation point the received symbol is sent from. The QSR algorithm will introduce errors in LLR calculation because LLR is only related to the constellation point closest to the received symbol. The quadrant condensed search reduction (QCSR) algorithm proposed in [9] combines the CSR algorithm and the QSR algorithm, but the problem when creating the subset still exists. An improved algorithm is proposed in [10] to solve this problem, and its subset is determined according to the constellation points that may have the minimum EDs with the received symbol, but the algorithm needs to recalculate EDs when calculating the LLR of each bit. Moreover, the subsets in the above-mentioned algorithms are determined based on statistical results, which may be inaccurate. The virtual-point-searching-based (VPS), further-approximated VPS (FAVPS), and condensed VPS (CVPS) algorithms proposed in [11,12] are not based on statistical results, and their subsets are determined in the polar coordinate system through projection operations, but these algorithms are only suitable for APSK-like 2D-NUCs designed for low and medium CRs. The lattice reduction algorithm proposed in [13] quantizes the 1D-NUC constellation points and received symbols into a lattice, and determines the subset according to the position of the received symbols. However, the lattice reduction algorithm is only suitable for specific 1D-NUCs, i.e., the distance between adjacent constellation points must be an integer multiple of the minimum distance.

This paper proposes an improved low-complexity NUC demapping algorithm, i.e., subset condensed search reduction (SCSR) algorithm, which can greatly reduce the number of EDs that need to be calculated and compared during the demapping process. SCSR algorithm is suitable for 2D-NUCs and 1D-NUCs designed for all CRs. The subsets of 2D-NUC are determined according to the quadrant of the received symbol, and the subsets of 1D-NUC are determined according to the sign of the in-phase (I)/quadrature (Q) component of the received symbol. These subsets are not determined based on statistical results, and the algorithm establishes different subsets for ED calculation and comparison. The algorithm also utilizes the condensation characteristic of NUC to further reduce the number of

constellation points contained in the subsets. The SCSR algorithm proposed in this paper is based on the NUCs in ATSC 3.0, but it is also applicable to other NUCs. Simulation results show that the SCSR algorithm can greatly reduce the demapping complexity of the Max-Log-MAP algorithm with negligible performance loss.

The rest of this paper is organized as follows. Section 2 introduces the characteristics of NUC. Section 3 describes the Max-Log-MAP algorithm and analyzes its complexity. Section 4 proposes the SCSR algorithm. Section 5 analyzes the complexity and bit error rate (BER) performance of the SCSR algorithm. Finally, the conclusions of this paper are summarized in Section 6.

2. Characteristics of NUC

NUC can be classified into 1D-NUC and 2D-NUC, and its design process has been described in [14–18]. Symmetry is the first characteristic of NUC. To reduce the complexity of design and demapping, NUC only optimized the constellation points in the first quadrant, constellation points in other quadrants are derived from the first quadrant. Moreover, only the I component needs to be considered during 1D-NUC optimization, and the Q component is obtained according to symmetry. Thus, the I and Q components of 1D-NUC are independent and can be decomposed into two NU-PAMs. LLRs of 1D-NUC can be calculated using the one-dimensional demapper. The I and Q components of 2D-NUC constellation points are jointly optimized, so they are dependent, and a two-dimensional demapper is required to calculate LLRs.

Condensation is another characteristic of NUC. Since NUC is designed according to its working SNR, and its working SNR is determined based on the waterfall area of FEC code, the shape of NUC is different for different CRs. In low and medium CRs, some constellation points are very close and form a constellation cluster. Especially, when CR is very low, the shape of a high-order NUC is closer to a low-order NUC. As CR increases, the constellation points are gradually dispersed. When CR is very high, the shape of NUC is closer to a uniform constellation.

The symmetry and condensation characteristics of NUC are shown in Figure 1. These two characteristics will be used in the design of the SCSR algorithm in Section 4.

