

Review

Control and Design of the Steel Continuous Casting Process Based on Advanced Numerical Models

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Abstract: The process of continuous casting of steel is a complex technological task, including issues related to heat transfer, the steel solidification process, liquid metal flow and phase transitions in the solid state. This involves considerable difficulty in creating the optimal process control system, which would include the influence of all the physico-chemical phenomena which may occur. In parallel, there is an intensive development of new mathematical models and an increase in computer performance, therefore complex numerical simulations requiring substantial computing time can be conducted. This paper presents a review of currently applied numerical methods allowing the phenomena accompanying the process of continuous casting of steel to be accurately represented. Special attention was paid to the selection of appropriate methods to solve the technological problem selected. The possibilities of applying selected numerical models were analysed in order to modify and improve the existing process or to design a new one linked to the implementation of new steel grades in the current production. The description of the method of defining the boundary conditions, initial conditions and material parameters as vital components ensuring that numerical calculations based upon them in the finite element method, which is that most frequently applied, are correct is an important element of the paper. The possibility of reliably defining the values of boundary parameters on the basis of information on the intensity of cooling in individual zones of the continuous casting machine was analysed.

Keywords: continuous casting process; numerical modelling; finite element method; process control

1. Introduction

Modelling the process of steel continuous casting is a very complex task, and can be accomplished using various types of mathematical models. The assumed computing objective and the required accuracy should be key in selecting the model. In many cases, the desired information is knowledge of the metallurgical length of the strand and the dynamics of changes in the shell thickness. This is the case when determining a place for carrying out the so-called soft reduction operation. As may happen when the strand casting speed needs to be changed, a procedure allowing a new cooling intensity to be determined is required. A problem like this does not require answering a series of questions related to stress occurring in the strand, the structure formed, or potential cast strand defects. Thus, it is understandable that the model is naturally simplified to a form, which still provides a credible answer to the questions that are primarily asked [1–4].

At the early stage of problem solving, the correct selection of the model type and related possibilities for its adaptation to the class of the solved problem is a difficult challenge. Theoretically, a more complex model (which is to say more “intelligent”) can easily answer questions about the primary technological parameters of the casting process. Yet in practice we encounter a number of limitations. Assuming hypothetically that a complex model has been verified as correct, in the best

case we face an unnecessary extension of the computing time. It results from the fact that the model calculates much more parameters than is needed to solve the defined problem.

The second danger caused by non-synchronising the complexity of the set problem with the “intelligence” of the used tool is the issue of the verification of model parameters and their correlation with the process data. The more theoretically elaborate the model, the more parameters and the higher risk of occurrence of unmeasurable parameters. The last comment concerns the problem of the strategy of acquiring knowledge of the value of the required model parameters. Many years of experience in modelling the continuous steel casting process show that the best choice is an experimental measurement of all measurable model parameters. It can be illustrated by parameters in the form of the specific heat of the steel cast as a function of temperature, heat conductivity for the steel, viscosity, etc. This paper will present considerations on the method of choosing priorities in planning the basic measurements of the properties of the material cast.

2. The Process of Steel Continuous Casting as an Object of Numerical Modelling

The process of steel continuous casting—due to its complexity—is accompanied by many physical phenomena. The steel solidification process within the mould and after leaving the mould—in the secondary cooling zone—should fall within the most important of those effects. In the primary cooling zone, the following partial processes should be distinguished [2–7]:

- Turbulent flow of liquid steel through a complex geometry area—a submerged entry nozzle (SEN) or a shroud—caused by convection
- Heat transfer within the liquid steel area
- Heat transfer in the mould between the forming shell and the mould wall
- Heat flow through the layer of solid and liquid slag
- Formation of thermal stress
- Shrinkage of the solidifying shell, related to transitions occurring during the steel solidification process
- Thermal effect accompanying the solidification phenomenon
- Mechanical impact of the mould walls on the solidifying strand
- The process of an air gap formation between the mould wall and the solidifying strand
- The formation of crystals within the solidification zone accompanied by element segregation effects
- Formation of surface defects

In the secondary cooling zone, the following processes should be distinguished:

- Heat transfer within the liquid core area (conduction and convection)
- Heat conduction in the solidified shell layer
- Thermal effect accompanying the solidification phenomenon
- Multi-stage heat transfer resulting from the strand cooling by the nozzle system, related to the number of spray zones and the applied cooling type
- Shrinkage of the solidifying strand, related to transitions occurring during the steel solidification process
- Formation of individual solidification zones (zone of dendritic crystals and zone of equiaxed crystals)
- Formation of stress related to the contact of rolls with the strand, and the possibility of bulging between the continuous casting machine rolls

At present it is not possible to simultaneously capture all of the above effects occurring during the whole process of steel continuous casting, and to present them in the form of a single comprehensive numerical model. The natural division applied in the continuous casting process modelling is related

to an attempt at identifying the ensuing problem during actual steel casting, or focusing on a selected section of the process in order to improve the existing technology. The use of results of numerical modelling of the continuous casting process in the industrial practice is most often related to the identification of defects forming during the steel casting process. A good example of the effective application of numerical models is the elimination of a problem concerning strand corner cracks immediately after leaving the mould for the strand size 240×240 mm, at the steelmaking shop of Ascometal, France [8]. Numerical simulations with the commercial software package THERCAST® have established the cause of the cracks. The analysis of the calculated stress distribution, along with the applied Yamanaka fracture criterion, has resulted in developing a specific recommendation to add an additional pair of rolls or to replace the mould with a higher one.

