buildings

MDPI

Article

Pricing and Coordinating the Lease-Oriented Closed-Loop Supply Chain for Construction Machinery in the Era of Carbon Tax

Jing Yin [1], Yifan Fang [1], Hengxi Zhang [2], Tingting Wang [3] and Shunyao Cai [4,*]

[1] Department of Industrial Engineering, School of Mechanical-Electronic and Vehicle Engineering, Beijing University of Civil Engineering and Architecture, Beijing 102616, China; yinjing@bucea.edu.cn (J.Y.), fangyifan07@hotmail.com (Y.F.)
[2] CCCC Infrastructure Maintenance Group Co., Ltd., Beijing 100011, China; zhanghengxi97@163.com
[3] School of Economics and Management, Beijing Institute of Graphic Communication, Beijing 102627, China; ttwang@bigc.edu.cn
[4] Department of Construction Management, Chongqing University, Chongqing 400045, China
* Correspondence: shunyao.cai@cqu.edu.cn; Tel.: +86-139-9604-5891

Abstract: Promoting sustainable production and consumption practices in the construction machinery industry is crucial for achieving energy savings and reducing carbon emissions. However, there is a lack of targeted studies addressing the challenge of scaling up leasing and recycling while maximizing economic benefits for enterprises. To fill this gap, this paper presents a lease-oriented closed-loop supply chain model that incorporates a carbon tax policy to investigate the impact of the carbon tax rate and consumer preferences for remanufactured products on the supply chain and introduces a leasing compensation–cost apportioning combined contract to achieve supply chain coordination in the construction machinery sector. The model considers differential selling and leasing prices for new and remanufactured products, as well as the recovery rate, under both centralized and decentralized decision-making approaches. The study explores the interrelationships between various parameters through sensitivity analysis and numerical simulation. The results demonstrate that within a certain range of the cost apportioning proportional coefficient and leasing compensation proportional coefficient, the combined contract can lead the supply chain to achieve Pareto optimality. As the carbon tax rate increases, it was observed that the profits for all parties in the supply chain tend to decrease. However, due to the increased demand for remanufactured products, the product recovery rate improves, resulting in a reduction in total carbon emissions in the closed-loop supply chain of construction machinery. Moreover, the profits of all parties and the total supply chain profits initially decrease and then increase with an increasing preference coefficient for remanufactured products among consumers. By leveraging these factors and adopting effective strategies, such as enhancing consumer recognition of remanufactured products and optimizing pricing and cost allocation, it is indeed possible for the profits of all parties and the total profits in the supply chain to surpass the initial values, even in the face of increasing carbon tax rates. This demonstrates the potential for aligning sustainability objectives with economic benefits in the construction machinery industry.

Keywords: construction machinery; carbon tax policy; lease-oriented closed-loop supply chain; the combined contract

Citation: Yin, J.; Fang, Y.; Zhang, H.; Wang, T.; Cai, S. Pricing and Coordinating the Lease-Oriented Closed-Loop Supply Chain for Construction Machinery in the Era of Carbon Tax. *Buildings* **2023**, *13*, 2145. https://doi.org/10.3390/buildings13092145

Academic Editor: Heap-Yih Chong

Received: 5 July 2023
Revised: 21 August 2023
Accepted: 22 August 2023
Published: 24 August 2023

1. Introduction

The building industry is a significant contributor to global energy consumption, accounting for over 40% of the total. Accordingly, it generates approximately 33% of greenhouse gas emissions throughout its entire life cycle, encompassing stages such as raw material production, transportation, construction, installation, operation, and eventual demolition [1]. Among these stages, the construction phase itself is particularly carbon intensive, contributing approximately 12.6% of the carbon dioxide emissions [2,3]. Although construction emissions may not constitute the largest share, they have a significant impact on the overall carbon footprint. As a result, there is a growing interest in studying and addressing the environmental impact of carbon emissions during the construction stage [4]. Construction machinery has been identified as a significant contributor to carbon emissions during the construction phase, primarily due to its extensive production investment, high energy consumption, and associated environmental pollution [5,6]. Despite its substantial impact, there has been limited research conducted on this specific aspect. Therefore, it is imperative to develop sustainable practices in the production and use of construction machinery to improve energy efficiency and reduce carbon emissions. By doing so, the building industry can make significant strides towards achieving its objective of low-carbon sustainable development.

Over the past two decades, China has witnessed a remarkable increase in the production output of construction machinery that has been driven by the rapid growth of the real estate and infrastructure sectors. Recognizing the environmental benefits, the China Recycling Economy Association has emphasized the potential advantages of leasing and recycling construction machinery. It is estimated that 55% of used parts, on average, can be reused for remanufacturing products, resulting in over 80% energy savings compared with manufacturing new products [7]. The industrial Internet platform (IIP) has been particularly instrumental in driving new business operational models in the construction machinery industry. This platform provides member enterprises with the life cycle data of construction machinery. The manufacturer retrieves the used products that are no longer functional to the lessors and then restores their performance through remanufacturing. The refurbished machinery is then sold again at a preferential price [8,9]. The integration of leasing, recycling, and remanufacturing services has formed a closed-loop supply chain. Although the lease-oriented model of construction machinery in China has been introduced relatively recently, it has shown promising growth potential. Consequently, there is a need for further exploration of key aspects such as the allocation of supply chain profits, enhancing collaboration among member companies, and facilitating the rapid development of leasing and recycling businesses within the lease-oriented closed-loop supply chain. However, there has been a limited amount of research conducted on this particular topic.

Researchers have extensively explored various facets of closed-loop supply chains through a range of methodologies. In the realm of inventory management within supply chains, certain scholars have delved into the intricacies of different closed-loop supply chain structures, employing heuristics, branch and bound algorithms, and more to optimize inventory management. Their goal is to minimize the overall cost of closed-loop supply chains [10,11]. Moreover, in the pursuit of optimal network design for product recovery, specific researchers have integrated parameters such as transportation costs and carbon emissions into their models. To achieve this, they have harnessed metaheuristic methods such as hyperheuristics and metaheuristics, striving to enhance both the economic and societal benefits of the supply chain [12–14]. Utilizing dynamic adjustments of factors such as recycling rates and product demand through system dynamics methodologies, other researchers have effectively projected shifts in supply chain performance. These insights contribute to the strategic management of supply chain operations [15–17]. In terms of enhancing the performance of supply chains, some scholars have explored various power structures or channels within supply chain models. By employing diverse strategies or coordination contracts, these studies aim to achieve supply chain coordination [18,19]. Simultaneously, capitalizing on the capabilities of industrial Internet platforms for data collection, processing, and sharing holds the potential to optimize closed-loop supply chains more effectively. This approach can foster remanufacturing and facilitate the recycling of discarded products [20,21].

In the context of closed-loop supply chains for construction machinery, researchers have delved into the interconnections between pricing strategies, remanufacturing costs, recovery rates, and overall supply chain performance [22–24]. The findings suggest that by implementing appropriate pricing strategies and increasing consumer preferences for remanufactured products, the closed-loop supply chain in the construction machinery industry can be optimized, leading to enhanced recycling efficiency and reduced carbon emissions throughout the supply chain.

The existing research on the closed-loop supply chain of construction machinery has primarily centered around the sales-oriented mode, with limited research on the lease-oriented mode. Nonetheless, valuable insights can be gained from studies on the leasing of durable goods given the similarities in terms of high purchase prices, extended design life, and repeated utilization. Previous research on durable goods leasing has predominantly focused on items such as automobiles and car batteries [25,26]. It is widely recognized that decision variables such as the selling and leasing prices are influenced by factors such as production costs and consumer preferences. These factors, in turn, exert a significant impact on the overall profitability of the supply chain and product demand [27–29].

To address the challenge of reducing carbon emissions, countries worldwide have implemented various carbon policies, including carbon taxes, carbon quotas, and carbon trading. Among these policy instruments, carbon tax is widely recognized as an effective tool to improve energy efficiency and mitigate environmental pollution [30–32]. However, the introduction and implementation of carbon tax policies have imposed significant economic pressure on manufacturers of high-carbon products, potentially hindering their acceptance and implementation [33,34]. To effectively mitigate the economic losses associated with carbon tax policies, it is necessary to explore ways to strengthen cooperation among enterprises within the supply chain.

Several studies have explored the coordination of supply chains under carbon tax policies [35]. For instance, Han et al. developed a Stackelberg model that incorporates consumers' low-carbon preferences and the cost of emission reduction under a carbon tax policy. They proposed a revenue sharing contract to incentivize manufacturers to reduce carbon emissions, leading to improved overall supply chain benefits [36]. Similarly, Zhu et al. investigated a two-channel supply chain operating under carbon tax policy and proposed a two-part tariff coordination contract that resulted in a Pareto improvement of supply chain profits [37]. These studies highlight the effectiveness of contract design in compensating for the reduction in supply chain profits caused by carbon taxes.

In certain situations, a single contract may not be sufficient to achieve effective supply chain coordination, leading to the need for combined contracts. Deng et al. conducted research on selecting the optimal coordination contract in the presence of a carbon tax and found that individual contracts such as wholesale price contracts, revenue sharing contracts, and green cost sharing contracts were not able to fully coordinate the supply chain. However, when these contracts were combined with a two-part tariff contract, supply chain coordination was achieved [38]. Similarly, Zou et al. examined the impact of retailers' low-carbon investments on the supply chain under carbon tax and carbon trading policies. They proposed a combined benefit sharing–cost sharing contract to encourage manufacturers' emission reductions and retailers' low-carbon investments [39]. Additionally, Yu and Han investigated the influence of carbon tax on carbon emissions and retail prices in the supply chain. They introduced optimal decision making through a modified wholesale price contract and a cost sharing contract that were combined with a two-part tariff contract and a fixed payment fee for retailers to achieve supply chain coordination [40]. These studies indicate that the combined contracts are more effective in addressing the double marginalization effects and mitigating the economic losses incurred by enterprises due to carbon taxes. As for supply chain pricing and coordination, prior research has primarily tackled this challenge by devising both centralized and decentralized decision models. These models often involve the formulation of Hessian matrices to facilitate solution processes, allowing for the comparison of supply chain performance across these divergent decision-making paradigms. Moreover, the attainment of supply chain coordination has been pursued through the design of appropriate contracts. Subsequently, through the application of numerical simulations, researchers have meticulously scrutinized the impact of various parameters on supply chain performance. This comprehensive approach aids in unraveling the intricacies of pricing and coordination mechanisms within supply chain dynamics. Therefore, this study will also adopt a similar approach.

Currently, studies focused on closed-loop supply chains under carbon tax policies are relatively limited [41–43]. Previous research has primarily analyzed the impact of carbon tax rates and consumers' preferences on the profitability of closed-loop supply chains. These studies have highlighted the positive impact of increasing consumer preferences for remanufactured products on the demand for such products. Additionally, the implementation of carbon tax policies has been found to effectively reduce carbon emissions within the closed-loop supply chain. However, it is worth noting that an inappropriate carbon tax rate may hinder the development of remanufacturing processes. Despite these findings, there exists a research gap when it comes to conducting a comprehensive examination of product pricing and contract design within the context of closed-loop supply chains. Specifically, in the construction machinery industry, no research has been identified on the topic of closed-loop supply chains under carbon tax policies. Therefore, there is a need for further investigation to explore the specific implications and potential strategies related to carbon tax policies in the construction machinery industry's closed-loop supply chains.

The primary objective of this article is to propose a lease-oriented closed-loop supply chain model that takes into account the carbon tax policy. The aim is to examine how the carbon tax rate and consumer preferences for remanufactured products influence the supply chain in the construction machinery industry. Furthermore, the article aims to design a leasing compensation–cost apportioning combined contract, known as the combined contract, to achieve coordination within the supply chain. The proposed research addresses the practical needs of China's construction machinery market by addressing pricing and coordination challenges in the lease-oriented closed-loop supply chain. It harnesses the technical advantages of the industrial Internet platform to promote the expansion of leasing and recycling businesses, facilitate the effective Implementation of carbon tax policies in the construction machinery industry, and promote low-carbon sustainable development while ensuring economic benefits.

This study introduces several key innovations that contribute to the field of supply chain management and sustainability. This study represents a pioneering effort in investigating closed-loop supply chain pricing and coordination within the construction machinery industry. Diverging from the majority of previous research that emphasizes sales-oriented closed-loop supply chains, this study introduces a two-stage closed-loop supply chain model tailored to the specific demands and future prospects of a leasing-oriented approach. Within this model, various parameters and decision variables are incorporated, including the preference coefficient for remanufactured products, differential pricing decisions, platform costs, differential carbon emissions, and carbon tax rates. These elements are systematically examined to uncover the effects of carbon tax policies and the influence of industrial Internet platforms on supply chain performance. To address the challenges of double marginalization within the supply chain, a novel combined contract is devised. This contract introduces cost apportioning proportional coefficients and leasing compensation proportional coefficients, effectively mitigating the double marginalization effect. As a result of these efforts, the supply chain achieves Pareto optimality, effectively striking a balance between the economic gains for enterprises and the environmental benefits for society, all while operating under the framework of carbon tax policies. In essence, this study provides a comprehensive framework for understanding and managing closed-loop supply chain dynamics in the context of the construction machinery industry. By considering leasing-oriented practices, carbon tax policies, and the potential of industrial Internet platforms, this research contributes to a more holistic understanding of supply chain operations that encompass economic, environmental, and societal factors.

2. Research Framework

The research framework of this study is illustrated in Figure 1. The study begins by defining the problem at hand and subsequently providing a detailed explanation of the formulated model. This includes a thorough exploration of the underlying assumptions that have been considered during the modeling process. Following this, the study proceeds to solve the model using both centralized and decentralized decision-making approaches. Through this analysis, the study derives differential pricing strategies for both new and remanufactured products, taking into account various decision-making paradigms. Additionally, a leasing compensation—cost apportioning combined contract is designed to tackle the issue of the double marginalization effect in the closed-loop supply chain. The subsequent section of the paper is dedicated to validating the effectiveness of this combined contract through a comprehensive numerical analysis. By following this framework, the study aims to provide a comprehensive analysis of the construction machinery leasing market and offer valuable insights into the pricing, coordination, and cooperative emission reduction aspects of the closed-loop supply chain.

