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Tracking Control of Uncertain Neural Network Systems with Preisach Hysteresis Inputs: A New Iteration-Based Adaptive Inversion Approach

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Abstract: To describe the hysteresis nonlinearities in smart actuators, numerous models have been presented in the literature, among which the Preisach operator is the most effective due to its capability to capture multi-loop or sophisticated hysteresis curves. When such an operator is coupled with uncertain nonlinear dynamics, especially in noncanonical form, it is a challenging problem to develop techniques to cancel out the hysteresis effects and, at the same time, achieve asymptotic tracking performance. To address this problem, in this paper, we investigate the problem of iterative inverse-based adaptive control for uncertain noncanonical nonlinear systems with unknown input Preisach hysteresis, and a new adaptive version of the closest-match algorithm is proposed to compensate for the Preisach hysteresis. With our scheme, the stability and convergence of the closed-loop system can be established. The effectiveness of the proposed control scheme is illustrated through simulation and experimental results.

Keywords: adaptive control; neural networks; stability analysis; piezo actuators; noncanonical nonlinear systems



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1. Introduction

Hysteresis widely occurs in smart material-based actuators [1–3], such as electromagnetic actuators [4] and piezoelectric actuators [5]. Experiments show that the system with hysteresis would exhibit poor tracking performance when the feedback control does not explicitly consider hysteresis [6]. To compensate for the hysteresis nonlinearity in control design, a mathematical operator that can describe the characteristics of the hysteresis nonlinearity is needed. In the literature, commonly used hysteresis models include the Preisach operator [7,8], the Duhem operator [9], the Prandtl–Ishlinskii (PI) operator [10], etc. Among these, the Preisach operator is considered the most effective due to its general and well-established mathematical structure and its ability to capture both multi-loop and asymmetric hysteresis curves, where the hysteresis nonlinearity is modeled using a superposition of infinity-weighted elementary relays. Consequently, the question naturally arises of how to compensate for the Preisach-type hysteresis nonlinearity.

It is well-known that traditional robust control methods are effective in accommodating nonlinearities in a controlled system [11–13]. However, such control approaches cannot adequately compensate for the hysteresis nonlinearity, which can lead to a significant degradation in the tracking performance of the system when the effects of the hysteresis nonlinearity are considerable. Therefore, it becomes necessary to employ advanced methods to compensate for the hysteresis nonlinearity. In this regard, one of the

fundamental approaches in effectively addressing the hysteresis nonlinearity is inverse compensation [14–16]. In [14], the inverse of the Krasnoselskii–Pokrovskii model was constructed using an inverse multiplicative structure. In [15], a direct hysteresis inverse model was constructed using clockwise relay operators. In [16], the parameters in a direct hysteresis inverse model were updated using an adaptive Kalman filter. These results aimed to reduce or eliminate the hysteresis effects by constructing an approximate or right-inverse hysteresis model. However, unlike some other hysteresis models, such as the PI operator (as a special case of the Preisach operator) and the Duhem operator, it is challenging to compute the analytical inverse of the Preisach operator. This difficulty arises due to the implicit involvement of the input signal within the operator [17].

To overcome the above challenge, Tan, Venkataraman, and Krishnaprasad proposed the closest-match algorithm [18], which is a classical iterative approximation algorithm for the Preisach inverse. In this algorithm, the number of iterations does not exceed the discretization degree of the input, and the state of the thermostat relay operator (1) changes only once for each solution, greatly saving computation time [18,19]. By requiring the piecewise monotonicity and Lipschitz continuity of the Preisach operator and allowing the density function to be non-negative and constant, an approximate inverse model based on the closest-match algorithm was proposed in [20] for calculating the inverse of the Preisach operator iteratively, and the convergence of the algorithm was proven. When the density function of the Preisach operator is unknown or not available for measurement, the previously mentioned open-loop inverse control is not possible. In this case, the feedback information obtained from the hysteresis output can be utilized to estimate the density function of the Preisach operator by developing an iterative algorithm with an adaptive estimator, ultimately reducing the inversion error [19]. The above-mentioned iterative adaptive inverse-control framework was established in [19,21]. For an individual Preisach operator, the compensation scheme has been studied in great depth. However, these results only consider the hysteresis nonlinearity while neglecting the influence of the plant. When the Preisach operator couples with some system dynamics (for example, smart material-based actuators can be modeled as a Preisach operator preceding linear dynamics [22] or when the hysteretic actuator modeled by the Preisach operator drives linear or nonlinear dynamics [19,23]), it remains unobserved for the output of hysteresis nonlinearity, which serves as the input to the system dynamics, and the adaptive scheme of the closest-match algorithm in [19] is not applicable. To overcome the above challenge, it is necessary to develop a new adaptive version of the closest-match algorithm to compensate for the Preisach hysteresis with complete convergence proof and stability analysis, especially when the system dynamics are described as a noncanonical nonlinear system with parametric uncertainties. Unlike the canonical nonlinear system, the noncanonical nonlinear system has no explicit relative structure, and the system output depends on several or all state variables. To address the presence of parameter uncertainties in the system, the predominant approach involves employing approximators for their approximation to construct a parameterized model. Neural networks, recognized as universal approximators, have recently experienced extensive and successful applications [24,25]. They are capable of approximating smooth functions with arbitrary accuracy in a desired compact set and effectively constructing dynamic system models. Leveraging this property, the method of utilizing neural network-based approximation techniques has been successfully applied to address the challenge of adaptive control for noncanonical nonlinear systems with parameter uncertainties [26–28]. In this regard, neural network approximation techniques will be used to construct a parameterized model in this paper, thus facilitating the design of an adaptive control scheme.

Motivated by the above observation, we have studied the adaptive inverse-control problem for uncertain noncanonical nonlinear systems with unknown input Preisach hysteresis. When the Preisach operator precedes the dynamics of an uncertain noncanonical nonlinear system, the hysteresis parameters, hysteresis output, and system parameters are all unknown, and the relative degree structure is also implicit. In this situation, we

propose an iterative adaptive inverse algorithm to effectively compensate for the hysteresis nonlinearity. This study makes the following contributions:

(1) A new adaptive version of the closest-match algorithm is proposed to address the inversion problem of the Preisach operator with unknown parameters and unobserved outputs. Based on the piecewise-monotonicity and Lipschitz-continuity properties of the adaptive Preisach operator, the convergence of the iteration algorithm for inverting the Preisach operator is successfully established.

(2) A Lyapunov-based adaptive inverse-control framework is proposed for uncertain noncanonical nonlinear systems with Preisach-type hysteresis inputs, with complete convergence proof and stability analysis.

The rest of the paper is organized as follows. In Section 2, we introduce the Preisach operator and formulate the control problem. In Section 3, by utilizing the feedback linearization technique, we derive a specific condition to define the relative degree of the neural network approximation system in noncanonical form. In Section 4, we propose an adaptive tracking control scheme containing an iterative adaptive inverse algorithm for an uncertain neural network approximation system with an unknown input Preisach hysteresis, which is the main focus of this paper. In Section 5, we provide a simulation example with corresponding results that validate the effectiveness of the control scheme. Finally, we provide the conclusions in Section 6.

2. Background and System Modeling

This section provides a concise review of the Preisach operator and applies it to effectively capture the complex hysteresis nonlinearity discussed in this paper, and the control problem is then formulated.

2.1. The Hysteresis Model

The Preisach operator stands out among various hysteresis models due to its ability to accurately represent complex hysteresis curves, including multi-loop and asymmetric hysteresis curves. It is constructed using the weighted superposition of infinite basic relay operators. Typically, the thermostat relay operators [19] are chosen as the fundamental components for constructing the Preisach operator, as shown in Figure 1.