The starting point of the numerical modelling of the continuous casting process—including effects occurring from the time of filling the tundish until leaving the secondary cooling chamber by the solidified strand—is the calculation of the correct temperature distribution. It is the basis for all numerical calculations and the starting point for conducting further considerations concerning the creation of patterns of liquid metal flow in the mould, assessment of the design and submersion level of the SEN, computing thermal stress occurring within the mould, predicting cast strand defects, simulations of the cast strand microstructure, assessment of the influence of design changes in the main structural components of the continuous casting machine, or changes in the technological parameters [1–11].

The vast majority of studies on numerical modelling of the continuous casting process is based upon the Finite Element Method (FEM) and Finite Volume Method (FVM). These methods are based on replacing a continuous description of the temperature fields analysed, stress state, etc., with an approximate discrete description. To obtain results from a numerical simulation, systems of algebraic equations are used. Their solutions serve for determining a discrete (discontinuous) approximation of the continuous solution of differential equations describing effects occurring in metals and alloys during solidification (heat flow, fluid movement, phase transitions, processes of structure formation, and others) [1,2,5,9–11]. The commercial packages ANSYS Fluent® and PHOENICS® [12–14] are most often used for numerical modelling of effects related to the liquid steel flow in the tundish. They are based upon numerical methods and algorithms applied in Computational Fluid Dynamics (CFD) [15].

Research conducted by many authors [12–14,16–21] shows a very good compliance between simulations of the flow of liquid steel through the tundish, using turbulent flow models $k-\epsilon$, and tests performed on water models and tests in industrial conditions. Most numerical calculations are conducted on the basis of the Reynolds-averaged Navier-Stokes equations (RANS) method, which allows the average rate of turbulent flow in the tundish to be determined. However, the application of the Large Eddy Simulation (LES) method, along with the Smagorinsky-Lilly turbulent model, allows us to determine the instantaneous velocity field of the steel inside the tundish [12,16,18,22]. Not only are the selection of the method of computing the rate of liquid steel flow through the tundish and the selection of the main model parameters key to the values of liquid steel velocity vectors in individual nodes of the finite element mesh, but they are also crucial to the temperature distribution within the whole tundish volume.

Computing made within the primary cooling zone is related mostly to the analysis of geometry, position and submersion depth of the SEN. The influence of the liquid steel turbulent flow from the tundish on the temperature distribution within the mould, and thus on the formation of the shell, is investigated. One should stress that obtaining a proper shell thickness after leaving the mould is key to the safety of the steel casting process. Most failures encountered during the actual continuous casting process are related to breakouts within the primary cooling zone. Ensuring a sufficient shell thickness is also related to the appropriate selection of the value of the mould wall taper, which is to compensate for the effect of steel shrinkage and to limit the occurrence of a gaseous gap. Many authors attempt to model effects occurring within the primary cooling zone using both commercial packages such as ANSYS Fluent®, ProCAST® and THERCAST® [3–6,12–14,16–20], and their own original codes

based on the FEM [1,2,11,23–26]. Numerical modelling of the primary cooling zone also includes an issue related to computing the thermal stress, which combined with a correct fracture criterion may be used for determining the correct mould wall taper in correlation with the casting speed.

At present, more and more numerical simulations of the continuous casting process are focused on taking into account the influence of additional casting support equipment e.g., electromagnetic stirrers (EMS) or the soft reduction system. Most studies—concerning a description of the Lorentz force impact on the liquid steel movement—are related to a numerical simulation of the influence of an electromagnetic stirrer or brake on the flow of the liquid steel in the mould [27–30]. The solution to such a problem is possible by computing the temperature distribution, along with the flow of liquid steel, and additionally taking into account the convection in the liquid steel generated by the impact of the electromagnetic force. One should emphasise that numerical models allowing the Maxwell equations to be solved are extremely complex, and often they require lengthy computing time. In this connection, computing the full temperature distribution including the effect of electromagnetic stirrers within the secondary cooling zone e.g., for a system of two S-EMS (strand electromagnetic stirrers) and F-EMS (final electromagnetic stirrers) on a 20 m long strand, which would allow the actual technology to be fully reproduced, remains problematic.