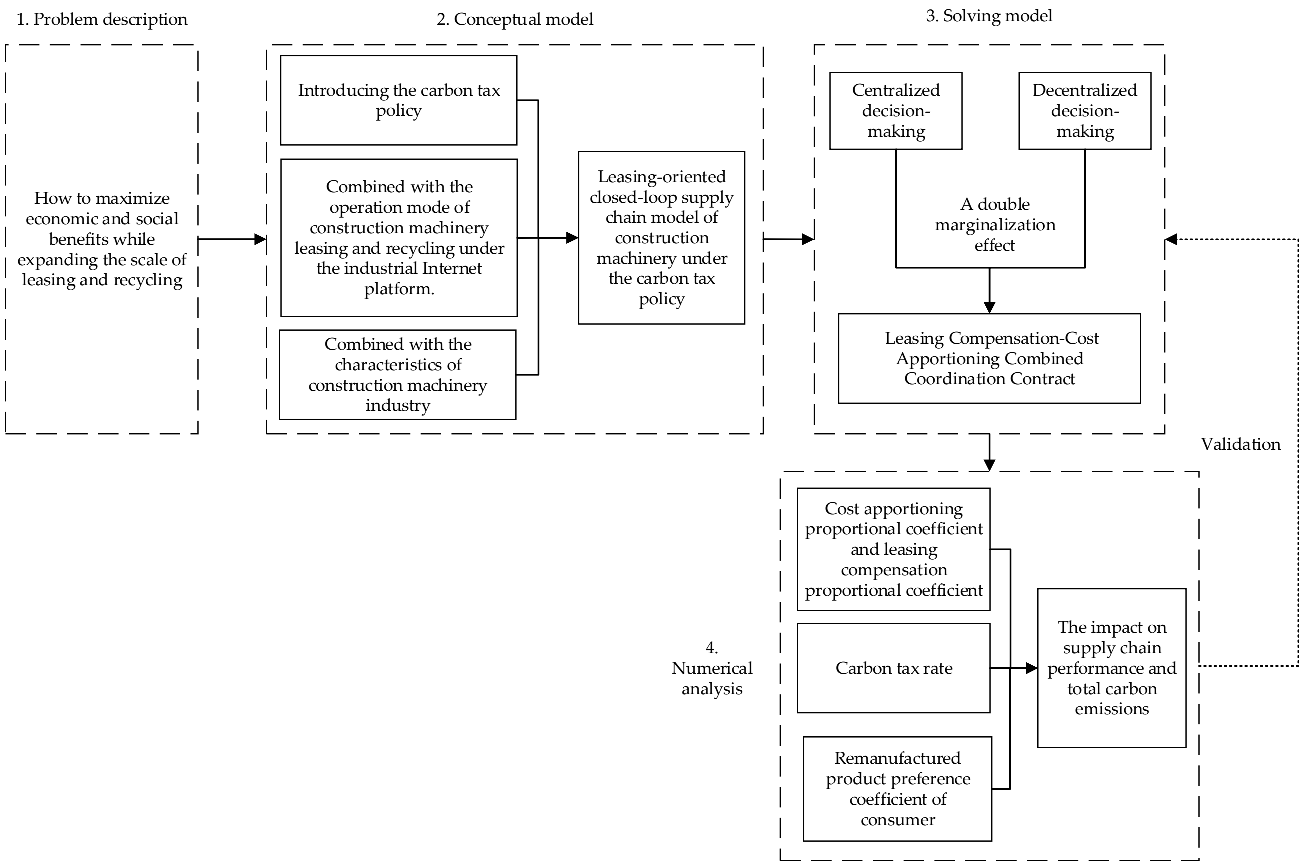

Figure 1. The framework of the study.

3. Model Framework

3.1. Model Description

This paper focuses on analyzing a closed-loop supply chain for construction machinery, incorporating leasing and remanufacturing practices under the carbon tax policy. The supply chain operates under a Stackelberg leadership model, where the manufacturer assumes the role of the leader and the lessor acts as the follower. Both parties have access to complete information to make their decisions.

The decision-making process begins with the manufacturer, who sets the selling prices for both the new product and the remanufactured product. Additionally, the manufacturer determines the product recovery rate for the remanufactured product and calculates the carbon tax payment based on the total production carbon emissions. Subsequently, the lessor responds by determining the leasing prices for the new product and the remanufactured product considering the manufacturer's strategies.

It is assumed that both the manufacturer and the lessor are risk neutral and make rational decisions. The overall flow of the leasing-oriented closed-loop supply chain for construction machinery under the carbon tax policy is illustrated in Figure 2.

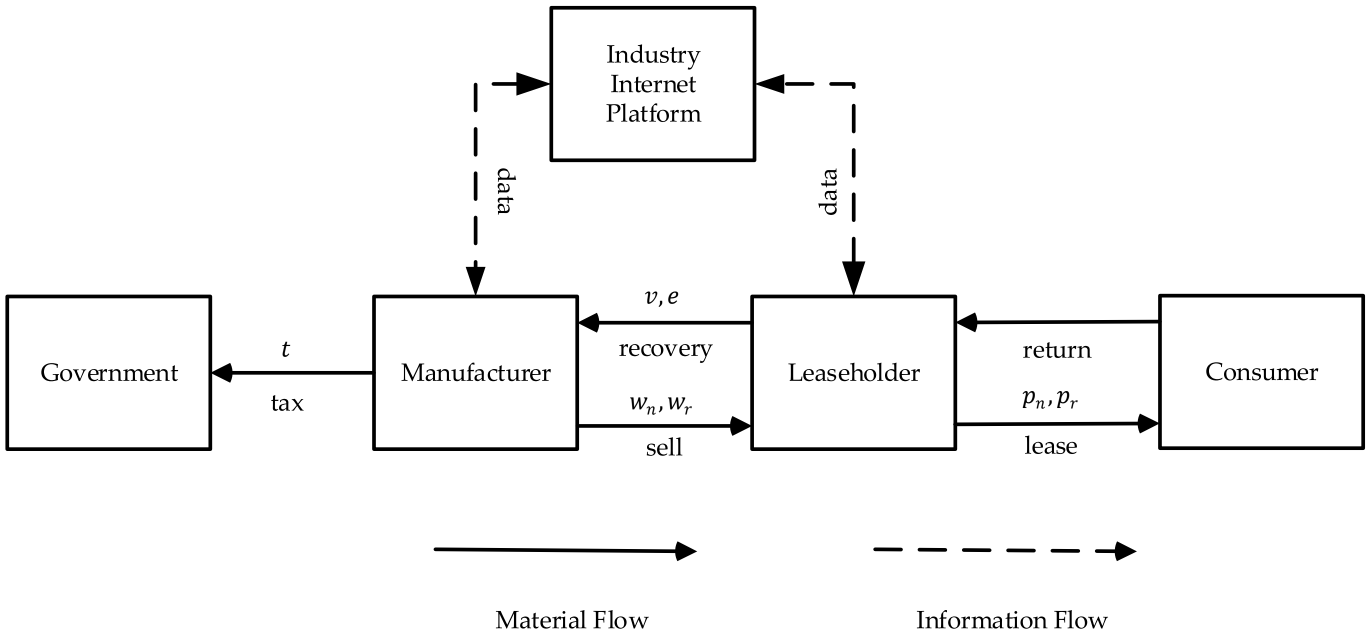

Figure 2. Flow chart of lease-oriented closed-loop supply chain for construction machinery under the carbon tax policy.

3.2. Definition of Symbols

The main notations are shown in Table 1.

Table 1. Notation.

Notation	Definition
Q_n^i	Demand for the new product under different decision-making situations, $i \in \{c, s, v\}$. c, s, and v represent centralized decision making, decentralized decision making, and combined contract, respectively.
Q_n^{i*}	The optimal value after solving for partial optimal decision variables
Q_r^i	Demand for the remanufactured product
π_m^i	Profit of the manufacturer
π_m^{i**}	The optimal value after further solving for all optimal decision variables
π_r^i	Profit of the lessor
π_t^i	Profit of the total supply chain
p_n	New product leasing price
p_r	Remanufactured product leasing price
w_n	New product selling price
w_r	Remanufactured product selling price
c_n	Unit new product cost
c_r	Unit remanufactured product cost
v	Unit recovery cost

Table 1. *Cont.*

Notation	Definition
β	Remanufactured product preference coefficient of consumer
t	Carbon tax rate
e	Product recovery rate
φ	Industrial Internet platform cost coefficient
h_n	Production carbon emissions of unit new product
h_r	Remanufactured carbon emissions of unit new product
λ	Cost apportioning proportional coefficient
f	Leasing compensation proportional coefficient
k	Unit revenue of parts that cannot be remanufactured
K	Total production carbon emission

In the forward supply chain, the manufacturer assumes the responsibility of producing new products and remanufacturing old products. The unit cost of producing a new product is denoted as c_n, and the unit cost of remanufacturing a product is represented by c_r. The manufacturer sells the new and remanufactured products to the lessor at prices w_n and w_r, respectively. The lessor then leases these products to consumers at prices p_n and p_r, respectively. Consumers have the flexibility to choose between new and remanufactured products based on their preferences.

In the reverse supply chain, the manufacturer engages in the recycling of used products through the industrial Internet platform. The manufacturer buys back the used products from the lessor at a price of v, and the recovery rate of these products is denoted as e. The manufacturer incurs a recovery cost of v for each unit of used product. Additionally, any parts that cannot be remanufactured are sold at a price of k.

In practical scenarios, the production activities of the manufacturer result in carbon emissions. The carbon emissions associated with the production of new and remanufactured products are denoted as h_n and h_r, respectively. The government imposes a carbon tax rate of t CNY/kg based on the total carbon emissions K, generated by the manufacturer's production activities.

3.3. Related Assumptions

Assumption 1. *To facilitate comparison, the leasing price and selling price of both the new and remanufactured products are averaged, representing the cost per unit of time that consumers spend on these products. In order to ensure profitability for both the lessor and the manufacturer, it is assumed that* $p_n > w_n$ *and* $p_r > w_r$.

Assumption 2. *The cost associated with the industrial Internet platform primarily encompasses two components: the initial expenses involved in constructing the platform and the ongoing operational costs, encompassing activities such as system maintenance, data processing, and information services. In this study, the recovery rate is utilized as a metric to gauge the extent of recycling activities. A higher recovery rate corresponds to a larger scale of recycling business operations, leading to increased costs borne by the platform. In accordance with the existing literature [44,45], the assumption is made that the cost of the industrial Internet platform exhibits a quadratic functional relationship with the recovery rate:* $S = \frac{1}{2}\varphi e^2$*, which satisfies* $\varphi > 0$ *and* $0 < e \leq 1$.

Assumption 3. *Consumers often exhibit distinct cognitive perceptions when considering new products compared with a remanufactured product, tending to display a preference for new products even when confronted with equivalent conditions. To address this consumer behavior, this study draws on the insights presented in reference [46] regarding consumer low-carbon preferences and assumes that the remanufactured product preference coefficient of consumer is* β*, which satisfies*

$0 < \beta < 1$, and the new product preference coefficient of consumer is 1, reflecting the baseline preference for new products.

Assumption 4. *It is assumed that the unit production cost and the unit carbon emission from the production of the new product are higher than that of the remanufactured product, denoted as $c_n > c_r$ and $h_n > h_r$, respectively.*

Assumption 5. *It is assumed that due to the higher material requirements for producing new products, the selling price and leasing price of new products are higher than that of remanufactured products, denoted as $w_n > w_r$ and $p_n > p_r$, respectively.*

Assumption 6. *During the remanufacturing process, certain parts that cannot be successfully remanufactured are sold at a price denoted as k. The revenue generated from selling these parts is assumed to be lower than the revenue from selling the remanufactured products.*

4. Model Solutions and Contract Coordination Design

4.1. Demand Functions and Profit Functions

According to the defined symbols and problem assumptions, the demand functions for the new product and the remanufactured product can be expressed as follows:

$$Q_n^i = \alpha - \frac{p_n - p_r}{1-\beta} \tag{1}$$

$$Q_r^i = \frac{\beta p_n - p_r}{\beta(1-\beta)} \tag{2}$$

Assumption 2 states that $e \in (0,1]$, which implies that the number of used products recycled by the manufacturer is higher than the demand for the remanufactured products. Therefore, we have $0 < Q_r^i \leq eQ_n^i$.

The total carbon emission after the end of production activities can be expressed as:

$$K_i = h_n Q_n^i + h_r Q_r^i \tag{3}$$

The profit functions of the manufacturer, lessor, and total supply chain can be expressed as follows:

$$\pi_m^i = (w_n - c_n - ev - th_n)Q_n^i + (w_r - c_r - th_r)Q_r^i + k(eQ_n^i - Q_r^i) - \frac{1}{2}\varphi e^2 \tag{4}$$

$$\pi_r^i = (p_n - w_n + ev)Q_n^i + (p_r - w_r)Q_r^i \tag{5}$$

$$\pi_t^i = (p_n - c_n - th_n)Q_n^i + (p_r - c_r - th_r) + k(eQ_n^i - Q_r^i) - \frac{1}{2}\varphi e^2 \tag{6}$$

Equation (4): The profit function of the manufacturer, which encompasses the profits from selling new products, remanufactured products, and the parts that cannot be remanufactured, while accounting for the cost of the industrial Internet platform.

Equation (5): The profit function of the lessor, representing the profits generated from leasing out both new products and remanufactured products.

Equation (6): The total supply chain profit function, obtained by summing the profit functions of the manufacturer and the lessor.