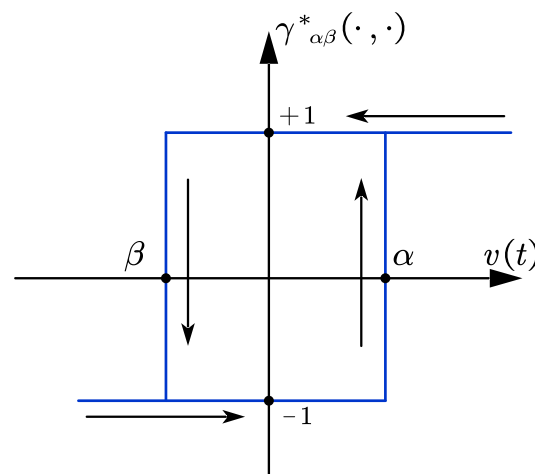


Figure 1. A thermostat relay operator $\gamma_{\alpha\beta}^*(\cdot, \cdot)$.

Thermostat relay operator: We first consider the Preisach plane as

$$T_0 = \{(\beta, \alpha) \in |\beta \geq \beta_0, \alpha \leq \alpha_0, \alpha \geq \beta\},$$

which is a right triangle area and consists of a vertex coordinate (β_0, α_0) and a portion of the line $\alpha = \beta$. For a visual representation, we present the geometric interpretation of the

Preisach plane T_0 in Figure 2. For any given point (β, α) on the Preisach plane T_0 , there is a corresponding thermostat relay operator

$$\gamma_{\alpha\beta}^*(v(t), \tau_0(\beta, \alpha)) = \begin{cases} +1, & \text{if } v(t) > \alpha \\ -1, & \text{if } v(t) < \beta \\ \gamma_{\alpha\beta}^*(v(t^-), \tau_0(\beta, \alpha)) & \text{if } v(t) \in [\beta, \alpha], \end{cases} \quad (1)$$

where $v(t) \in [0, t_m]$ is the input of the thermostat relay operator with continuity and piecewise monotonicity, $t^- = \lim_{\varepsilon > 0, \varepsilon \rightarrow 0} t - \varepsilon$, and $\tau_0(\beta, \alpha)$ represents the initial value of the thermostat relay operator $\gamma_{\alpha\beta}^*(v(0), \cdot)$. For example, $\tau_0(\beta, \alpha) = 1$, whereas $\forall (\beta, \alpha) \in T_0$ and $v(0) > \alpha_0$.

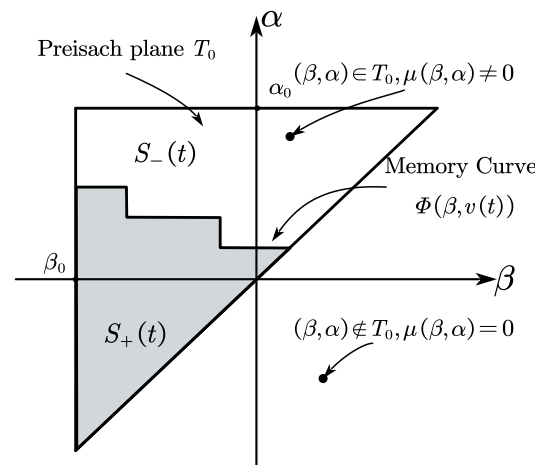


Figure 2. The Preisach plane T_0 and memory curve.

Preisach operator: The Preisach operator is constructed using the weighted superposition of infinite thermostat relay operators on the Preisach plane T_0 , which is expressed as follows

$$\begin{aligned} u(t) &= \mathcal{H}(v(t), \tau_0(\beta, \alpha)) \\ &= \iint_{T_0} \mu(\beta, \alpha) \gamma_{\alpha\beta}^*(v(t), \tau_0(\beta, \alpha)) d\beta d\alpha, \end{aligned} \quad (2)$$

where the weighting function $\mu(\beta, \alpha)$ is also referred to as the density function. According to the definition of the Preisach operator, all points $(\beta, \alpha) \in T_0$ have the corresponding density function $\mu(\beta, \alpha) \neq 0$, and when $(\beta, \alpha) \notin T_0$, the density function $\mu(\beta, \alpha) = 0$, as shown in Figure 2.

Memory curve: The memory effects of the Preisach operator can be captured using the memory curve in the Preisach plane T_0 (as illustrated in [19]). When the Preisach input increases monotonically, the output of the thermostat relay operator above the α threshold switches to +1 and forms an upward-shifting curve. Similarly, when the Preisach input decreases monotonically, the output of the thermostat relay operator below the β threshold switches to −1 and forms a leftward-shifting curve. Then, in the Preisach plane T_0 , a piecewise monotonic input signal $v(t)$ can create the memory curve $\Phi(\beta, v(t))$, as shown in Figure 2, where the memory curve divides the plane T_0 into two parts:

$$\begin{aligned} S_+(t) &= \{(\beta, \alpha) \in T_0 \mid \gamma_{\alpha\beta}^*(v(t), \cdot) = +1\}, \\ S_-(t) &= \{(\beta, \alpha) \in T_0 \mid \gamma_{\alpha\beta}^*(v(t), \cdot) = -1\}, \end{aligned} \quad (3)$$

and we can rewrite the integral (2) as

$$\begin{aligned} u(t) &= \mathcal{H}(v(t), \tau_0(\beta, \alpha)) \\ &= \iint_{S_+(t)} \mu(\beta, \alpha) d\beta d\alpha - \iint_{S_-(t)} \mu(\beta, \alpha) d\beta d\alpha \\ &= 2 \iint_{S_+(t)} \mu(\beta, \alpha) d\beta d\alpha - \iint_{T_0} \mu(\beta, \alpha) d\beta d\alpha, \end{aligned} \quad (4)$$

which is an essential form for analyzing the output range of the Preisach operator and proving the piecewise monotonicity of the adaptive Preisach operator.

Following the description of the Preisach operator, we proceed to introduce the considered plant model and formulate the adaptive control problem.

2.2. System Modeling

Consider the following uncertain noncanonical nonlinear system with an unknown input Preisach hysteresis:

$$\begin{aligned} \dot{x}(t) &= \mathcal{N}(x(t)) + Bu(t), \\ y(t) &= Cx(t), \quad u(t) = \mathcal{H}(v(t), \tau_0(\beta, \alpha)), \end{aligned} \quad (5)$$

where the $\mathcal{N}(x(t)) \in \mathbb{R}^n$ represents the unknown unparametrizable system nonlinearities, $x(t) \in \mathbb{R}^n$ denotes the system state vector, $y(t) \in \mathbb{R}$ denotes the system output, and $B \in \mathbb{R}^n, C \in \mathbb{R}^{1 \times n}$ are the unknown system parameters. The control input $v(t) \in \mathbb{R}^n$ is implicitly involved in the Preisach operator, and $u(t)$ is the Preisach output, which directly affects the system. Since the Preisach hysteresis parameters $\mu(\beta, \alpha)$ are unknown, the output of the Preisach operator $u(t)$ is not available for measurement, which poses a challenge in compensating for the hysteresis nonlinearity.

An approximation system: In our research, the system nonlinearities $\mathcal{N}(\cdot)$ in (5) cannot be fully parameterized and are considered to be unknown. This poses a challenge in designing the control scheme for the original system (5) due to the lack of an explicit characterization of these nonlinearities. To overcome this challenge, we construct a parametrizable neural network approximation system, which serves as an equivalent representation of the original system (5) over any desired compact set $\Psi \in \mathbb{R}^n$ [29] and takes the following form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \mathcal{W}^* S(x(t)) + Bu(t), \\ y(t) &= Cx(t), \quad u(t) = \mathcal{H}(v(t), \tau_0(\beta, \alpha)), \end{aligned} \quad (6)$$

where $A \in \mathbb{R}^{n \times n}$ is a stable matrix, and $\mathcal{W}^* \in \mathbb{R}^{n \times l}$ and $S(x(t)) \in \mathbb{R}^l$ are an unknown connection weight matrix and a known activation functions vector, respectively.

