actuators

Article

Fault-Estimation Design Based on an Iterative Learning Scheme for Interconnected Multi-Flexible Manipulator Systems with Arbitrary Initial Value

Li Feng [1], Guangxi Chen [1], Shuiqing Xu [2,*] and Kenan Du [3]

[1] College of Traffic and Transportation, Chongqing Jiaotong University, Chongqing 400074, China; fengli@cqjtu.edu.cn (L.F.); cgxcqjtu@126.com (G.C.)
[2] College of Electrical and Automation Engineering, Hefei University of Technology, Hefei 230009, China
[3] College of Automation, Chongqing University, Chongqing 401331, China; dukenan@mails.cqjtu.edu.cn
* Correspondence: xsqanhui@hfut.edu.cn

Abstract: This paper reports the design of an iterative-learning-scheme-based fault-estimation method for interconnected nonlinear multi-flexible manipulator systems with arbitrary initial value. For state estimation, observers are employed to reconstruct the state. The proposed scheme ensures that each flexible manipulator subsystem's states can track their desired reference signals within a finite time. In the next step, an iterative learning fault-estimation law is proposed to track the actual fault signal. In contrast to the previous literature, this approach utilizes potential information from previous iterations to enhance the accuracy of the estimation in the current iteration. Based on these efforts, the obstacle caused by the arbitrary initial value is circumvented, and addressing the fault-estimation errors of each flexible manipulator subsystem are uniformly ultimately bounded is successfully achieved. Then, the λ-norm is developed to explore the convergence conditions of the presented methods. Finally, the effectiveness and feasibility of the proposed approach are verified through assessment of simulation results.

Keywords: fault estimation; interconnected multi-flexible manipulator systems; iterative learning scheme; arbitrary initial value

Citation: Feng, L.; Chen, G.; Xu, S.; Du, K. Fault-Estimation Design Based on an Iterative Learning Scheme for Interconnected Multi-Flexible Manipulator Systems with Arbitrary Initial Value. *Actuators* **2023**, *12*, 443. https://doi.org/10.3390/act12120443

Academic Editors: Giovanni Boschetti and João Miguel da Costa Sousa

Received: 9 October 2023
Revised: 23 November 2023
Accepted: 24 November 2023
Published: 28 November 2023

1. Introduction

With the rapid advancement of industrialization, control systems have become increasingly intricate. Interconnected multi-flexible manipulator systems, which can achieve mutual coordination and cooperation among multi-flexible manipulator and complete complex parallel tasks, have received widespread attention. This technology has found widespread application across various domains, including industrial automation, healthcare, aerospace and more. In these applications, when the flexible manipulator suffers from faults, these tasks are difficult to complete, and can cause significant security incidents and economic losses. It is well-known that there is information transmission among subsystems within interconnected multi-flexible manipulator systems, implying that a malfunction in one subsystem can lead to anomalies in another [1–3]. In other words, the impact of faults and the resultant losses in interconnected multi-flexible manipulator systems far exceed those in individual flexible manipulator systems. Therefore, the safety and reliability requirements of flexible manipulators have become more stringent, and it is now necessary to undertake research on fault estimation for interconnected multi-flexible manipulators.

Fault estimation [4–6] as a significant means of fault diagnosis and prediction not only enables early fault detection but also provides precise fault information, including the fault magnitude and shape. This provides a basis for system control decisions, effectively reducing the accident occurrence rate. In recent years, the design and analysis of fault-estimation methods have produced fruitful results within interconnected systems and flexible manipulator systems. In ref. [7], a distributed fault-estimation and fault-tolerant control system for continuous-time interconnected systems is proposed. This method fully utilizes the associated information between subsystems to improve the accuracy of fault estimation. Reference [8] reports the proposal of a new distributed fault-estimation observer with adjustable parameters for a class of nonlinear interconnected systems. It can achieve a lower performance level in quantitative analysis compared with existing fault-estimation approaches. In [9], a robust fuzzy observer design method is proposed to estimate both the state and faults for the considered class of a single-link flexible joint manipulator. In reference [10], a hidden Markov model method is used to represent the asynchronous mode variations between the original systems and the fault estimator. Although these existing methods exhibit good performance in fault estimation for interconnected systems or flexible manipulator systems, they do not consider the repetitive operation characteristics of flexible manipulator systems. A number of works have drawn attention to interconnected multi-flexible manipulator systems that can perform repetitive operations.

An iterative learning scheme [11,12] has been proposed to achieve the perfect tracking of a prescribed reference trajectory for systems that operate repetitively. This scheme utilizes important information from past experiences to improve the system's current behavior, making it a popular topic in existing research. In reference [13], an iterative learning observer is developed to achieve fault estimation and state reconstruction simultaneously for nonlinear systems with varying trial lengths. In reference [14], aiming at a class of nonlinear systems that contains faults, a novel iterative learning scheme is applied to fault detection, and a novel algorithm for fault detection is used to achieve the goal of fault detection in these systems. In reference [15], a state/fault simultaneous estimation observer based on iterative learning methods is designed for random repetitive systems with Brownian motion. It accurately estimates both the state and faults through iterations. However, the authors did not consider the initial value problem of the system. Traditional iterative learning fault-estimation methods require strict consistency between the initial state and the expected initial state for each iteration. However, with regard to the fault-estimation problem of flexible joint robotic arm systems that perform repetitive operations, the influence of the material properties of flexible robotic arms leads to deviations in the initial state in each iteration, resulting in failure to meet the strict consistency requirement between the initial state and the expected initial state. Currently, research on the initial value problem includes the following four cases in terms of scientific studies: (1) the case where the ideal initial value is exactly equal to the system initial value; (2) the case where the system initial value is not equal to the ideal initial value, but the system initial value remains fixed at the same value for each iteration; (3) the case where the system initial value can be any value at each iteration, but its magnitude varies within a certain range of the ideal initial value; and (4) the case where the system initial value can be any value. The existing literature focuses more on iterative learning control methods for systems with inconsistent initial states and ideal initial states. Reference [16] describes iterative learning control for a fractional order nonlinear system with a fixed initial value. In reference [17], a novel practical ILC updating law is proposed to improve the path-tracking accuracy of nonholonomic mobile robots with a fixed initial value. In reference [18], a different initial state that shifts the rectifying schemes and solves the problem of iterative learning control for high-order nonlinear systems with arbitrary initial state error is described. However, there are few studies that have simultaneously considered the initial shifts and faults. Regarding the case of a single-link flexible robotic arm experiencing a driver failure, ref. [19] proposed an adaptive boundary fault-tolerant control method, ref. [20] introduced an adaptive compensatory control method, and [21] suggested an adaptive PID control strategy. These control methods assume that the type of mechanical arm fault is known. However, during the operation of the mechanical arm, the type of driver failure is often unknown. Among these fault estimations, the tracking error and the system state error in previous iterations are not considered in the current iteration. Therefore, it is a challenging and necessary research task to design a fault-estimation method based on an iterative learning strategy to detect faults in a timely manner and to accurately estimate the fault signals.

Based on the above discussion, a novel iterative-learning-based fault-estimation method is proposed in this paper to solve the problem of fault estimation for interconnected nonlinear flexible manipulator systems with arbitrary initial value.

The main contributions of this paper are as follows:

(1) To address the problem of arbitrary initial value offsets in each subsystem of an interconnected multi-flexible manipulator system, a novel initial value reconstruction method based on an iterative learning strategy is proposed, which eliminates the adverse effects induced by arbitrary initial value offsets within interconnected multi-flexible manipulator systems.

(2) Considering the interconnections among the flexible manipulator subsystems, an iterative learning fault-estimation method is designed. This method can quickly and accurately estimate the fault signals occurring in each subsystem.

The paper is organized as follows: Section 2 provides a description of the observer system and the proposed iterative learning method. Section 3 describes the convergence conditions for this method, and Section 4 presents simulation examples of an interconnected dual-flexible manipulator system with arbitrary initial value to illustrate the theoretical findings of the paper. Finally, the conclusions are presented in Section 5.