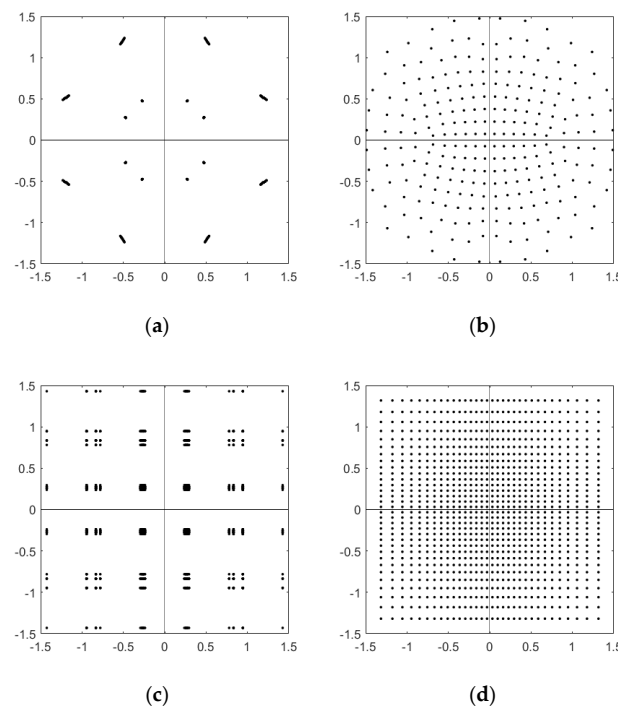


Figure 1. 2D-256NUC and 1D-1024NUC designed for different CRs in ATSC 3.0. (a) 2D-256NUC designed for CR = 3/15; (b) 2D-256NUC designed for CR = 13/15; (c) 1D-1024NUC designed for CR = 3/15; (d) 1D-1024NUC designed for CR = 13/15.

3. Soft Demapping Algorithm

The BICM system model is shown in Figure 2. A cascaded FEC encoder, bit interleaver, and constellation mapper are included in the transmitter. And in the receiver, the received symbols are first equalized, and then fed to the constellation demapper. After the demapping, values which correspond to the bit interleaved codewords in the transmitter are fed to the bit deinterleaver, and then to the FEC decoder. Assuming that the transmitted symbol is t , the received symbol r can be represented by

$$r = ht + n \quad (1)$$

where h is the channel fading coefficient, n is additive white Gaussian noise (AWGN).

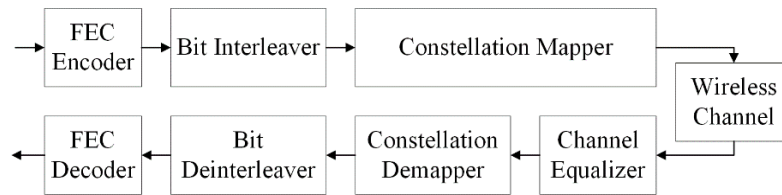


Figure 2. BICM system block diagram.

For optimal performance in decoding, a soft demapping algorithm is usually used to calculate the LLRs of the received symbol. Using the Log-MAP algorithm, the LLR of the i -th bit can be calculated by

$$\text{LLR}(b_i) = \log \frac{\Pr(b_i = 0|r)}{\Pr(b_i = 1|r)} = \log \frac{\sum_{x \in \chi_i^0} \exp(-\frac{|r-hx|^2}{N_0})}{\sum_{x \in \chi_i^1} \exp(-\frac{|r-hx|^2}{N_0})} \quad (2)$$

where x is the possible transmitted symbol, N_0 is the noise variance, χ_i^0 and χ_i^1 denote the sets of constellation points for which $b_i = 0$ and $b_i = 1$ respectively, and $\log$ represents the natural logarithm. Using the Max-Log approximation, the Log-MAP algorithm can be simplified to the Max-Log-MAP algorithm. Then, the approximate LLR can be obtained as

$$\text{LLR}(b_i) = \frac{1}{N_0} (\min_{x \in \chi_i^1} |r - hx|^2 - \min_{x \in \chi_i^0} |r - hx|^2) \quad (3)$$

Since demapping is performed after equalization, with ideal channel estimation and zero forcing equalization, the received symbol y after equalization can be expressed as

$$y = \frac{r}{h} = t + \frac{n}{h} \quad (4)$$

Substitute Equation (4) into Equation (3), we can obtain

$$\text{LLR}(b_i) = \frac{|h|^2}{N_0} (\min_{x \in \chi_i^1} |y - x|^2 - \min_{x \in \chi_i^0} |y - x|^2) \quad (5)$$