A strand cast obtained in the process of steel continuous casting is characterised by a diverse crystalline structure consisting of the chill zone, columnar zone and equiaxed zone. Contemporary numerical solutions use the Cellular Automata (CA) method to model processes of formation of the cast strand structure, taking nucleation and grain growth into account [31]. An attempt to model processes of formation of the cast strand structure, including nucleation and grain growth, was made in the studies [32,33]. A micro-macro type numerical model was used in these studies. In the micro-macro modelling the issue of heat flow is investigated on the scale of the whole strand (macro). The mathematical model of nucleation and grain growth on the micro scale allows us to describe the process of structure formation and to determine the crystallisation heat emission rate. Modelling the internal structure of dendritic grains in a two-dimensional space with the cellular automata method is presented in [34]. The paper [34,35] presents the capabilities of a model like this to describe the mechanism of forming a narrow area with an increased sulphur content in the strand immediately under the chill zone. However, as a cellular automaton with a mesh of 1 μm must be applied, it is not suitable to model crystallisation in large spaces. The Cellular Automata Finite Element (CAFE) model, which is now available as a ProCAST[®] software module, was used in the papers [32–35] to describe the process of nucleation and grain growth of the primary phase in the cast strand. In this modelling, mathematical methods of cellular automata are used. The cellular automata method enables both the stochastic grain nucleation on the strand surface and within its volume to be simulated, and the growth of individual primary phase grains to be analysed. The modelling results in the history of changes of the interface position, and the determination of the grain boundaries after completion of the strand solidification. The CA method computations are coupled with the simulation of the strand temperature field.

3. Development of the Numerical Model for the Continuous Casting of Steel Process

Describing the heat transfer model in the continuous steel casting process is an extremely complex task, as it involves all three mechanisms of heat transfer: conduction, radiation and convection [1–7,36–38]. The following processes influence heat transfer in the continuous steel casting process: conduction and convection in the liquid steel, conduction in the solidified shell, heat transport between the outer layer of the solidified shell and the mould wall surface through the air gap forming within the mould, heat conduction within the mould, heat transfer within the mould between the channel walls and the cooling water, heat transfer within the secondary cooling zone by convection and radiation, heat transfer between the solidifying strand and the rolls by conduction. Additionally, thermal effects related to the accompanying phase transitions have a significant impact on the heat transfer model.

Discretisation of the domain modelled is the first step in creating an FEM model. It is breaking down a given domain into finite elements, which are elements of a FEM mesh. The elements share nodes and additionally they contain information on their mutual neighbourhood. Discretisation is the key issue, as the correctness, accuracy and numerical computing time depends on the quality of the finite element mesh. Increasing mesh density extends the computing time and leads to an increase in demand for the size of the required operating memory used during computing, the number of carriers required for archiving the results, and the result access time. However, by increasing the mesh density a higher accuracy of the obtained simulation results is achieved.

The basic element of an irregular computing mesh, which should be applied when modelling the majority of typical casting processes, is a linear four-node element—tetrahedron. A tetrahedron mesh is generated automatically on the basis of the surface mesh, with a triangle as its main element. The possibility and often the necessity of optimisation of the created finite element mesh is an important aspect. Most often it involves refinement of the mesh in the area of the boundary condition occurrence. For continuous casting of steel, those will primarily be the areas at the mould walls and on the strand surface within the secondary cooling zone. Generally, as a rule, the mesh should be refined (using a shorter side length) in the areas where during the problem solving the highest temperature gradients are expected.

3.1. Strategy of the Boundary Conditions Calculation

The temperature field can be determined by solving the generalised Fourier equation, which also describes the heat transfer. In the general form this equation is written as follows [3–6]:

$$\frac{\partial(\rho c_p T)}{\partial t} = \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial z} \right) + Q \quad (1)$$

where ρ is density, $\text{kg} \cdot \text{m}^{-3}$; c_p is specific heat, $\text{kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$; λ is thermal conductivity, $\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$; t is time, s; T is temperature, K; Q is the heat source term, $\text{W} \cdot \text{m}^{-3}$; x, y, z are 3D coordinate axes. The solution to the thermal problem is the T vector, representing the temperature values in the individual nodes of the finite element mesh. The solution to the generalised diffusion equation—the Fourier equation—should meet the boundary conditions declared on the cast strand surface. The boundary conditions may be described with a general equation in the form [3–6]:

$$-\lambda \frac{\partial T_{wl}}{\partial n} = \alpha(T_{wl} - T_a) \quad (2)$$

where T_a —temperature of the surrounding medium, K; T_{wl} —temperature of the cast strand surface, K; α —heat transfer coefficient (HTC), $\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$.

Obtaining a unique solution to the heat transfer equation involves declaring the boundary conditions of the heat transfer. In the source literature the boundary conditions are divided into three basic groups:

- The first-type boundary condition, when the surface temperature at the edge of the domain is specified.
- The second-type boundary condition, when the heat flux density transferred to the environment at the edge of the domain is specified.
- The third-type boundary condition, when the method of heat transfer at the edge of the domain expressed numerically by the heat transfer coefficient, and the ambient temperature are determined.