2. Problem Formulation and Preliminaries

2.1. Interconnected Nonlinear System

This paper considers an interconnected system consisting of N single-link flexible manipulators. The system repetitively executes a given trajectory within the finite time interval $[0, T]$. A dynamic model of the subsystem i is as follows [22]:

$$\begin{aligned} &I\ddot{q}_{i,k} + K(q_{i,k} - \theta_{i,k}) + Mgl\sin q_{i,k} = 0 \\ &J\ddot{\theta}_{i,k} - K(q_{i,k} - \theta_{i,k}) = u_{i,k} + f_{i,k} \end{aligned} \tag{1}$$

where k denotes the index of the iterative number, $i \in \{1, 2, \cdots, N\}$, $q_{i,k} \in R^1$, $\theta_{i,k} \in R^1$ $u_{i,k} \in R^1$ and $f_{i,k} \in R^1$ correspond to the rotational angle of the manipulator arm, the motor angle, the motor input torque and the motor fault of the i-th flexible manipulator subsystem, respectively. I represents the moment of inertia of the manipulator arm, J denotes the inertia matrix of the motor, K represents the joint's elastic stiffness coefficient, M is the mass of the manipulator arm, l denotes the center of mass position of the manipulator arm, and g represents the gravitational acceleration.

To facilitate designing subsequent fault-estimation methods, the dynamic equation of the i-th single-link flexible manipulator system can be expressed based on the interconnected flexible manipulators system (1) as follows:

$$\begin{aligned} \dot{x}_{i,k}(t) = &\ A_i x_{i,k}(t) + B_i u_{i,k}(t) + g_1(x_{i,k}(t)) + E_i f_{i,k}(t) + \sum_{j=1, j\neq i}^{N} H_{ij} g_2\Big(x_{j,k}(t)\Big) \\ y_{i,k}(t) = &\ C_i x_{i,k}(t) \end{aligned} \tag{2}$$

where $x_{i,k}(t) = \begin{bmatrix} q_{i,k} & \dot{q}_{i,k} & \theta_{i,k} & \dot{\theta}_{i,k} \end{bmatrix}^T$ is the state vector, $u_{i,k}(t) \in R^1$ represents the input vector, $f_{i,k}(t) \in R^1$ denotes the bounded actuator fault signal and satisfies $f_{i,k+1}(t) = f_{i,k}(t)$, and $y_{i,k}(t) \in R^4$ is the output vector. The parameter matrices A_i, B_i, E_i, H_{ij} and C_i are constant real and with appropriate dimensions. The system matrices satisfy Assumption 1. It is noted that H_{ij} is the interconnected matrix between the subsystems i and j, and t is the time index. Specially, the vectors $g_1(x_{i,k}(t))$ and $g_2(x_{i,k}(t))$ denote the continuous Lipschitz nonlinear terms satisfying the following Assumption 2.

Assumption 1. *The pairs* (C_i, B_i) *and* (A_i, C_i) *are controllable and observable, respectively. The matrix* C_i *and* E_i *are of full rank.*

Assumption 2. *There exists a Lipschitz constant* ρ*; thus, the nonlinear functions satisfy*

$$\left\|g_j(\hat{x}_{i,k}(t)) - g_j(x_{i,k}(t))\right\| \le \rho_j \left\|\hat{x}_{i,k}(t) - x_{i,k}(t)\right\|, j = 1, 2 \tag{3}$$

2.2. Fault-Estimation Design Based on Iterative Learning Control

For the interconnected nonlinear system (1), the following observer for the i-th subsystem is constructed:

$$\begin{cases} \dot{\hat{x}}_{i,k}(t) = & A_i\hat{x}_{i,k}(t) + B_i u_{i,k}(t) + g_1(\hat{x}_{i,k}(t)) + E_i\hat{f}_{i,k}(t) \\ & + \sum\limits_{j=1, j\neq i}^{N} H_{ij} g_2\left(\hat{x}_{j,k}(t)\right) + L(y_{i,k}(t) - \hat{y}_{i,k}(t)) \\ \hat{y}_{i,k}(t) = & C_i\hat{x}_{i,k}(t) \end{cases} \tag{4}$$

where $\hat{x}_{i,k}(t) \in R^{4\times1}$, $\hat{y}_{i,k}(t) \in R^{4\times1}$ and $\hat{f}_{i,k}(t) \in R^{1\times1}$ denote the state estimation, the output estimation, and the fault estimation, respectively. $L \in R^{4\times4}$ represents the observer gain matrix to be determined. The fault-estimation law based on the iterative learning scheme is represented by Equation (5). The term $\hat{f}_{i,k}(t)$ denotes the fault estimate based on the iterative learning scheme, which is presented as follows:

$$\hat{f}_{i,k\,+\,1}(t) \;=\; \hat{f}_{i,k}(t) \;+\; K_{i,1}\Delta y_{i,k}(t) \;+\; K_{i,2}\Delta\dot{y}_{i,k}(t) \tag{5}$$

where $K_{i,1}$ and $K_{i,2}$ are the estimating gain matrices, $e_{i,k}(t)$ is the state estimation error, $\Delta y_{i,k}(t)$ is the output estimation error, $\Delta\dot{y}_{i,k}(t)$ is the derivative of the output estimation error, and $r_{i,k}(t)$ is the fault-estimation error.

$$\begin{cases} e_{i,k}(t) \;=\; \hat{x}_{i,k}(t) - x_{i,k}(t) \\ \Delta y_{i,k}(t) \;=\; \hat{y}_{i,k}(t) - y_{i,k}(t) \\ r_{i,k}(t) \;=\; \hat{f}_{i,k}(t) - f_{i,k}(t) \end{cases} \tag{6}$$

Based on the observer and the estimator designed above, the main purpose of this paper is to find the appropriate parameters to ensure convergence of the two error terms.

$$\begin{cases} \lim\limits_{k\to\infty} e_{i,k}(t) = 0 \\ \lim\limits_{k\to\infty} r_{i,k}(t) = 0 \end{cases} \tag{7}$$

Remark 1. *The flexible manipulator system runs on a finite time interval of* $[0, T]$*, and then repeatedly runs k times. The convergence result we seek is that the fault estimation fully tracks the actual fault throughout the interval of* $[0, T]$ *after k iterations.*

Lemma 1. *If* a_k *is the sequence of real numbers, which satisfies*

$$\|a_{k+1}\| \le \alpha\|a_k\| + \beta \quad 0 \le \alpha < 1, \beta > 0 \tag{8}$$

then, $\lim\limits_{k\to\infty}\|a_k\| \le \frac{\beta}{1-\alpha}$.

2.3. Problem Analysis

Due to the material properties of flexible joint robotic arms, which cause the system to deviate from the initial state in each iteration, there is a failure to meet the strict requirement of iterative learning strategies for the initial value to be consistently identical to the ideal initial value. This significantly constrains the application of an iterative-learning-scheme-based fault-estimation approach for flexible manipulator systems.

In this paper, the system initial value can be any value at each iteration, but its magnitude varies within a certain range of the ideal initial value and satisfies $\hat{x}_{i,k}(0) = x_{i,k}(0) + \delta_{i,k}$, where $\delta_{i,k}$ is an arbitrary number within a certain range of $x_{i,k}(0)$. Hence, the main problems considered in this paper are the following:

(1) How to track the fault signal well?
(2) How to eliminate the effect of initial value changes?

To address the first question, we design the state observer and fault-estimation law shown in Equations (5) and (6). The next step is to obtain the convergence conditions of the proposed method. For the second question, the initial state estimation is presented as

$$\hat{x}_{i,k+1}(0) \;=\; \hat{x}_{i,k}(0) + m_i \Delta y_{i,k}(0) \tag{9}$$

where $m_i \in R^{4\times 4}$ are the estimating gain matrices.

Then, the main questions addressed in this paper include finding out the appropriate parameters of the designed iterative-learning-scheme-based estimating law for fault signal reconstruction and the initial value estimation.

3. Convergence Analysis

In this section, the behavior of the proposed observers for the interconnected nonlinear system (1) are analyzed and the convergence condition for fault reconstruction and the initial state estimation are obtained.

Theorem 1. *Consider the system (2) satisfying Assumptions 1 and 2. If there exist the gain matrices $K_{i,1}$ and $K_{i,2}$, such that*

$$\Gamma = \max_{1\le i\le N}\{\|(I + K_{i,2}C_iE_i)\|\} < 1 \tag{10}$$

then, the fault-estimation errors $r_{i,k}(t)$ of each subsystem are uniformly ultimately bounded over the entire time interval $[0,T]$ under the action of the learning scheme (5).