It can be seen from Equation (5) that when using the Max-Log-MAP algorithm to calculate LLRs, it is necessary to calculate the EDs between the received symbol and all constellation points, and then find the minimum ED in χ_i^0 and χ_i^1 respectively. The I and Q components of 1D-NUC are independent,

so 1D-NUC can be decomposed into two independent NU-APSKs, only one-dimensional EDs need to be considered when calculating LLRs

$$\text{LLR}(b_p) = \frac{|h|^2}{N_0} \left[\min_{x \in \chi_p^1} |y_I - x_I|^2 - \min_{x \in \chi_p^0} |y_I - x_I|^2 \right] \quad (6)$$

$$\text{LLR}(b_q) = \frac{|h|^2}{N_0} \left[\min_{x \in \chi_q^1} |y_Q - x_Q|^2 - \min_{x \in \chi_q^0} |y_Q - x_Q|^2 \right] \quad (7)$$

where index p refers to the bits demapped from the I components and index q from the Q components. The I and Q components of 2D-NUC are dependent, so two-dimensional EDs need to be considered when calculating LLRs

$$\text{LLR}(b_i) = \frac{|h|^2}{N_0} \left[\min_{x \in \chi_i^1} (|y_I - x_I|^2 + |y_Q - x_Q|^2) - \min_{x \in \chi_i^0} (|y_I - x_I|^2 + |y_Q - x_Q|^2) \right] \quad (8)$$

The complexity of the Max-Log-MAP algorithm is related to two factors, i.e., the number of EDs that need to be calculated and compared to demapping each received symbol. Since multiplication operations are inevitable in ED calculation, the complexity of ED calculation is much greater than ED comparison. In the demapping process, ED between the received symbol and each constellation point only needs to be calculated once, and only ED comparison is required when calculating $\text{LLR}(b_i)$. For 1D-NUC and 2D-NUC with modulation order M , when demapping each received symbol, the number of one-dimensional EDs required to be calculated are $2\sqrt{M}$ and $2M$, and the number of EDs required to be compared are $\sqrt{M} \log_2 M$ and $M \log_2 M$.

To reduce the number of EDs that need to be calculated and compared, especially the number of EDs that need to be calculated, this paper proposes an improved algorithm named SCSR. The details of the SCSR algorithm are described in the next section.

4. Proposed Low-Complexity Demapping Algorithm

This section proposes an improved low-complexity NUC demapping algorithm named SCSR, which consists of two processes. The first process is to create the subsets, and the second process is to calculate the LLRs of the received symbol according to the subsets. It should be noted that the process of creating subsets is performed off-line. This process only needs to be executed once and the obtained subsets are stored in the receiver. Therefore, it does not increase the demapping complexity.

4.1. Create the Subsets

Taking 2D-NUC as an example, and assume that the received symbol is in the first quadrant, the creation process of the subsets is as described in Algorithm 1. Since 2D-NUC is quadrant symmetric, the subsets can be derived from the subsets of the first quadrant when the received symbol is in other quadrants.

Algorithm 1 Subsets Creation Process of SCSR Algorithm**Input:**

x : constellation (with modulation order M)

Output:

