The Unified Creep-Fatigue Equation for Stainless Steel 316

Dan Liu [1], Dirk John Pons [1],* and Ee-hua Wong [2]

[1] Department of Mechanical Engineering, University of Canterbury, Christchurch 8140, New Zealand; dan.liu@pg.canterbury.ac.nz

[2] Energy Research Institute, Nanyang Technological University, Singapore 637553, Singapore; ehwong@ntu.edu.sg

* Correspondence: dirk.pons@canterbury.ac.nz; Tel.: +64-021-069-0900

Abstract: Background—The creep-fatigue properties of stainless steel 316 are of interest because of the wide use of this material in demanding service environments, such as the nuclear industry. Need—A number of models exist to describe creep-fatigue behaviours, but they are limited by the need to obtain specialized coefficients from a large number of experiments, which are time-consuming and expensive. Also, they do not generalise to other situations of temperature and frequency. There is a need for improved formulations for creep-fatigue, with coefficients that determinable directly from the existing and simple creep-fatigue tests and creep rupture tests. Outcomes—A unified creep-fatigue equation is proposed, based on an extension of the Coffin-Manson equation, to introduce dependencies on temperature and frequency. The equation may be formulated for strain as $\varepsilon_p = C_0 c\left(T,\ t, \varepsilon_p\right) N^{-\beta_0}$, or as a power-law $\varepsilon_p = C_0 c\left(T, t\right) N^{-\beta_0 b(T,t)}$. These were then validated against existing experimental data. The equations provide an excellent fit to data ($r^2 = 0.97$ or better). Originality—This work develops a novel formulation for creep-fatigue that accommodates temperature and frequency. The coefficients can be obtained with minimum experimental effort, being based on standard rather than specialized tests.

Keywords: creep-fatigue; creep-rupture; unified equation; fatigue model

1. Introduction

The life of nuclear power plants has been a major issue because it strongly relates to safety and economy [1]. Stainless steel 316 is widely used for the making of components, such as turbine blades and piping, because of its excellent corrosion resistance.

As the two main fatigue evaluation and design methods in the nuclear industry, the linear damage rule and the crack growth law have been used for many years. However, microstructural characteristics lead to imperfect prediction of fatigue failure. The coefficients in these models are obtained from a large number of experiments, which are time-consuming and expensive for industry to perform. When these models are employed, the coefficients are normally obtained from specific temperature and frequency, so these coefficients cannot be used to predict fatigue life in other situations. Therefore, there is a need to develop a creep-fatigue model that can largely avoid the influence of microstructure, present the influence of creep effects on fatigue behaviour, and be generalised to other situations of temperature and frequency. Ideally, the parameters in this model should be easy to obtain from empirical tests with minimum effort.

In this paper, the strain-form unified creep-fatigue equation and power-law form will be introduced and will be verified on stainless steel 316. As part of the validation, the simple experimental methods of extracting coefficients will be presented.

2. Existing Approaches

In the case of pure fatigue, three general fatigue models are used to predict the fatigue life of this material: the Basquin equation (Equation (1)) [2], the Coffin-Manson equation (Equation (2)) [3,4] and Morrow's energy-based equation (Equation (3)) [5,6].

$$\frac{\Delta \sigma}{2} = \sigma'_f \left(2N_f \right)^b \tag{1}$$

$$\frac{\Delta \varepsilon_p}{2} = \varepsilon'_f \left(2N_f \right)^c \tag{2}$$

$$\Delta W_P = W'_f \left(2N_f \right)^\beta \tag{3}$$

where $\Delta \sigma$ is the stress amplitude, σ'_f is the fatigue strength coefficient, b is the fatigue strength exponent, $\Delta \varepsilon_p$ is the plastic strain amplitude, ε'_f is the fatigue ductility coefficient, c is the fatigue ductility exponent, ΔW_p is the plastic energy, W'_f is the energy coefficient, β is the energy exponent and N_f is the number of cycles. The coefficients in Equations (1) and (2) can be related through the compatibility (Equation (4)) [7] and cyclic stress-strain relation (Equation (5)) [8].

$$K' = \frac{\sigma'_f}{\left(\varepsilon'_f \right)^{n'}} \ n' = \frac{b}{c} \tag{4}$$

$$\varepsilon_p = \left(\frac{\sigma_a}{K'} \right)^{\frac{1}{n'}} \tag{5}$$

where σ_a is the stress amplitude, K' is the strain hardening coefficient and n' is the strain hardening exponent.

In nuclear power plants, some components which are made of stainless steel 316 are subjected to fatigue at elevated temperature, at which the mechanism of creep is active. The failure of these components is caused by the combination of fatigue damage and creep damage. Two major rules are used to evaluate creep-fatigue life: the linear damage rule and the crack growth law.

2.1. The Linear Damage Rule

The linear damage theory was proposed by Palmgren [9] in 1924; it was further developed by Miner [10] in 1945 and called the Palmgren-Miner rule. This rule is widely used in the nuclear industry to design and evaluate the life of nuclear power plants [11–14]. According to this rule, damage can be calculated through using Equation (6), and the engineering structure fails when D equals 1.

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{6}$$

where D is the accumulated fatigue damage, k is the number of block loading, n_i is the number of constant amplitude cycles under the ith strain/stress range, and N_i is the number of cycles to fatigue failure under the ith strain/stress range.