Proof of Theorem 1. Based on the definition in (6), it can be obtained that

$$\begin{aligned} \dot{e}_{i,k}(t) =&\; \dot{\hat{x}}_{i,k}(t) - \dot{x}_{i,k}(t) \\ =&\; (A_i - LC_i)e_{i,k}(t) + E_i r_{i,k}(t) + g_1(\hat{x}_{i,k}(t)) - g_1(x_{i,k}(t)) \\ &+ \sum_{j=1,j\neq i}^{N} H_{ij}\Big(g_2\Big(\hat{x}_{j,k}(t)\Big) - g_2\Big(x_{j,k}(t)\Big)\Big) \end{aligned} \tag{11}$$

Integrating both sides of the above expression over $[0,T]$, it can be obtained that

$$\begin{aligned} e_{i,k}(t) =&\; e_{i,k}(0) \;+\; \int_0^t (A_i - LC_i)e_{i,k}(\tau)d\tau \\ &+ \int_0^t E_i r_{i,k}(\tau)d\tau + \int_0^t (g_1(\hat{x}_{i,k}(t)) - g_1(x_{i,k}(t)))d\tau \\ &+ \sum_{j=1,j\neq i}^{N} H_{ij}\int_0^t \Big(g_2\Big(\hat{x}_{j,k}(\tau)\Big) - g_2\Big(x_{j,k}(\tau)\Big)\Big)d\tau \end{aligned} \tag{12}$$

Then, the following Equation (13) can be computed by taking the Euclidean norm on both sides of Equation (12):

$$\begin{aligned}\|e_{i,k}(t)\| \leq& \|e_{i,k}(0)\| + \int_0^t (\|A_i - LC_i\|)\|e_{i,k}(\tau)\|d\tau \\ &+ \int_0^t \|E_i\|\|r_{i,k}(\tau)\|d\tau + \int_0^t (\|g_1(\hat{x}_{i,k}(t)) - g_1(x_{i,k}(t))\|)d\tau \\ &+ \sum_{j=1,j\neq i}^{N} H_{ij} \int_0^t \left(\left\|g_2\left(\hat{x}_{j,k}(\tau)\right) - g_2\left(x_{j,k}(\tau)\right)\right\|\right)d\tau\end{aligned} \tag{13}$$

Based on Assumption 2, one has

$$\begin{aligned}\|e_{i,k}(t)\| \leq& \|e_{i,k}(0)\| + \int_0^t (\|A_i - LC_i\| + \rho_1)\|e_{i,k}(\tau)\|d\tau \\ &+ \int_0^t \|E_i\|\|r_{i,k}(\tau)\|d\tau + \sum_{j=1,j\neq i}^{N} \|H_{ij}\| \int_0^t \rho_2\left\|e_{j,k}(\tau)\right\|d\tau\end{aligned} \tag{14}$$

Then, multiplying both sides by a function $e^{-\lambda t}$, it can be obtained

$$\begin{aligned}e^{-\lambda t}\|e_{i,k}(t)\| \leq& e^{-\lambda t}\|e_{i,k}(0)\| + e^{-\lambda t}\int_0^t (\|A_i - LC_i\| + \rho_1)\|e_{i,k}(\tau)\|d\tau \\ &+ e^{-\lambda t}\int_0^t \|E_i\|\|r_{i,k}(\tau)\|d\tau + e^{-\lambda t}\sum_{j=1,j\neq i}^{N} \|H_{ij}\| \int_0^t \rho_2\left\|e_{j,k}(\tau)\right\|d\tau\end{aligned} \tag{15}$$

Considering the definition of λ-norm, one obtains

$$\begin{aligned}\|e_{i,k}(t)\|_\lambda \leq& \|e_{i,k}(0)\|_\lambda + \frac{1-e^{-\lambda t}}{\lambda}\Big((\|A_i - LC_i\| + \rho_1)\|e_{i,k}(\tau)\|_\lambda \\ &+ \|E_i\|\|r_{i,k}(\tau)\|_\lambda + \sum_{j=1,j\neq i}^{N} \rho_2\|H_{ij}\|\left\|e_{j,k}(\tau)\right\|_\lambda\Big)\end{aligned} \tag{16}$$

Taking the sum of the above expression for i from 1 to N, one can obtain

$$\begin{aligned}\sum_{i=1}^{N}\|e_{i,k}(t)\|_\lambda \leq& \sum_{i=1}^{N}\|e_{i,k}(0)\|_\lambda + \frac{1-e^{-\lambda t}}{\lambda}\Big((\|A_i - LC_i\| + \rho_1)\sum_{i=1}^{N}\|e_{i,k}(\tau)\|_\lambda \\ &+ \|E_i\|\sum_{i=1}^{N}\|r_{i,k}(\tau)\|_\lambda + \sum_{j=1,j\neq i}^{N}\|H_{ij}\|\sum_{i=1}^{N}\rho_2\left\|e_{j,k}(\tau)\right\|_\lambda\Big) \\ \leq& \sum_{i=1}^{N}\|e_{i,k}(0)\|_\lambda + \frac{1-e^{-\lambda t}}{\lambda}\Big(c_1\sum_{i=1}^{N}\|e_{i,k}(\tau)\|_\lambda \\ &+ c_2\sum_{i=1}^{N}\|r_{i,k}(\tau)\|_\lambda + \sum_{j=1,j\neq i}^{N} c_3\sum_{i=1}^{N}\left\|e_{j,k}(\tau)\right\|_\lambda\Big) \\ \leq& \sum_{i=1}^{N}\|e_{i,k}(0)\|_\lambda + \frac{1-e^{-\lambda t}}{\lambda}\Big(c_4\sum_{i=1}^{N}\|e_{i,k}(\tau)\|_\lambda \\ &+ c_2\sum_{i=1}^{N}\|r_{i,k}(\tau)\|_\lambda\Big)\end{aligned} \tag{17}$$

where $c_1 = \max\limits_{1\le i\le N}\{(\|A_i - LC_i\| + \rho_1)\}$, $c_2 = \max\limits_{1\le i\le N}\{(\|E_i\|)\}$, $c_3 = \max\limits_{1\le i\le N}\{(\rho_2\|H_{ij}\|)\}$, and $c_4 = c_1 + (N-1)c_3$. It is noted that the condition $\frac{1-e^{-\lambda t}}{\lambda}c_4 < 1$ is satisfied when the parameter λ is large enough. Hence, it can be concluded that

$$\sum_{i=1}^{N}\|e_{i,k}(t)\|_\lambda \le \frac{1}{c_5}\sum_{i=1}^{N}\|e_{i,k}(0)\|_\lambda + \frac{c_2}{c_5}\frac{1-e^{-\lambda t}}{\lambda}\sum_{i=1}^{N}\|r_{i,k}(\tau)\|_\lambda \tag{18}$$

where $c_5 = 1 - \frac{1-e^{-\lambda t}}{\lambda}c_4$

Then, based on the fault-estimation law (5) and the estimating error (6), one can obtain that

$$\begin{aligned} r_{i,k+1}(t) &= \hat{f}_{i,k+1}(t) - f_{i,k}(t) \\ &= \hat{f}_{i,k}(t) + K_{i,1}\Delta y_{i,k}(t) + K_{i,2}\Delta\dot{y}_{i,k}(t) - f_{i,k}(t) \\ &= r_{i,k}(t) + K_{i,1}\Delta y_{i,k}(t) + K_{i,2}\Delta\dot{y}_{i,k}(t) \end{aligned} \tag{19}$$

Together with (11), we have

$$\begin{aligned} r_{i,k+1}(t) &= r_{i,k}(t) + K_{i,1}\Delta y_{i,k}(t) + K_{i,2}\Delta\dot{y}_{i,k}(t) \\ &= (K_{i,1}C_i + K_{i,2}C_i(A_i - LC_i))e_{i,k}(t) \\ &\quad + (I + K_{i,2}C_iE_i)r_{i,k}(t) + K_{i,2}C_i(g_1(\hat{x}_{i,k}(t)) - g_1(x_{i,k}(t))) \\ &\quad + K_{i,2}C_i\sum_{j=1,j\ne i}^{N}H_{ij}\left(g_2\left(\hat{x}_{j,k}(\tau)\right) - g_2\left(x_{j,k}(\tau)\right)\right) \end{aligned} \tag{20}$$

The Euclidean norm can be obtained as follows:

$$\begin{aligned} \|r_{i,k+1}(t)\| \le& (\|(K_{i,1}C_i + K_{i,2}C_i(A_i - LC_i))\| + \rho_1)\|e_{i,k}(t)\| \\ &+ \|(I + K_{i,2}C_iE_i)\|\|r_{i,k}(t)\| + \sum_{j=1,j\ne i}^{N}\|K_{i,2}C_iH_{ij}\rho_2\|\left\|e_{j,k}(t)\right\| \end{aligned} \tag{21}$$