C : subset used for ED calculation; C_i^k : subset used for ED comparison; $\tilde{x}$: new constellation

```

1:  for  $i = 0$  to  $\log_2 M - 1$  do
2:    for  $k = 0$  to 1 do
3:      determine the constellation points contained in  $\chi_i^k$ 
4:      delete the constellation points in  $\chi_i^k$  that can never have the minimum ED from the received symbol,
      and obtain a reduced subset  $S_i^k$ 
5:    end for
6:  end for
7:  calculate the union of  $S_i^k$ , and obtain a subset  $S$ 
8:  regard a constellation cluster containing several constellation points in  $x$  as a virtual point, and its
      coordinate is set to the mean value of these constellation points
9:  use virtual points to replace the corresponding constellation points in  $x$ , and obtain a new constellation  $\tilde{x}$ 
10: use virtual points to replace the corresponding constellation points in  $S_i^k$ , and obtain the subset  $C_i^k$  used for
      ED comparison
11: use virtual points to replace the corresponding constellation points in  $S$ , and obtain the subset  $C$  used for
      ED comparison

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To facilitate the understanding of the creation process of the subsets, we take 2D-256NUC designed for CR = 6/15 as an example. Assuming that the most significant bit of 2D-256NUC is b_7 and the least significant bit is b_0 . Then, b_7 and b_6 determine the quadrant where the constellation point is located, and b_5 to b_0 determine the position of the constellation point in a specific quadrant.

The subsets creation process of b_7 and b_6 is similar. Taking b_6 as an example, the constellation points contained in χ_6^0 and χ_6^1 of the Max-Log-MAP algorithm are shown in Figure 3a. Where the red circles represent the constellation points contained in χ_6^0 , and the blue circles represent the constellation points contained in χ_6^1 . It can be seen that the constellation points contained in χ_6^0 are all located in the first and fourth quadrants, and the constellation points contained in χ_6^1 are all located in the second and third quadrants. When the received symbol is in the first quadrant, it is obvious that the constellation points in the fourth quadrant in χ_6^0 can never have the minimum ED with the received symbol. Therefore, only the constellation points in the first quadrant in χ_6^0 need to be retained and then obtain the subset S_6^0 . Since χ_6^1 does not contain the constellation points in the first quadrant, it is necessary to keep the constellation points closest to the boundary of the first quadrant in χ_6^1 , and then obtain the subset S_6^1 . The constellation points contained in S_6^0 and S_6^1 are shown in Figure 3b.

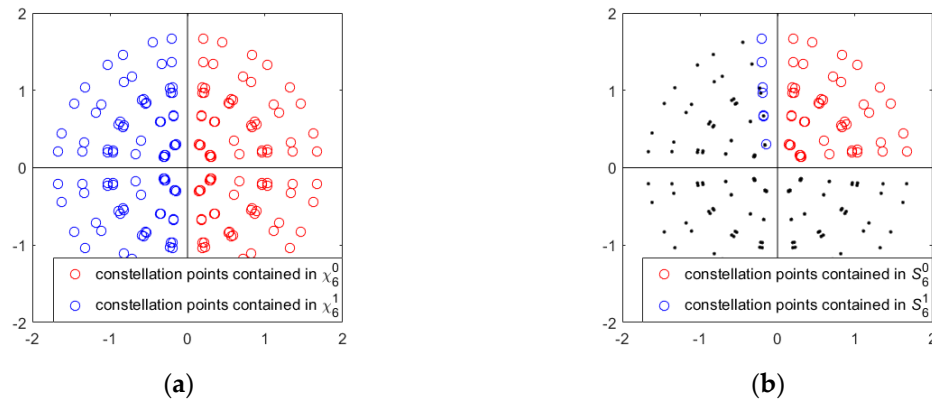