Remark 1. The nonlinear term $\mathcal{W}^* S(\cdot)$ in (6) is considered a parameterizable uncertainty, which is capable of approximating unparametrizable uncertainties with arbitrary accuracy within a desired compact set. Hence, the proposed control scheme for the approximation system (6) in this paper is valid for the general noncanonical nonlinear system (5) with unparametrizable nonlinearities. By leveraging the neural network approximation system as an equivalent representation, our control scheme provides a practical and viable solution for achieving the desired control performance with an unknown input Preisach hysteresis.

Considering the constructed approximation system (6), our control objective is to design a control input signal $v(t)$ by coupling the Lyapunov method with the iterative algorithm to ensure that the signals within the closed-loop system are bounded and achieve asymptotic tracking performance.

3. Relative Degree Conditions and Stability of Zero Dynamics Subsystem

In this paper, our main focus is on addressing the control problem for noncanonical nonlinear systems with input hysteresis using adaptive control techniques, specifically in the relative-degree-one case. It should be pointed out that the relative degree greater than one case remains open for future research and will be considered in our future work. This noncanonical neural network system can be considered a general nonlinear system, allowing the feedback linearization theory to be used to define its relative degree. In a later section, we provide the specific conditions for the relative-degree-one case.

Relative Degree Conditions: By combining the definition of relative degree [30] with the noncanonical nonlinear system (6), we establish the following necessary condition for the cases where the system has a relative degree of one.

Lemma 1 ([29]). *The approximation system (6) preceded by the Preisach operator has a relative degree $\varrho = 1$ if and only if*

$$CB \neq 0. \quad (7)$$

The approximation system (6) can be equivalently transformed into the general nonlinear system $\dot{x}(t) = f_0(t) + g_0(t)u(t)$, $y(t) = Cx(t)$, and based on the feedback linearization conclusions, Lemma 1 can be straightforwardly proven.

Lemma 2 ([29]). *Suppose the approximation system (6) has a relative degree ϱ on the compact set Ψ . To facilitate analysis and control design, we employ a diffeomorphism $\Omega(x) = [T_c^T(x), T_z^T(x)]^T \in \mathbb{R}^n$ where*

$$\begin{aligned} T_c(x) &= \xi(t) = [\xi_1(t), \xi_2(t), \dots, \xi_\varrho(t)]^T \\ &= [h_0(x), L_{f_0}h_0(x), \dots, L_{f_0}^{\varrho-1}h_0(x)] \in \mathbb{R}^\varrho, \\ T_z(t) &= \eta(t) \in \mathbb{R}^{n-\varrho}, \end{aligned}$$

which can transform the system into two subsystems [31]. The first subsystem, known as the tracking dynamics subsystem, is dedicated to achieving accurate tracking of a desired reference signal, and it is defined as follows

$$\begin{aligned} \dot{\xi}_k(t) &= \xi_{k+1}(t), k = 1, 2, \dots, \varrho - 1, \\ \dot{\xi}_\varrho(t) &= L_f^\varrho h(x) + L_g L_f^{\varrho-1} h(x)u(t). \end{aligned} \quad (8)$$

The second subsystem, referred to as the zero dynamics subsystem, is of great importance for ensuring the convergence and stability of the system's internal dynamics. It takes the following form:

$$\dot{\eta}(t) = \Xi(\xi(t), \eta(t)). \quad (9)$$

Stability of the zero dynamics system: By utilizing the feedback linearization technique, the approximate system (6) can be divided into two subsystems (as illustrated in Lemma 2). The zero dynamics subsystem does not contain control inputs. Therefore, the stability of the zero dynamics subsystem needs to be guaranteed to ensure that the control scheme developed for the noncanonical nonlinear system with input hysteresis in this paper is available. The following assumption satisfies our requirements.

Assumption 1. *The partial derivatives of the zero dynamics subsystem with respect to $\xi(t)$ (9) are bounded, and the zero dynamics subsystem satisfies the following inequality:*

$$\eta^T(t)\Xi(0, \eta(t)) \leq -\lambda_0 \eta^T(t)\eta(t) + \lambda_m(t), \quad (10)$$

where λ_0 is a positive constant, and $\lambda_m(t)$ is a bounded function [32].

Remark 2. Based on Assumption 1, we can establish the following inequality

$$\|\eta(t)\| \leq \mathcal{K}_1 \|\xi(t)\| + \mathcal{K}_2, \quad (11)$$

where $\mathcal{K}_1, \mathcal{K}_2 > 0$ are the proper constants. Inequality (11) indicates that the state vector $\eta(t)$ in (9) is bounded, along with the bounded input vector $\xi(t)$. Such a conclusion is called bounded-input bounded-state (BIBS) stability [33], which indicates that the response of the system remains within a specific range in the presence of disturbances or external inputs.

4. Adaptive Inverse-Control Scheme for Relative-Degree-One Case with Preisach Hysteresis

This section proposes a control scheme for the relative-degree-one case of the uncertain noncanonical nonlinear neural network system (6) with input Preisach hysteresis, for which the necessary condition is given in (7). The procedure for designing the control scheme is detailed below.

4.1. System Parameterization

According to Lemma 1, the relative degree of the approximation system (6) is one when it satisfies $CB \neq 0$, which leads to the formulation of the tracking control dynamics subsystem, which can be expressed as follows

$$\begin{aligned} \dot{y}(t) &= CAx(t) + CW^*S(x(t)) + CBu(t), \\ u(t) &= \mathcal{H}(v(t), \tau_0(\beta, \alpha)), \end{aligned} \quad (12)$$

where the system parameters A, B, C , and $\mathcal{W}^*$ are all unknown. For the tracking control study, the following basic assumption is needed.

Assumption 2. The sign of the control gain CB in (12) is known and positive [26].

This assumption guarantees that the design procedure of the control scheme is free from any unknown control direction problems.

For ease of designing an adaptive control scheme, the system (12) needs to be reparameterized. We introduce some new parameters to transform the system into a more suitable form for adaptive control scheme design. Let $\vartheta_1^* = [CA, CW^*]^T$ represent a parameter vector, $\omega_1(t) = [x(t), S^T(x(t))]^T$ denote the state vector, and $\mu^*(\beta, \alpha) = CB\mu(\beta, \alpha)$ represent the modified density function. Then, the system (12) can be expressed as follows

$$\begin{aligned} \dot{y}(t) &= \vartheta_1^{*T} \omega_1(t) + \mathcal{H}^*(v(t), \tau_0(\beta, \alpha)), \\ \mathcal{H}^*(v(t), \tau_0) &= \iint_{T_0} \mu^*(\beta, \alpha) \gamma_{\alpha\beta}^*(v(t), \tau_0) d\beta d\alpha. \end{aligned} \quad (13)$$

Assumption 3. The modified density function $\mu^*(\beta, \alpha)$ defined on a finite right triangle plane T_0 takes values between two known non-negative bounded values $\mu_a(\beta, \alpha)$ and $\mu_b(\beta, \alpha)$, implying that $\mu_a(\beta, \alpha) \leq \mu^*(\beta, \alpha) \leq \mu_b(\beta, \alpha)$.

Assumption 3 is used later in a projection design to equip the adaptive estimate $\hat{\mu}(\beta, \alpha, t)$ of $\mu^*(\beta, \alpha)$ with non-negativity and boundedness properties.