Combined with the creep effects, the total damage is divided into fatigue damage and creep damage at the elevated temperature (Equation (7)) [15], which shows that the accumulation of fatigue and creep damage happens at different stages.

$$D = D_f + D_c \tag{7}$$

where D_f is the fatigue damage and D_c is the creep damage.

However, as one of the simplified methods which are used to predict life in the nuclear industry, the linear damage rule can lead to inaccurate results because of the neglect of loading sequences [13].

This problem was also realized in other industries. Therefore, many studies were conducted to improve the accuracy of this rule. For example, Richard and Newmark [16] proposed a power-law damage rule. Manson [17] demonstrated that failure can still happen when *D* is less than 1 and the linear damage rule was modified to double linear damage rule. Although these models can improve the results, these modifications do not change the character of linear accumulation of damage. Therefore, when creep is active, the inaccuracy which comes from the linear accumulation of creep damage and fatigue damage still cannot be solved, because the linear addition of damage is inconsistent with the microstructural characteristics. To be specific, cyclic strain/stress causes the slips between lattices, which can lead to the persistent slip bands. These deformations then lead to cracks. Meanwhile, the damage caused by creep comes from diffusion and dislocation along the grain boundaries and within the lattice, which leads to the accumulation of voids.

2.2. The Linear Damage Rule

The crack growth law is also used in the nuclear industry to predict fatigue life of some components [18,19], such as piping and tanks. The crack growth law shows that the fatigue life is the number of loading cycles which is required to achieve the final crack size, and this process is divided into two stages: initiation and propagation. Therefore, the total crack size can be presented as the linear addition of initial crack size and propagative crack size. Normally, the initial crack is identified as the real crack size in the structures before loading, and the propagative crack (Equation (8)) [19,20] can be obtained through Paris's model [21]:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C \left(\Delta J_{\mathrm{eff}} \right)^l \tag{8}$$

where $\frac{\mathrm{d}a}{\mathrm{d}N}$ is the total crack growth per cycle, ΔJ_{eff} is the effective range of *J*-integral, and *C* is a material constant obtained from experiments. When creep is active, the total damage is calculated through the sum of fatigue and creep crack growth (Equations (9a) and (9b)) [21]:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \left(\frac{\mathrm{d}a}{\mathrm{d}N} \right)_f + \left(\frac{\mathrm{d}a}{\mathrm{d}N} \right)_c \tag{9a}$$

$$\left(\frac{\mathrm{d}a}{\mathrm{d}N} \right)_f = C \left(\Delta J_{\mathrm{eff}} \right)^l \text{ and } \left(\frac{\mathrm{d}a}{\mathrm{d}N} \right)_c = \int_0^{t_h} AC^{*q} \mathrm{d}t \tag{9b}$$

where $\left(\frac{\mathrm{d}a}{\mathrm{d}N} \right)_f$ is the crack growth per cycle due to cyclic load changes, $\left(\frac{\mathrm{d}a}{\mathrm{d}N} \right)_c$ is the crack growth per cycle due to hold time, t_h is the hold time, C^* is the time-dependent fracture parameter, and *l*, *A* and *q* are material constants obtained from experiments.

The crack growth law provides a good physical explanation of damage. However, the quantitative summation between the cracks caused by fatigue and the cracks caused by creep does not consider the directions of these cracks. This means that two parallel cracks can cause the same damage as two non-parallel cracks, which is inconsistent with the microstructure, because the angle between two cracks plays an important role in the total damage.

2.3. Recent Developments towards a Unified Creep-Fatigue Equation

As identified above, the limitations of these methods are that they do not fully accommodate the observed microstructural characteristics, they require extensive testing to determine the coefficients, and the results cannot be generalised to other situations of temperature and frequency.

In an attempt to address these problems, Wong and Mai [22] proposed a formalism to accommodate fatigue and creep-fatigue, which they called a unified creep-fatigue equation (hereafter WM equation); see Equation (10). The unified creep-fatigue equation takes into account the influence

of temperature and frequency on fatigue life. This equation was developed by extension of the Coffin-Manson equation.

$$\varepsilon_p = C_0 s\,(\sigma)\, c\,(T, t)\, N^{-\beta_0 b(T,t)} \tag{10}$$

where σ is the stress, T is the temperature, t is the cyclic time ($1/$frequency), C_0 is the fatigue ductility coefficient, β_0 is the fatigue ductility exponent, ε_p is the plastic strain and N is the fatigue life. The basic premise of the WM formulation is that 'all fatigue phenomenon are indeed creep-fatigue, and "pure fatigue" is just a special case of creep-fatigue' [22]. They reasoned that the Coffin-Manson equation was "a special case of a unified creep-fatigue equation".

The general principles of this were shown for the case of SnPb solder [22]. However there are several issues with the WM formulation. They did not provide the method to get the stress function $s\,(\sigma)$. This means that this unified equation still cannot be used to predict fatigue life. A related issues is that they assumed that functions $c\,(T, t)$ and $b\,(T, t)$ share the same pattern and characteristics, such that internal coefficients $c_1/c_2 = b_1/b_2$, but no reason was provided for this assumption. Also, the method of extracting the coefficients of function $c\,(T, t)$ and $b\,(T, t)$ was not proposed.