4.2. Centralized Decision-Making Scenario Analysis

Under centralized decision making, the manufacturer and lessor collaborate closely and prioritize the maximization of supply chain profits over their individual benefits. They share information and jointly make decisions. The profit functions of the different stakeholders within the supply chain encompass a range of decision variables. In order to establish the Hessian matrix, it becomes imperative to calculate the partial derivatives for each decision variable. Through the assessment of principal minors facilitated by the

Hessian matrix computation, it is feasible to ascertain whether the profit functions of supply chain participants exhibit convexity or concavity under specific circumstances. By equating the first-order partial derivatives of each decision variable to zero, it is possible to derive the optimal values for these decision variables. This methodology ensures that in the context of centralized decision making or later introduced decentralized decision making and the combined contract model, the supply chain attains the optimal values for demand, overall profit, and carbon emissions across all stakeholders. In this context, the Hessian matrix is constructed to solve for the optimal values of p_n, p_r, and e.

$$H_1 = \begin{bmatrix} \frac{\partial^2 \pi_t^c}{\partial p_n^2} & \frac{\partial^2 \pi_t^c}{\partial p_n \partial p_r} & \frac{\partial^2 \pi_t^c}{\partial p_n \partial e} \\ \frac{\partial^2 \pi_t^c}{\partial p_r \partial p_n} & \frac{\partial^2 \pi_t^c}{\partial p_r^2} & \frac{\partial^2 \pi_t^c}{\partial p_r \partial e} \\ \frac{\partial^2 \pi_t^c}{\partial e \partial p_n} & \frac{\partial^2 \pi_t^c}{\partial e \partial p_r} & \frac{\partial^2 \pi_t^c}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -\frac{2}{1-\beta} & \frac{2}{1-\beta} & -\frac{k}{(1-\beta)} \\ \frac{2}{1-\beta} & -\frac{2}{\beta(1-\beta)} & \frac{k}{(1-\beta)} \\ -\frac{k}{(1-\beta)} & \frac{k}{(1-\beta)} & -\varphi \end{bmatrix}$$

The Hessian matrix exhibits negativity when $\Delta_1 = -2/(1-\beta) < 0$, $\Delta_2 = 4(1-\beta) > 0$, $\Delta_3 = 2(k^2 - 2\varphi + 2\beta\varphi)/\beta(1-\beta)^2 < 0$, indicating that the objective function, π_t^c, is strictly concave with respect to p_n, p_r, and e. By solving the equations $\frac{\partial \pi_t^c}{\partial p_n} = 0$, $\frac{\partial \pi_t^c}{\partial p_r} = 0$, and $\frac{\partial \pi_t^c}{\partial e} = 0$, we can derive the following equations:

$$p_n^{c*} = \frac{-kc_r + 2\alpha\varphi - k^2(2\alpha + k) - 2\beta\varphi(c_n - \alpha - th_n) + 2\varphi(c_n + th_n) + k^2(\beta\alpha - th_r)}{2(2\varphi - 2\beta\varphi - k^2)} \tag{7}$$

$$p_r^{c*} = \frac{c_r + k + \beta\alpha + th_r}{2} \tag{8}$$

$$e_c^* = \frac{k(c_r - c_n + \alpha(1-\beta) + k - t(h_n - h_r))}{2\varphi - 2\beta\varphi - k^2} \tag{9}$$

By substituting Equations (7)–(9) into Equations (1)–(3) and (6), we can derive the optimal values for the new product demand, the remanufactured product demand, and the total supply chain profit under centralized decision making as follows:

$$Q_n^{c*} = \frac{\varphi(c_r - c_n + k + \alpha(1-\beta) - t(h_n - h_r))}{2\varphi - 2\beta\varphi - k^2}$$

$$Q_r^{c*} = \frac{(k^2 - 2\varphi)(c_r + k) + \beta(2\varphi c_n - \alpha k^2 + 2\varphi th_n) - th_r(2\varphi - k)}{2(2\varphi - 2\beta\varphi - k^2)}$$

$$K_c^* = \frac{A_1}{2\beta(2\varphi - 2\beta\varphi - k^2)},\ \pi_t^{c*} = \frac{A_2}{4(2\varphi - 2\beta\varphi - k^2)}$$

where:

$$A_1 = 2\beta\varphi(c_n(h_r - h_n) + h_n(c_r + k + \alpha - \beta\alpha - th_n + 2th_r)) + (k^2h_r - 2\varphi h_r)(k + th_r + c_r) - \beta\alpha k^2 h_r$$

$$\begin{aligned} A_2 &= 4\beta^2\varphi\alpha(c_n + th_n) - \beta^2\alpha(k^2 + 2\alpha\varphi) + 2\varphi c_r(c_r + 2th_r - 2k) + (k^2 - 2\varphi)(t^2h_r^2 + 2kth_r + k^2) \\ &+ 2\beta\varphi(\alpha^2 + c_n(c_n + 2(th_n - c_r - th_r - k - \alpha)) + th_n(th_n - 2(c_r + th_r + k + \alpha))) \\ &+ 2\beta\alpha k^2(c_r + th_r + k) - c_r k^2(c_r + 2k + 2th_r) \end{aligned}$$

Proposition 1. *Under centralized decision making, the demand for new products exhibits a negative correlation with the carbon tax rate, whereas it shows a positive correlation with the leasing price of remanufactured products. However, the correlation between the total profit of the supply chain, the carbon emissions from total production, the demand for remanufactured products, and the leasing price of new products varies across different parameter ranges in relation to the carbon tax rate.*

Proof of Proposition 1. By calculating the partial derivatives $\frac{\partial Q_n^{c*}}{\partial t}$, $\frac{\partial Q_r^{c*}}{\partial t}$, $\frac{\partial K_c^{*}}{\partial t}$, $\frac{\partial p_n^{c*}}{\partial t}$, $\frac{\partial p_r^{c*}}{\partial t}$, and $\frac{\partial \pi_t^{c*}}{\partial t}$, we can derive the following results:

$$\frac{\partial Q_n^{c*}}{\partial t} = -\frac{\varphi(h_n - h_r)}{2\varphi - 2\beta\varphi - k^2} < 0,\ \frac{\partial Q_r^{c*}}{\partial t} = \frac{k^2 h_r - 2\varphi h_r + 2\beta\varphi h_n}{2\beta(2\varphi - 2\beta\varphi - k^2)}$$

$$\frac{\partial K_c^{*}}{\partial t} = \frac{2\beta\varphi h_n(h_n - 2h_r) - h_r^2(k^2 - 2\varphi)}{2\beta(2\varphi - 2\beta\varphi - k^2)},\ \frac{\partial p_n^{c*}}{\partial t} = -\frac{k^2 h_r - 2\varphi h_n + 2\beta\varphi h_n}{2(2\varphi - 2\beta\varphi - k^2)}$$

$$\frac{\partial p_r^{c*}}{\partial t} = \frac{h_r}{2} > 0,\ \frac{\partial \pi_t^{c*}}{\partial t} = \frac{A_3}{2\beta(2\varphi - 2\beta\varphi - k^2)}$$

where

$$A_3 = (k^2 h_r - 2\varphi h_r)(k + c_r + th_r) - \beta\alpha k^2 h_r + 2\beta\varphi(c_n(h_r - h_n) + h_n(c_r + k + \alpha - \beta\alpha - th_n + 2th_r))$$

When $k^2 h_r - 2\varphi h_r + 2\beta\varphi h_n > 0$, there is $\frac{\partial Q_r^{c*}}{\partial t} > 0$.
When $2\beta\varphi h_n(h_n - 2h_r) - h_r^2(k^2 - 2\varphi) < 0$, there is $\frac{\partial K_c^{*}}{\partial t} < 0$.
When $k^2 h_r - 2\varphi h_n + 2\beta\varphi h_n > 0$, there is $\frac{\partial p_n^{c*}}{\partial t} < 0$.
When $A_3 < 0$, there is $\frac{\partial \pi_t^{c*}}{\partial t} < 0$. □

Proposition 1 reveals that as the carbon tax rate increases, several outcomes are observed. Firstly, the demand for new products decreases, whereas the demand for remanufactured products increases. Secondly, the leasing prices of both new products and remanufactured products decrease. However, Assumption 5 states that the leasing price of new products is higher than that of remanufactured products, resulting in a decrease in the total supply chain profit. Additionally, Assumption 4 assumes that carbon emissions from the production of remanufactured products are lower than those from new products, leading to a decrease in the total supply chain carbon emissions. This highlights the need for enterprises to take on more social responsibility by adjusting the carbon tax rate. To enhance supply chain profits, manufacturers should improve their industrial Internet platform operation and maintenance technology as well as remanufacturing techniques. Simultaneously, lessors should focus on promoting the benefits of remanufactured products to expand the market demand.

Proposition 2. *Under centralized decision making, the correlation between the total supply chain profit, the total production carbon emissions, the demand for new products, and the demand for remanufactured products is influenced by the remanufactured product preference coefficient of consumers, which varies across different parameter ranges.*

Proof of Proposition 2. By calculating the partial derivatives $\frac{\partial Q_n^{c*}}{\partial \beta}$, $\frac{\partial Q_r^{c*}}{\partial \beta}$, $\frac{\partial K_c^{*}}{\partial \beta}$, and $\frac{\partial \pi_t^{c*}}{\partial \beta}$, we can obtain:

$$\frac{\partial Q_n^{c*}}{\partial \beta} = \frac{\varphi(k(\alpha k + 2\varphi) - 2\varphi(c_n - c_r) - 2\varphi t(h_n - h_r))}{(2\varphi - 2\beta\varphi - k^2)^2},\ \frac{\partial Q_r^{c*}}{\partial \beta} = \frac{A_4}{2\beta^2(2\varphi - 2\beta\varphi - k^2)^2}$$

$$\frac{\partial K_c^{*}}{\partial \beta} = \frac{A_5}{2\beta^2(2\varphi - 2\beta\varphi - k^2)^2},\ \frac{\partial \pi_t^{c*}}{\partial \beta} = \frac{A_6}{4\beta^2(2\varphi - 2\beta\varphi - k^2)^2}$$

where

$$A_4 = 4\varphi(\varphi + \beta k^2 - 2\beta\varphi)(c_r + k + th_r) - 2\varphi k^2(2k + \beta\alpha\varphi) + 4\beta^2\varphi^2(c_n + th_n) + k^2(k^2 - 4\varphi)(th_r + c_r)$$

$$\begin{aligned} A_5 &= h_r(k^4 + 4\varphi(\varphi - 2\beta\varphi + \beta k^2 - 1))(k + th_r + c_r) + 2\beta^2\varphi\alpha k^2(h_n - h_r) \\ &+4\beta^2\varphi^2(c_n(h_r - h_n) + h_n(c_r + k + 2th_r - th_n)) \end{aligned}$$

$$A_6 = -(k^2h_r - 2\varphi h_r)(k + th_r + c_r) + \beta\alpha k^2h_r - 2\beta\varphi(c_n(h_r - h_n) + h_n(c_r + k + \alpha - \beta\alpha - th_n + 2th_r))$$

When $\varphi(k(\alpha k + 2\varphi) - 2\varphi(c_n - c_r) - 2\varphi t(h_n - h_r)) < 0$, there is $\frac{\partial Q_n^{c*}}{\partial\beta} < 0$.
When $A_4 > 0$, there is $\frac{\partial Q_r^{c*}}{\partial\beta} > 0$.
When $A_5 > 0$, there is $\frac{\partial K_c^*}{\partial\beta} < 0$; otherwise, there is $\frac{\partial K_c^*}{\partial\beta} > 0$.
When $A_6 > 0$, there is $\frac{\partial \pi_t^{c*}}{\partial\beta} > 0$; otherwise, there is $\frac{\partial \pi_t^{c*}}{\partial\beta} < 0$. □

Proposition 2 indicates that in a centralized decision-making setting, when the remanufactured product preference coefficient of consumers is low, the market demand is primarily driven by new products, resulting in higher total production carbon emissions. However, as consumer awareness of low-carbon options increases, the remanufactured product preference coefficient also rises, leading to a gradual increase in demand for remanufactured products and a reduction in carbon emissions. Ultimately, this improves the overall efficiency of the supply chain.

4.3. Decentralized Decision-Making Scenario Analysis

Under decentralized decision making, the manufacturer and lessor aim to maximize their own profit. Based on the reverse solution method, the Hessian matrix is constructed to solve for the optimal values of p_n and p_r.

$$H_2 = \begin{bmatrix} \frac{\partial^2\pi_r^s}{\partial p_n^2} & \frac{\partial^2\pi_r^s}{\partial p_n\partial p_r} \\ \frac{\partial^2\pi_r^s}{\partial p_r\partial p_n} & \frac{\partial^2\pi_r^s}{\partial p_r^2} \end{bmatrix} = \begin{bmatrix} -\frac{2}{1-\beta} & \frac{2}{1-\beta} \\ \frac{2}{1-\beta} & -\frac{2}{\beta(1-\beta)} \end{bmatrix}$$

The Hessian matrix is found to be negative when $\Delta_1 = -2/(1-\beta) < 0$, $\Delta_2 = 4(1-\beta) > 0$. As a result, π_r^s is strictly concave with respect to p_n and p_r. By solving $\frac{\partial \pi_t^{c*}}{\partial p_n} = 0$ and $\frac{\partial \pi_t^{c*}}{\partial p_r} = 0$, we can obtain the following equations:

$$p_n^{s*} = \frac{\alpha + w_n - ev}{2} \tag{10}$$

$$p_r^{s*} = \frac{w_r + \beta\alpha}{2} \tag{11}$$

By substituting Equations (10) and (11) into Equation (4), we can obtain the following result:

$$\pi_m^{s*} = \frac{B_1}{2\beta(1-\beta)} \tag{12}$$

where

$$\begin{aligned} B_1 &= k(w_r - \beta w_n(1+e) + \beta e(\alpha + v + w_r - \beta\alpha + ev)) \\ &+2(w_r - \beta w_n + \beta ev)(c_r - w_r + th_r) - \varphi e^2\beta(1-\beta) \\ &-\beta(c_n - w_n + ev + th_n)(\alpha - w_n + w_r - \beta\alpha + ev) \end{aligned}$$

In order to analyze the impact of w_n, e, and w_r on π_m^{s*}, the Hessian matrix is constructed.

$$H_3 = \begin{bmatrix} \frac{\partial^2\pi_m^{s*}}{\partial w_n^2} & \frac{\partial^2\pi_m^{s*}}{\partial w_n\partial w_r} & \frac{\partial^2\pi_m^{s*}}{\partial w_n\partial e} \\ \frac{\partial^2\pi_m^{s*}}{\partial w_r\partial w_n} & \frac{\partial^2\pi_m^{s*}}{\partial w_r^2} & \frac{\partial^2\pi_m^{s*}}{\partial w_r\partial e} \\ \frac{\partial^2\pi_m^{s*}}{\partial e\partial w_n} & \frac{\partial^2\pi_m^{s*}}{\partial e\partial w_r} & \frac{\partial^2\pi_m^{s*}}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{1-\beta} & \frac{1}{1-\beta} & -\frac{k-2v}{2(1-\beta)} \\ \frac{1}{1-\beta} & -\frac{1}{\beta(1-\beta)} & \frac{k-2v}{2(1-\beta)} \\ -\frac{k-2v}{2(1-\beta)} & \frac{k-2v}{2(1-\beta)} & \frac{v(k-v)-\varphi(1-\beta)}{1-\beta} \end{bmatrix}$$

The Hessian matrix is found to be negative when $\Delta_1 = -1/(1-\beta) < 0$, $\Delta_2 = 1-\beta > 0$, $\Delta_3 = (k^2 - 4\varphi + 4\beta\varphi)/4\beta(1-\beta)^2 < 0$. As a result, π_m^{s*} is strictly concave with respect to w_n, w_r, and e. By solving $\frac{\partial \pi_m^{s*}}{\partial w_n} = 0$, $\frac{\partial \pi_m^{s*}}{\partial w_r} = 0$, and $\frac{\partial \pi_m^{s*}}{\partial e} = 0$, we can obtain the following equations:

$$w_n^{s*} = \frac{B_2}{2(4\varphi - 4\beta\varphi - k^2)} \tag{13}$$

$$w_r^{s*} = \frac{c_r + k + \beta\alpha + th_r}{2} \tag{14}$$

$$e_s^* = \frac{k(k - c_n + c_r + \alpha - \beta\alpha - th_n + th_r)}{4\varphi - 4\beta\varphi - k^2} \tag{15}$$

where

$$\begin{aligned} B_2 &= k^2(2v - c_r + \beta\alpha - k - th_r - 2\alpha) + 4\varphi(\alpha + c_n - \beta\alpha + th_n) \\ &\quad -4\beta\varphi(c_n + th_n) + 2kv(c_r - c_n + \alpha - \beta\alpha - th_n + th_r) \end{aligned}$$