4.2. Implicit Controller Equation

To compensate for the input hysteresis nonlinearity $\mathcal{H}(v(t), \tau_0)$ and to construct a tracking error system with asymptotic convergence properties, we develop an adaptive Preisach inverse implicit controller as follows

$$\begin{aligned} \iint_{T_0} \hat{\mu}(\beta, \alpha, t) \gamma_{\alpha\beta}^*(v(t), \tau_0) d\beta d\alpha \\ = -\iota(y(t) - y_m(t)) - \vartheta_1^T(t) \omega_1(t) + \dot{y}_m(t), \end{aligned} \quad (14)$$

where $\hat{\mu}(\beta, \alpha, \cdot)$ and $\vartheta_1(\cdot)$ are the estimates of $\mu^*(\beta, \alpha)$ and ϑ_1^* , respectively, and ι is a positive constant. The implicit controller equation (14) consists of an adaptive Preisach operator, with $\hat{\mu}(\beta, \alpha, t)$ as the adaptive estimate of $\mu^*(\beta, \alpha)$ on the left sides and the desired output of the adaptive Preisach operator on the right sides. Then, we define

$$\hat{\mathcal{H}}(v(t), \tau_0) = \iint_{T_0} \hat{\mu}(\beta, \alpha, t) \gamma_{\alpha\beta}^*(v(t), \tau_0) d\beta d\alpha, \quad (15)$$

$$u_d(t) = -\iota(y(t) - y_m(t)) - \vartheta_1^T(t) \omega_1(t) + \dot{y}_m(t), \quad (16)$$

and the implicit controller equation (14) is rewritten as

$$\hat{\mathcal{H}}(v(t), \tau_0) = u_d(t). \quad (17)$$

The next task is to solve the implicit controller equation (17) so that we can compute the control input $v(t)$ in real time. This is essentially equivalent to constructing the inverse function $v(t) = \hat{\mathcal{H}}^{-1}(u_d(t), \tau_0)$. Next, we propose an inverse iterative algorithm to solve it.

A closest-match algorithm for solving the implicit controller equation (17) and its convergence proof: Given that the desired output of the adaptive Preisach operator $u_d(t)$ exhibits continuous, piecewise monotonic behavior over the defined time interval $[0, t_E]$, where the partition is

$$0 = t_0 < t_1 < \dots < t_{N-1} < t_N = t_E, \quad (18)$$

for a positive integer $N \geq 1$, and during each sub-interval $(t_i, t_{i+1}]$, $i = 0, 1, 2, \dots, N-1$, $u_d(t)$ is monotonic. Then, the implicit controller Equation (17) will be solved on each sub-interval $(t_i, t_{i+1}]$. It is shown in the analysis in Remark 4 that the adaptive estimate density function $\hat{\mu}(\beta, \alpha, t)$ changes slowly with time. In this sense, we can assume that $\hat{\mu}(\beta, \alpha, t) = \hat{\mu}(\beta, \alpha, t_i)$ during each sub-interval $(t_i, t_{i+1}]$, where $i = 0, 1, 2, \dots, N-1$. With this in mind, the adaptive Preisach operator $\hat{\mathcal{H}}(v(t), \tau_0)$ can be expressed as

$$\begin{aligned} \hat{\mathcal{H}}(v(t), \tau_0) = \iint_{T_0} \hat{\mu}(\beta, \alpha, t_i) \gamma_{\alpha\beta}^*(v(t), \tau_0) d\beta d\alpha, \\ t \in (t_i, t_{i+1}], i = 0, 1, 2, \dots, N-1. \end{aligned} \quad (19)$$

With the later projection design, $\hat{\mu}(\beta, \alpha, t)$ is ensured boundedness as $\mu_a(\beta, \alpha) \leq \hat{\mu}(\beta, \alpha, t) \leq \mu_b(\beta, \alpha)$, and non-negativity for $\forall t > 0$ and $\forall (\beta, \alpha) \in T_0$. Then, using the third equality in (4), it is not hard to prove that the adaptive Preisach operator $\hat{\mathcal{H}}(v(t), \tau_0)$ has monotonicity on each sub-interval $(t_i, t_{i+1}]$, and the output range of $\hat{\mathcal{H}}(v(t), \tau_0)$ can be obtained using the following equation during $(t_i, t_{i+1}]$:

$$H_{i,min} = - \iint_{T_0} \hat{\mu}(\cdot) d\beta d\alpha, \quad H_{i,max} = \iint_{T_0} \hat{\mu}(\cdot) d\beta d\alpha.$$

For the implicit controller equation (17) to have a solution, the following constructed saturation condition is necessary:

$$H_{i,min} \leq u_d(t) \leq H_{i,max} \quad \text{for} \quad \forall t \in (t_i, t_{i+1}], \quad (20)$$

where $i = 0, 1, 2, \dots, N-1$. The limitation of the output range (20) stems from the fact that the Preisach operator $\mathcal{H}(v(t), \tau_0)$ is a saturated hysteresis model, and the saturation occurs when the control input $v(t)$ is above the upper threshold α_0 or below the lower threshold β_0 .

Suppose that Condition (20) is satisfied. There are two discretization steps involved: the discretization of the time interval $[0, t_E]$ has been described in (18), and the discretization range $R = [v_{\min}, v_{\max}]$ of the adaptive Preisach operator (15) input $v(t)$ is uniformly divided into L segments as $V_L = \{\bar{v}_j, j = 1, 2, \dots, L+1\}$, where $\bar{v}_j = v_{\min} + (j-1)\Delta v$, $\Delta v = (v_{\max} - v_{\min})/L$, and L is called the discretization level. The result of discretizing the input range R is that the Preisach plane T_0 is divided into cells. Considering the plane T_0 with discretization degree L , and within each discretization cell, assuming that the density function $\hat{\mu}(\beta, \alpha, t)$ in (15) is non-negative and remains constant. The inversion problem is as follows: given the desired instantaneous value of $u_d(t)$ and the memory curve $\Phi(\beta, v(t_d))$ generated by the previous input, find the corresponding input signal $v^*(t)$, such that the equality $u_d(t) = \hat{\mathcal{H}}(v^*(t), \tau_0)$ is satisfied, which can be calculated using the following Algorithm 1:

Algorithm 1 Closest-Match Algorithm For Adaptive Preisach Operator [20].

Input: The memory curve $\Phi(\beta, v(t_d))$ and the desired value of $u_d(t)$

Output: Control input $v_L^*(t)$

(Step 1) Set $m = 0, v^{(m)} = v_{\min}$

(Step 2)

if $v^{(m)} = \bar{v}_{L+1}$ then

go to Step 5.

else

$v^{(m+1)} = v^{(m)} + \Delta v$;

$\tilde{\Phi} = \Phi(\beta, v^{(m)})$ (backup the memory curve);

$m = m + 1$;

go to Step 3.

end if

(Step 3) Calculate $u_d^{(m)} = \hat{\mathcal{H}}(v^{(m)}, \tau_0)$, and update the memory curve to $\Phi(\beta, v^{(m)})$.

if $u_d^{(m)} = u_d(t)$ then

go to Step 5.

else if $u_d^{(m)} < u_d(t)$ then

go to Step 2.

else

go to Step 4.

end if

(Step 4)

if $|u_d^{(m)} - u_d(t)| \leq |u_d^{(m-1)} - u_d(t)|$ then

go to Step 5.

else

$v_L^*(t) = v^{(m-1)}$;

$\Phi(\beta, v_L^*(t)) = \tilde{\Phi}$;

Exit.

end if

(Step 5)

$v_L^*(t) = v^{(m)}$;

$\Phi(\beta, v_L^*(t)) = \Phi(\beta, v^{(m)})$;

Exit.

The algorithm is based on the piecewise-monotonicity property of the adaptive Preisach operator $\hat{\mathcal{H}}(v(t), \tau_0)$, and it is not hard to see that the algorithm obtains the solution $v_L^*(t)$ in at most L times. The convergence of the above iterative algorithm is provided below.