The WM equation has potential, but the concept needs further development. It has not been applied other than to solder, so its universal applicability is uncertain. There is a need to further validate or modify and the relationships between the coefficients, or improve the formulation.

3. Methods

3.1. Research Question

The purpose of this paper was to extend and modify the WM unified creep-fatigue equation. The WM equation provided some helpful initial starting points for the present work. Firstly, they showed that the creep-fatigue behaviour is negatively influenced by temperature, frequency and stress. Secondly, the unified creep-fatigue equation could be deduced from the Coffin-Manson equation and the experimental data of Shi [23] on solder, and the reference condition could be introduced into this unified equation. Thirdly, function $c\,(T, t)$ and $b\,(T, t)$ could be related to Manson-Haferd parameter, at least numerically.

3.2. Approach

Work in progress towards a further conceptual development has been to show how plastic strain (ε_p) may be theoretically related, in the creep-fatigue situation, to conditions at the reference temperature (T_{ref}) and reference cycle time (t_{ref}) at which tests are performed [24]. This line of thinking results in two forms of the unified creep-fatigue equation. The first form (strain form) represents the plastic strain as a function of cycle time, number of cycles, and temperature: $\varepsilon_p = C_0 c\,(T, t, \varepsilon_p)\, N^{-\beta_0}$. The second form (power-law form) does the same, but is simplified to a power-law relationship: $\varepsilon_p = C_0 c\,(T, t)\, N^{-\beta_0 b(T,t)}$. Definitions of variables are provided below.

In this paper both forms are verified by application to stainless steel 316. As part of the validation, the simple experimental methods of extracting coefficients are presented. The approach is to take published empirical data for creep rupture tests and creep-fatigue tests. The experimental data for creep-fatigue are from [25] and the creep rupture data are from [26] for stainless steel 316. From the data which were obtained at an arbitrary temperature and reference cycle time, we extract coefficients for the strain-form unified creep-fatigue equation (Section 4.3.1), and validate them through the empirical data at other conditions (Section 4.3.2). Then, the empirical data at two temperatures and reference cycle time are used to extract the coefficients for the power-law form (Sections 4.4.1.1 and 4.4.2.1), and validate it through the empirical data at other conditions (Sections 4.4.1.2 and 4.4.2.2).

The unified creep-fatigue equations were originally developed for solder, a material that is very different to stainless steel. Results show that the two forms of the unified creep-fatigue equation provide excellent representation of the stainless steel 316 creep fatigue data. A temperature modified

Coffin-Manson equation was derived from the combination of the power-law unified creep-fatigue equation and the frequency modified Coffin-Manson equation.

4. Results: Theory and Calculation

4.1. Introduction to the Unified Creep-Fatigue Equation

At this point, the term "fatigue capacity" needs to be introduced to describe the relation between pure fatigue and creep. For creep-fatigue, the fatigue capacity is reduced because of the increasing influence of creep damage. This can be seen in Figure 1: the creep-fatigue curves between pure fatigue curve and pure creep curve show the residual fatigue capacities, and the reductions are caused by creeps at different plastic strains/stresses.
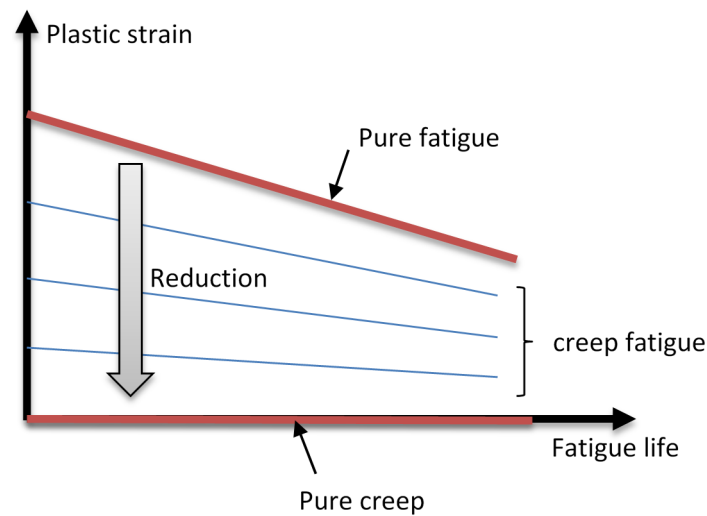

Figure 1. The relation between pure fatigue and pure creep.

Therefore, given the contribution of creep to the reduction of fatigue capacity, it is reasonable that the time-temperature-stress relationship, such as the Sherby-Dorn Parameter (Equation (11)) [27], the Larson-Miller Parameter (Equation (12)) [28] and the Manson-Haferd Parameter (Equation (13)) [29], is involved in the unified creep-fatigue equation.

$$P_{SD}\left(\sigma\right) = te^{-Q/RT} \tag{11}$$

$$P_{LM}\left(\sigma\right) = T\left(\log t + C\right) \tag{12}$$

$$P_{MH}\left(\sigma\right) = \frac{T - T_a}{\log t - \log t_a} \tag{13}$$

where P_{SD} is the Sherby-Dorn Parameter, P_{LM} is the Larson-Miller Parameter, P_{MH} is the Manson-Haferd Parameter, Q is the activation energy of the creep mechanism, R is the Boltzmann's constant, t is the time, T is the absolute temperature, C is a constant and $(\log t_a, T_a)$ is the point of convergence of the $\log t$–T lines. Among these three parameters, the Manson-Haferd Parameter is regarded as the best description of stress-time-temperature relation [30].