By substituting Equations (13)–(15) into Equations (10) and (11), we can obtain the following result:

$$p_n^{s**} = \frac{k^2((\beta\alpha - th_r) - 4\alpha - (c_r + k)) + 4\varphi(3\alpha(1-\beta) + (1-\beta)(c_n + th_n))}{4(4\varphi - 4\beta\varphi - k^2)} \tag{16}$$

$$p_r^{s**} = \frac{c_r + k + 3\beta\alpha + th_r}{4} \tag{17}$$

By substituting Equations (13)–(17) into Equations (1)–(6), we can obtain the optimal values of the new product demand, the remanufactured product demand, the total production carbon emission, the manufacturer profit, the lessor profit, and the total supply chain profit under decentralized decision making as follows:

$$Q_n^{s*} = \frac{\varphi(c_r - c_n + k + \alpha - th_n + th_r)}{4\varphi - 4\beta\varphi - k^2},$$

$$Q_r^{s*} = \frac{k^2(c_r + k - \beta\alpha + th_r) + 4\varphi(\beta c_n - c_r - k - th_r + \beta th_n)}{4\beta(4\varphi - 4\beta\varphi - k^2)}$$

$$K_s^* = \frac{B_3}{4\beta(4\varphi - 4\beta\varphi - k^2)},\ \pi_m^{s**} = \frac{B_4}{8\beta(4\varphi - 4\beta\varphi - k^2)}$$

$$\pi_r^{s**} = \frac{B_5}{16\beta(4\varphi - 4\beta\varphi - k^2)} + \frac{B_6}{4(4\varphi - 4\beta\varphi - k^2)^2}$$

$$\pi_t^{s*} = \frac{2B_4 + \beta B_5}{16\beta(4\varphi - 4\beta\varphi - k^2)} + \frac{B_6}{4(4\varphi - 4\beta\varphi - k^2)^2} = N$$

where

$$B_3 = 4\beta\varphi(c_n(h_r - h_n) + h_n(c_r + k + \alpha - th_n + 2th_r - \beta\alpha)) - \beta\varphi k^2 h_r + (k^2 h_r - 4\varphi h_r)(k + th_r + c_r)$$

$$\begin{aligned} B_4 &= 2\beta(4\beta\varphi\alpha(c_n + th_n) + 4\varphi(c_n(th_n - c_r - th_r - k - \alpha) - th_n(k + \alpha)) \\ &\quad +\alpha k^2(c_r + th_r + k) + 4\beta\varphi(t^2 h_n^2 + \alpha^2 - \beta\alpha^2 + c_n^2))) \\ &\quad +k^2(4\varphi - k^2 - \beta^2\varphi^2 - th_r(2k + th_r) - c_r(c_r + 2k + 2th_r)) \\ &\quad +4\varphi(c_r(c_r + 2th_r + 2k) + th_r(th_r + 2k)) \end{aligned}$$

$$B_5 = (-c_r - k + \beta\alpha - th_r)((k^2 - 4\varphi)(c_r + k) + 4\varphi(\beta c_n - th_r + \beta th_n) + k^2(th_r - \beta\alpha))$$

$$B_6 = \varphi(c_r - c_n + k + \alpha(1-\beta) - t(h_n - h_r))(4\varphi(1-\beta)(\alpha - c_n - th_n) + k^2(c_r + k - \beta\alpha - th_r))$$

Proposition 3. *Under decentralized decision making, the demand for new products exhibits a negative correlation with the carbon tax rate, whereas it shows a positive correlation with the leasing price and selling price of remanufactured products. However, the correlation between the* *manufacturer's profit, the lessor's profit, the total production carbon emissions, the demand for remanufactured products, the selling price of new products, and the leasing price of new products with the carbon tax rate varies across different parameter ranges.*

Proof of Proposition 3. Finding $\frac{\partial Q_n^{s*}}{\partial t}$, $\frac{\partial Q_r^{s*}}{\partial t}$, $\frac{\partial K_s^{*}}{\partial t}$, $\frac{\partial p_n^{s*}}{\partial t}$, $\frac{\partial p_r^{s*}}{\partial t}$, $\frac{\partial w_n^{s*}}{\partial t}$, $\frac{\partial w_r^{s*}}{\partial t}$, $\frac{\partial \pi_m^{s**}}{\partial t}$, and $\frac{\partial \pi_r^{s**}}{\partial t}$, we can obtain:

$$\frac{\partial Q_n^{s*}}{\partial t} = -\frac{\varphi(h_n - h_r)}{4\varphi - 4\beta\varphi - k^2} < 0,\ \frac{\partial Q_r^{s*}}{\partial t} = \frac{F}{4\beta(4\varphi - 4\beta\varphi - k^2)}$$

$$\frac{\partial K_s^{*}}{\partial t} = \frac{4\beta\varphi h_n(2h_r - h_n) + h_r^2(k^2 - 4\varphi)}{4\beta(4\varphi - 4\beta\varphi - k^2)},\ \frac{\partial p_n^{s*}}{\partial t} = \frac{4\varphi h_n(1-\beta) - h_r k^2}{4(4\varphi - 4\beta\varphi - k^2)}$$

$$\frac{\partial p_r^{s*}}{\partial t} = \frac{h_r}{4} > 0,\ \frac{\partial w_n^{s*}}{\partial t} = \frac{-2kv(h_n - h_r) - F}{2(4\varphi - 4\beta\varphi - k^2)},\ \frac{\partial w_r^{s*}}{\partial t} = \frac{h_r}{2} > 0$$

$$\frac{\partial \pi_m^{s**}}{\partial t} = \frac{B_7}{4\beta(4\varphi - 4\beta\varphi - k^2)},\ \frac{\partial \pi_r^{s**}}{\partial t} = \frac{B_8}{8\beta(4\varphi - 4\beta\varphi - k^2)^2}$$

where

$$B_7 = (k^2 h_r - 4\varphi h_r)(k + th_r + c_r) - \beta\alpha(k^2 h_r + 4\beta\varphi h_n) + 4\beta\varphi(h_r(c_n + 2th_n) + h_n(c_r - c_n + k + \alpha - th_n))$$

$$\begin{aligned} B_8 &= k^4 h_r(k + th_r + c_r - \beta\alpha) + 16\varphi^2 h_r(c_r + k + th_r) + 16\beta\varphi(\varphi h_n(c_n - \beta^2\alpha) \\ &+ \varphi(t(h_n^2 - h_r^2) - h_r(c_n + k) - c_r(h_n + h_r) - h_n(k + \alpha + 2th_r)) \\ &+ \beta\varphi(c_n(h_r - h_n) + h_n(c_r + k + 2\alpha - th_n + 2th_r))) \\ &+ 8k^2\varphi h_r(1-\beta)(-c_r - th_r + \beta\alpha - k) \end{aligned}$$

$$F = 4\varphi h_n(1-\beta) - h_r k^2$$

When $F > 0$, there is $\frac{\partial Q_r^{s*}}{\partial t} > 0$.
When $4\beta\varphi h_n(2h_r - h_n) + h_r^2(k^2 - 4\varphi) < 0$, there is $\frac{\partial K_s^{*}}{\partial t} < 0$.
When $4\varphi h_n(1-\beta) - h_r k^2 > 0$, there is $\frac{\partial p_n^{s*}}{\partial t} < 0$.
When $-2kv(h_n - h_r) - F > 0$, there is $\frac{\partial w_n^{s*}}{\partial t} > 0$.
When $B_7 < 0$, there is $\frac{\partial \pi_m^{s**}}{\partial t} < 0$.
When $B_8 < 0$, there is $\frac{\partial \pi_r^{s**}}{\partial t} < 0$. □

Proposition 3 shows that under centralized decision making, with an increase in the carbon tax rate, the production cost of the new products increases and the production of new products has to be reduced. The proportion of the remanufactured products in the market continues to increase, thereby reducing the total production carbon emission. The high carbon tax rate and the change in the market product structure lead to the fact that the manufacturer has to increase the selling price of the new product and the remanufactured product to maintain profits. The increase in the selling price also makes the lessor increase the leasing price. When the carbon tax policy is implemented in the early stage in order to achieve the goal of reducing carbon emissions and accelerating industrial transformation, the manufacturer and lessor must assume more carbon emission reduction tasks at the expense of their part of the profits.

Proposition 4. *Under centralized decision making, the correlation between the remanufacturer's profit, the lessor's profit, the total production carbon emission, the new product demand, and the remanufactured product demand with the remanufactured product preference coefficient of the consumer varies under different parameter ranges.*

Proof of Proposition 4. By calculating the partial derivatives $\frac{\partial Q_n^{s*}}{\partial \beta}$, $\frac{\partial Q_r^{s*}}{\partial \beta}$, $\frac{\partial K_s^*}{\partial \beta}$, $\frac{\partial \pi_m^{s**}}{\partial \beta}$, and $\frac{\partial \pi_r^{s**}}{\partial \beta}$, we can obtain:

$$\frac{\partial Q_n^{s*}}{\partial \beta} = \frac{\varphi(\alpha k^2 - 4\varphi((c_n - c_r - k) - t(h_n - h_r)))}{(4\varphi - 4\beta\varphi - k^2)^2}, \ \frac{\partial Q_r^{s*}}{\partial \beta} = \frac{B_9}{4\beta^2(4\varphi - 4\beta\varphi - k^2)^2}$$

$$\frac{\partial K_s^*}{\partial \beta} = \frac{B_{10}}{4\beta(4\varphi - 4\beta\varphi - k^2)^2}, \ \frac{\partial \pi_m^{s**}}{\partial \beta} = \frac{B_{11}}{8\beta^2(4\varphi - 4\beta\varphi - k^2)}$$

$$\frac{\partial \pi_r^{s**}}{\partial \beta} = \frac{B_{12}}{16\beta^2(4\varphi - 4\beta\varphi - k^2)} + \frac{4\beta B_{13} - B_{14}(4\varphi - 4\beta\varphi - k^2)}{4\beta(4\varphi - 4\beta\varphi - k^2)^3} = M$$

where

$$B_9 = (k^4 + 16\varphi^2(1 - 2\beta) + 8\varphi(\beta k^2 - k))(c_r + k + th_r) + 4\beta^2\varphi(4\varphi(c_n + th_n) - \alpha k^2)$$

$$\begin{aligned} B_{10} &= h_r(k + th_r + c_r)(k^4 + 8\varphi k^2(\beta h_r - h_r) + 16\varphi^2(1 - 2\beta h_r)) \\ &+ 4\beta^2\varphi(4\varphi(c_n(h_r - h_n) + h_n(c_r + k - th_n + 2th_r)) + \alpha k^2(h_n - h_r)) \end{aligned}$$

$$\begin{aligned} B_{11} &= -(16\beta^2\varphi^2(c_n(2c_r - c_n - 2t(h_n - h_r) + 2k) + h_n(2c_r t - t^2(h_n - 2h_r) + 2tk)) \\ &+ 8\beta^2\varphi k^2(\alpha(th_n - c_r - th_r - k + c_n) - 4) - 32\beta\varphi^2(c_r(c_r + 2th_r + 2k) + th_r(2k + th_r)) \\ &+ 8\beta\varphi k^2(c_r(2k + c_r + 2th_r) + th_r(th_r + 2k) + k^2) - 8\varphi c_r k^2(c_r + c_r k + 2k) \\ &+ 16\varphi^2(th_r(2k + th_r + 2c_r) + c_r(2k + c_r)) - 8\varphi th_r k^2(2c_r + 2k + th_r) \\ &+ k^4(c_r(2k + 2th_r + c_r) + th_r(th_r + 2k) + (k^2 - 8\varphi)) + k^2(16\varphi^2 - \beta^2\alpha^2 k^2)) \end{aligned}$$

$$\begin{aligned} B_{12} &= (c_r + k - \beta\alpha + th_r)(k^2(c_r + k - \beta\alpha + th_r) + 4\varphi(\beta c_n - c_r - k - th_r + \beta th_n)) \\ &+ \beta\alpha(k^2(c_r + k - \beta\alpha + th_r) + 4\varphi(\beta c_n - c_r - k - th_r + \beta th_n)) \\ &+ \beta(\alpha k^2 - 4\varphi(c_n + th_n))(c_r + k - \beta\alpha + thr) \end{aligned}$$

$$\begin{aligned} B_{13} &= 2\varphi^2 k^2(c_r - c_n + k + \alpha - \beta\alpha - th_n + th_r)(c_r + k - \beta\alpha + th_r) \\ &+ 4\varphi(\beta(c_n - \alpha + th_n) - th_n - c_n + \alpha)) \end{aligned}$$

$$\begin{aligned} B_{14} &= \beta\varphi\alpha(k^2(c_r + k - \beta\alpha + th_r) + 4\varphi(\beta(c_n - \alpha + th_n) - c_n - th_n + \alpha)) \\ &- \varphi(c_r + k - \beta\alpha + th_r)(k^2(c_r + k - \beta\alpha + th_r) + 4\varphi(\beta c_n - c_r - k - th_r + \beta th_n)) \\ &+ \beta\varphi(\alpha k^2 + 4\varphi(\alpha - c_n - th_n))(c_r - c_n + k + \alpha - \beta\alpha - th_n + th_r) \end{aligned}$$

When $\varphi(\alpha k^2 - 4\varphi((c_n - c_r - k) - t(h_n - h_r))) < 0$, there is $\frac{\partial Q_n^{s*}}{\partial \beta} < 0$.
When $B_9 > 0$, there is $\frac{\partial Q_r^{s*}}{\partial \beta} > 0$.
When $B_{10} > 0$, there is $\frac{\partial K_s^*}{\partial \beta} > 0$; otherwise, there is $\frac{\partial K_s^*}{\partial \beta} < 0$.
When $B_{11} > 0$, there is $\frac{\partial \pi_m^{s**}}{\partial \beta} > 0$; otherwise, there is $\frac{\partial \pi_m^{s**}}{\partial \beta} < 0$.
When $M > 0$, there is $\frac{\partial \pi_r^{s**}}{\partial \beta} > 0$; otherwise, there is $\frac{\partial \pi_r^{s**}}{\partial \beta} < 0$. □

The conclusions of Proposition 4 are similar to Proposition 2, with the difference being that under decentralized decision making the manufacturer and the lessor are two independent individuals who prioritize their own benefit improvement. However, similar to Proposition 2, when the remanufactured product preference coefficient of consumers is low, the market demand is dominated by new products. Despite the higher selling and leasing prices of new products compared with remanufactured products, both the manufacturer and lessor experience declining profits and effectively reducing total product carbon emissions becomes challenging. As the remanufactured product preference coefficient of consumers increases, the demand for remanufactured products rises in the market. The manufacturer responds to the market demand by reducing new product production and allocating more resources to remanufacturing. This indicates that the enhancement of consumer low-carbon awareness in the market benefits enterprise profits and facilitates the implementation of low-carbon emission policies.