Proposition 1. Under Assumption 1–3, suppose that Condition (20) is satisfied. Then, the iterative algorithm can find a solution $v_L^*(t) = \bar{v}_j \in V_L$, such that

$$|\hat{\mathcal{H}}(v_L^*(t), \tau_0) - u_d(t)| = \min_{\bar{v}_j \in V_L} |\hat{\mathcal{H}}(\bar{v}_j, \tau_0) - u_d(t)|. \quad (21)$$

In addition, as the discretization degree tends to infinity, we can find the exact solution of the inverse problem, i.e., $\lim_{L \rightarrow \infty} v_L^*(t) = v^*(t)$.

Proof. Our task is to prove the piecewise monotonicity and Lipschitz continuity of the adaptive Preisach operator (15). Based on this property, we can follow the arguments in Proposition 5.1 in [20] to prove Proposition 1.

As previously demonstrated, it has been established that the adaptive Preisach operator (15) can be represented in the form of (19) during the sub-intervals $(t_i, t_{i+1}]$ with the adaptive density function $\hat{\mu}(\beta, \alpha, t_i)$ being non-negative. Then, from the third equality of (4), the following inequality holds

$$(\hat{\mathcal{H}}(v(t_2), \tau_0) - \hat{\mathcal{H}}(v(t_1), \tau_0))(v(t_2) - v(t_1)) \geq 0, \quad (22)$$

for $\forall t_1, t_2 \in (t_i, t_{i+1}]$. Hence, the adaptive Preisach operator (15) is piecewise monotonic on $[0, t_E]$ for the continuous, piecewise monotonic control input signal $v(t)$ on $(0, t_E]$. In addition, with the later projection design, the adaptive density function $\hat{\mu}(\beta, \alpha, t_i)$ is guaranteed to be non-negative and bounded for $\forall t_i \geq 0$, and $\forall (\beta, \alpha) \in T_0$. Then, based on the piecewise expression (19), we can obtain the following Lipschitz continuity property:

$$\|\hat{\mathcal{H}}(v(\mathcal{T}_2), \tau_0) - \hat{\mathcal{H}}(v(\mathcal{T}_1), \tau_0)\| \leq K_L \|v(\mathcal{T}_2) - v(\mathcal{T}_1)\|, \quad (23)$$

for $\forall \mathcal{T}_1, \mathcal{T}_2 \in [0, t_E]$, where K_L is a Lipschitz constant.

Based on the piecewise monotonicity (22) and Lipschitz continuity (23) of the adaptive Preisach operator (15), we can follow the arguments in Proposition 5.1 in [20] to prove Proposition 1. $\square$

So far, we have provided an iterative algorithm through which the control input signal $v(t)$ in the implicit controller equation (17) can be computed iteratively, and finally, the convergence of this iterative algorithm has been proven. Next, we analyze the performance of the adaptive control scheme.

4.3. Performance Analysis

Due to the limitations of computation time and the efficiency of the iterative algorithm, obtaining an exact solution within a finite number of iterations is challenging. Therefore, the implicit control equation (14) can be reformulated as follows

$$\begin{aligned} & \iint_{T_0} \hat{\mu}(\beta, \alpha, t) \gamma_{\alpha\beta}^*(v_L^*(t), \tau_0) d\beta d\alpha + \delta(t) \\ & = -l(y(t) - y_m(t)) - \vartheta_1^T(t) \omega_1(t) + \dot{y}_m(t), \end{aligned} \quad (24)$$

where $\delta(t)$ is the bounded iteration error. Using the iteration results $v_L^*(t)$ as the control input and by substituting (13) into (24), we have the tracking error equation as follows

$$\begin{aligned} \dot{e}(t) & = - \iint_{T_0} \tilde{\mu}(\beta, \alpha, t) \gamma_{\alpha\beta}^*(v_L^*(t), \tau_0) d\beta d\alpha \\ & \quad - \delta(t) - l e(t) - \tilde{\vartheta}_1^T(t) \omega_1(t), \end{aligned} \quad (25)$$

where $e(t) = y(t) - y_m(t)$, and the adaptive parameter errors are $\tilde{\mu}(\beta, \alpha, \cdot) = \hat{\mu}(\beta, \alpha, \cdot) - \mu^*(\beta, \alpha)$ and $\tilde{\vartheta}_1^T(\cdot) = \vartheta_1(\cdot) - \vartheta_1^*$.

Remark 3. In practical engineering applications, a proper bounded discretization degree L ensures that $|\delta(t)| \leq \epsilon$, where ϵ is an acceptable minor positive constant in engineering applications. Hence, we consider the iterative error $\delta(t)$ as an external disturbance and use the following tracking error Equation (26) for the next analysis in this paper.

$$\begin{aligned} \dot{e}(t) = & - \iint_{T_0} \tilde{\mu}(\beta, \alpha, t) \gamma_{\alpha\beta}^*(v_L^*(t), \tau_0) d\beta d\alpha \\ & - \iota e(t) - \tilde{\vartheta}_1^T(t) \omega_1(t). \end{aligned} \quad (26)$$

By considering the tracking error Equation (26), we choose the positive definite function as

$$V(e, \tilde{\mu}) = \frac{1}{2\phi} \iint_T \tilde{\mu}^2(\beta, \alpha, \cdot) d\beta d\alpha + \frac{1}{2} e^2 + \frac{1}{2} \tilde{\vartheta}_1^T \Gamma_1^{-1} \tilde{\vartheta}_1, \quad (27)$$

where $\Gamma_1 = \Gamma_1^T > 0$ and $\phi > 0$ are the adaptive parameters for adaptive laws. Then, the time derivation of $V(e, \tilde{\mu})$ is

$$\begin{aligned} \dot{V} = & -\frac{1}{\phi} \iint_T \tilde{\mu}(\beta, \alpha, t) \left(\phi \gamma_{\alpha\beta}^*(v_L^*(t), \tau_0) e(t) - \frac{\partial}{\partial t} \hat{\mu}(\beta, \alpha, t) \right) d\beta d\alpha \\ & - \iota e^2(t) - \tilde{\vartheta}_1^T \Gamma_1^{-1} (\Gamma_1 \omega_1(t) e(t) - \dot{\vartheta}_1(t)). \end{aligned} \quad (28)$$

Lyapunov-based adaptive control scheme: To ensure that $\dot{V} \leq 0$, the update laws for the estimates $\vartheta(t)$ and $\hat{\mu}(\beta, \alpha, t)$ are chosen as

$$\dot{\vartheta}_1(t) = \Gamma_1 \omega_1(t) e(t), \quad (29)$$

$$\frac{\partial}{\partial t} \hat{\mu}(\beta, \alpha, t) = \begin{cases} \phi \gamma^*(t) e(t) & \text{if } \hat{\mu} \in (\mu_a, \mu_b), \text{ or} \\ & \text{if } \hat{\mu} = \mu_a, \gamma^*(t) e(t) \geq 0, \text{ or} \\ & \text{if } \hat{\mu} = \mu_b, \gamma^*(t) e(t) \leq 0, \\ 0, & \text{otherwise,} \end{cases} \quad (30)$$

where $\gamma^*(t)$, $\hat{\mu}$, μ_a , and μ_b are the brief representations of $\gamma_{\alpha\beta}^*(v_L^*(t), \tau_0)$, $\hat{\mu}(\beta, \alpha, t)$, $\mu_a(\beta, \alpha)$, and $\mu_b(\beta, \alpha)$, respectively. By choosing the initial value of $\hat{\mu}(\beta, \alpha, t)$ within the range $[\mu_a(\beta, \alpha), \mu_b(\beta, \alpha)]$, the projection design (30) ensures that $\mu_a(\beta, \alpha) \leq \hat{\mu}(\beta, \alpha, t) \leq \mu_b(\beta, \alpha)$ and $\tilde{\mu}(\beta, \alpha, t) \left(\phi \gamma_{\alpha\beta}^*(v_L^*(t), \tau_0) e(t) - \frac{\partial}{\partial t} \hat{\mu}(\beta, \alpha, t) \right) \geq 0$ for $\forall t \geq 0$. Therefore, we have the following results for $\lim_{t \rightarrow \infty} e(t) = 0$.