It can be found that the unified creep-fatigue equation (Equations (14) and (15)) [24] shows a linear relationship between T and $\log t$:

$$\varepsilon_p = C_0 c\left(T, t, \varepsilon_p\right) N^{-\beta_0} \tag{14}$$

with

$$c\left(T, t, \varepsilon_p\right) = 1 - c_1\left(\varepsilon_p\right)\left(T - T_{ref}\right) - c_2 \log\left(t/t_{ref}\right) \ \left(T > T_{ref}, \ t > t_{ref}\right) \tag{15}$$

Note that this relationship is consistent with the form of the P_{MH} parameter.

The *reference condition* refers to the threshold temperature T_{ref} at which creep first occurs (below this temperature no creep occurs), and cycle time (period) t_{ref} (arbitrary set as 1 second for comparing different data sets). The reference temperature and cyclic time for stainless steel 316 are identified as $T_{ref} = 670$ K and $t_{ref} = 1$ s. This temperature is 0.4 of the melting temperature and corresponds to the widely held assumption that below this temperature no creep occurs.

We then determine the plastic strain at reference condition, by transforming Equation (14):

$$\varepsilon_{p,ref} = C_0 N^{-\beta_0} = \frac{\varepsilon_p}{c\left(T, t, \varepsilon_p\right)} \tag{16}$$

This transformation would result in all ε_p: N data reducing into one single $\varepsilon_{p,ref}$: N curve if the creep function (Equation (15)) could describe the influence of creep on fatigue well.

However, the strain form returns an equation that is not power-law. Because power-law relation is expressed as a straight line on a log-log plot, it can provide an easy and clear way to present the creep-fatigue behaviours between different temperatures and cyclic times through translation and rotation. For this reason, the unified creep-fatigue equation is represented as a power-law form (Equation (17)) [24] through fitting the ε_p-N data with a power-law relation.

$$\varepsilon_p = C_0 c\left(T, t\right) N^{-\beta_0 b(T,t)} \tag{17}$$

with

$$c\left(T,\ t\right) = 1 - c_1\left(T - T_{ref}\right) - c_2 \log\left(t/t_{ref}\right)\ \left(T > T_{ref},\ t > t_{ref}\right) \tag{18}$$

$$b\left(T,\ t\right) = 1 - b_1\left(T - T_{ref}\right) - b_2 \log\left(t/t_{ref}\right)\ \left(T > T_{ref},\ t > t_{ref}\right) \tag{19}$$

Then, the plastic strain at reference condition is determined by transforming Equation (17):

$$\varepsilon_{p,ref} = C_0 N^{-\beta_0} = C_0 \left[\frac{\varepsilon_p}{C_0 c\left(T, t\right)}\right]^{1/b(T,t)} \tag{20}$$

This transformation would cause all ε_p:N data to collapse into one $\varepsilon_{p,ref}$:N curve if the creep function (Equation (18)) and stress function (Equation (19)) could present the influence of creep on fatigue well.

It can be seen that the unified creep-fatigue equation is restored to the Coffin-Manson equation at reference condition (T_{ref}, t_{ref}), which builds a bridge between pure fatigue and creep fatigue. The deduction of coefficients (Equations (A1)–(A13)) in the strain-form unified creep-fatigue equation and power-law form is shown in Appendix A.

Next, the relatively simple methods of obtaining the coefficients will be verified on stainless steel 316.

4.2. Extracting the Creep-Rupture Properties of Stainless Steel 316

The Manson–Haferd parameter was extracted through plotting creep-rupture data [26] as log *(time)* vs. *temperature*. According to the Manson-Haferd parameter, all log*t*–*T* lines at different stresses should converge to one point, where the temperature is regarded as the Creep Initiation Temperature, below which no creep occurs. We determine this temperature as 40% of the melting temperature. This corresponds to the reference temperature. In this case the reference temperature is found to be $T_{ref} = 670$ K. At this temperature, the rupture times at different stresses were found. According to Manson-Haferd parameter, these rupture times should be the same. However, because the accuracy of experiments is influenced by many factors, such as facilities and environment, the points (rupture time, reference temperature) at different stresses cannot be collected into one point. Therefore, the average of these rupture times is employed to define the rupture time at the point of convergence, and the point of convergence is identified to be (670, 9.54) (shown in Figure 2). The Manson-Haferd parameter

is stress dependent, and the values are obtained as the inverse of the slope from the fitted lines. Then, the relationship between Manson-Haferd parameter and stress can be extracted from Figure 2, and Equation (21) is given through curve-fitting:

$$-\frac{1}{P_{\mathrm{MH}}(\sigma)} = 4.50 \times 10^{-3} + 1.26 \times 10^{-5}\sigma + 1.70 \times 10^{-7}\sigma^2 \tag{21}$$
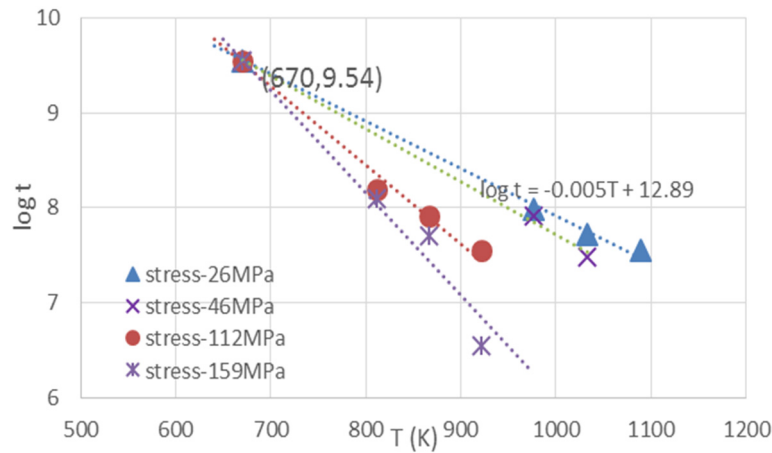
Figure 2. The $P_{\mathrm{MH}}(\sigma, T, t)$ parameter of AISI stainless steel 316. Data extracted from [26].

To convert $P_{\mathrm{MH}}(\sigma)$ to $P_{\mathrm{MH}}(\varepsilon_p)$, the data of stress, total strain, temperature and Young's modulus are extracted from [31]. Because the time of tension tests is much less than the creep-rupture time, the influence of time (creep) on tension tests can be neglected. The temperature-dependent plastic strain-stress relationship (Equations (22)–(24)) can be developed through curve-fitting.

$$\varepsilon_p = \left(\frac{\sigma}{K(T)}\right)^{1/n(T)} \tag{22}$$

$$K(T) = -0.4875T + 769.79 \tag{23}$$

$$n(T) = 7.04 \times 10^{-2} + 1.94 \times 10^{-4}T - 1.22 \times 10^{-7}T^2 \tag{24}$$

Then, substituting Equations (22)–(24) into Equation (21) gives Equation (25):

$$-\frac{1}{P_{\mathrm{MH}}(\varepsilon_p,\,T)} = 4.50 \times 10^{-3} + 1.26 \times 10^{-5}K(T)\,\varepsilon_p^{n(T)} + 1.70 \times 10^{-7}K(T)\,\varepsilon_p^{2n(T)} \tag{25}$$

4.3. Evaluation of the Coefficients of the Strain-Form Unified Creep-Fatigue Equation $\varepsilon_p = C_0 c\,(T, t, \varepsilon_p)\,N^{-\beta_0}$ for the Stainless Steel 316 and Validation

4.3.1. Evaluation of Creep-Fatigue Coefficients

Based on the Equations (A1)–(A6) shown in Appendix A, the coefficients of the creep function, $c\,(T, t, \varepsilon_p)$, for the stainless steel 316 are established as follows.

Substituting $\log t_a = 9.54$ and $t_{\mathrm{ref}} = 1$ s into Equation (A2), the coefficient c_2 is evaluated as:

$$c_2 = \frac{1}{\log(t_a/t_{\mathrm{ref}})} = \frac{1}{\log(10^{9.54}/1)} = 0.105 \tag{26}$$

Substituting Equation (26) into Equation (A4), coefficient c_1 (σ) and c_1 (ε_p) are developed respectively:

$$c_1(\sigma, T) = 4.73 \times 10^{-4} + 1.32 \times 10^{-6}\sigma(T) + 1.79 \times 10^{-8}\sigma^2(T) \tag{27}$$

$$c_1(\varepsilon_p, T) = 4.73 \times 10^{-4} + 1.32 \times 10^{-6}K(T)\ \varepsilon_p^{n(T)} + 1.79 \times 10^{-8}K^2(T)\ \varepsilon_p^{2n(T)} \tag{28}$$

The fatigue coefficients C_0 and β_0 of the stainless steel 316 can be extracted from fatigue test at an arbitrary temperature at the reference cycle time. Selecting the data point ($T = 873$ K, t_{ref}), at which ε_p ($T = 873$ K, t_{ref}, $N = 1$) = 2.1705, C_o is extracted through using Equation (A6), which is 2.95, and β_0 is given as 0.663. However, a big error ($\sum \left(N_{f-exp} - N_{f-equ} \right)^2 = 49.85$) is given through comparing the fatigue life obtained from experiments with the fatigue life obtained from unified creep-fatigue. Therefore, the C_0 is resolved as 0.959 through minimizing the error (0.312).