Figure 3. The subsets of ED comparison to calculate $\text{LLR}(b_6)$ when the received symbol is in the first quadrant. (a) Subsets of Max-Log-MAP algorithm; (b) Subsets obtained after some constellation points are removed.

The subsets creation process of b_5 to b_0 is simpler. Since both χ_i^0 and χ_i^1 ($i = 0, 1, \dots, 5$) contain constellation points in the first quadrant, the subsets S_i^0 and S_i^1 can be obtained by retaining the constellation points in the first quadrant in χ_i^0 and χ_i^1 respectively. Taking b_3 as an example, the constellation points contained in χ_3^0 and χ_3^1 are shown in Figure 4a, and the constellation points contained in S_3^0 and S_3^1 are shown in Figure 4b.

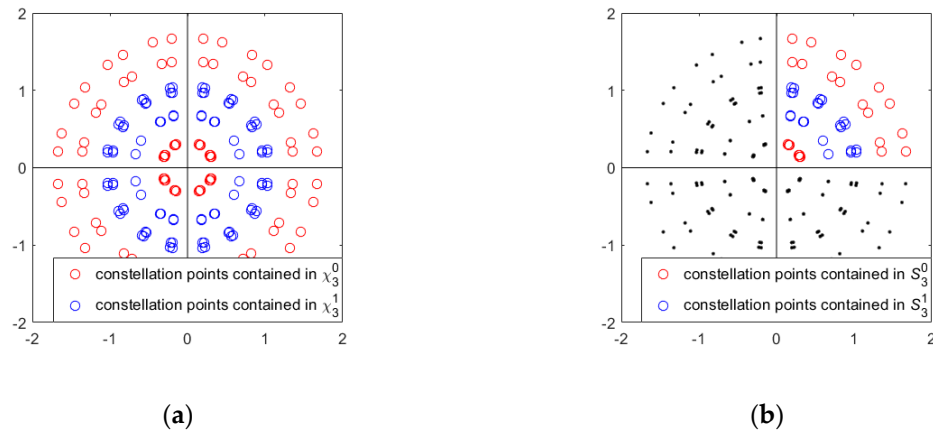


Figure 4. The subsets of ED comparison to calculate $\text{LLR}(b_3)$ when the received symbol is in the first quadrant. (a) Subsets of Max-Log-MAP algorithm; (b) Subsets obtained after some constellation points are removed.

Calculate the union of S_i^k , i.e., $S = \bigcup_{i=0}^7 S_i^k$, ($k = 0, 1$). Then, S is the subset used for ED calculation when the received symbol is in the first quadrant, as shown in Figure 5a. The constellation points contained in S are represented by green circles.

To further reduce the number of constellation points contained in S and S_i^k , the condensation characteristic of NUC can be used. A constellation cluster containing several constellation points can be regarded as a virtual point, and its coordinate is set to the mean value of these constellation points. Using virtual points to replace the corresponding constellation points in S and S_i^k , the reduced subset C used for ED calculation and the reduced subsets C_i^k used for ED comparison are obtained. The subset C is shown in Figure 5b.

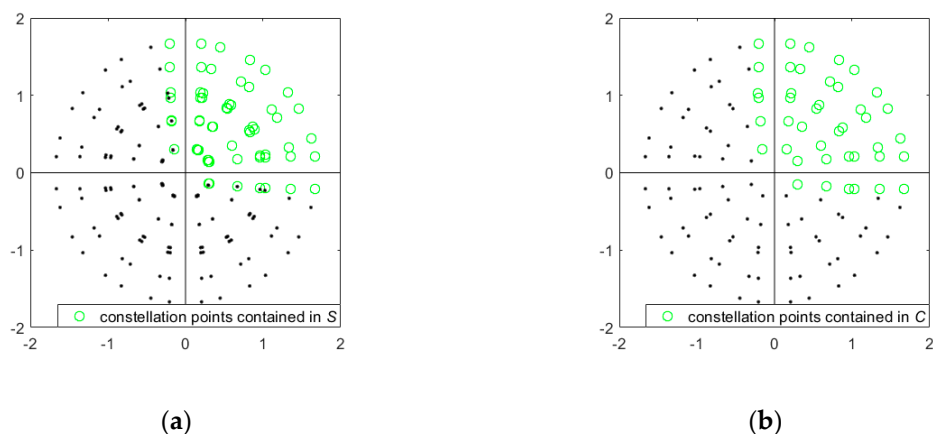


Figure 5. The subset of ED calculation when the received symbol is in the first quadrant. (a) Subset before using condensation characteristic; (b) Subset after using condensation characteristic.

The above is the process of creating the subsets for 2D-NUC. Since 1D-NUC can be decomposed into two independent NU-PAMs, the process of creating its subsets is simpler. Only the subsets when the I component of the received symbol is positive need to be created, and the subsets when the I

component is negative can be derived based on symmetry. Since the Q component of 1D-NUC is also obtained according to symmetry, the same subsets can be used for the Q component and the I component.

4.2. LLR Calculation

This section takes 2D-NUC as an example to illustrate the LLR calculation process of the SCSR algorithm, as described in Algorithm 2.

Algorithm 2 LLR Calculation Process of SCSR Algorithm

Input:

y : received symbols after equalization; h : channel fading coefficient; N_0 : noise variance; $\tilde{x}$: new constellation (with modulation order M); CR: code rate

Output:

LLR calculation result