Theorem 1. Under Assumptions 1–3 and Proposition 1, all signals in the closed-loop system consisting of the noncanonical nonlinear system (6), the Preisach operator (2), the iterative inverse algorithm, and the implicit controller (14), which is updated using the adaptive laws (29) and (30), are bounded, and the tracking error $e(t)$ satisfies

$$\lim_{t \rightarrow \infty} e(t) = 0.$$

Proof. By substituting the adaptive laws (29) and (30) into the derivation of V (28), we can derive that

$$\dot{V} \leq -\iota e^2(t). \quad (31)$$

Since ι is a positive constant, we have $\dot{V} \leq 0$. Then, $e(t)$, $\vartheta_1(t)$, and $\hat{\mu}(\beta, \alpha, t)$ are bounded, which implies that $y(t)$ is bounded. From Assumption 1, we can establish the inequality that $\|\eta(t)\| \leq \mathcal{K}\|y(t)\| + \mathcal{K}$ for a proper constant $\mathcal{K}$, thus $\eta(t)$ and $x(t)$ are bounded. From the desired output of the adaptive Preisach operator (16), we can derive the boundedness of $u_d(t)$. Then, the boundedness of all the closed-loop signals is established. Next, we show the properties that $e(t) \in L^2$ and $\lim_{t \rightarrow \infty} e(t) = 0$. Integrating both sides of the first inequality in the derivation of V in (31) yields $\int_0^\infty e^2(t) dt < \infty$, so $e(t) \in L^2$.

From the tracking error equation (26), it is evident that $\dot{e}(t)$ is bounded. Therefore, using Barbalat's Lemma, we can conclude that $\lim_{t \rightarrow \infty} e(t) = 0$. $\square$

Remark 4. Since $e(t) \in L^2 \cap L^\infty$ and $\lim_{t \rightarrow \infty} e(t) = 0$, using the projection design in (30), it is not hard to derive that

$$\frac{\partial}{\partial t} \hat{\mu}(\beta, \alpha, t) \in L^2 \cap L^\infty, \text{ and } \lim_{t \rightarrow \infty} \frac{\partial}{\partial t} \hat{\mu}(\beta, \alpha, t) = 0, \quad (32)$$

which means that $\hat{\mu}(\beta, \alpha, t)$, the adaptive estimate of $\mu^*(\beta, \alpha)$, changes very slowly and eventually converges to a time-independent value $\hat{\mu}^*(\beta, \alpha)$. Besides, the adaptive estimate $\hat{\mu}(\beta, \alpha, t)$ is limited to a compact set $[\mu_a(\beta, \alpha), \mu_b(\beta, \alpha)]$ by the projection design (30), which means that the $\hat{\mu}(\beta, \alpha, t)$ would not be large. Then, during each sub-interval $(t_i, t_{i+1}]$, $i = 0, 1, 2, \dots, N-1$, we can consider $\hat{\mu}(\beta, \alpha, t) = \hat{\mu}(\beta, \alpha, t_i)$ on the iterative algorithm for $\forall t \in (t_i, t_{i+1}]$, and would not affect the performance of the system.

5. Simulation Study

This section presents the simulation results for the relative-degree-one case of a non-canonical nonlinear approximation system (6) with unknown parameters and preceded by the Preisach hysteresis operator. The purpose of this section is to provide strong evidence for the proposed adaptive control scheme in achieving the desired tracking performance, as illustrated in Theorem 1, which guarantees that the tracking error converges to zero as time tends to infinity.

5.1. Experimental Equipment

We developed a piezo actuator-driven stage as the experimental platform, which mainly consists of four parts: (1) an E01 piezoelectric ceramic controller, including a communication module E18.i3, a sensor control module E09.S3/L3, and a power amplifier module E03.00, which has a voltage output range of 0–150 V; (2) a piezoelectric actuator, which has a displacement output of 0–40 μm ; (3) a vibration isolation table, which serves the purpose of isolating the experimental equipment from external vibration; and (4) a computer with MATLAB R2020a installed (see Figure 3).

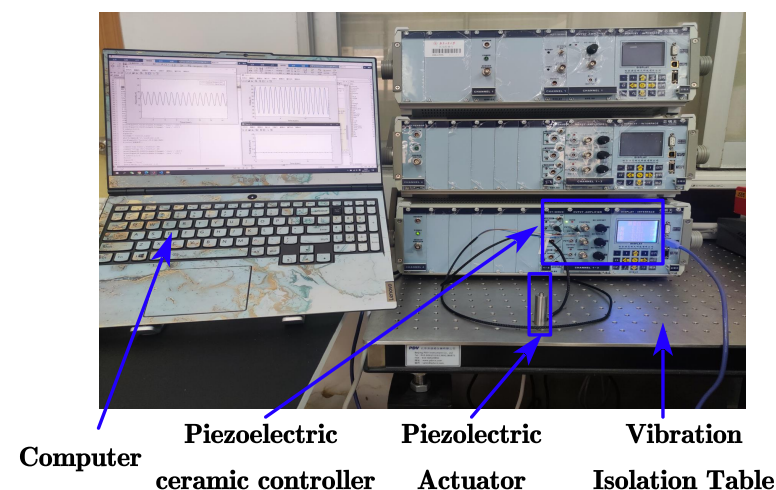


Figure 3. Experimental platform.

5.2. Hysteresis Identification

As the only parameter of the Preisach operator, the upper and lower bounds of the density function $\mu(\beta, \alpha)$ play an important role in ensuring the convergence of the iterative algorithm. Unreasonable settings of these bounds can seriously affect output accuracy. Therefore, it is necessary to perform systematic identification of the actual piezoelectric

actuator and then determine the upper and lower bounds of the estimator $\hat{\mu}(\beta, \alpha, t)$ based on the identification results. With this in mind, we employed a gradient descent algorithm to identify the density function. In this experiment, a triangular wave was chosen as the voltage input signal $v(t)$ at 5 Hz, with a range from 0 V to 115 V, and we set the sampling rate at 1 kHz. The identified density function is shown in Figure 4a. To evaluate the hysteresis curve matching degree between the one generated by the Preisach operator with the identified density function and the experiment measurement, the voltage input was chosen as a triangular wave signal at 5 Hz within a range of -55 V to 55 V, and the results are shown in Figure 4b, demonstrating a close resemblance between the two curves.