4.3.2. Validations

Using the fatigue and the creep coefficients evaluated in Section 4.3.1, namely, $C_0 = 0.959$, $\beta_0 = 0.663$, $c_2 = 0.105$, and c_1 (ε_p, T) as described by Equation (28), the generated raw fatigue data (ε_p–N) obtained from [25] ($T = 723$ K, 873 K and 973 K) are transformed to the reference condition ($\varepsilon_{p,ref}$–N) through Equation (16). The transformed data are plotted in Figure 3.
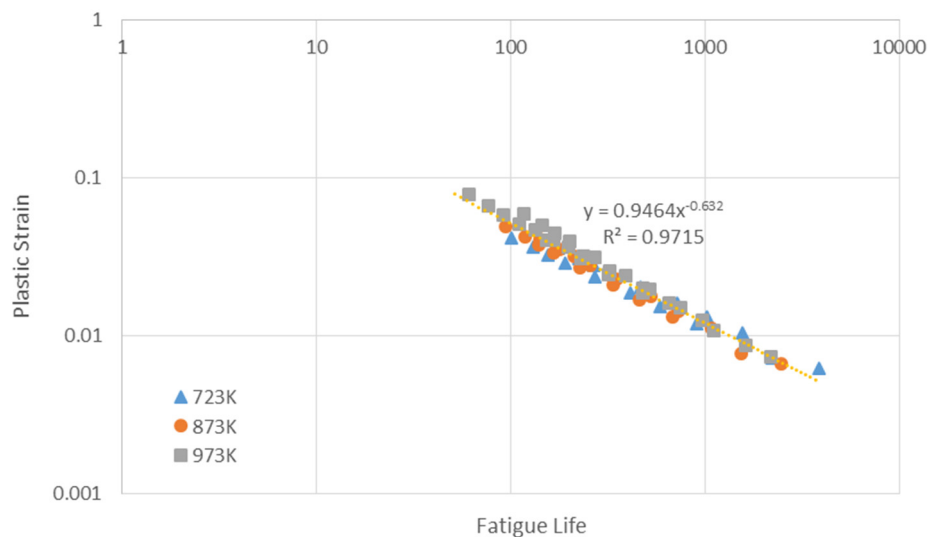
Figure 3. Transformed $\varepsilon_{p,ref}$-N data of stainless steel 316 using Equation (14) with creep function c (T, t, ε_p).

Significantly, these data are collected into a power-law curve of $\varepsilon_{p,ref} = 0.9464N^{-0.632}$ with the quality of fit as $R^2 = 0.9715$. This good transformation verifies the unified equation, $\varepsilon_p = C_0 c\left(T,\ t, \varepsilon_p\right) N^{-\beta_0}$, the form of creep function c (T, t, ε_p), and the methods of extracting the fatigue and the creep coefficients. The error between fatigue life from experiments and creep-fatigue equation is 0.312.

4.3.3. Application

This shows that the mathematical representation provided by the strain-form unified creep-fatigue equation accommodates the data for multiple temperatures, fatigue life, and plastic strain.

$$\varepsilon_p = 0.959\ \left[1 - c_1\left(\varepsilon_p, T\right)\left(T - 670\right) - 0.105\log\left(t\right)\right] N^{-0.663} \tag{29}$$

with

$$
\begin{aligned}
c_1\left(\varepsilon_p, T\right) = 4.73 \quad & \times 10^{-4} + 8.712 \times 10^{-7} \\
& \times\left(769.79 - 0.4875T\right)\ \varepsilon_p^{7.04 \times 10^{-2} + 1.94 \times 10^{-4}T - 1.22 \times 10^{-7}T^2} + 7.797 \\
& \times 10^{-9} \times\left(769.79 - 0.4875T\right)^2 \varepsilon_p^{0.1408 + 3.88 \times 10^{-4}T - 2.44 \times 10^{-7}T^2}
\end{aligned}
\tag{30}
$$

This could be used to determine fatigue life for given plastic strain, temperature and cycle time. Alternatively, to determine plastic strain by numerical solution of the equation.

4.4. Evaluation of the Coefficients of the Power-Law Unified Creep-Fatigue Equation $\varepsilon_p = C_0 c\left(T, t\right) N^{-\beta_0 b(T,t)}$ for the Stainless Steel 316 and Validation

The fatigue behaviour of stainless steel 316 presents an inflection point at the temperature of 873 K [25]. Below this temperature, the fatigue life decreases with the increasing temperature, while, increases above this temperature. Given that the power-law unified creep-fatigue equation is built on the assumption of continual increasing/decreasing fatigue life with increasing temperature, the evaluation and validation of the power-law unified creep-fatigue equation are conducted at two temperature regimes (below 873 K and above 873 K).

4.4.1. Evaluation of Creep-Fatigue Coefficients and Validation below 873 K

4.4.1.1. Evaluation of Creep-Fatigue Coefficients

Based on the Equations (A7)–(A13) shown in Appendix A, the coefficients of c function and b function for the stainless steel 316 are established as follows.

The creep coefficient c_2 is identical as 0.105.

Substituting Equation (18) with the data points ($T = 873$ K, $t = t_{\text{ref}}$) and ($T = 873$ K, $t = 10$ s), where ε_p ($T = 873$ K, $t = t_{\text{ref}}$, $N = 1$) = 1.0296 (Equation (A8)) and ε_p ($T = 873$ K, $t = 10$ s, $N = 1$) = 0.5987 (Equation (A9)), gives $C_0 c_2 = 0.4309$. Then, C_0 is solved as 4.1038. In addition, substituting Equation (19) with these two data points, where $\beta_0 b$ (873 K, $t = t_{\text{ref}}$) = 0.663 (Equation (A10)) and $\beta_0 b$ ($T = 873$ K, $t = 10$ s) = 0.651 (Equation (A11)), gives $\beta_0 b_2 = 0.012$.

Substituting Equation (18) with the data points ($T = 723$ K, $t = t_{\text{ref}}$) and ($T = 873$ K, $t = t_{\text{ref}}$), where ε_p ($T = 723$ K, $t = t_{\text{ref}}$, $N = 1$) = 2.1705 (Equation (A12)) and ε_p ($T = 873$ K, $t = t_{\text{ref}}$, $N = 1$) = 1.0296 (Equation (A8)), gives $c_1 = 0.001853$. Then, substituting Equation (19) with these two data points, where $\beta_0 b$ (723 K, $t = t_{\text{ref}}$) = 0.634 (Equation (A13)) and $\beta_0 b$ (873 K, $t = t_{\text{ref}}$) = 0.663 (Equation (A10)), gives $\beta_0 = 0.624$ and $b_1 = -0.00031$, then b_2 is solved as 0.01924.