Multiplying both sides by a function $e^{-\lambda t}$ and applying the λ-norm, one can obtain that

$$\begin{aligned} \|r_{i,k+1}(t)\|_\lambda \le& (\|(K_{i,1}C_i + K_{i,2}C_i(A_i - LC_i))\| + \rho_1)\|e_{i,k}(t)\|_\lambda \\ &+ \|(I + K_{i,2}C_iE_i)\|\|r_{i,k}(t)\|_\lambda + \sum_{j=1,j\ne i}^{N}\|K_{i,2}C_iH_{ij}\rho_2\|\left\|e_{j,k}(t)\right\|_\lambda \\ \le& c_6\|e_{i,k}(t)\|_\lambda + \Gamma\|r_{i,k}(t)\|_\lambda + \sum_{j=1,j\ne i}^{N}c_7\left\|e_{j,k}(t)\right\|_\lambda \end{aligned} \tag{22}$$

where $c_6 = \max\limits_{1\le i\le N}(\|(K_{i,1}C_i + K_{i,2}C_i(A_i - LC_i))\| + \rho_1)$, $\Gamma = \max\limits_{1\le i\le N}(\|(I + K_{i,2}C_iE_i)\|)$, and $c_7 = \max\limits_{1\le i\le N}(\|K_{i,2}C_iH_{ij}\rho_2\|)$.

Then, taking the sum of both sides of Equation (22), it can be derived that

$$\begin{aligned} \sum_{i=1}^{N}\|r_{i,k+1}(t)\|_\lambda \le& c_6\sum_{i=1}^{N}\|e_{i,k}(t)\|_\lambda + \Gamma\sum_{i=1}^{N}\|r_{i,k}(t)\|_\lambda + \sum_{j=1,j\ne i}^{N}c_7\sum_{i=1}^{N}\left\|e_{j,k}(t)\right\|_\lambda \\ \le& c_8\sum_{i=1}^{N}\|e_{i,k}(t)\|_\lambda + \Gamma\sum_{i=1}^{N}\|r_{i,k}(t)\|_\lambda \end{aligned} \tag{23}$$

where $c_8 = c_6 + (N-1)c_7$. Substituting (18) into (23) results in

$$
\begin{aligned}
\sum_{i=1}^{N}\left\|r_{i,k+1}(t)\right\|_{\lambda} &\le c_8\left(\frac{1}{c_5}\sum_{i=1}^{N}\left\|e_{i,k}(0)\right\|_{\lambda}+\frac{c_2}{c_5}\frac{1-e^{-\lambda t}}{\lambda}\sum_{i=1}^{N}\left\|r_{i,k}(\tau)\right\|_{\lambda}\right)+\Gamma\sum_{i=1}^{N}\left\|r_{i,k}(t)\right\|_{\lambda} \\
&\le \frac{c_8}{c_5}\sum_{i=1}^{N}\left\|e_{i,k}(0)\right\|_{\lambda} + \left(\frac{c_8c_2}{c_5}\frac{1-e^{-\lambda t}}{\lambda}+\Gamma\right)\sum_{i=1}^{N}\left\|r_{i,k}(t)\right\|_{\lambda} \\
&\le (c_9+\Gamma)\sum_{i=1}^{N}\left\|r_{i,k}(t)\right\|_{\lambda}+c_{10}
\end{aligned}
\tag{24}
$$

where $c_9 = \frac{c_8c_2}{c_5}\frac{1-e^{-\lambda t}}{\lambda}$, $c_{10} = \frac{c_8 2N \max\limits_{1\le i\le N}\{\|\delta_{i,k}\|\}}{c_5}$.

One can determine $c_9+\Gamma<1$ as λ is sufficiently large. According to Lemma 1, it can be concluded that the fault estimation can be satisfied (25) if the condition (10) holds.

$$
\lim_{k\to\infty}\left\|r_{i,k}(t)\right\|_{\lambda} \le G_1 \tag{25}
$$

where $G_1 = \frac{c_{10}}{1-(c_9+\Gamma)}$.

Thus, the fault estimation $\hat{f}_{i,k}(t)$ converges to a small neighborhood of the actual fault $f_{i,k}(t)$ after enough iterations. □

Theorem 2. *Consider the system (1) and the initial state estimation law (9). If there exist the gain matrices m_i with the condition*

$$
\Upsilon = \max_{1\le i\le N}\{\|(I + m_iC_i)\|\} < 1 \tag{26}
$$

then, the initial state estimation errors $e_{i,k}(0)$ of each subsystem are uniformly ultimately bounded over the entire time interval $[0,T]$ under the action of the learning scheme (9).

Proof of Theorem 2. To solve the convergence problem of the initial state estimation, one can obtain the following description by utilizing the definition (6):

$$
e_{i,k}(0) = \hat{x}_{i,k}(0) - x_{i,k}(0) \tag{27}
$$

Subtracting $x_{i,k}(0)$ on both sides of the initial state estimation law (9), it can be obtained that

$$
\begin{aligned}
&\hat{x}_{i,k+1}(0) - x_{i,k}(0) = \hat{x}_{i,k}(0) - x_{i,k}(0) + m_i\Delta y_{i,k}(0) \\
&\Rightarrow \hat{x}_{i,k+1}(0) - x_{i,k+1}(0) + \delta_{i,k+1} - \delta_{i,k} = \hat{x}_{i,k}(0) - x_{i,k}(0) + m_i\Delta y_{i,k}(0) \\
&\Rightarrow e_{i,k+1}(0) = e_{i,k}(0) + m_iC_ie_{i,k}(0) + \delta_{i,k} - \delta_{i,k+1}
\end{aligned}
\tag{28}
$$

That is,

$$
e_{i,k+1}(0) = (I+m_iC_i)e_{i,k}(0) + \delta_{i,k} - \delta_{i,k+1} \tag{29}
$$

Taking the Euclidean norm on both sides of Equation (29)

$$
\left\|e_{i,k+1}(0)\right\| \le \|(I+m_iC_i)\|\left\|e_{i,k}(0)\right\| + \Delta\delta \tag{30}
$$

where $\Delta\delta = \max\limits_{1\le i\le N}\{\|\delta_{i,k}\|+\|\delta_{i,k+1}\|\}$. Then, multiplying both sides by a function $e^{-\lambda t}$, it can be obtained

$$
e^{-\lambda t}\left\|e_{i,k+1}(0)\right\| \le e^{-\lambda t}\|(I+m_iC_i)\|\left\|e_{i,k}(0)\right\| + e^{-\lambda t}\Delta\delta \tag{31}
$$

Considering the definition of the λ-norm, one obtains,

$$
\left\|e_{i,k+1}(0)\right\|_{\lambda} \le \Upsilon\left\|e_{i,k}(0)\right\|_{\lambda} + e^{-\lambda t}\Delta\delta \tag{32}
$$

where Y is defined in Equation (26). According to Lamma 1, it can be concluded that the initial state estimation can satisfy (33) if condition (26) holds.

$$\lim_{k\to\infty} \left\|e_{i,k+1}(0)\right\|_\lambda \le G_2 \tag{33}$$

where $G_2 = \frac{e^{-\lambda t}\Delta\delta}{1-\Upsilon}$.

Thus, the initial state estimation $\hat{x}_{i,k}(0)$ converges to a small neighborhood of the actual initial state $x_{i,k}(0)$ after enough iterations. □

4. Simulation Results and Discussion

To validate the feasibility and effectiveness of the proposed method in this paper, this section presents a numerical analysis for an interconnected dual-flexible manipulator system.