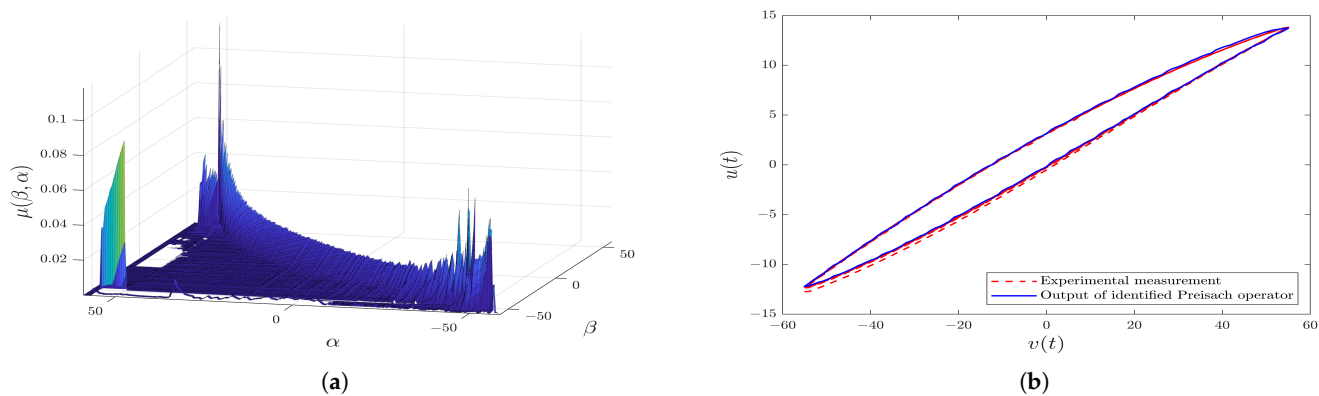


Figure 4. The identification results of the piezoelectric actuator. (a) The identified density function $\mu(\beta, \alpha)$ of the Preisach operator. (b) Matching degree of the Preisach operator with the identified density function with the experimental measurement of the hysteresis curve.

5.3. Simulation System Modeling

In the simulation experiments, we follow the noncanonical nonlinear approximation system (6) for the system parameter design, where $B = [2, 1]^T$, $C = [1, -1]$,

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \quad \mathcal{W}^* = \begin{bmatrix} 2.0 & -0.6 \\ 1.2 & 0.2 \end{bmatrix}, \quad (33)$$

and the activation functions vector $S(x) = [S_1(x), S_2(x)]^T$ with

$$S_1(x) = \frac{3}{1 + e^{-2x_2}} - 1.5, \quad S_2(x) = \left(\frac{4}{1 + e^{-2x_2}} - 2 \right) \left(\frac{3}{1 + e^{-2x_1}} - 1.5 \right). \quad (34)$$

The Preisach operator plane is defined by the thresholds $\beta_0 = -59$ and $\alpha_0 = 59$.

The initial control input is chosen as $v(0) = 0$, and considering the definition of the memory curve, we have $\Phi(\beta, 0) = 0$. The initial output of the Preisach operator is $\mathcal{H}(v(0), \tau_0(\beta, \alpha)) = 0.558$ with the density function $\mu(\beta, \alpha)$ obtained from the identification. The initial value of the state vector is chosen as $x(0) = [0.6, 0]^T$, and a basic sinusoidal function $y_m(t) = 2 \sin(2t) + 2.5$ is chosen as the reference signal.

Remark 5. For a general nonlinear system (5) in noncanonical form, it does not have an explicit relative degree, neither do its approximation systems [29], and its relative degree depends entirely on the unknown unparametrizable nonlinear functions in $\mathcal{N}(x(t))$. In this regard, the system matrix A , the connection weighted matrix $\mathcal{W}^*$, and the activation functions vector $S(x(t))$ in Equations (33) and (34) are in noncanonical form. An example of the general nonlinear system (5) in canonical form is expressed as follows:

$$\mathcal{N}(x(t)) = \begin{bmatrix} x_2 \\ -50 \sin(x_1) - 0.1x_2 \end{bmatrix}, \quad B = [0, 50]^T, \quad C = [1, 0].$$

5.4. Simulation Results

Initial parameters and design parameters: It is not hard to conclude that the simulation system satisfies the condition $CB \neq 0$, and then the adaptive scheme for the relative-degree-one case in Section 4 can be used to control this system. With the diffeomorphism $\Omega(x) = [\zeta, \eta]^T = [x_1 - x_2, -x_1 + 2x_2]^T$, the noncanonical nonlinear approximation system (6) can be transformed into a tracking dynamics subsystem and a zero dynamics subsystem with BIBS stability, thereby satisfying Assumption 1. Using a simple calculation, the nominal parameters are $\vartheta_1^* = [2, 0, 0, 0.8]^T$, and $\mu^*(\beta, \alpha) = CB\mu(\beta, \alpha)$, which are unknown for the control design and are estimated using $\vartheta_1(t)$ and $\hat{\mu}(\beta, \alpha, t)$, respectively. The lower and upper bounds of $\mu^*(\beta, \alpha)$ are chosen as $\mu_a(\beta, \alpha) = 0$ and $\mu_b(\beta, \alpha) = 1.1\mu(\beta, \alpha)$ for the projection design, where $\mu(\beta, \alpha)$ is obtained from the identification results. The initial parameters are chosen as $\vartheta_1(0) = 0.5\vartheta_1^*$ and $\hat{\mu}(\beta, \alpha, 0) = 0.7\mu(\beta, \alpha)$. Other design parameters are chosen as $\iota = 4$, $\phi = 0.08$, and $\Gamma_1 = \text{diag}\{1, 0.8, 0.8, 1.2\}$.

Simulation results and analysis: We employed the proposed adaptive control scheme in the simulation system, and the tracking performance is depicted in Figure 5a, which confirms the desired behavior of the control scheme and shows that the output $y(t)$ converges to the reference signal $y_m(t)$ over time. Figure 5b shows that the tracking error $e(t)$ gradually diminishes and eventually converges to zero over time. Furthermore, Figure 5c shows the boundedness of the system input $u(t)$ and control input $v(t)$. As an example to confirm that the estimate $\hat{\mu}(\beta, \alpha, t)$ changes very slowly and eventually converges to a time-independent value $\hat{\mu}^*(\beta, \alpha)$, as described in Remark 4, Figure 5d shows the trajectories of $\hat{\mu}(30, 50)$ and $\hat{\mu}(-50, -30)$ vs. time. With a discretization level of $L = 118$, Figure 6a shows that the iteration error $\hat{\mathcal{H}}(v_L^*(t), \tau_0) - u_d(t)$ is within the range of $\pm 0.2 \mu\text{m}$, where the control input $v_L^*(t)$ is calculated using the iterative algorithm in the implicit control Equation (17). The results confirm the convergence of the iterative algorithm established in Proposition 1. The effectiveness of the iterative algorithm is shown in Figure 6b, where we can see that the desired value $u_d(t)$ is achieved using the adaptive operator $\hat{\mathcal{H}}(v(t), \tau_0)$. For comparison, the PID control scheme is considered with the controller

$$v(t) = K_p e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + \lambda_d \frac{de(t)}{dt}, \quad (35)$$

where $K_p = 0.8$, $T_i = 0.067$, $\lambda_d = 0.4$. The closed-loop system tracking performance of the PID scheme is depicted in Figure 7, which shows that the PID scheme with meticulously adjusted parameters can also constrain the tracking error within a certain range. However, it is not clear how to further minimize the tracking error by adjusting the parameters K_p , T_i , and λ_d . With our scheme, the complete convergence proof and stability analysis of the closed-loop system are well-established, and the tracking error can converge to zero over time (as shown in Figure 5b), which effectively illustrates its advantages compared to the PID scheme.

Remark 6. The limitation of the proposed method lies in the fact that the proposed adaptive inverse compensation algorithm cannot be extended to other hysteresis models because the algorithm is designed specifically to compensate for hysteresis nonlinearities modeled by the Preisach operator. However, this limitation does not have an impact on this study. In the future, the authors intend to investigate a new adaptive version of the closest-match algorithm to extend it to other hysteresis models, aiming to eliminate this limitation.

Remark 7. The controlled plant in this article is modeled as a cascade of hysteresis nonlinearity and noncanonical nonlinear systems, which can describe various practical systems, such as smart material-based actuators (SMAs) [1], atomic force microscopes (AFMs) [34], and the end actuators of macro- and micro-manipulation robotic arms [35]. They can perform tasks such as corrosion measurement [36], polysaccharide microscopic analysis [37], intracytoplasmic sperm injections [38], and have other practical industrial applications. Consequently, the proposed control scheme presented in this article holds the potential for application in various real-world industrial scenarios.