The error between the fatigue life from experiments and creep-fatigue equation is 52.88. As shown in the evaluation of coefficients for strain-form unified creep-fatigue equation, this poor prediction is caused by the inaccuracy of C_0. Therefore, the C_0 is resolved as 0.876 through minimizing the error (0.792).

4.4.1.2. Validations

Using the fatigue and creep coefficients found in Section 4.4.1.1, namely, $C_0 = 0.876$, $\beta_0 = 0.624$, $c_1 = 0.001853$, $c_2 = 0.105$, $b_1 = -0.0003094$ and $b_2 = 0.01924$, the raw fatigue data (ε_p–N) obtained from [25] ($T = 723$ K and 873 K) are transformed to the reference condition ($\varepsilon_{p,\text{ref}}$–$N$) through Equation (20). The transformed data are plotted in Figure 4, which shows that these data can be collapsed into a power-law curve of $\varepsilon_{p,\text{ref}} = 0.5959 N^{-0.556}$ with the quality of fit as $R^2 = 0.9583$. This transformation has verified the unified equation, $\varepsilon_p = C_0 c\left(T, t\right) N^{-\beta_0 b(T,t)}$, the form of creep function $c\left(T, t\right)$ and stress function $b\left(T, t\right)$, and the methods of extracting the coefficients. The error between fatigue life from experiments and creep-fatigue equation is 0.792.
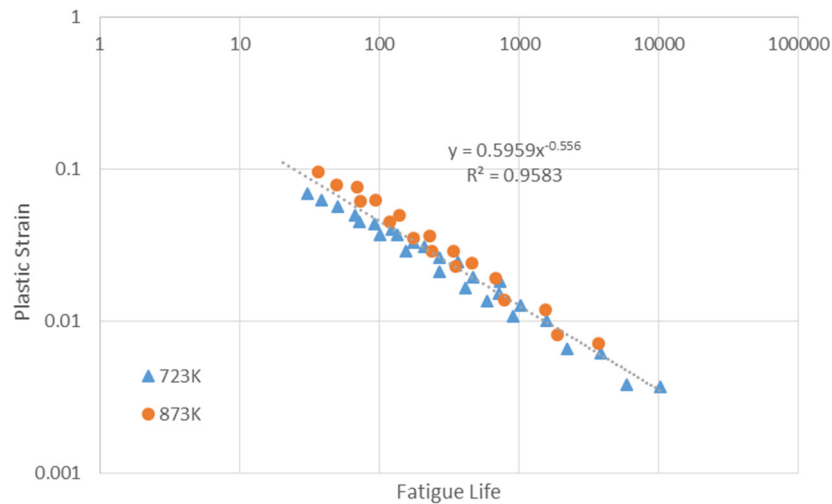
Figure 4. Transformed $\varepsilon_{p,\mathrm{ref}}$–$N$ data (below 873 K) of stainless steel 316 using Equation (17) with functions c (T, t) and b (T, t).

4.4.2. Evaluation of Creep-Fatigue Coefficients and Validation above 873 K

4.4.2.1. Evaluation of Creep-Fatigue Coefficients

The data points (T = 873 K, t = t_{ref}), (T = 873 K, t = 10 s) and (T = 973 K, t = t_{ref}) are selected to evaluate the coefficients. These coefficients are obtained through the same method shown in Section 4.4.1.1: C_0 = 0.879, β_0 = 0.807, c_1 = 0.00146, c_2 = 0.105, b_1 = 0.00088 and b_2 = 0.01487.

4.4.2.2. Validations

Using the fatigue and creep coefficients found in Section 4.4.2.1, the raw fatigue data (ε_p–N) obtained from [25] (T = 873 K and 973 K) are transformed to the reference condition ($\varepsilon_{p,\mathrm{ref}}$–$N$) through Equation (20). The transformed data plotted in Figure 5, which shows that these data can be collapsed into a power-law curve of $\varepsilon_{p,\mathrm{ref}} = 0.9439N^{-0.82}$ with a quality of fit of R^2 = 0.9797. This transformation has verified the unified equation, $\varepsilon_p = C_0 c\,(T,t)\,N^{-\beta_0 b(T,t)}$, the form of creep function c (T, t) and stress function b (T, t), and the methods of extracting the coefficients. The error between fatigue life from experiments and creep-fatigue equation is 0.300.
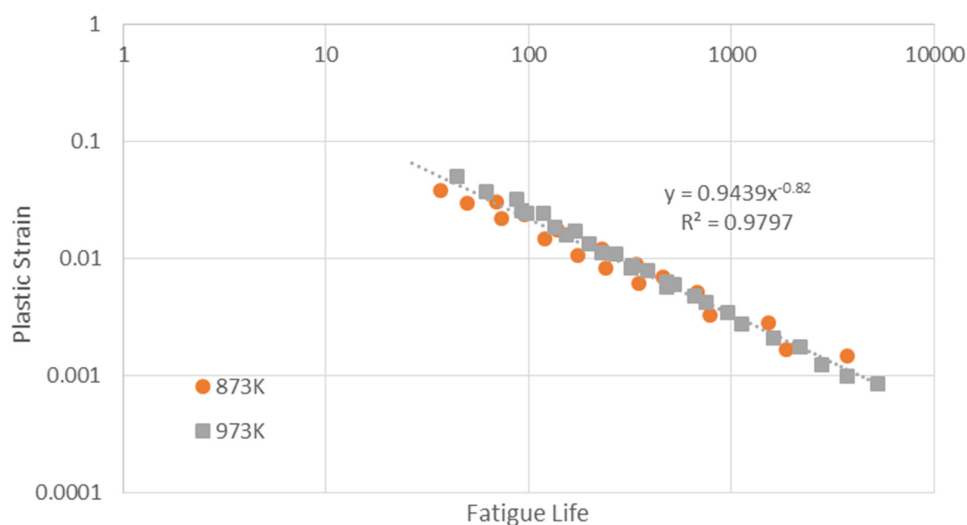
Figure 5. Transformed $\varepsilon_{p,\mathrm{ref}}$–$N$ data (above 873 K) of stainless steel 316 using Equation (17) with functions c (T, t) and b (T, t).