```

1: for  $n = 1$  to size( $y$ ) do
2:     determine the quadrant  $Q$  of  $y_n$ 
3:     select the corresponding subsets  $C$  and  $C_i^k$  according to  $Q$  and CR
4:     for  $m = 1$  to size( $C$ ) do
5:         calculate the ED between  $y_n$  and the  $m$ -th constellation point in  $C$ 
6:     end for
7:     for  $i = 0$  to  $\log_2 M - 1$  do
8:         compare the EDs between  $y_n$  and the constellation points in  $C_i^0$  and  $C_i^1$  respectively, and obtain the minimum EDs
9:         calculate  $\text{LLR}(b_i)$ 
10:    end for
11: end for

```

The 1D-NUC demapping process is similar to Algorithm 2, except that the I component and Q component of the received symbol need to be decomposed first, and the subsets are selected according to the sign of the I/Q component.

It can be seen from Section 4.1 that the subsets of the SCSR algorithm are not obtained by statistical results, but are obtained by removing the constellation points that never have the minimum ED with the received symbol from χ_i^k . Therefore, if the condensation characteristic is not used, the SCSR algorithm can achieve the same performance as the Max-Log-MAP algorithm. In fact, the condensation characteristic is used, so the SCSR algorithm has a performance loss compared with the Max-Log-MAP algorithm, but its complexity is greatly reduced. The complexity and BER performance of the SCSR algorithm will be analyzed in detail in the next section.

5. Performance Evaluation

In this section, the complexity and BER performance of the Max-Log-MAP algorithm, QCSR algorithm, and SCSR algorithm are compared. The NUCs used in this section are 2D-256NUCs and 1D-1024NUCs in ATSC 3.0. The length of the LDPC code is 64,800 bits, and the CR is 2/15, 6/15, 10/15, 13/15.

5.1. Complexity Comparison

We compare the complexity of three algorithms from two aspects, i.e., the number of one-dimensional EDs to be calculated and the number of EDs to be compared when calculating the LLRs of each received symbol. Table 1 shows the complexity comparison when demapping 2D-256NUC. When CR = 2/15, 2D-256NUC becomes 2D-16NUC after condensation characteristic is used, so the complexity of QCSR and SCSR algorithms is greatly reduced compared with the Max-Log-MAP algorithm. Moreover, the subsets of the QCSR algorithm are obtained based on

statistical results, and the lower the SNR, the more constellation points are contained in the subsets. Therefore, when $CR = 2/15$, the complexity of the SCSR algorithm is lower than that of the QCSR algorithm. As CR increases, the number of constellation points that can be condensed gradually decreases. When $CR = 6/15$ and $10/15$, using the condensation characteristic only reduces the number of constellation points in NUC, but cannot turn it into a low-order constellation. Therefore, the complexity of QCSR and SCSR algorithms is increased, but the complexity of the SCSR algorithm is still lower than that of the QCSR algorithm. When CR is increased to $13/15$, there are no condensed constellation points in 2D-256NUC, and neither the QCSR algorithm nor the SCSR algorithm can use condensation characteristics. Further, because of the high SNR, the number of constellation points contained in the subsets of the QCSR algorithm is greatly reduced. Therefore, the number of EDs that the QCSR algorithm needs to calculate is slightly less than that of the SCSR algorithm. However, as described in Section 4, C_i^k is a subset of C. Therefore, the number of EDs that the SCSR algorithm needs to compare is still far less than the QCSR algorithm. For all CRs, the number of EDs to be calculated for the SCSR algorithm is reduced by 67.58% to 96.48%, compared with the Max-Log-MAP algorithm. Then, the number of EDs to be compared is reduced by 74.07% to 98.97%.

Table 1. Complexity comparison when demapping 2D-256NUC.