Consider two identical single-link flexible manipulator subsystems as described in Equation (2). The parameter matrices are set as follows:

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21} & 0 & a_{22} & 0 \\ 0 & 0 & 0 & 1 \\ a_{41} & 0 & a_{42} & 0 \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_{43} \end{bmatrix}, C_i = diag(1,1,1,1), E_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_{43} \end{bmatrix},$$

where $i = 1,2$, $a_{21} = -K/I$, $a_{22} = K/I$, $a_{23} = -Mgl/I$, $a_{41} = K/J$, $a_{42} = -K/J$, $a_{43} = 1/J$, $M = 2.3$ kg, $l = 1$ m, $I = 2.3$ kg·m^2, $J = 0.5$ kg·m^2, $K = 15$ N·m/rad, and $g = 9.8$ m/s^2. The fault signals are simulated as:

$$\begin{cases} f_{1,k}(t) = 5\sin(0.5\pi t) & 0 \le t \le 7 \\ f_{1,k}(t) = 0 & 7 < t \le 10 \end{cases}, \begin{cases} f_{2,k}(t) = 5\sin(0.5\pi t) & 0 \le t \le 7 \\ f_{2,k}(t) = 0 & 7 < t \le 10 \end{cases}.$$

The desired trajectories are set as:

$$\ddot{q}_{1d} = 2\sin 1.5t - 0.5q_{1d} - 1.5\dot{q}_{1d}, \ddot{q}_{2d} = 2\sin 1.5t - 0.5q_{2d} - 1.5\dot{q}_{2d}.$$

The nonlinear terms are set as:

$$g_1(x_{1,k}(t)) = \begin{bmatrix} 0 & a_{23}\sin q_{1,k} & 0 & 0 \end{bmatrix}^T, g_1(x_{2,k}(t)) = \begin{bmatrix} 0 & a_{23}\sin q_{2,k} & 0 & 0 \end{bmatrix}^T.$$

The interconnected term is represented as: $\sum\limits_{j=1, j\ne i}^{N} diag(0.1, 0.1, 0.1, 0.1)x_{j,k}(t)$, $i = 1,2$.

The ideal initial values are set as: $x_{1,d}(0) = x_{2,d}(0) = \begin{bmatrix} 0.2 & 0 & 0.1 & 0 \end{bmatrix}^T$.

For comparative analysis, it is initially assumed that the system's initial state coincides with the ideal initial value. Based on engineering experience, the convergence effects, and the convergence condition stated in Theorem 1, the gain matrices are chosen as $K_{1,1} = K_{2,1} = [-6, -0.4, -4.0, -0.4]$, and $K_{1,2} = K_{2,2} = [-1.5, -0.1, -1.0, -0.1]$, which imply that the convergence condition in Theorem 1 is $\Gamma = \max\limits_{1\le i\le N}\{\|(I + K_{i,2}C_iE_i)\|\} = 0.8 < 1$. The simulation results of system (2) applying the proposed iterative-learning-based fault-estimation method (5) are shown in Figures 1 and 2.

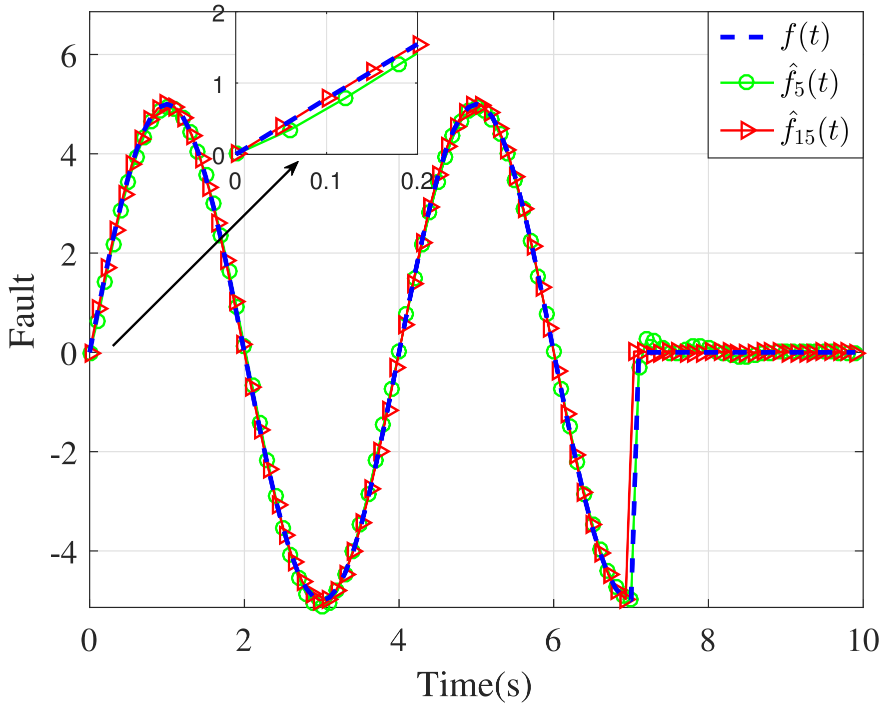

Figure 1. Fault-estimation curve for Subsystem 1 under the ideal initial value.

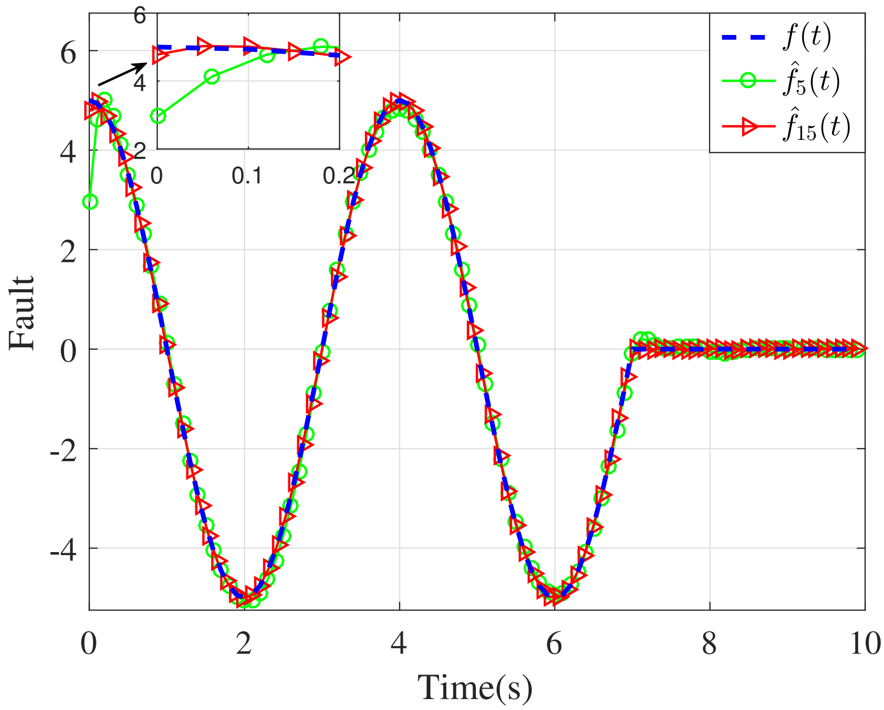

Figure 2. Fault-estimation curve for Subsystem 2 under the ideal initial value.

From Figures 1 and 2, it can be observed that the iterative-learning-based fault-estimation method can effectively and accurately track the fault signal when the system's initial state matches the ideal initial state. However, due to the influence of the material properties of the flexible manipulator, it is difficult to achieve equivalence between the actual initial state and the ideal initial state. Therefore, assuming a set of arbitrary initial values such that $x_{1,k}(0) = x_{2,k}(0) = \begin{bmatrix} 0.2 + 2\text{rand} & 0 & 0.1 & 0 \end{bmatrix}^T$, where rand is a random number between 0 and 1. The simulation results are presented in Figures 3 and 4.

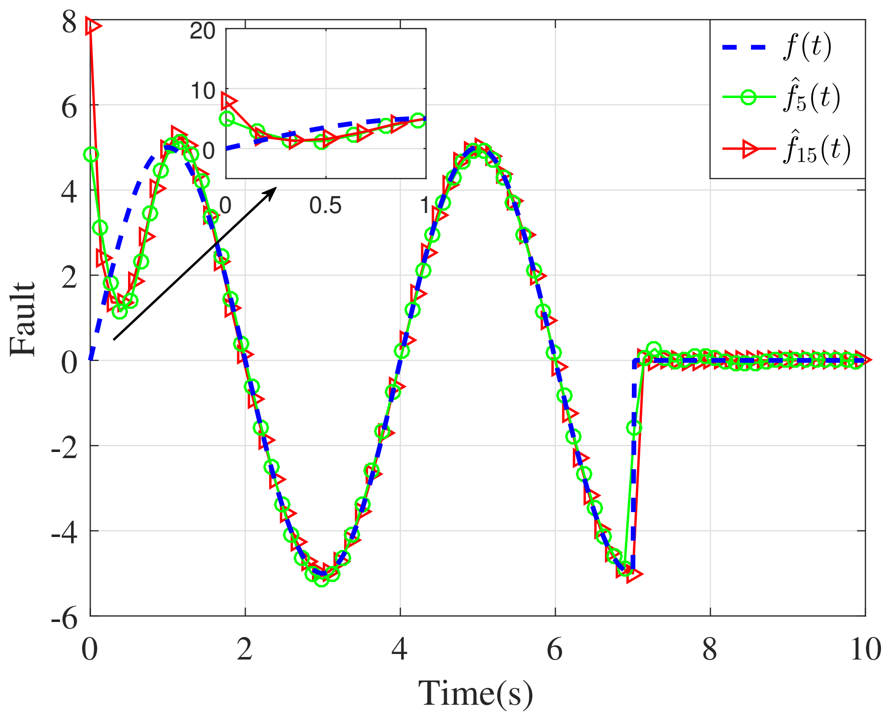

Figure 3. Fault-estimation curve for Subsystem 1 under the arbitrary initial value.