The biggest difficulty as regards computing reliable boundary conditions is related to the selection of a model describing the heat transfer in the primary and secondary cooling zones, and to the collection of a database of actual process parameters. The contact of the face of the solidifying cast strand with the inner side of the mould and the heat transfer accompanying this process can be implemented by

determining the value of the heat transfer coefficient as a function of the strand surface temperature. The faces of the 3D model with the applied finite element mesh, for which the heat transfer coefficient should be computed, can be divided into four groups:

1. The mould outer side
2. The contact of the solidifying strand face with the inner side of the mould
3. The surface of the liquid steel meniscus
4. The secondary cooling zone (divided into a set number of spray zones)

The authors of the papers [3–6] provide average values of the heat transfer coefficient from 18,000 to 24,000 $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$, which corresponds to the value of heat received by the water flowing within the mould channels—the outer side of the mould. The above value was calculated on the basis of process data related to the cooling water temperature at the inlet and outlet of the mould, and the rate of cooling water flow for each of the mould walls (slab casting machine, size 220×1100 mm). The share of the water-cooled mould surface is an essential parameter taken into account in computing.

The boundary condition related to the contact of the solidifying strand face with the mould walls is amongst the most difficult to determine. Many models have been presented in the literature. Basically, they can be classified into the following groups: heat transfer models based upon the average value of heat transfer coefficient, and models presenting a functional relationship between the strand surface temperature and the heat transfer coefficient. In addition, heat transfer models may assume that an air gap forms during casting. The air gap preventing contact of the solidifying strand with the mould wall disturbs the heat transfer process. In the paper [6], the influence of a change in the coefficient of the heat transfer between the strand and the mould was examined in detail by calculating the temperature distribution in the primary and secondary cooling zones. The authors compared 11 literature models and developed a new one, empirical model (No. 12) of heat transfer between the solidifying strand and the mould wall, assuming the occurrence of an air gap. The authors analysed models based upon the selected average values of heat transfer coefficient—group 1, next models taking into account a step change in the value of the heat transfer coefficient—group 2, and two cases, where the heat transfer coefficient changed linearly—group 3. The biggest group—group 4—comprised complex models allowing the temperature distribution to be calculated for the heat transfer coefficient as a function of the strand surface temperature. Data concerning the average and maximum values of the heat transfer coefficient is presented in Table 1.

Table 1. The heat transfer coefficient in the primary cooling zone—a comparison of the models (adopted from [6], with permission from Polish Academy of Science, 2018).

Number of the <i>HTC</i> Model	Type of the <i>HTC</i> Model	Average or Maximum Value of the <i>HTC</i> [$\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$]
1	The average value of the <i>HTC</i>	$HTC = 1200$
2	The average value of the <i>HTC</i>	$HTC = 1300$
3	The average value of the <i>HTC</i>	$HTC = 1500$
4	Two values of the <i>HTC</i>	$HTC = 1163$ for $z \leq 0.6$ m $HTC = 1395.6$ for $z > 0.6$ m
5	A linear values of the <i>HTC</i>	$HTC = 800\text{--}2000$
6	A linear values of the <i>HTC</i>	$HTC = 600\text{--}1500$
7	The values of the <i>HTC</i> as a function	$HTC_{max} = 1300$
8	The values of the <i>HTC</i> as a function	$HTC_{max} = 2500$
9	The values of the <i>HTC</i> as a function	$HTC_{max} = 2000$
10	The values of the <i>HTC</i> as a function	$HTC_{max} = 1300$
11	The values of the <i>HTC</i> as a function	$HTC_{max} = 3097$
12	The values of the <i>HTC</i> as a function	$HTC_{max} = 1600$

z —height of the mould (m).

The tests showed that the maximum difference for the temperature measured at the surface of the strand leaving the mould between the models (models No. 3 and No. 10) investigated was as high as 327 °C. The foregoing value shows a discrepancy that may occur when the various type of the heat transfer in the primary cooling zone are applied. In order to calculate the maximum difference for the temperature one measured point, at the surface of the strand was selected, for all implemented heat transfer models. The authors of the above-mentioned paper also compared the thickness of the shell leaving the mould and obtained about 1 cm difference between the extreme variants (models No. 3 and No. 10). Table 2 presents the results of simulations conducted for the *HTC* values presented in Table 1.

Table 2. Comparison of the shell thickness and temperature for those selected models of *HTC* (adopted from [6], with permission from Polish Academy of Science, 2018).

Number of the <i>HTC</i> Model	Thickness of the Shell after Leaving the Mould, cm	Temperature, °C
1	2	836
2	2.3	812
3	2.75	772
4	2.27	833
5	2.38	900
6	1.94	956
7	1.98	976
8	1.92	1050
9	1.86	1090
10	1.82	1099
11	2.51	874
12	2.52	938

The obtained results clearly show that the *HTC* is a very important parameter of the continuous casting model and lack of information about the scale of its impact leads to significant deviations of model calculations from the actual strand temperature distribution. The results of the conducted numerical simulations have also shown the importance of proper selection and verification of the adopted concept of description of the heat transfer within the continuous caster mould.