Proposition 5. *Under specific parameter conditions, the leasing price, demand, and profits for new and remanufactured products are superior under centralized decision making compared with decentralized decision making.*

Proof of Proposition 5. By calculating $p_n^{c*} - p_n^{s**}$, $p_r^{c*} - p_r^{s**}$, $Q_n^{c*} - Q_n^{s*}$, $Q_r^{c*} - Q_r^{s*}$, and $\pi_t^{c*} - \pi_t^{s*}$, we can obtain:

$$\begin{aligned} p_n^{c*} - p_n^{s**} &= \frac{-kc_r + 2\alpha\varphi - k^2(2\alpha + k) - 2\beta\varphi(c_n - \alpha - th_n) + 2\varphi(c_n + th_n) + k^2(\beta\alpha - th_r)}{2(2\varphi - 2\beta\varphi - k^2)} \\ &- \frac{-k^2(c_r + k) + 12\alpha\varphi(1-\beta) + 4\varphi(1-\beta)(c_n + th_n) + k^2\left(\beta\alpha - th_r\right) - 4\alpha k^2}{4(4\varphi - 4\beta\varphi - k^2)} \end{aligned}$$

$$p_r^{c*} - p_r^{s**} = \frac{c_r + k - \beta\alpha + th_r}{4}$$

$$Q_n^{c*} - Q_n^{s*} = \frac{\varphi(c_r - c_n + k + \alpha(1-\beta) - t(h_n - h_r))}{2\varphi - 2\beta\varphi - k^2} - \frac{\varphi(c_r - c_n + k + \alpha - th_n + th_r)}{4\varphi - 4\beta\varphi - k^2}$$

$$\begin{aligned} Q_r^{c*} - Q_r^{s*} &= \frac{(k^2 - 2\varphi)(c_r + k) + \beta(2\varphi c_n - \alpha k^2 + 2\varphi th_n) - th_r(2\varphi - k)}{2(2\varphi - 2\beta\varphi - k^2)} \\ &- \frac{k^2 c_r + k^3 - 4\varphi c_r - 4\varphi k + 4\beta\varphi c_n - 4\varphi th_r - \beta\alpha k^2 + k^2 th_r + 4\beta\varphi th_n}{4\beta(4\varphi - 4\beta\varphi - k^2)} \end{aligned}$$

$$\pi_t^{c*} - \pi_t^{s*} = \frac{A_2}{4(2\varphi - 2\beta\varphi - k^2)} - N$$

To facilitate subsequent narration, we make the following assumption:

$$\pi_t^{c*} - \pi_t^{s*} = U_1,\ p_n^{c*} - p_n^{s**} = U_2,\ p_r^{c*} - p_r^{s**} = U_3,\ Q_n^{c*} - Q_n^{s*} = U_4,\ Q_r^{c*} - Q_r^{s*} = U_5$$

Under centralized decision making, the manufacturer and the lessor strive to maximize the profit of the supply chain. As a result, the total profit of the supply chain, as well as the demand for both new products and remanufactured products, are greater than for decentralized decision making. In contrast, the leasing price of new products and remanufactured products is lower under centralized decision making than under decentralized decision making. Therefore, we can conclude that:

$$U_1 > 0,\ U_2 < 0,\ U_3 < 0,\ U_4 > 0, \text{ and } U_5 > 0$$

□

Hence, the presence of a double marginalization effect is observed in the leasing-oriented closed-loop supply chain for the construction machinery industry under the carbon tax policy. Therefore, it becomes necessary to design a contract that can optimize the supply chain.

4.4. Leasing Compensation–Cost Apportioning Combined Coordination Contract Model

In practice, the management of the supply chain tends to be closer to decentralized decision making. Proposition 5 highlights the presence of a double marginalization effect in the closed-loop supply chain of construction machinery. To enhance the overall efficiency of the supply chain, this section proposes the design of a leasing compensation–cost apportioning contract to facilitate coordination within the supply chain.

In the absence of a contract, the manufacturer establishes an industrial Internet platform for recycling old products and bears the associated costs. However, as the manufacturer acts rationally, they may hesitate to make unilateral investments and reduce their investment in the industrial Internet platform cost. This reduction in investment leads to a decrease in the scale of recycling and hinders efforts to reduce carbon emissions within the supply chain. To incentivize the manufacturer to actively engage in recycling, it is essential to design a leasing compensation–cost apportioning combined contract. Under this combined contract, both the manufacturer and the lessor share the responsibility of the industrial Internet platform cost, with a cost apportioning proportional coefficient of 1-λ for the manufacturer and λ for the lessor, respectively. However, as the lessor is the weaker party in the supply chain, assuming the industrial Internet platform cost could potentially impact their leasing operations and, consequently, the satisfaction of consumer demand. Therefore, the manufacturer needs to provide the lessor with a leasing compensation f to enhance their profits.

Under the combined contract, the profit functions of the manufacturer and the lessor can be expressed as follows:

$$\pi_m^v = (w_n - c_n - ev - th_n)Q_n + (w_r - c_r - th_r)Q_r + k(eQ_n - Q_r) - \frac{1}{2}(1-\lambda)\varphi e^2 - f \quad (18)$$

$$\pi_r^v = (p_n - w_n + ev)Q_n + (p_r - w_r)Q_r - \frac{1}{2}\lambda\varphi e^2 + f \quad (19)$$

Using the inverse solution method, we can solve for p_n and p_r in order to construct the Hessian matrix.

$$H_4 = \begin{bmatrix} \frac{\partial^2 \pi_r^v}{\partial p_n^2} & \frac{\partial^2 \pi_r^v}{\partial p_n \partial p_r} \\ \frac{\partial^2 \pi_r^v}{\partial p_r \partial p_n} & \frac{\partial^2 \pi_r^v}{\partial p_r^2} \end{bmatrix} = \begin{bmatrix} -\frac{2}{1-\beta} & \frac{2}{1-\beta} \\ \frac{2}{1-\beta} & -\frac{2}{\beta(1-\beta)} \end{bmatrix}$$

The Hessian matrix is found to be negative when $\Delta_1 = -2/(1-\beta) < 0$, $\Delta_2 = 4(1-\beta) > 0$. As a result, π_r^v is strictly concave with respect to p_n and p_r. By solving $\frac{\partial \pi_r^v}{\partial p_n} = 0$ and $\frac{\partial \pi_r^v}{\partial p_r} = 0$, we can obtain the following equations:

$$p_n^{v*} = \frac{\alpha + w_n - ev}{2} \quad (20)$$

$$p_r^{v*} = \frac{w_r + \beta\alpha}{2} \quad (21)$$

By substituting Equations (20) and (21) into Equation (18):

$$\pi_m^{v*} = \frac{C_1}{2\beta(1-\beta)} - f \quad (22)$$

where

$$\begin{aligned} C_1 \quad &= -\beta(c_n - w_n + ev + th_n)(\alpha - w_n + w_r - \beta\alpha + ev) + (1-\lambda)\beta\varphi e^2(\beta - 1) \\ &+k(w_r - \beta w_n + \beta\alpha e + \beta ev - \beta e w_n + \beta e w_r - \beta^2\alpha e + \beta e^2 v) \\ &+2(w_r - \beta w_n + \beta ev)(c_r - w_r + th_r) \end{aligned}$$

To determine the relationship between π_m^{v*} and w_n, w_r, and e, the Hessian matrix is constructed.

$$H_5 = \begin{bmatrix} \frac{\partial^2 \pi_m^{v*}}{\partial w_n^2} & \frac{\partial^2 \pi_m^{v*}}{\partial w_n \partial w_r} & \frac{\partial^2 \pi_m^{v*}}{\partial w_n \partial e} \\ \frac{\partial^2 \pi_m^{v*}}{\partial w_r \partial w_n} & \frac{\partial^2 \pi_m^{v*}}{\partial w_r^2} & \frac{\partial^2 \pi_m^{v*}}{\partial w_r \partial e} \\ \frac{\partial^2 \pi_m^{v*}}{\partial e \partial w_n} & \frac{\partial^2 \pi_m^{v*}}{\partial e \partial w_r} & \frac{\partial^2 \pi_m^{v*}}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{1-\beta} & \frac{1}{1-\beta} & -\frac{k-2v}{2(1-\beta)} \\ \frac{1}{1-\beta} & -\frac{1}{\beta(1-\beta)} & \frac{k-2v}{2(1-\beta)} \\ -\frac{k-2v}{2(1-\beta)} & \frac{k-2v}{2(1-\beta)} & \frac{v(k-v)-\varphi(1-\lambda)(1-\beta)}{1-\beta} \end{bmatrix}$$

The Hessian matrix is found to be negative when $\Delta_1 = -1/(1-\beta) < 0$, $\Delta_2 = 1-\beta > 0$, $\Delta_3 = (k^2 - 4(1-\lambda)(\varphi - \beta\varphi))/4\beta(1-\beta)^2 < 0$. As a result, π_m^{v*} is strictly concave with respect to w_n, w_r, and e. By solving $\frac{\partial \pi_m^{v*}}{\partial w_n} = 0$, $\frac{\partial \pi_m^{v*}}{\partial w_r} = 0$, and $\frac{\partial \pi_m^{v*}}{\partial e} = 0$, we can obtain the following equations:

$$w_n^{v*} = \frac{C_2}{2(4(1-\lambda)(\varphi - \beta\varphi) - k^2)} \tag{23}$$

$$w_r^{v*} = \frac{c_r + k + \beta\alpha + th_r}{2} \tag{24}$$

$$e_v^* = \frac{k(k - c_n + c_r + \alpha - \beta\alpha - th_n + th_r)}{4(1-\lambda)(\varphi - \beta\varphi) - k^2} \tag{25}$$

where

$$\begin{aligned} C_2 \;&= k^2(2v - c_r + \beta\alpha - k - th_r - 2\alpha) + 4\varphi(\alpha + c_n - \beta\alpha + th_n) \\ &-4\beta\varphi(c_n + th_n) + 2kv(c_r - c_n + \alpha - \beta\alpha - th_n + th_r) \\ &-4\varphi(1-\lambda)(c_n + \alpha + th_n)(1-\beta) \end{aligned}$$

By substituting Equations (23)–(25) into Equations (20) and (21):

$$p_n^{v**} = \frac{\left(k^2(c_r + 4\alpha - \beta\alpha + th_r + k) + 4(1-\lambda)\varphi(\beta(c_n + 3\alpha + th_n) - c_n - th_n - 3\alpha)\right)}{4(4(1-\lambda)(\varphi - \beta\varphi) - k^2)} \tag{26}$$

$$p_r^{v**} = \frac{c_r + k + 3\beta\alpha + th_r}{4} \tag{27}$$

By substituting Equations (23)–(27) into Equations (1)–(3), (6), (18), and (19), the optimal values of the new product demand, the remanufactured product demand, the total production carbon emission, the manufacturer's profit, the lessor's profit, and the total supply chain profit under the combined contract can be obtained:

$$Q_n^{v*} = \frac{\varphi(1-\lambda)(c_r - c_n + k + \alpha - th_n + th_r)}{4(1-\lambda)(\varphi - \beta\varphi) - k^2}$$

$$Q_r^{v*} = \frac{k^2(c_r + k - \beta\alpha + th_r) + 4\varphi(1-\lambda)(\beta c_n - c_r - k - th_r + \beta th_n)}{4\beta(4(1-\lambda)(\varphi - \beta\varphi) - k^2)}$$

$$K_v^* = \frac{C_3}{4\beta(4(1-\lambda)(\varphi - \beta\varphi) - k^2)},\; \pi_m^{v**} = \frac{C_4}{8\beta(4(1-\lambda)(\varphi - \beta\varphi) - k^2)} - f$$

$$\pi_r^{v**} = \frac{C_5(4(1-\lambda)(\varphi - \beta\varphi) - k^2) + 4C_6}{16(4(1-\lambda)(\varphi - \beta\varphi) - k^2)} + f$$

$$\pi_t^{v*} = \frac{C_4}{8\beta(4(1-\lambda)(\varphi - \beta\varphi) - k^2)} + \frac{C_5(4(1-\lambda)(\varphi - \beta\varphi) - k^2) + 4C_6}{16(4(1-\lambda)(\varphi - \beta\varphi) - k^2)^2} = N_2$$

where

$$\begin{aligned} C_3 \;&= (h_r k^2 - 4\lambda\varphi h_r)(k + th_r + c_r) - \beta\alpha(4\lambda\beta\varphi h_n^2 + h_r k^2) \\ &+4\lambda\beta\varphi(c_n h_r + h_n(c_r - c_n - th_n + k + \alpha + 2th_r)) \end{aligned}$$

$$\begin{aligned} C_4 \;&= -(+4\lambda\beta\varphi(c_n(2c_r - c_n - 2th_n + 2th_r + 2k + 2\alpha) + th_n(2c_r - th_n + 2th_r + 2k + 2\alpha)) \\ &+4\lambda\beta^2\alpha\varphi(\alpha + 2c_n - 2th_n) + \beta\alpha k^2(\beta\alpha - 2th_r - 2k - 2c_r) + k^2(t^2h_r^2 + 2th_r k + k^2)) \\ &+c_r k^2(2k + c_r + 2th_r) - 4\lambda\varphi(c_r(c_r + 2th_r + 2k) + t^2h_r^2 + 2th_r k + k^2 + \beta\alpha^2) \end{aligned}$$

$$C_5 = (\beta\alpha - c_r - k - th_r)(k^2(c_r + k - \beta\alpha + th_r) + 4\lambda\varphi(\beta c_n - c_r - k - th_r + \beta th_n))$$

$$
\begin{aligned}
C_6 &= \varphi(2k^2(\lambda-1)+\lambda k^2(c_r+k-\beta\alpha+th_r)+4\lambda(\beta(c_n-\alpha+th_n)\\
&+\alpha-c_n-th_n))(c_r-c_n+k+\alpha-\beta\alpha-th_n+th_r)
\end{aligned}
$$

To ensure the effectiveness of the combined contract, two conditions need to be met. Firstly, the overall supply chain profit must increase compared with the decentralized decision-making scenario. Secondly, the profits of both the manufacturer and the lessor should be higher than those under decentralized decision making. Therefore, we have the following conditions:

$$\pi_t^{v*} > \pi_t^{s*} \tag{28}$$

$$\pi_m^{v**} > \pi_m^{s**} \tag{29}$$

$$\pi_r^{v**} > \pi_r^{s**} \tag{30}$$

By substituting the values of π_t^{v*} and π_t^{s*} into Equation (28), the range of the cost apportioning proportional coefficient λ can be obtained. Similarly, using Equations (29) and (30), the range of the leasing compensation proportional coefficient f can be determined. However, due to the complexity of the profit functions under the combined contract, finding the precise interval for λ and f is challenging. Therefore, a numerical analysis is conducted to explore the model in more detail.