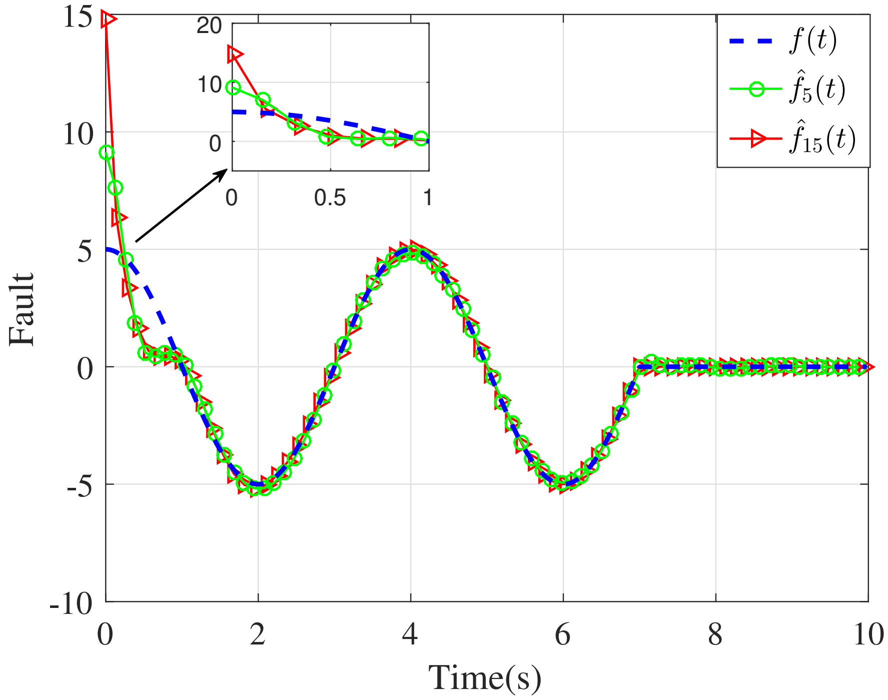

Figure 4. Fault-estimation curve for Subsystem 2 under the arbitrary initial value.

As depicted in Figures 3 and 4, even after 15 iterations, the fault-estimation trajectory fails to precisely track the actual fault signal trajectory. It can be observed that there exists a significant fault-estimation error while the initial value is arbitrary. The arbitrary initial value diminishes the accuracy of the iterative-learning-based fault-estimation method. To accurately track the fault signals within the system and to eliminate the adverse effects of the arbitrary initial value, we employ the iterative-learning-based fault-estimation method described in Equation (5), combined with the initial state learning law (9) proposed in this paper. Based on engineering experience, the convergence effects, and the convergence condition stated in Theorem 2, the gain matrices are chosen as $m_i = diag(-0.8, -0.8, -0.8, -0.8)$, which imply that the convergence condition in Theorem 2 is $\Upsilon = \max_{1 \le i \le N} \{\|(I + m_i C_i)\|\} = 0.2 < 1$. The comparative simulation results based on the proposed method and the VWIL-based fault-estimation method described in reference [23] are shown in Figures 5–10.

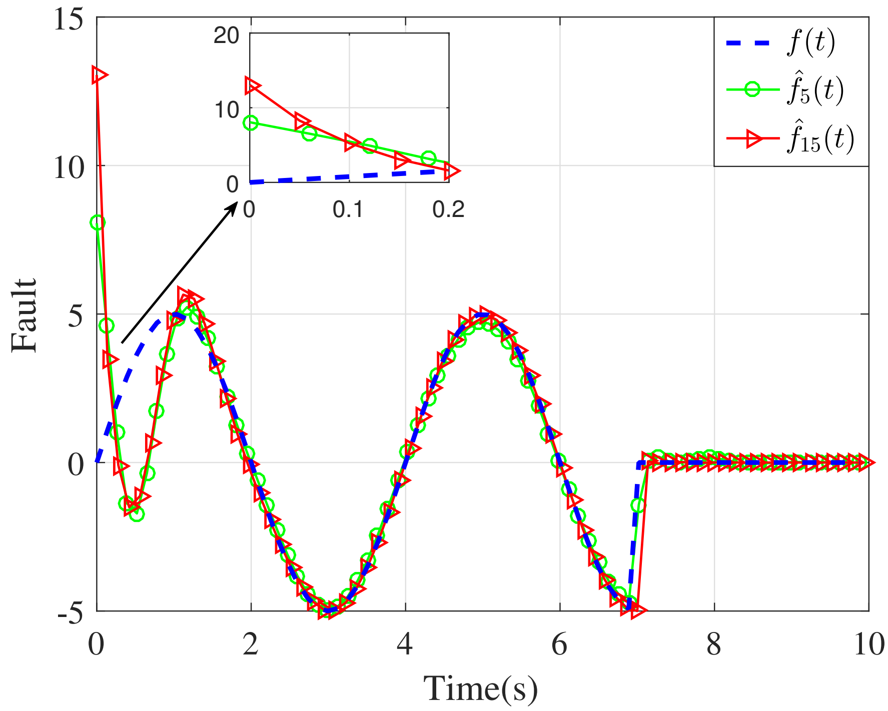

Figure 5. Fault-estimation results based on the method reported in reference [23] for different iterations for Subsystem 1.

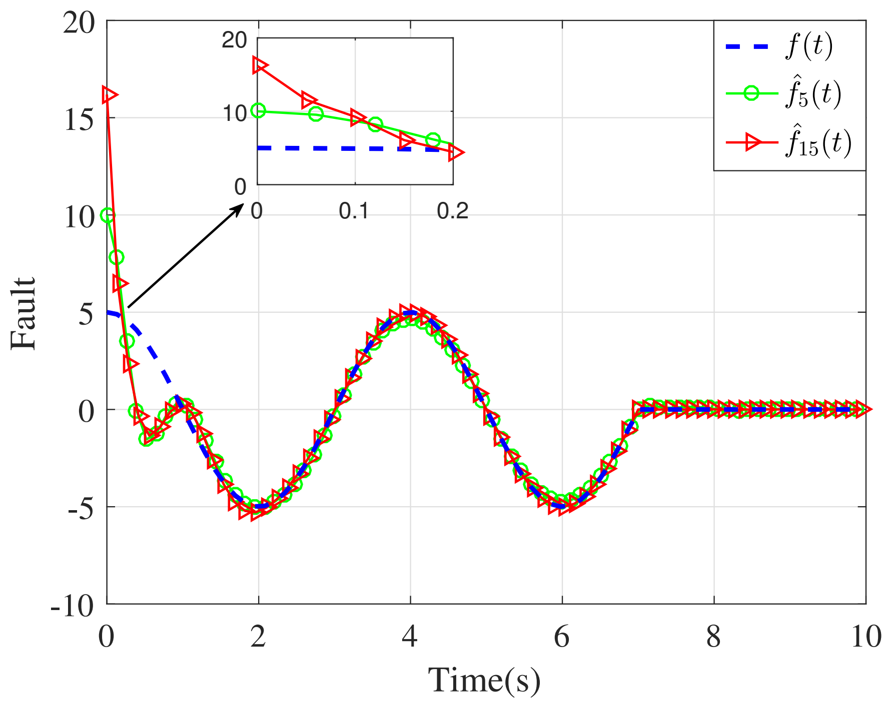

Figure 6. Fault-estimation results based on the method reported in reference [23] for different iterations for Subsystem 2.