If the solidifying strand surface temperature is not known, a simplified formula may be applied to calculate the heat transfer coefficient for individual spray zones within the secondary cooling zone [3–6]:

$$\alpha_{\text{spray}} = 10v + (107 + 0.688v)w \quad (3)$$

where v is water drop velocity, $\text{m}\cdot\text{s}^{-1}$; w is water flux density, $\text{dm}^3\cdot\text{m}^{-2}\cdot\text{s}^{-1}$. For each of the spray zones the average value of heat transfer coefficient can be computed on the basis of water flux density. It does not take into account places of contact of rolls with the strand. The main technological parameter—necessary to compute the heat transfer coefficient—is the cooling intensity, most often given in $\text{dm}^3\cdot\text{min}^{-1}$, which divided by the area of an individual spray zone allows the water flux density to be computed for the selected casting speed. In the literature [1,2,7,11,39–43], we can find many models, which allow the heat transfer coefficient to be computed in a selected spray zone as a function of the solidifying strand surface temperature.

One needs to emphasise that for the primary and secondary cooling zones there is no single universal heat transfer model, which would allow the correct value of heat transfer coefficient to be computed. Each time when a selected heat transfer model is implemented, it needs to be adjusted to the actual conditions of the process, starting with the mould type, its height and the place of the air gap formation. The database containing technological data on cooling in the continuous casting process is key to computing the values of the declared boundary conditions. It must contain data concerning water flow rates within individual spray zones and flow rates through the mould walls, cooling water

temperature both for the primary and the secondary zone. The difference in temperature between the incoming and outgoing water from the mould cooling channels is very important information for computing the thermal balance. For the secondary cooling zone, it is necessary to analyse the flow rates of the cooling water in the individual spray zones in correlation to the casting speed. In order to formulate a reliable model of heat transfer within the secondary cooling zone, also information on the cooling nozzles, their number, spray angle and diameters in the individual spray zones will be needed.

3.2. Material-Related Parameters

Chemical composition is the basic feature that determines the classification of steels. It defines both the casting process conditions and the other thermophysical and rheological properties of the material tested. The liquidus temperature is calculated on the basis of the steel chemical composition with a very good accuracy. Well known formulas which are applied in industrial practice, e.g., the Roeser and Wensel formula, can be used for this purpose. Also experimental methods (e.g., Differential Scanning Calorimetry—Differential Thermal Analysis analysis) or thermodynamic databases can be applied. The latter can be used to determine other material-related parameters. The Fourier equation, Formula (3), must be complemented with the following thermophysical parameters: specific heat or enthalpy, thermal conductivity, density. To calculate the temperature distribution in the continuous casting process it is also necessary to know the solidus temperature, while the value of the viscosity of the selected steel grade is necessary for modelling the flows. The movement of liquid steel is described by Navier-Stokes equations resulting from the momentum balance, supplemented with mass conservation equations [1,2,11,44]. They contain both the dynamic viscosity coefficient and the kinematic viscosity coefficient [45–47]. Modelling the stress distribution during casting involves the need to declare the rheological parameters i.e., the Young modulus, the Poisson's ratio, yield stress, ultimate yield stress, plastic strain and thermal expansion.

The correctness of the numerical calculation results strictly depends on the correctness and accuracy of material-related data implemented in the model. The specific heat, along with the enthalpy accompanying transitions occurring within the solidifying cast strand, are amongst the most important parameters. The relationship between enthalpy and specific heat, at a set temperature for metal alloys is defined as follows [3–6,48]:

$$H(t) = \int_{t_0}^t c_p(t) dt + (1 - f_s)L \quad t_{sol} \leq t \leq t_{liq} \quad (4)$$

where c_p is specific heat, $\text{kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$; L —latent heat, kJ ; f_s —solid fraction (0–1). Formula (4) presents also the method for declaring the latent heat between the liquidus and solidus temperatures.

It should be emphasised that specific heat values can be implemented in the generalised Fourier equation (Equation (1)), as well as the values of enthalpy calculated on the basis of specific heat values. The enthalpy approach allows the heat accompanying transitions occurring in the solidifying steel to be presented accurately. Paper [44] presents a numerical model of the primary cooling zone with a submerged entry nozzle. A series of numerical calculations were performed to show the difference in the temperature distribution within the mould zone for the declared values of specific heat and enthalpy. It was shown that the method of implementation of the values of specific heat and enthalpy in the numerical model of the continuous casting process is key to the correct calculation of the solidifying strand temperature distribution and the prediction of shell thickness. The differences in the temperature distribution in the primary cooling zone for the selected values of specific heat and enthalpy (including values computed with thermodynamic databases) were as high as 200 °C. The presented results showed the influence of material-related values implemented in a numerical model on the final results.

The authors of papers [10,24,25,49–52] apply average values of specific heat in modelling the continuous casting process. It seems to be a good trade-off when no experimental data is available.