5. Numerical Analysis

This section focuses on solving the value ranges of λ and f under the combined contract. It also explores the effects of the carbon tax rate and the remanufactured product preference coefficient of consumers on various factors, including the leasing price, selling price, recovery rate, product demand, total production carbon emission, and supply chain profit. The selected parameter values for the analysis are as follows: c_n = 100, c_r = 50, h_n = 500, h_n = 250, α = 15,000, k = 10, v = 50, β = *0.8*, and t < 0.2, which align with the current carbon tax policy in China [47].

5.1. Determining the Value Range of λ and f

In this section, the carbon tax rate is assumed to be constant, with a value of t = 0.1, whereas the other parameters remain unchanged. By applying Equation (28), we can determine that the value range for λ is 0.01 < λ < 0.49. Furthermore, by utilizing Equations (29) and (30), we can obtain the range of the compensation proportional coefficient:

$$F_1 < f < F_2$$

where

$$
\begin{aligned}
F_1 &= -\frac{85867500\lambda-(8025\lambda-8025)(342000\lambda-339685)}{4(120\lambda-119)^2}\\
&+509644-\frac{1215375000\lambda-1081394375}{38400\lambda-38080}
\end{aligned}
$$

$$F_2 = \frac{780495000\lambda-775135775}{768\lambda-761.6}-1017773$$

Table 2 is utilized to evaluate whether the implementation of the combined contract leads to an improvement in supply chain profitability. It compares the difference ($\Delta\pi_t$) between the total supply chain profits achieved under the combined contract and those obtained under decentralized decision making.

Table 2. The f and the decision variables under different values for λ.

λ	f	p_n^{v**}	w_n^{v**}	e_v	$\Delta\pi_t$
0.05	(0.03, 15.3)	2286.3	1596.3	0.47	15.3
0.15	(45.3, 267.9)	2286.2	1598.8	0.53	222.6
0.25	(165.8, 506.7)	2286.0	1602.1	0.60	340.9
0.35	(438.5, 820.0)	2285.8	1606.3	0.69	381.5
0.45	(1022.6, 1248.9)	2285.4	1612.0	0.82	226.2

Table 2 and Figure 3 demonstrates that, for the range of $0.01 < \lambda < 0.49$, $\Delta\pi_t > 0$, indicating that the implementation of the combined contract leads to an increase in the total supply chain profit and represents a Pareto improvement. As the cost apportioning proportional coefficient (λ) increases, the leasing compensation proportional coefficient (f) also increases. This suggests that the lessor assumes a larger proportion of the industrial Internet platform cost, resulting in the need for a higher leasing compensation fee from the manufacturer. Furthermore, the manufacturer, with increased financial resources, can actively engage in recycling activities and enhance the recovery rate.

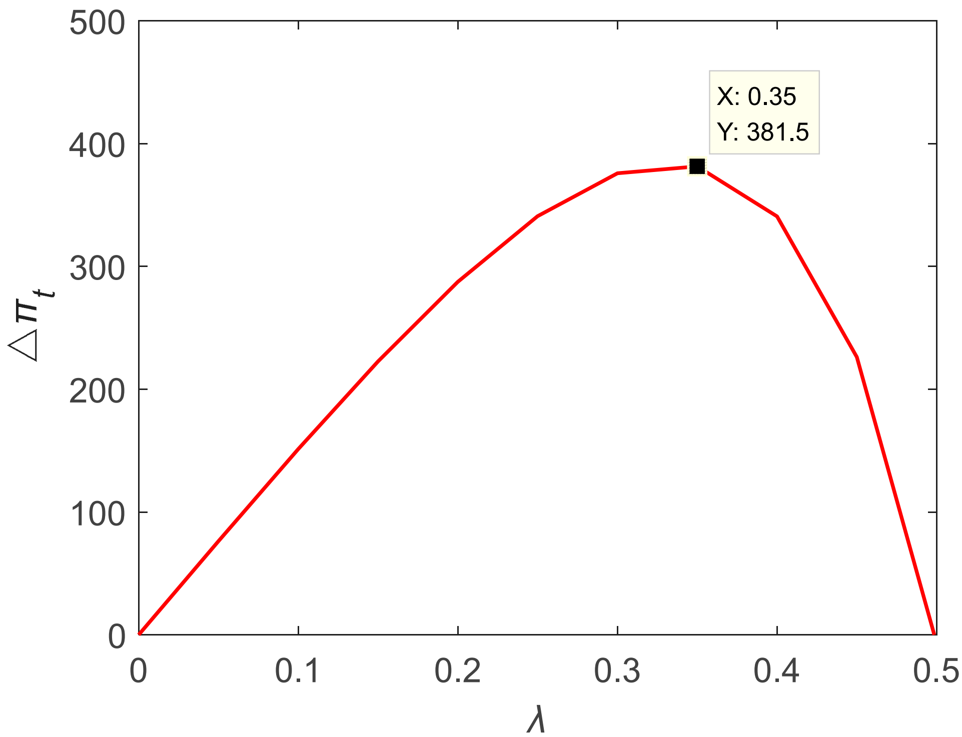

Figure 3. The impact of the cost apportioning proportional coefficient (f) on the profit increment ($\Delta\pi_t$).

It can be observed from Figure 3 that the combined contract demonstrates the most significant improvement in total closed-loop supply chain profits when λ is set to 0.35.

5.2. The Impact of the Carbon Tax Rate t

To analyze the impact of the carbon tax rate on the leasing price, selling price, demand, total production carbon emissions, and supply chain profits under different decision-making scenarios (centralized decision making, decentralized decision making, and combined contract), we conducted a study with a specified range for the carbon tax rate $0.02 < t < 0.2$ and kept other parameters constant ($\lambda = 0.35$ and $f = 600$). This analysis led to the generation of Figures 4 and 5, as well as Tables 3–5.

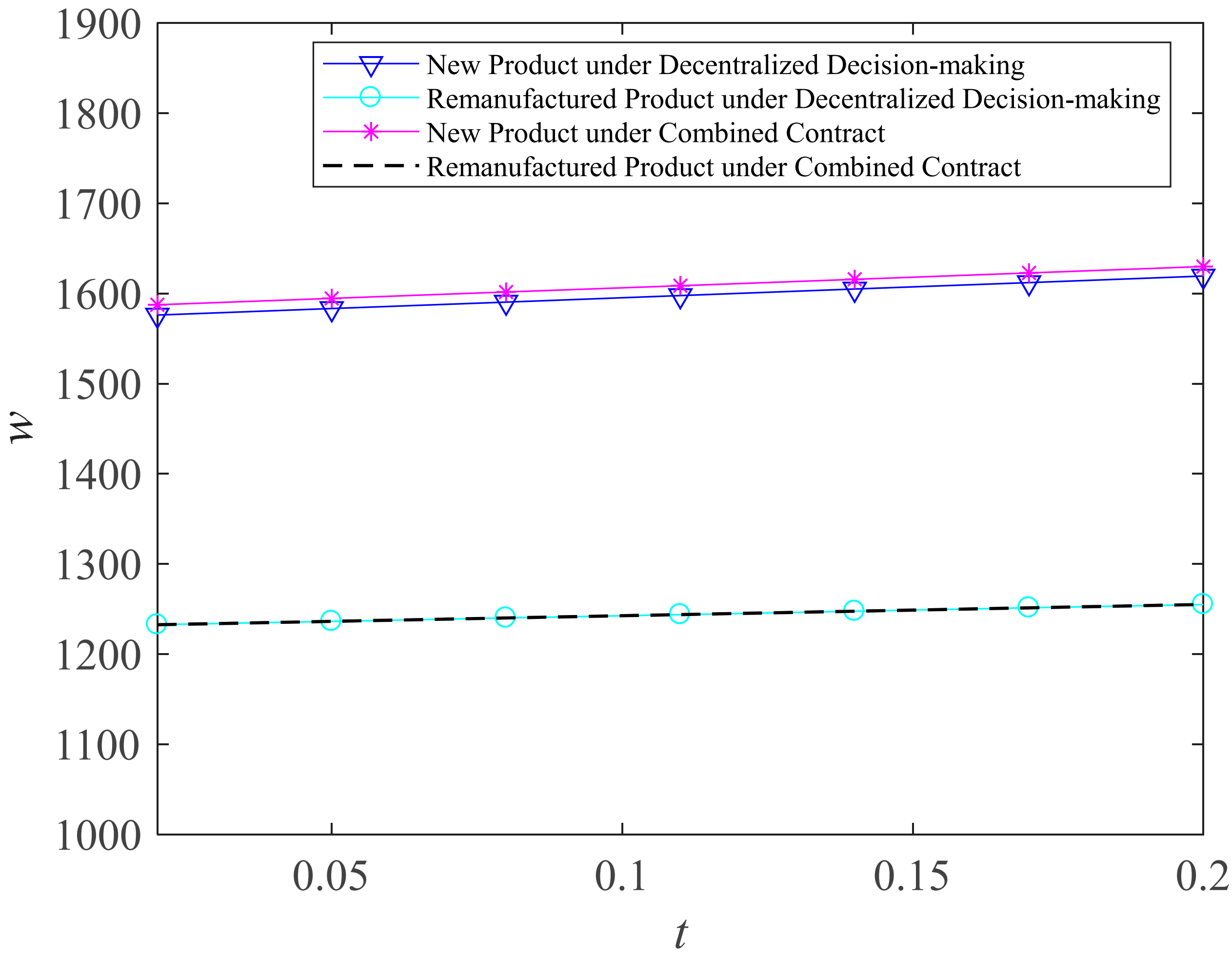

Figure 4. The impact of the carbon tax rate (t) on the selling price (w).

As seen in Table 3, as the carbon tax rate increases, we observe a gradual decrease in the total supply chain profits across all decision-making approaches. Additionally, the profits of both the manufacturer and the lessor decrease under decentralized decision making and the combined contract. It is important to note that the total supply chain profits are the highest under centralized decision making, where the goal is to maximize overall profitability. Furthermore, the profits of the manufacturer and the lessor are consistently higher under the combined contract than under decentralized decision making, which further validates the findings presented in Table 2.

Based on Figure 3 and Table 4, we observe that as the carbon tax rate increases, the leasing prices of the new product and the remanufactured product rise under all decision-making scenarios. Additionally, the selling prices of the new product and the remanufactured product increase under decentralized decision making and the combined contract. This price increase is a result of the higher production costs incurred by the manufacturer due to the carbon tax. To maintain profitability, the manufacturer needs to raise the selling prices, with the new product having a higher price compared with the remanufactured product.

The higher selling prices of both products lead to an increase in the leasing costs borne by the lessor. Consequently, the lessor needs to adjust the lease prices to maintain profitability. Under the combined contract, where the manufacturer and the lessor share the industrial Internet platform cost, the manufacturer has more financial resources to invest in improving construction machinery manufacturing technology. This investment contributes to a higher selling price for the new product.

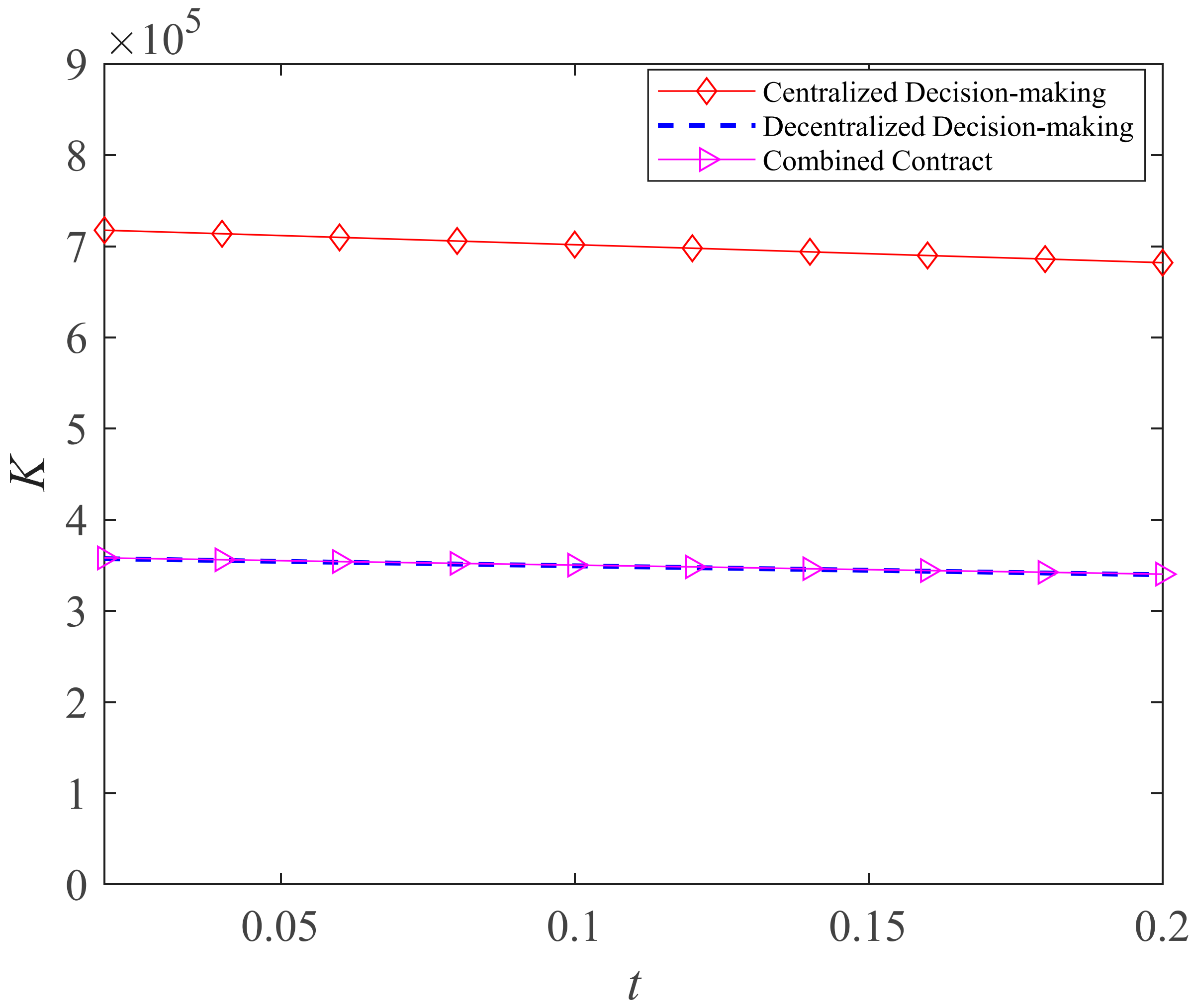

Figure 5. The impact of the carbon tax rate (t) on the total production carbon emission (K).