Figures 5–8 depict the estimated results of the fault signal for different iterations. It is noteworthy that the proposed method successfully tracks the fault signal after only a few iterations, whereas the method described in reference [23] fails to track the fault after 15 iterations. Figures 9 and 10 show the comparative fault-estimation error results based on the method reported in reference [23] and the proposed method for 15 iterations. It is evident that the proposed method achieves better fault-estimation error results. From the above simulation results, it can be concluded that the proposed method is effective in ensuring convergence for interconnected multi-flexible manipulator systems with arbitrary initial value.

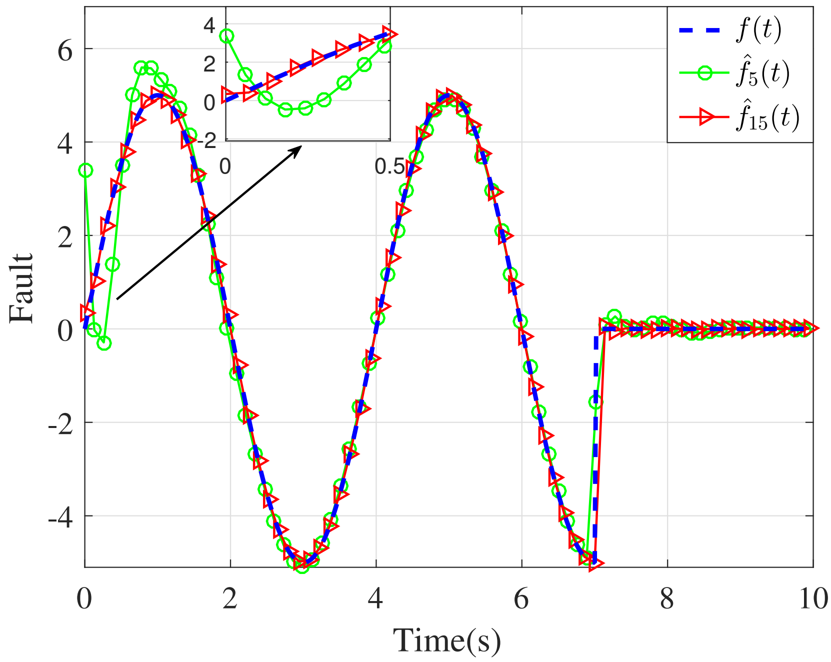

Figure 7. Fault-estimation results based on the proposed method for different iteration index for Subsystem 1.

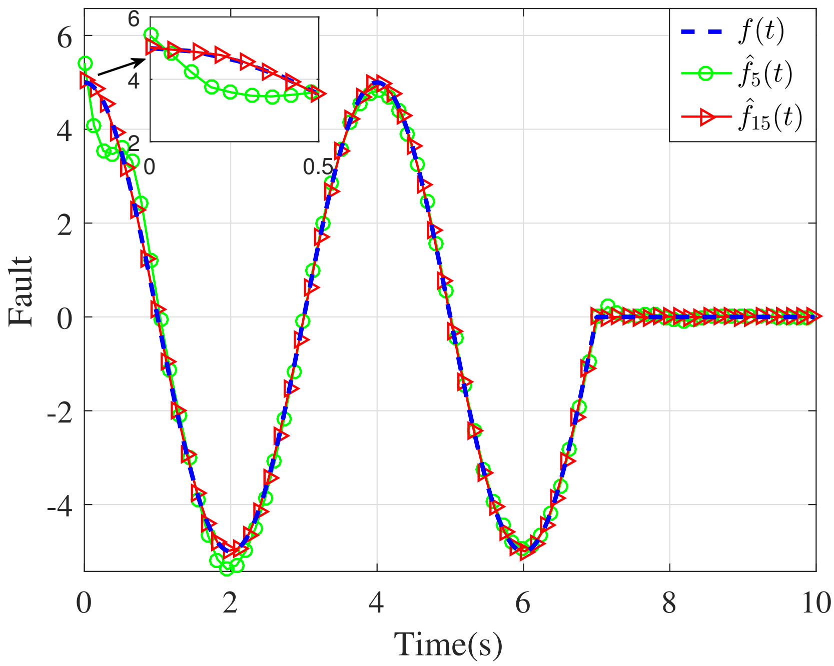

Figure 8. Fault-estimation results based on the proposed method for different iterations for Subsystem 2.

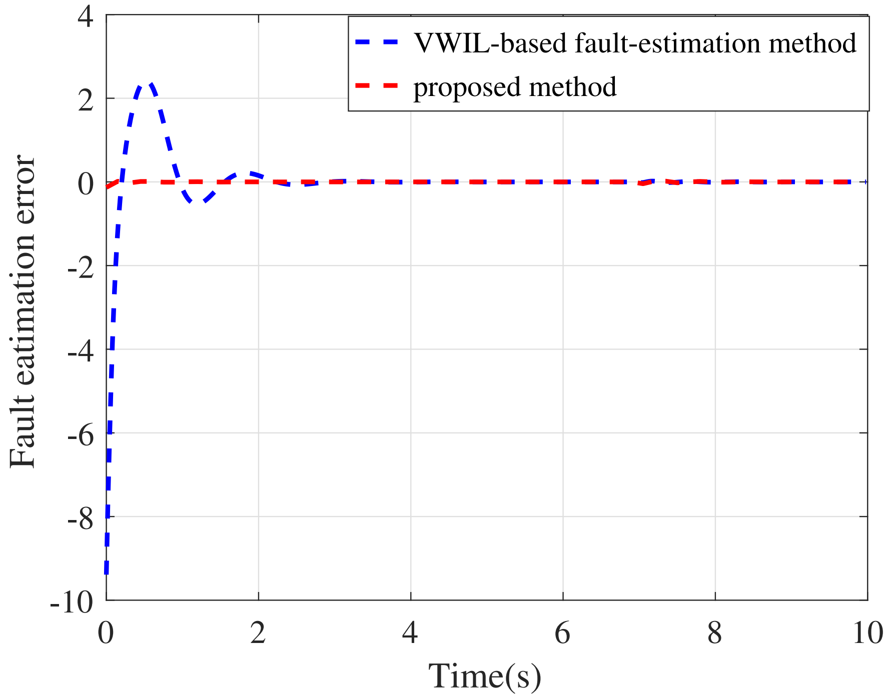

Figure 9. Fault-estimation error curve for Subsystem 1 at the 15th iteration under arbitrary initial value.

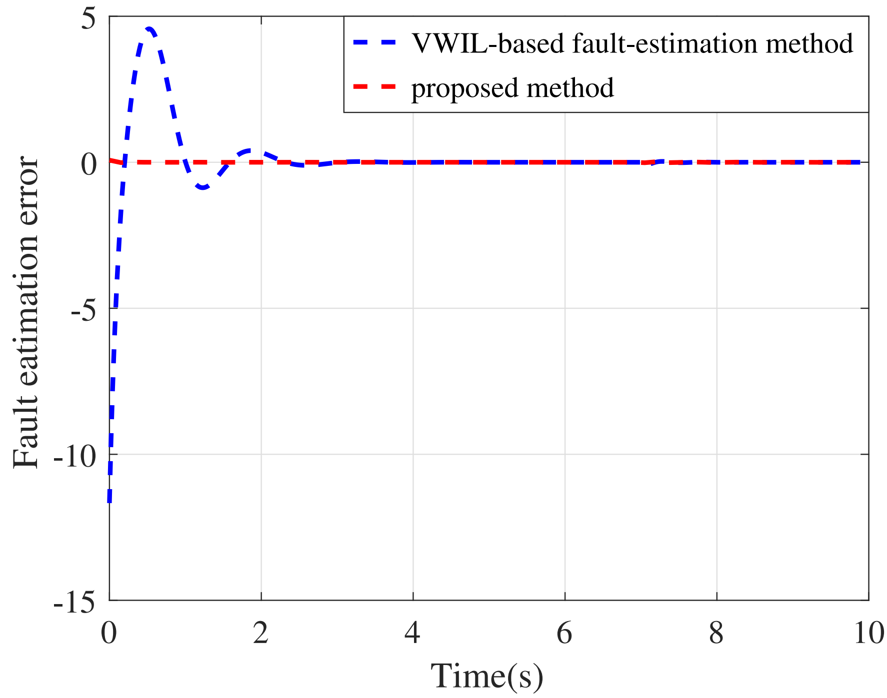

Figure 10. Fault-estimation error curve for Subsystem 2 at the 15th iteration under arbitrary initial value.