To compute the values of enthalpy as a function of temperature, thermodynamic databases can be used, e.g., being an integral part of the Thermo-Calc[®] or LCC Computherm[®] software databases supplied together with ProCAST[®] software. It is a frequently applied approach in designing numerical models. It is recommended if the results of experimental tests are obtained at a temperature lower than the solidus temperature e.g., for tests of thermal conductivity. Material-related parameters computed with thermodynamic databases can complement experiments conducted to determine the functional relationship of temperature and the values of the thermophysical parameter. It should be emphasised that the reliability of material-related parameters is another extremely valid factor which influences the correctness of numerical computing results.

3.3. Inverse Modeling

In Sections 3.1 and 3.2 a set of necessary boundary conditions is presented, along with the thermophysical parameters that are implemented in the numerical model of the continuous casting process. The determination of their values, most often as a function of temperature, is a time consuming and expensive task. The simplest method for defining the set of model input data is to use the literature data. Unfortunately, the available data concerns a limited number of materials for steels they are practically limited to those steels that were developed in the 1970s at the latest. Additionally, it is very difficult to find data concerning material tests conducted at very high temperatures—close to the solidus temperature of the steel grade tested. The problem is additionally complicated by a broad range of steel grades that are manufactured nowadays and the high cost of some material tests. Experimental methods allowing specific heat, viscosity, or thermal conductivity to be determined require developing a measurement methodology and carrying out multiple measurements—including calibration measurements.

A similar problem appears when we implement data related to the boundary conditions in the numerical model—with heat transfer coefficients within the primary and secondary cooling zones. Very often, access to the full characteristics of the continuous casting machine or industrial data recording casting parameters for a selected sequence is hampered. Note that part of the control data—primarily changes made by the machine operator during casting (adjustment of cooling intensity, emergency mode casting, etc.)—is often not recorded together with the control data. A difficulty in determining the boundary conditions is also related to developing and adjusting the selected heat transfer model to a specific technological solution.

When the boundary conditions cannot be determined in a classic way—directly from the process data—it is possible to use inverse analysis to find the searched heat transfer coefficient. In the Inverse method, contrary to the direct solution, where the equation parameters are known, the input data are the results of the conducted tests. The main objective of the Inverse method is to determine the missing model parameters or the range of their changes. Inverse methods use temperature measurements at a single or a few points of the volume of a sensor either heated or cooled at a known initial temperature distribution. The boundary condition is searched on the basis of the changes of temperature. After substituting the boundary condition into the analytical or numerical solution of the heat conduction equation, the temperature field is achieved that is the best reflection of the changes measured during the experiment. It is often defined by the heat transfer coefficient between the interface and the environment. When the ambient temperature is known, it is easy to compute the equivalent condition expressed by the heat flux or its density. The Inverse method can be applied to determine the heat transfer coefficient between the mould wall and the solidifying strand thanks to the knowledge of the readings of temperature values—most often at places of thermocouple installation in the mould walls. At present, elaborate detection systems for shell adhesion in the primary cooling zone are applied in continuous casting machines. They are based upon continuous monitoring of the mould wall temperature at reference points. In modern moulds, systems allowing temperature to be monitored along the whole wall height are installed—both for wide and narrow walls—along with computing the density of heat flux received from the solidified steel. The temperature data

obtained this way provides a very good foundation to determine the average value of the heat transfer coefficient (or its values as a function of temperature) within the zone of contact of the strand with the mould wall.

At present, the authors of papers [53–55] in their computing based on the Inverse method, use both their original source codes and commercial computing packages. The presented papers show that using the inverse analysis in the numerical modelling of the continuous casting process is very widely applied, due to the possibility of precise determination of boundary conditions and the selected material-related parameters.

4. Sensitivity Analysis

The subject of the sensitivity analysis is to investigate the impact of input parameters on the variability of output parameters of a model. The definition formulated this way allows us to introduce classification, which enables the sensitivity analysis to be adjusted to the needs of the continuous casting modelling. Formally, the analysis of possibilities should answer, for instance, the following questions:

- What factors cause the biggest changes in the metallurgical length of the strand and the shell thickness under the mould?
- Are there any factors whose influence on the above mentioned parameters is negligible?
- Are there any interactions, which enhance or suppress the variability caused by individual factors?

Formally, various kinds of sensitivity analysis can be distinguished depending on the method used to obtain an answer to the question asked above. Unlike the continuous casting process models, we define the key division of these methods as local or global sensitivity analysis. Local sensitivity analysis takes into account output variability related to the changes of input parameters around the nominal value determined. On the other hand, global sensitivity analysis takes into account changes in the whole space of the variability of input parameters. With respect to the continuous casting process modelling problem concerned, the local sensitivity analysis is much more useful. It arises from the fact that for the verification of the process model, industrial data of an actual continuous casting machine is most often used, for which nominal values of the model input parameters are approximately known. An attempt to use the global method involves the necessity to define the scope of acceptable variability of all input parameters. This may cause errors that are difficult to estimate.