Table 3. The supply chain profits of parties under different carbon tax rates (t).

t	π_t^{c*}	π_t^{s*}	π_t^{v*}	π_m^{s**}	π_r^{s**}	π_m^{v**}	π_r^{v**}
0.02	2,095,400	1,569,900	1,570,300	1,046,000	523,840	1,046,300	523,970
0.05	2,073,900	1,553,800	1,554,200	1,035,400	518,480	1,035,600	518,620
0.08	2,052,700	1,537,900	1,538,300	1,024,800	513,160	1,025,000	513,320
0.11	2,031,600	1,522,200	1,522,600	1,014,300	507,890	1,014,500	508,060
0.14	2,010,700	1,506,500	1,506,900	1,003,900	202,670	1,004,100	502,840
0.17	1,990,000	1,491,000	1,491,400	993,600	497,490	993,700	497,670
0.20	1,969,400	1,475,700	1,476,000	983,300	492,350	983,500	492,550

Table 4. The leasing price (p) under different carbon tax rates (t).

t	p_n^{c*}	p_r^{c*}	p_n^{s**}	p_r^{s**}	p_n^{v**}	p_r^{v**}
0.02	1550.3	1232.5	2276.3	1816.2	2275.7	1816.2
0.05	1557.9	1236.3	2280.1	1818.1	2279.5	1818.1
0.08	1565.4	1240.0	2283.9	1820.0	2283.2	1820.0
0.11	1573.0	1243.7	2287.6	1821.9	2287.0	1821.9
0.14	1580.6	1247.5	2291.4	1823.7	2290.8	1823.7
0.17	1588.1	1251.2	2295.2	1825.6	2294.6	1825.6
0.20	1595.7	1255.0	2298.9	1827.5	2298.3	1827.5

Table 5. The demand (Q) under different carbon tax rates (t).

t	Q_n^{c*}	Q_r^{c*}	Q_n^{s*}	Q_r^{s*}	Q_n^{v*}	Q_r^{v*}
0.02	1410.0	48.4	699.6	30.1	702.8	26.9
0.05	1391.9	62.7	690.1	37.2	693.3	34.1
0.08	1372.9	77.1	680.7	44.3	683.8	41.2
0.11	1353.8	91.5	671.2	51.4	674.3	48.4
0.14	1334.7	105.9	661.8	58.5	664.8	55.5
0.17	1315.7	120.3	652.3	65.7	655.3	62.7
0.20	1296.6	134.6	642.8	72.8	645.8	69.8

Table 4 demonstrates that the leasing price of the new product under the combined contract is consistently lower than under decentralized decision making. This indicates that the combined contract effectively reduces the leasing price of the new product, allowing consumers to enjoy the same product services at a more affordable price.

Based on Figure 4 and Table 5, we observe that as the carbon tax rate (t) increases, the demand for the new product and the total production carbon emissions decrease under all decision-making scenarios. Conversely, the demand for the remanufactured product increases.

As stated in Assumption 4, the production carbon emissions of the new product are higher than those of the remanufactured product. With the increase in t, the manufacturer starts reducing the production of the new product and shifting focus towards remanufacturing. This strategic shift ultimately leads to a reduction in the carbon emissions of the overall supply chain.

Although the implementation of carbon tax policies may result in reduced profits for all parties involved in the supply chain, it has effectively improved the recovery rate of used products, decreased the total production carbon emissions, and enabled enterprises to fulfill their social responsibilities more effectively.

5.3. The Impact of the Remanufactured Product Preference Coefficient of Consumer β

To analyze the impact of the remanufactured product preference coefficient of consumer β on the product demand, the total production carbon emissions, and the supply chain profits of all parties under different decision-making scenarios (centralized decision making, decentralized decision making, and the combined contract), we conducted a study with $0.05 < \beta < 0.95$ and kept other parameters constant ($\lambda = 0.35$, $f = 600$ and $t = 0.1$). Figures 6 and 7, as well as Table 6, are obtained.

Table 6. The supply chain profits under different remanufactured product preference coefficients of consumers (β).

β	π_t^{c*}	π_t^{s*}	π_t^{v*}	π_m^{s**}	π_r^{s**}	π_m^{v**}	π_r^{v**}
0.05	2,069,400	1,550,300	1,550,700	1,032,900	517,350	1,033,900	517,430
0.2	2,042,500	1,530,100	1,530,500	1,019,500	516,020	1,020,400	510,700
0.35	2,038,800	1,527,400	1,527,800	1,017,600	509,710	1,018,600	509,800
0.5	2,037,600	1,526,500	1,526,900	1,017,000	509,410	1,018,000	509,510
0.65	2,037,500	1,526,400	1,526,800	1,017,100	509,360	1,018,000	906,480
0.8	2,038,600	1,527,400	1,527,800	1,017,800	509,640	1,018,600	509,810
0.95	2,050,600	1,527,300	1,537,400	1,024,600	512,640	1,025,000	513,060

From Table 6, it can be observed that as the remanufactured product preference coefficient of consumers increases, the total supply chain profits initially decrease and then increase for both centralized decision making and decentralized decision making, as well as under the combined contract. Similarly, the profits of the manufacturer and lessor also follow a decreasing and then increasing trend under decentralized decision making and the combined contract. Additionally, Figure 6 reveals that when the remanufactured product preference coefficient of consumers is low, there is a lack of consumer preference for the remanufactured product. Consequently, despite a gradual decline in the demand for new products, they still dominate the construction machinery market. However, as the remanufactured product preference coefficient of consumers increases, the demand for remanufactured products begins to rise. Despite the higher selling and leasing prices of new products compared with remanufactured products, the significant increase in demand for the remanufactured product results in higher profits for both the manufacturer and the lessor.

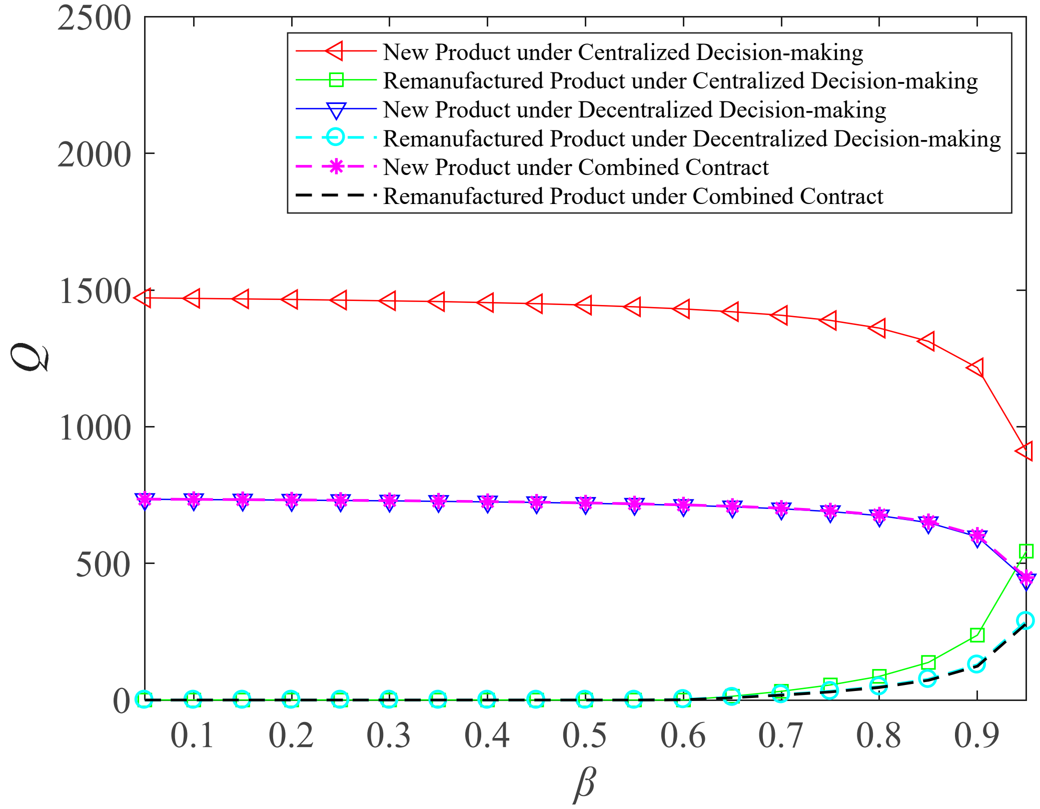

Figure 6. The impact of the remanufactured product preference coefficient of consumers (β) on the product demand (Q).

Based on Figure 7, as the remanufactured product preference coefficient of consumers increases, the total production carbon emissions initially increase and then decrease for centralized decision making, decentralized decision making, and the combined contract. Referring to the analysis in Figure 6, when the consumer preference coefficient for the remanufactured product is low, only the new product is present in the market. Since the new product has a higher production carbon emissions than the remanufactured product, the total production carbon emissions of the supply chain increase. However, as the remanufactured product preference coefficient of consumers reaches a certain value, the proportion of the remanufactured product in the construction machinery market becomes significant. As a result, the total production carbon emissions decrease substantially due to the lower production carbon emissions associated with the remanufactured product.

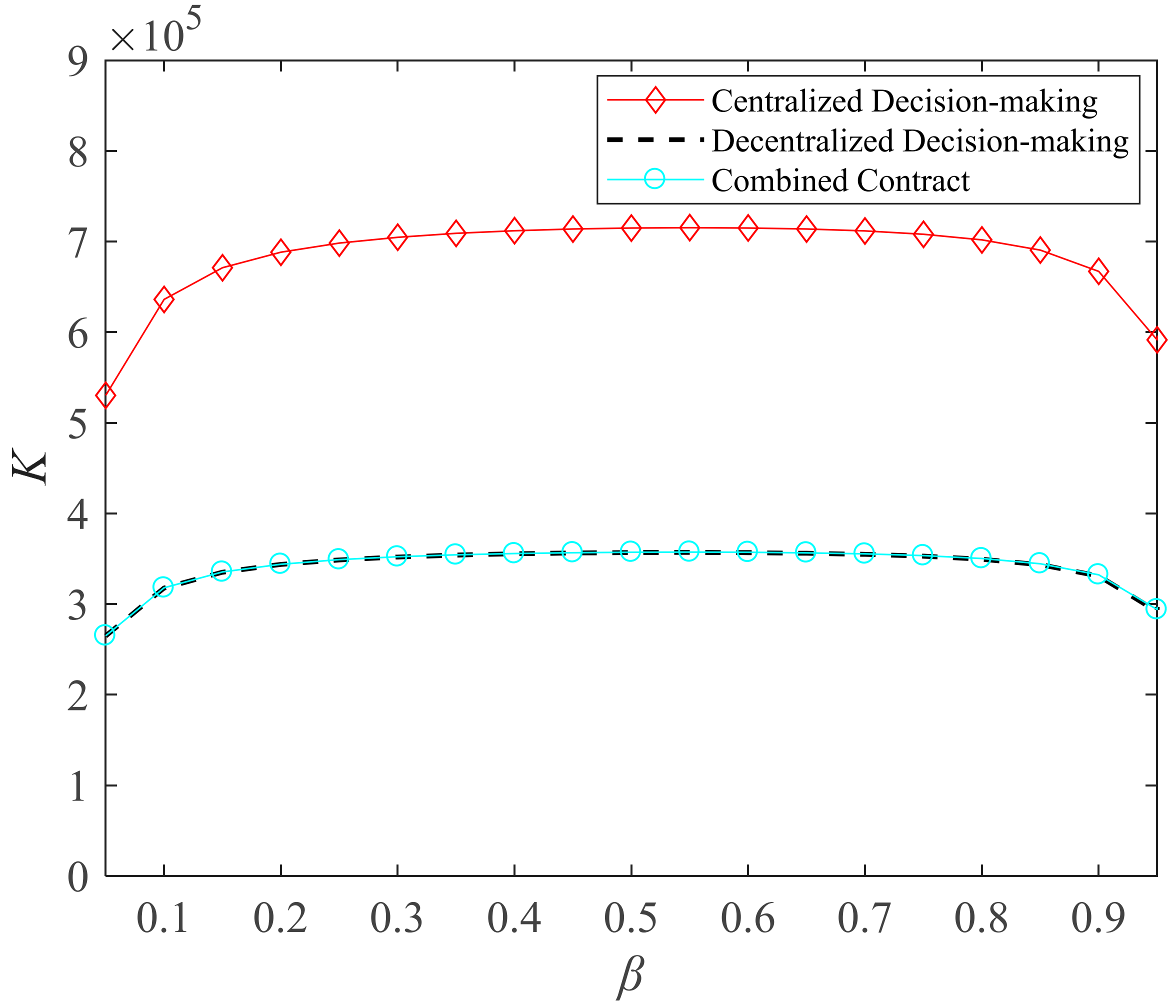

Figure 7. The impact of the remanufactured product preference coefficient of consumers (β) on the total production carbon emissions (K).

6. Conclusions, Discussion, Limitations, and Implications

This paper examines the impact of carbon tax policy on the lease-oriented closed-loop supply chain for the construction machinery industry. It incorporates carbon tax rates and consumer preferences for remanufactured products into decision-making approaches. The study also designs a leasing compensation–cost apportioning combined contract to improve supply chain performance and conducts sensitivity and numerical analyses to assess its effectiveness. The relevant discussion and insights are outlined below.

6.1. Discussion

(1) In the context of a leasing compensation–cost apportioning combined contract, as the cost apportioning proportional coefficient (λ) increases, the corresponding leasing compensation proportional coefficient (f) also experiences an increment. This increase in the allocation of platform construction costs to the lessor necessitates higher compensation fees to ensure the viability of operations. Nevertheless, the most substantial enhancement in total closed-loop supply chain profits is observed when λ is set at 0.35;

(2) As the carbon tax rate (t) increases, the total profits of the supply chain exhibit a decline across all decision-making scenarios, including both the centralized and decentralized decision-making approaches as well as the combined contract. Notably, when considering the combined contract, both manufacturers and lessors achieve comparatively higher profits compared with the decentralized decision-making strat-

egy. Furthermore, the increase in t is found to result in higher selling and leasing prices for both new and remanufactured products, spanning all scenarios. Specifically, under the combined contract, the leasing prices of new products remain consistently lower than those under decentralized decision making. Additionally, the increase in t leads to a decrease in the demand for new products across all scenarios, whereas the demand for remanufactured products and total carbon emissions also exhibit a downward trend;

(3) As consumer awareness and acceptance of remanufactured products grow, there is an increase in demand for these products. This results in a decline in profits for all parties in the supply chain initially, followed by an eventual increase. However, the final profit level is higher than the initial profit level. Additionally, the production of remanufactured products contributes to lower production carbon emissions, leading to an overall reduction in carbon emissions within the closed-loop supply chain.