5. Conclusions

This paper describes the employment of iterative learning strategies to design a fault-estimation method for estimating faults in interconnected flexible manipulator systems with arbitrary initial value. The simulation results obtained demonstrate that the proposed iterative learning fault-estimation algorithm with initial state learning effectively addresses the problem of fault-estimation trajectories which do not closely track the actual fault signal trajectories caused by initial state errors in the interconnected system. Compared to the VWIL-based fault-estimation method, the iterative learning law with initial state learning introduced in this paper enables the rapid and accurate estimation of faults occurring in flexible manipulator interconnected systems, eliminating the adverse effects of an arbitrary initial value on the iterative learning fault-estimation method. The convergence conditions and the parameter ranges are obtained through an analysis of Bellman–Gronwall theory and λ-norm theory. The algorithm's effectiveness is validated through simulation experiments on an interconnected dual-flexible manipulator system as the controlled object.

As previously commented, the presented method is appropriate for interconnected multi-flexible manipulator systems without uncertainties, time-delay, and so on. Hence, the application range is limited. In future work, all these influences will be taken into consideration. As a result, the proposed iterative-learning-scheme-based fault-estimation method can be applied to more general flexible manipulator systems. Furthermore, only the simulation results were utilized to illustrate the effectiveness of the method presented, How to apply the iterative-learning-scheme-based fault-estimation method to actual flexible manipulator systems is also an important future issue to be addressed.

Author Contributions: Conceptualization, L.F. and G.C.; methodology, L.F. and G.C.; software, L.F. and G.C.; validation, G.C. and K.D.; formal analysis, L.F. and K.D.; investigation, G.C.; resources, L.F.; data curation, L.F. and G.C.; writing—original draft preparation, L.F. and G.C.; writing—review and editing, L.F., G.C. and K.D.; visualization, G.C. and K.D.; supervision, L.F. and S.X.; project administration, S.X.; funding acquisition, L.F. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Natural Science Foundation of Chongqing (grant number CSTB2022NSCQ-MSX1456), the Science and Technology Research Program of Chongqing Municipal Education Commission (grant number KJZD-K202300705), the National Key Research and Development Project (grant number 2020YFB2009405), the State Key Laboratory of Industrial Control Technology (grant number ICT2022B08), and the Research Project on Teaching Reform of Graduate Education (grant number yjg213094).

Data Availability Statement: The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Mu, Y.; Zhang, H.; Yan, Y.; Xie, X. Distributed observer-based robust fault estimation design for discrete-time interconnected systems with disturbances. *IEEE Trans. Cybern.* **2023**, *53*, 6737–6747. [CrossRef] [PubMed]
2. Li, X.; Yan, J.; Yang, G. Adaptive fault estimation for T-S fuzzy interconnected systems based on persistent excitation condition via reference signals. *IEEE Trans. Cybern.* **2018**, *49*, 2822–2834. [CrossRef] [PubMed]
3. Zhang, C.; Yang, H.; Jiang, B. Fault estimation and accommodation of fractional-order nonlinear, switched, and interconnected systems. *IEEE Trans. Cybern.* **2020**, *52*, 1443–1453. [CrossRef] [PubMed]
4. Zhu, F.; Shan, Y.; Zhang, J.; Wang, F. Observer-based fault reconstructions and fault-tolerant control designs for uncertain switched systems with both actuator and sensor faults. *IET Control Theory Appl.* **2020**, *14*, 2017–2019. [CrossRef]
5. Shan, Y.; Zhu, F. Interval observer fault tolerant control strategy with fault estimation and compensation. *Asian J. Control* **2022**, *24*, 895–906. [CrossRef]
6. Xu, S.; Wang, L.; Feng, L.; Yang, X.; Chai, Yi.; Du, H.; Zheng, W. Self-Correcting Iterative Learning-Based Fault Estimation for Parabolic Distributed Parameter Systems. *IEEE Trans. Circuits Syst. II Express Briefs* **2023**, 1–5. [CrossRef]
7. Zhang, K.; Jiang, B.; Chen, M.; Yan, X. Distributed fault estimation and fault-tolerant control of interconnected systems. *IEEE Trans. Cybern.* **2019**, *51*, 1230–1240. [CrossRef]
8. Zhang, K.; Jiang, B.; Shi, P. Distributed fault estimation observer design with adjustable parameters for a class of nonlinear interconnected systems. *IEEE Trans. Cybern.* **2018**, *49*, 4219–4228. [CrossRef]
9. Makni, S.; Bouattour, M.; El Hajjaji, A.; Chaabane, M. Robust fault estimation and fault-tolerant tracking control for uncertain Takagi–Sugeno fuzzy systems: Application to single link manipulator. *Int. J. Adapt. Control Signal. Process.* **2021**, *35*, 846–876. [CrossRef]
10. Che, J.; Zhu, Y.; Zhou, D. Hidden Markov model-based robust H_∞ fault estimation for Markov switching systems with application to a single-link robot arm. *Asian J. Control* **2021**, *23*, 2227–2238. [CrossRef]

11. Wang, L.; Shen, Y.; Li, B.; Yu, J.; Zhang, R.; Gao, F. Hybrid iterative learning fault-tolerant guaranteed cost control design for multi-phase batch processes. *Can. J. Chem. Eng.* **2017**, *96*, 521–530. [CrossRef]
12. Tao, H.; Paszke, W.; Rogers, E.; Yang, H.; Galkowski, K. Iterative learning fault-tolerant control for differential time-delay batch processes in finite frequency domains. *J. Process Control* **2017**, *56*, 112–128. [CrossRef]
13. Feng, L.; Xu, S.; Chai, Y.; Zhang, K. Iterative learning scheme-based fault estimation design for nonlinear systems with varying trial lengths and specified constraints. *Int. J. Robust Nonlinear Control* **2018**, *28*, 4850–4864. [CrossRef]
14. Cao, W.; Cong, W.; Sun, M. Iterative learning based fault detection and estimation in nonlinear systems. *J. Syst. Eng. Electron.* **2012**, 23, 419–424. [CrossRef]
15. Li, L.; Yao, L.; Wang, H.; Gao, Z. Iterative learning fault diagnosis and fault tolerant control for stochastic repetitive systems with Brownian motion. *ISA Trans.* **2022**, *121*, 171–179. [CrossRef] [PubMed]
16. Zhou, F.; Wang, Y. Iterative learning control for fractional order nonlinear system with initial shift. *Nonlinear Dyn.* **2021**, *106*, 3305–3314.
17. Zhao, Y.; Zhou, F.; Li, Y.; Wang, Y. A novel iterative learning path-tracking control for nonholonomic mobile robots against initial shifts. *Int. J. Adv. Robot. Syst.* **2017**, *14*, 123–140. [CrossRef]
18. Li, G.; Lu, T.; Han, Y.; Xu, Z. Adaptive iterative learning control for high-order nonlinear systems with random initial state shifts. *ISA Trans.* **2022**, *130*, 205–215. [CrossRef]
19. Ma, Y.; He, X.; Zhang, S.; Sun, Y.; Fu, Q. Adaptive compensation for infinite number of actuator faults and time-varying delay of a flexible manipulator system. *IEEE Trans. Ind. Electron.* **2022**, *69*, 13141–13150. [CrossRef]
20. Abd Latip, S.F.; Rashid Husain, A.; Mohamed, Z.; Mohd Basri, M.A. Adaptive PID actuator fault tolerant control of single-link flexible manipulator. *Trans. Inst. Meas. Control* **2019**, *41*, 1019–1031. [CrossRef]
21. Li, Y.; Tong, S.; Li, T. Fuzzy adaptive dynamic surface control for a single-link flexible-joint robot. *Nonlinear Dyn.* **2012**, *70*, 2035–2048. [CrossRef]
22. Sun, L. Decentralized Adaptive Iterative Learning Control for a Class of Interconnected Systems and Applications. Ph.D. Thesis, Zhejiang University, Hangzhou, China, 2014.
23. Xu, S.; Dai, H.; Feng, L.; Chen, H.; Chai, Y.; Zheng, W. Fault Estimation for Switched Interconnected Nonlinear Systems with External Disturbances via Variable Weighted Iterative Learning. *IEEE Trans. Circuits Syst. II Express Briefs* **2023**, *70*, 2011–2015. [CrossRef]