The problem of sensitivity analysis of the model applied to simulate the continuous steel casting process needs to be discussed in detail. The classic approach to the problem, known for example from the economy field, is of little use to the continuous steel casting process. To clearly define the problem, one should distinctly emphasise that when talking about the sensitivity of a continuous casting process model, it is recommended to take into account the sensitivity analysis both with regard to the control parameters and the process state parameters, as well as model parameters determined at the stage of its verification. Improper care in both mentioned cases very often leads to the occurrence of critical errors in the model, disqualifying the possibility of its application. The research findings discussed in Section 3.1, presented in Table 1, are a good illustration of this problem.

The analysis of various mathematical models of the continuous steel casting process allows us to propose the following classification of their parameters.

- I. Technological process parameters: strand casting speed and cooling intensity in individual zones of the continuous casting machine.
- II. Parameters characteristic of the properties of the material cast:
 - chemical composition of the steel cast, (%)
 - initial temperature of the steel cast (in the tundish), (K)
 - liquidus temperature of the steel cast, (K)
 - solidus temperature of the steel cast, (K)

- viscosity of the steel cast as a function of temperature, (mP)
- specific heat of the steel cast as a function of temperature, ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)
- melting heat of the steel, ($\text{J}\cdot\text{kg}^{-1}$)
- steel density as a function of temperature ($\text{kg}\cdot\text{m}^{-3}$)
- thermal conductivity, ($\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$)

Before starting a detailed analysis of the mentioned parameters regarding their impact on the final result of computing, note that determining the values of some of them is very precise even in industrial conditions. It means that conducting sensitivity analysis for those parameters is not appropriate, as the problem does not concern them. In the first group of technological parameters, the strand casting speed is an example of a parameter that is determined very accurately. With the current state of measurement technology in this area, one may assume that the accuracy of determination of the strand withdrawal speed is absolutely sufficient for model calculations to be accurate.

The second of the mentioned control parameters is cooling intensity, which in each model is represented by the boundary conditions. In practice it is one of the most difficult problems related to the mathematical modelling of the continuous casting process. There may be various methods of converting the intensity of the supplied cooling medium to the heat transfer coefficient *HTC*. However, in any case, they constitute a potential source of errors. Such a state simply forces the need to assess the sensitivity of the continuous casting process model applied to the value of the heat transfer coefficient *HTC* [5,6].

In the second group defined as “material-related properties of the steel cast”, the only parameter which can be very accurately measured is the temperature of the steel cast in the tundish, which is treated as the initial value for the conducted simulations of the casting process. Note that the value of this temperature may fluctuate and the sensitivity analysis for this parameter may only be omitted if a reliable system of steel temperature measurement in the tundish is in place.

For other parameters of the second group, classic tests based upon the local sensitivity analysis method were conducted. The justification of the selection of this particular method has already been provided above. The results of the conducted analysis allowed us, as expected, to sort all parameters by the criterion testing their influence on the final result of computing the shell thickness and the strand metallurgical length.

Summing up the conducted sensitivity analysis of continuous steel casting process models, one should state that three parameters are the most important in terms of sensitivity:

- *HTC* in the mould
- *HTC* in the secondary cooling zone
- Steel specific heat as a function of temperature $c_p(T)$

5. Evaluation of Possibilities for Using Numerical Models in Order to Design or Modify the Existing Technology

The necessary condition for the correct selection of parameters of strand cooling within the continuous caster is to have a reliable process model, taking into account the design of the machine and its relevant technical parameters. A verified and tested model allows changes in the existing casting process to be made and processes for new steel grades to be engineered.

5.1. Development of the Cooling Program for a New Steel Grade

The method of creating a process manual for a specific steel group includes determining the cooling intensity in the individual zones of the machine for a standard casting speed and modification of the cooling intensity for a changed speed. A set of information about cooling water intensity in machine sections depending on the casting speed is called a cooling schedule. The strategy of developing a new cooling schedule for a selected steel grade, assuming the existence of a correctly

verified process model, can be accomplished according to, for instance, the procedure presented below. It comprises six stages:

- The simulation of the casting process for various strand withdrawal speeds, taking into account the technical parameters of the continuous casting machine without changing the values of the cooling parameters.
- The assessment of the impact of a speed change on changes of basic parameters of the strand cast i.e., the metallurgical length, shell thickness under the mould, and the value of the strand surface temperature at the reference points.
- The estimation of the necessary change in values of heat transfer coefficients in the individual cooling zones of the continuous casting machine allowing the assumed metallurgical length and shell thickness under the mould to be achieved for the changed casting speed.
- The conversion of new heat transfer coefficients to water consumption in individual secondary cooling zones.
- The experimental check of the correctness of the calculated temperature of the strand surface at the selected reference points.