CR	Demapping Algorithm	Number of One-Dimensional EDs to be Calculated and Reduction Percentage	Number of EDs to be Compared and Reduction Percentage
All	Max-Log-MAP	512	2048
2/15	QCSR	32 (93.75%)	64 (96.88%)
	SCSR	16 (96.48%)	21 (98.97%)
6/15	QCSR	132 (74.22%)	528 (74.22%)
	SCSR	84 (83.59%)	274 (86.62%)
10/15	QCSR	160 (68.75%)	640 (68.75%)
	SCSR	150 (70.70%)	475 (76.81%)
13/15	QCSR	160 (68.75%)	640 (68.75%)
	SCSR	166 (67.58%)	531 (74.07%)

Table 2 shows the complexity comparison when demapping 1D-1024NUC. Since the QCSR algorithm is not designed for 1D-NUC, only the Max-Log-MAP algorithm and SCSR algorithm are compared. 1D-NUC can be decomposed into two independent NU-PAMs, and only one-dimensional demapping is required. Therefore, the demapping complexity of 1D-NUC is greatly reduced compared with that of 2D-NUC. Similar to 2D-NUC, 1D-NUC becomes a low-order constellation in low CR, so the complexity of the SCSR algorithm is greatly reduced. With the CR increases, the number of constellation points that can be condensed gradually decreases, so the complexity of the SCSR algorithm gradually increases, but the complexity of the SCSR algorithm is always lower than that of the Max-Log-MAP algorithm. For all CRs, the number of EDs to be calculated for the SCSR algorithm is reduced by 46.88% to 90.63%, compared with the Max-Log-MAP algorithm. Then, the number of EDs to be compared is reduced by 49.38% to 96.88%.

Table 2. Complexity comparison when demapping 1D-1024NUC.

CR	Demapping Algorithm	Number of One-Dimensional EDs to be Calculated and Reduction Percentage	Number of EDs to be Compared and Reduction Percentage
All	Max-Log-MAP	64	320
2/15	SCSR	6 (90.63%)	10 (96.88%)
6/15	SCSR	20 (68.75%)	104 (67.50%)
10/15	SCSR	28 (56.25%)	138 (56.87%)
13/15	SCSR	34 (46.88%)	162 (49.38%)

5.2. BER Simulation Results

According to the description in Section 4, if condensation characteristic is not used, SCSR algorithm can achieve the same performance as Max-Log-MAP algorithm, so the condensation characteristic is the factor that causes the performance difference between SCSR algorithm and Max-Log-MAP algorithm. However, the constellations obtained by the SCSR algorithm and QCSR algorithm after using the condensation characteristic are the same, so the factor that causes the performance difference between the two algorithms is the constellation points contained in subsets.

Figure 6 compares the BER performance of the Max-Log-MAP algorithm, QCSR algorithm, and SCSR algorithm over i.i.d Rayleigh channel. 2D-256NUCs are used in the simulation. Figure 6a shows the BER performance comparison when $CR = 2/15$. Combined with Table 1, it can be seen that the subsets of the QCSR algorithm contain all 16 constellation points. For the received symbols in a certain quadrant, although some constellation points in the subsets are redundant for LLR calculation, it can ensure that all constellation points that may have the minimum ED with the received symbol are contained in the subsets. The subsets of the SCSR algorithm remove these redundant constellation points. Therefore, the complexity of the SCSR algorithm is lower than that of the QCSR algorithm, but it can achieve the same BER performance as the QCSR algorithm. Moreover, due to the use of condensation characteristic, the performance of these two algorithms is 0.006 dB worse than that of the Max-Log-MAP algorithm. Figure 6b shows the BER performance comparison when $CR = 6/15$. As described in Section 1, the subsets of the QCSR algorithm are determined based on statistical results, which contain some redundant constellation points, and some constellation points that may have the minimum ED with the received symbol are lost. Therefore, the QCSR algorithm has the worst performance. The subsets of the SCSR algorithm contain all