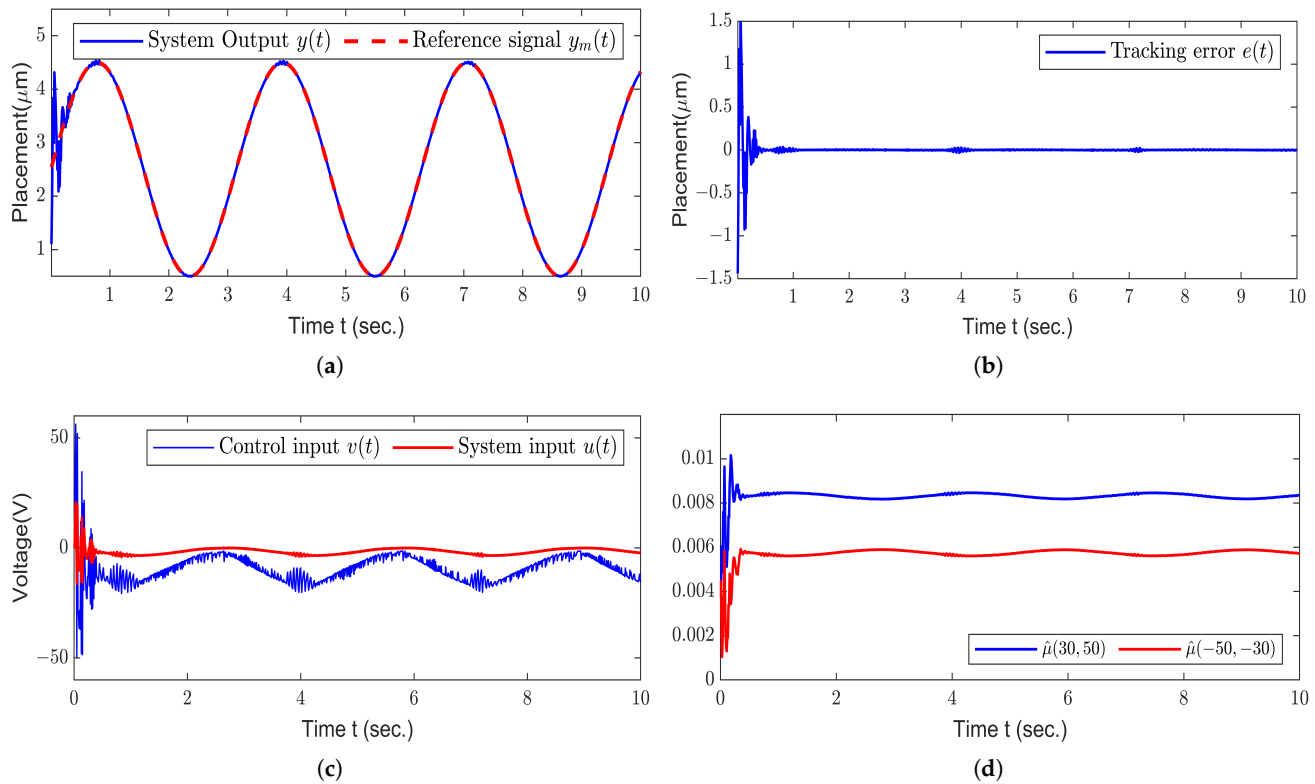


Figure 5. System response with $y_m(t) = 2\sin(t) + 2.5$. (a) Tracking performance $y(t)$ and $y_m(t)$ versus time (s). (b) Tracking error $e(t)$ versus time (s). (c) Control input $v(t)$ and system input $u(t)$ versus time (s). (d) Estimators $\hat{\mu}(30, 50)$ and $\hat{\mu}(-50, -30)$ versus time (s).

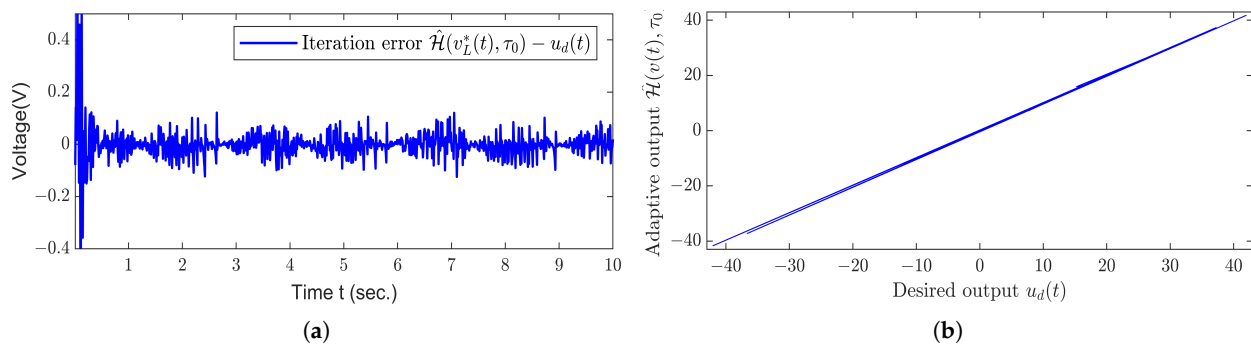


Figure 6. Performance of iterative algorithm.

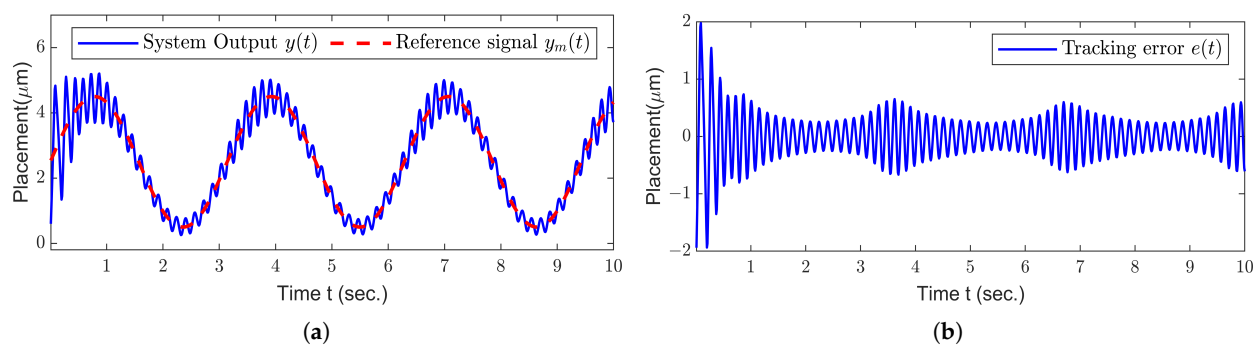


Figure 7. The system response of the PID control scheme. (a) Tracking performance $y(t)$ and $y_m(t)$ versus time (s). (b) Tracking error $e(t)$ versus time (s).

6. Conclusions

We have developed an iterative inverse-based adaptive control scheme for an uncertain nonlinear system in a noncanonical form with unknown input Preisach hysteresis. The control scheme utilizes a new adaptive version of the closest-match algorithm to effectively compensate for the unknown hysteresis effects. The convergence of the iterative algorithm was established by demonstrating the piecewise monotonicity and Lipschitz continuity of the adaptive Preisach operator $\hat{\mathcal{H}}(v(t), \tau_0)$. Furthermore, we conducted a complete stability analysis of the closed-loop system. The simulation results serve as strong evidence for the effectiveness of the proposed control scheme in achieving the desired tracking performance.

In our future work, for the completeness of the control scheme proposed in this article, we will attempt to develop an adaptive control scheme for a controlled plant constructed using the coupling of hysteresis nonlinearity and noncanonical nonlinear dynamics systems with relative degrees greater than one.

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