5.2. Method of Selection of Cooling Parameters

In Section 5.1, an algorithm of proceeding during the development of a new cooling schedule was proposed. It requires, among others, computing the necessary change of the values of the heat transfer coefficients in the individual cooling zones of the continuous casting machine, to guarantee the assumed metallurgical length. The length of the liquid core—metallurgical length—measured from the level of liquid steel meniscus in the mould to the time of total solidification of the cast strand—is a very important parameter in the continuous casting process. Often maintaining a similar metallurgical length for various casting speeds is a challenge during the actual steel casting process. In industrial practice, it is aimed at total strand solidification at the place where it leaves the secondary cooling chamber or exactly at the point of the mechanical “soft reduction” zone. In addition, maintaining comparable metallurgical lengths for various strand casting speeds guarantees that safety is ensured during the continuous casting process as the strand is fully solidified before shearing. Another important aspect is the strand straightening points within the secondary cooling zone. It is also important that the liquid fraction is left at their position. In the papers [3,6], the authors presented the utilisation of a numerical model of the continuous steel casting process for determining new cooling parameters in the secondary zone by optimisation of the liquid core length. The method presented in the paper is based on determining the percentage contribution of the casting speed into the metallurgical length at constant cooling rates. Next, on the basis of the heat transfer coefficients for each of the spray zones, the mean heat transfer coefficient is computed for the whole secondary cooling zone. The next stage is computing the new, average heat transfer coefficient, and the transition to average values of the coefficient within the spray zones, and thus the flow rates within individual spray zones. Confirming that the proposed method is right has been proven by computing the temperature distribution in the primary and secondary cooling zones. The template of the described solution, for the secondary cooling zone comprising seven spray zones, is presented in Figure 1.

One should stress that the correctness of model calculations was accurately verified, and based upon the following parameters: thickness of the shell leaving the mould, strand metallurgical length, and comparing the computed temperature values at the measurement points with the values measured with optical pyrometers in the secondary cooling zone. Additionally, the authors presented a solution, which could be directly applied in the industrial conditions thanks to determining new cooling water flow rates within the secondary cooling zone.

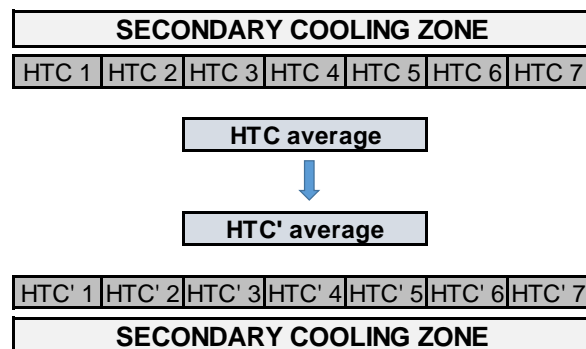


Figure 1. Models of heat transfer coefficient versus temperature on the strand surface (adopted from [3], with permission from Polish Academy of Science, 2018).

6. Conclusions

The control of the steel continuous casting process is one of the most difficult tasks in the steel production line. The conditions assumed in the casting process, influence both the safe process course and the required product properties. Evaluating the problem from a historical perspective, we notice that the advantages resulting from the application of the continuous casting process have been obvious long before. Technical problems and imperfections of available materials have prevented the implementation of the expected solution for a long time. From the time when H. Bessemer was granted the patent for continuous steel casting in 1846 to the beginning of the 1970s about 130 years passed. This was the time needed to overcome the technical challenges and to roll out the process on an industrial scale.

Controlling a modern steel continuous casting process requires full control of the strand solidification process. The measurement system of a continuous casting machine provides a lot of process information. However, key information, such as changes in the shell thickness at individual points of the machine and the metallurgical length (the length of the liquid core), is missing. It arises from the described reasons that the mathematical model of the process is extremely important in the control system of the continuous casting process. Its accuracy allows it to be used in making technological decisions.

Observing the development of scientific research on the described problem, one can show two compatibility groups which characterise current art and the level of practical solutions. The first group concerns the selection of the type of model describing the continuous casting process. Except for the differences resulting from the adopted specific solutions, one needs to state that models based upon the finite element method prevail. The possibilities of models of this class seem to be sufficient in solving any existing or future challenges. The other group of problems seem to be not fully solved yet. Generally, it is about a reliable transformation of essential process parameters to the boundary conditions of a mathematical model. Indeed, the existing solutions with this regard already provide satisfactory results, but there is still plenty of room for the continuation of research, especially in the experimental domain. An additional motivation to carry this research out is the sensitivity of continuous casting process models to the values of HTC , established beyond any doubts. Other parameters, whose significance was shown by the sensitivity analysis, should be determined experimentally. Moreover, the use of inverse analysis is very helpful. As computer capacity increases, it becomes more and more important.

Nowadays, the development of numerical models of the continuous casting process definitely improves the ability to predict cast strand defects and the possibilities for determining the influence of additional devices, such as electromagnetic stirrers or the location of the “soft reduction” zone. The right development trend includes tests conducted on modelling the natural effect of element segregation during continuous casting. Any studies using numerical models, which contributed to the development of technological guidelines resulting in an increase in steel product quality,

the elimination of steel semi-product defects, and reducing the risk of carrying out the so-called hot tests, properly justify ongoing research on the numerical modelling of the continuous casting process.

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