6.2. Conclusions

(1) The leasing compensation–cost apportioning combined contract serves as an effective solution to mitigate the challenges posed by dual marginal effects in a decentralized decision-making environment. This approach significantly enhances the overall profitability of the supply chain. Moreover, it leads to reduced leasing prices, thereby enabling consumers to enjoy improved services and benefits;

(2) Although the carbon tax policy does exert a certain influence on enterprise profitability, it concurrently generates positive societal outcomes by effectively curbing carbon emissions within the lease-oriented closed-loop supply chain of construction machinery. This policy also incentivizes manufacturers to prioritize the production of remanufactured products, leading to higher utilization rates of machinery equipment;

(3) As the carbon tax rate increases, the initial decline in profits for various stakeholders in the supply chain, owing to the enhanced consumer acceptance of remanufactured products, is eventually offset by the subsequent increase in overall supply chain profits. This shift is accompanied by a reduction in carbon emissions within the closed-loop supply chain. Furthermore, the trend fosters a higher substitution rate for remanufactured products over new ones, reflecting the positive impact of consumer preference changes on the sustainability of the supply chain.

6.3. Limitations

However, it should be noted that this paper only focuses on the study of the carbon tax policy, whereas other carbon policies such as carbon trading and carbon quotas remain unexplored. Comparing different carbon policies would provide valuable insights into their economic and social benefits. Furthermore, this paper examines the impact of combined contract coordination on various variables, including the recovery rate, total carbon emissions, and supply chain performance improvement. However, it is important to acknowledge that the dynamics of carbon tax and the recovery rate over time were not considered in this analysis. Therefore, future research could incorporate the time variable to investigate the optimal decision-making problem under dynamic conditions.

6.4. Implications

(1) The notable effectiveness of the combined contract underscores its substantial managerial significance. This highlights the importance of collaborative efforts between manufacturers and lessors in boosting the overall efficiency of the supply chain. Moreover, this collaboration not only fosters environmental sustainability but also aligns with the industry's trajectory towards a low-carbon transformation and progress;

(2) The carbon tax policy, recognized as a potent instrument for emissions reduction, has showcased considerable effectiveness in curtailing carbon emissions. Nonetheless, it is crucial to recognize that setting carbon tax rates at excessively low levels might prompt manufacturers to curtail their recycling efforts. Conversely, excessively high

rates could potentially impede corporate profitability and dampen the incentive for pursuing energy conservation and emissions reduction. Consequently, in establishing carbon tax rates, governments must undertake comprehensive research and formulate a well-calibrated carbon tax policy. This policy would play a pivotal role in steering environmentally conscious development within enterprises;

(3) The enhancement of consumer environmental awareness and the execution of governmental low-carbon policies play equally pivotal roles. Governments ought to bolster consumer eco-consciousness via low-carbon campaigns, whereas manufacturing enterprises should remain committed to advancing remanufacturing technologies. This endeavor ensures the maintenance of product quality and fosters increased consumer acceptance of remanufactured goods.

Author Contributions: Conceptualization, J.Y. and S.C.; funding acquisition, J.Y.; formal analysis, Y.F. and H.Z.; methodology, S.C., Y.F. and H.Z.; project administration, J.Y., S.C. and Y.F.; writing—original draft, J.Y., S.C. and T.W.; writing—review and editing, J.Y., S.C. and Y.F. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Beijing Social Science Fund General Project (No. 19GLB011) and the Fundamental Research Funds for the Central Universities (2021CDJSKJC15).

Data Availability Statement: The data presented in this study are available from the first and corresponding author upon reasonable request.

Acknowledgments: Our deepest gratitude goes to the anonymous referees for detailed reviews and insightful comments that have helped to substantially improve this research.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Li, L.; Chen, K. Quantitative assessment of carbon dioxide emissions in construction projects: A case study in Shenzhen. *J. Clean. Prod.* **2016**, *141*, 394–408. [CrossRef]
2. Peng, C. Calculation of a building's life cycle carbon emissions based on Ecotect and building information modeling. *J. Clean. Prod.* **2016**, *112*, 453–465. [CrossRef]
3. Wang, W.; Wang, X.; Sun, J. The construction of carbon emissions control theory research. In Proceedings of the 2015 International Forum on Energy, Environment Science and Materials, Shenzhen, China, 25–26 September 2015; pp. 1348–1352.
4. Chen, Y.; Wu, Y.; Chen, N.; Kang, C.; Du, J.; Luo, C. Calculation of Energy Consumption and Carbon Emissions in the Construction Stage of Large Public Buildings and an Analysis of Influencing Factors Based on an Improved STIRPAT Model. *Buildings* **2022**, *12*, 2211. [CrossRef]
5. Son, H.; Kim, C. Integrated worker detection and tracking for the safe operation of construction machinery. *Autom. Constr.* **2021**, *126*, 103670. [CrossRef]
6. Fan, W.; Jiang, W.; Chen, J.; Yang, F.; Qian, J.; Ye, H. Exhaust emission inventory of typical construction machinery and its contribution to atmospheric pollutants in Chengdu, China. *J. Environ. Sci.* **2023**, *125*, 761–773. [CrossRef]
7. China Association of Circular Economy. The Current Situation and the Future Development of the Remanufacturing in the Construction Machinery Industry. Available online: https://www.chinacace.org/news/view?id=9653 (accessed on 15 August 2023).
8. Liu, Y.; Wang, L.; Xu, X.; Zhang, L.; Wang, X.V. Industrial internet for manufacturing. *Robot. Comput. Integr. Manuf.* **2021**, *70*, 1–2. [CrossRef]
9. Tian, G.F.; Zhang, Z.S.; Jin, F.Z.; Qiao, Y.Y. Remanufacturing Technology Analysis of Construction Machinery and Equipment. *Adv. Mater. Res.* **2011**, *421*, 43–46. [CrossRef]
10. Parsa, M.; Nookabadi, A.S.; Atan, Z.; Malekian, Y. An optimal inventory policy for a multi-echelon closed-loop supply chain of postconsumer recycled content products. *Oper. Res.* **2020**, *22*, 1887–1938. [CrossRef]
11. Fokouop, R.; Sahin, E.; Jemai, Z.; Dallery, Y. A heuristic approach for multi-echelon inventory optimisation in a closed-loop supply chain. *Int. J. Prod. Res.* **2023**, 1–25. [CrossRef]
12. Nayeri, S.; Paydar, M.M.; Asadi-Gangraj, E.; Emami, S. Multi-objective fuzzy robust optimization approach to sustainable closed-loop supply chain network design. *Comput. Ind. Eng.* **2020**, *148*, 106716. [CrossRef]
13. Salçuk, K.; Şahin, C. A novel multi-objective optimization model for sustainable supply chain network design problem in closed-loop supply chains. *Neural Comput. Appl.* **2022**, *34*, 22157–22175. [CrossRef]
14. Hajipour, V.; Tavana, M.; Di Caprio, D.; Akhgar, M.; Jabbari, Y. An optimization model for traceable closed-loop supply chain networks. *Appl. Math. Model.* **2019**, *71*, 673–699. [CrossRef]

15. Kazancoglu, Y.; Ekinci, E.; Mangla, S.K.; Sezer, M.D.; Kayikci, Y. Performance evaluation of reverse logistics in food supply chains in a circular economy using system dynamics. *Bus. Strat. Environ.* **2020**, *30*, 71–91. [CrossRef]
16. Miao, S.; Wang, T.; Chen, D. System dynamics research of remanufacturing closed-loop supply chain dominated by the third party. *Waste Manag. Res.* **2017**, *35*, 379–386. [CrossRef]
17. Katsoras, E.; Georgiadis, P. An integrated System Dynamics model for Closed Loop Supply Chains under disaster effects: The case of COVID-19. *Int. J. Prod. Econ.* **2022**, *253*, 108593. [CrossRef]
18. Jie, J.; Bin, L.; Nian, Z.; Su, J. Decision-making and coordination of green closed-loop supply chain with fairness concern. *J. Clean. Prod.* **2021**, *298*, 126779.
19. Zheng, B.; Yang, C.; Yang, J.; Zhang, M. Dual-channel closed loop supply chains: Forward channel competition, power structures and coordination. *Int. J. Prod. Res.* **2017**, *55*, 3510–3527. [CrossRef]
20. Stock, T.; Seliger, G. Opportunities of Sustainable Manufacturing in Industry 4.0. *Procedia CIRP* **2016**, *40*, 536–541. [CrossRef]
21. Wang, Y.; Yu, Z.; Shen, L.; Jin, M. Operational modes of E-closed loop supply chain considering platforms' services. *Int. J. Prod. Econ.* **2022**, *251*, 108551. [CrossRef]
22. Deng, Q.-W.; Liao, H.-L.; Xu, B.-W.; Liu, X.-H. The Resource Benefits Evaluation Model on Remanufacturing Processes of End-of-Life Construction Machinery under the Uncertainty in Recycling Price. *Sustainability* **2017**, *9*, 256. [CrossRef]
23. Yi, P.; Huang, M.; Guo, L.; Shi, T. Dual recycling channel decision in retailer oriented closed-loop supply chain for construction machinery remanufacturing. *J. Clean. Prod.* **2016**, *137*, 1393–1405. [CrossRef]
24. Huang, M.; Yi, P.; Shi, T. Triple Recycling Channel Strategies for Remanufacturing of Construction Machinery in a Retailer-Dominated Closed-Loop Supply Chain. *Sustainability* **2017**, *9*, 2167. [CrossRef]
25. Xu, F.-X.; Liu, X.-H.; Chen, W.; Zhou, C.; Cao, B.-W. An Ontology and AHP Based Quality Evaluation Approach for Reuse Parts of End-of-Life Construction Machinery. *Math. Probl. Eng.* **2018**, *2018*, 3481030. [CrossRef]
26. Cheng, L.; Guo, X.; Wang, B. Manufacturer's Leasing Channel Management in a Car Supply Chain. *Asia-Pacific J. Oper. Res.* **2020**, *37*, 2050026. [CrossRef]
27. Long, Y.; Xu, D.; Li, X. Channel coordination of battery supplier and battery swap station of micro-grid with uncertain leasing demand. *Soft Comput.* **2019**, *23*, 9733–9745. [CrossRef]
28. Bulow, J.I. Durable-goods monopolists. *J. Political Econ.* **1982**, *90*, 314–332. [CrossRef]
29. Zhang, Y.; Tan, D.; Liu, Z. Leasing or Selling? Durable Goods Manufacturer Marketing Model Selection under a Mixed Carbon Trading-and-Tax Policy Scenario. *Int. J. Environ. Res. Public Health* **2019**, *16*, 251. [CrossRef] [PubMed]
30. Kalantari, S.S.; Esmaeili, M.; Taleizadeh, A.A. Selling by clicks or leasing by bricks? A dynamic game for pricing durable products in a dual-channel supply chain. *J. Ind. Manag. Optim.* **2023**, *19*, 1107. [CrossRef]
31. Liu, B.; Holmbom, M.; Segerstedt, A.; Chen, W. Effects of carbon emission regulations on remanufacturing decisions with limited information of demand distribution. *Int. J. Prod. Res.* **2014**, *53*, 532–548. [CrossRef]
32. Yang, H.; Chen, W. Retailer-driven carbon emission abatement with consumer environmental awareness and carbon tax: Revenue-sharing versus Cost-sharing. *Omega* **2018**, *78*, 179–191. [CrossRef]
33. Shakya, S.R.; Kumar, S.; Shrestha, R.M. Co-benefits of a carbon tax in Nepal. *Mitig. Adapt. Strateg. Glob. Chang.* **2012**, *17*, 77–101. [CrossRef]
34. Liu, Z.; Hu, B.; Zhao, Y.; Lang, L.; Guo, H.; Florence, K.; Zhang, S. Research on Intelligent Decision of Low Carbon Supply Chain Based on Carbon Tax Constraints in Human-Driven Edge Computing. *IEEE Access* **2020**, *8*, 48264–48273. [CrossRef]
35. Xu, J.; Wei, W. Would carbon tax be an effective policy tool to reduce carbon emission in China? Policies simulation analysis based on a CGE model. *Appl. Econ.* **2021**, *54*, 115–134. [CrossRef]
36. Han, C.-Q.; Gu, H.-Y.; Sui, L.-H.; Shao, C.-P. Carbon Emission Reduction Decision and Revenue-Sharing Contract with Consumers' Low-Carbon Preference and CER Cost under Carbon Tax. *Math. Probl. Eng.* **2021**, *2021*, 3458607. [CrossRef]
37. Zhu, X.; Ding, L.; Guo, Y.; Zhu, H. Decisions and Coordination of Dual-Channel Supply Chain considering Retailers' Bidirectional Fairness Concerns under Carbon Tax Policy. *Math. Probl. Eng.* **2022**, *2022*, 4139224. [CrossRef]
38. Deng, K.; Gao, B. Selection of optimal coordination contract for sustainable supply chain under carbon tax. *J. Physics: Conf. Ser.* **2019**, *1303*, 012135. [CrossRef]
39. Zou, F.; Zhou, Y.; Yuan, C. The Impact of Retailers' Low-Carbon Investment on the Supply Chain under Carbon Tax and Carbon Trading Policies. *Sustainability* **2020**, *12*, 3597. [CrossRef]
40. Yu, W.; Han, R. Coordinating a Two-Echelon Supply Chain under Carbon Tax. *Sustainability* **2017**, *9*, 2360. [CrossRef]
41. Lyu, S.; Chen, Y.; Wang, L. Optimal Decisions in a Multi-Party Closed-Loop Supply Chain Considering Green Marketing and Carbon Tax Policy. *Int. J. Environ. Res. Public Health* **2022**, *19*, 9244. [CrossRef]
42. Luo, R.; Zhou, L.; Song, Y.; Fan, T. Evaluating the impact of carbon tax policy on manufacturing and remanufacturing decisions in a closed-loop supply chain. *Int. J. Prod. Econ.* **2022**, *245*, 108408. [CrossRef]
43. Shu, T.; Huang, C.; Chen, S.; Wang, S.; Lai, K.K. Trade-Old-for-Remanufactured Closed-Loop Supply Chains with Carbon Tax and Government Subsidies. *Sustainability* **2018**, *10*, 3935. [CrossRef]
44. Zhang, Z.; Zhang, Q.; Liu, Z.; Zheng, X. Static and Dynamic Pricing Strategies in a Closed-Loop Supply Chain with Reference Quality Effects. *Sustainability* **2018**, *10*, 157. [CrossRef]
45. Wang, Y.; Yu, Z.; Shen, L. A Study of "Letting Go" Fair Concern Recycling Decision Making in E-commerce Closed-Loop Supply Chains. *China Manag. Sci.* **2021**, *28*, 87–97.

46. Wang, Q.; Zhao, D.; He, L. Contracting emission reduction for supply chains considering market low-carbon preference. *J. Clean. Prod.* **2016**, *120*, 72–84. [CrossRef]
47. Sun, Y.; Mao, X.; Yin, X.; Liu, G.; Zhang, J.; Zhao, Y. Optimizing carbon tax rates and revenue recycling schemes: Model development, and a case study for the Bohai Bay area, China. *J. Clean. Prod.* **2021**, *296*, 126519. [CrossRef]