constellation points that may have the minimum ED with the received symbol, so its performance is 0.004 dB better than that of the QCSR algorithm. Then, due to the use of condensation characteristic, the performance of the SCSR algorithm is 0.009 dB worse than that of the Max-Log-MAP algorithm. Figure 6c shows the BER performance comparison when $CR = 10/15$. For the QCSR algorithm, although some constellation points in its subsets are redundant, the subsets still contain all constellation points that may have the minimum ED with the received symbol. Therefore, the performance of the QCSR algorithm and the SCSR algorithm are the same. Due to the use of condensation characteristic, the performance of these two algorithms is 0.006 dB worse than that of the Max-Log-MAP algorithm. Figure 6d shows the BER performance comparison when $CR = 13/15$. Since the condensation characteristic cannot be used, the SCSR algorithm has the same performance as the Max-Log-MAP algorithm. And some constellation points that may have the minimum ED with the received symbol are lost in the subsets of the QCSR algorithm, so the QCSR algorithm has the worst performance. The performance of the SCSR algorithm is 0.05 dB better than that of the QCSR algorithm.

When using 1D-1024NUCs, the performance comparison between the SCSR algorithm and the Max-Log-MAP algorithm is shown in Figure 7. Similar to Figure 6, when the condensation characteristic can be used, the SCSR algorithm has a slight performance loss compared with the Max-Log-MAP algorithm. When the CR is 2/15, 6/15, 10/15, the performance of the SCSR algorithm is worse than that of the Max-Log-MAP algorithm by 0.003 dB, 0.004 dB, and 0.0001 dB, respectively. When CR is 13/15, the condensation characteristic cannot be used, so the performance of the SCSR algorithm is the same as that of the Max-Log-MAP algorithm.

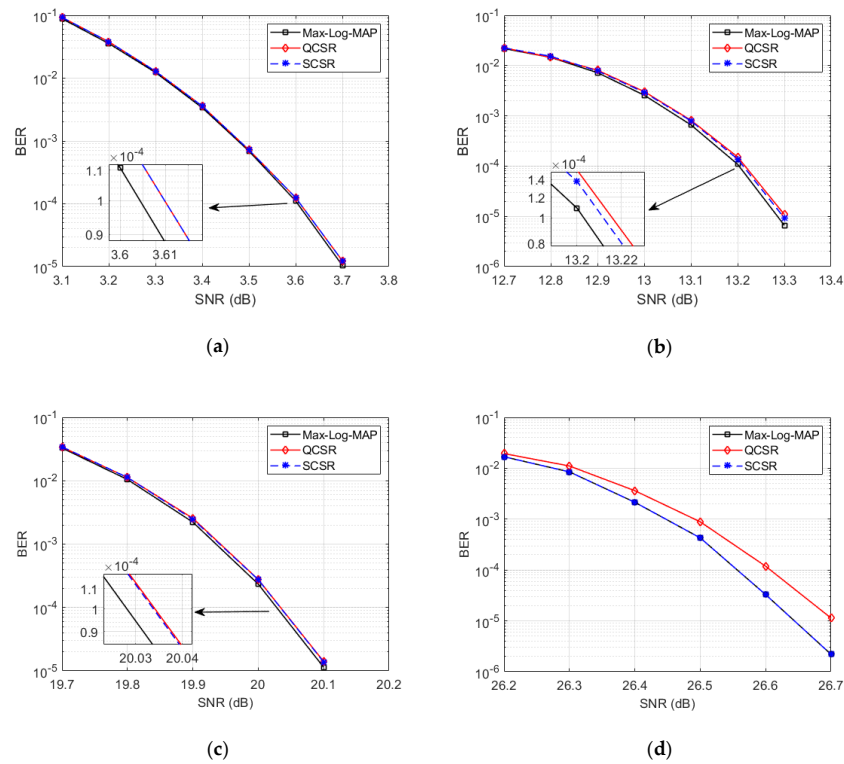


Figure 6. BER Performance comparison of Max-Log-MAP, QCSR and SCSR algorithms over i.i.d Rayleigh channel. Results presented for 2D-256NUCs. (a) CR = 2/15; (b) CR = 6/15; (c) CR = 10/15; (d) CR = 13/15.

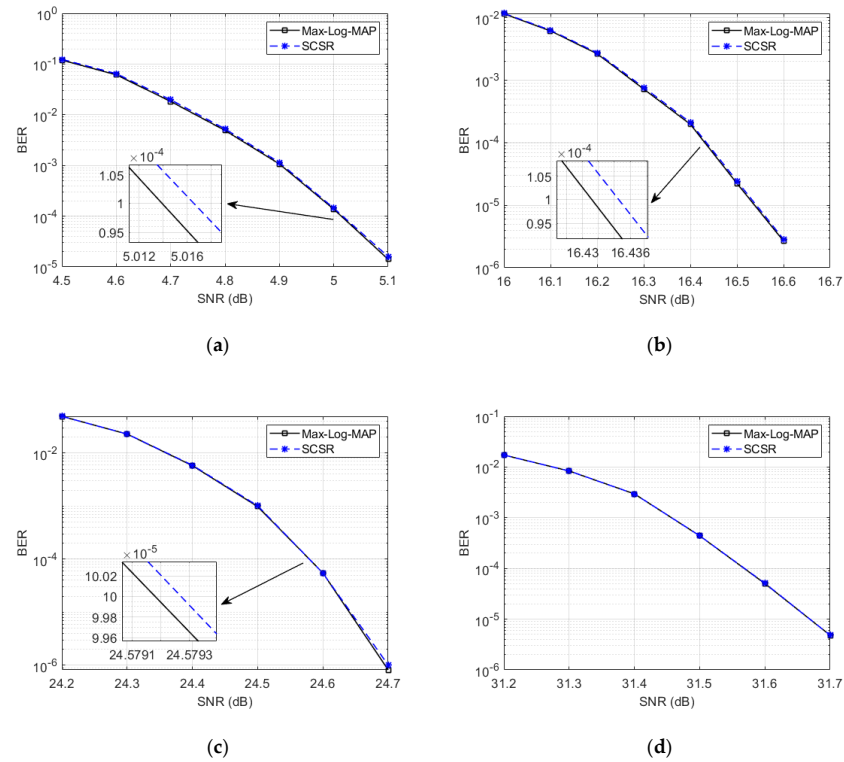


Figure 7. BER Performance comparison of Max-Log-MAP and SCSR demapping algorithms over i.i.d Rayleigh channel. Results presented for 1D-1024NUCs. (a) CR = 2/15; (b) CR = 6/15; (c) CR = 10/15; (d) CR = 13/15.

6. Conclusions

This paper proposes a low-complexity NUC demapping algorithm, i.e., the SCSR algorithm. SCSR algorithm creates the subsets based on the quadrant of the 2D-NUC received symbol or the sign of the I/Q component after the 1D-NUC received symbol is decomposed. The number of constellation points contained in the subsets is further reduced through the condensation characteristic. When demapping the received symbols, EDs calculation and comparison only need to be performed on the constellation points contained in the subsets. Compared with the Max-Log-MAP algorithm, when demapping 2D-256NUC, the number of EDs that need to be calculated can be reduced by 67.58% to 96.48%, and the number of EDs that need to be compared can be reduced by 74.07% to 96.88%. When demapping 1D-1024NUC, the number of EDs that need to be calculated can be reduced by 46.88% to 90.63%, and the number of EDs that need to be compared can be reduced by 49.38% to 96.88%. The BER simulation results show that, compared with the Max-Log-MAP algorithm, the performance loss of the SCSR algorithm is less than 0.01 dB. This paper also compares the complexity and performance of the SCSR algorithm and QCSR algorithm. The results show that the SCSR algorithm can always achieve the same or better BER performance as the QCSR algorithm, and the complexity of the SCSR algorithm is almost always lower than that of the QCSR algorithm.

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