4.4.3. Application

The mathematical representation (Equation (31) for below 873 K and Equation (32) for above 873 K) provided by the unified creep-fatigue equation accommodates the data for multiple temperatures, fatigue life, and plastic strain. This could be used to determine fatigue life for given plastic strain, temperature and cycle time. Alternatively, to determine plastic strain by numerical solution of the equation:

$$\varepsilon_p = 0.876[2.2415 - 0.001853\text{T} - 0.105\log{(t)}]N^{-0.624[0.7927+0.0003094T-0.01924\log(t)]}$$
$$(T < 873\text{K}) \tag{31}$$

$$\varepsilon_p = 0.879[1.9782 - 0.00146\text{T} - 0.105\log{(t)}]N^{-0.807[1.5896+0.00088T-0.01487\log(t)]}$$
$$(T \geq 873\text{K}) \tag{32}$$

5. Discussion

5.1. The Moderating Factor

As shown above, C_0 in the strain-form unified creep-fatigue equation was obtained from data point (T = 723 K, t_{ref}). However, this result is different from C_0 obtained from data point (T = 873 K, t_{ref}) and (T = 973 K, t_{ref}). To improve the accuracy of C_0, these three data points are used to regress the magnitude of C_0, which is 2.517 (Equation (33)).

$$\varepsilon_p\left(T, t_{\text{ref}}, N = 1\right) = C_0' = C_0\left[1 - c_1\left(C_0'\right)\left(T - T_{\text{ref}}\right)\right] = 2.517\left[1 - 0.00278\left(T - T_{\text{ref}}\right)\right] \tag{33}$$

At data point (T = 723 K, t_{ref}), substituting C_0' into the Equation (28) cannot yield to 2.1705, and the magnitude of c_1 (2.1705) is bigger than 0.00278. Thus, according to the Equation (27), it appears that the big contribution of stress leads to higher magnitude of c_1 function. Therefore, mathematically, the amplitude of stress should be compressed in order to reduce the magnitude of c_1 (C_0') into the result of regression (0.00278). Then, a moderating factor, f, is introduced into Equation 27 to modify stress, and Equations (27) and (28) can be expressed as:

$$c_1\left(\sigma, f\right) = 4.73 \times 10^{-4} + 1.32 \times 10^{-6}f\sigma + 1.79 \times 10^{-8}f^2\sigma^2 \tag{34}$$

$$c_1\left(\varepsilon_p, T, f\right) = 4.73 \times 10^{-4} + 1.32 \times 10^{-6}fK\left(T\right)\varepsilon_p^{n(T)} + 1.79 \times 10^{-8}f^2K\left(T\right)\varepsilon_p^{2n(T)} \tag{35}$$

This moderating factor is solved as 0.69, and C_0 is given as 0.846 through minimizing the error (0.276).

Using the fatigue and the creep coefficients, namely, C_0 = 0.846, β_0 = 0.663, c_2 = 0.105, $c_1\left(\varepsilon_p,T,f\right)$ as described by Equation (35) and f = 0.69, the generated raw fatigue data (ε_p–N) obtained from [25] (T = 723 K, 873 K and 973 K) are transformed to the reference condition ($\varepsilon_{p,\text{ref}}$–$N$) through Equation (16). The transformed data are plotted in Figure 6.

Figure 6 illustrates how the transformed data are collected into a power-law curve of $\varepsilon_{p,\text{ref}} = 0.6633N^{-0.592}$ with the quality of fit R^2 = 0.9759. The error between fatigue life from experiments and creep-fatigue equation is 0.276. Comparing this result with the transformation in Section 4.3.2 shows that the introduction of moderating factor can provide a better description of creep effect through c_1 function and prediction of creep-fatigue behaviour.

The research conducted by Gary shows that the stress vs. creep rupture time curves under cyclic loading lie above the curves under constant loading [32]. This means that cyclic stress is higher than constant stress at the same rupture time. Because c_1 function is based on the time-temperature relation under constant stress (Manson-Haferd parameter), the creep effect is enlarged when the cyclic stress is

imposed. Therefore, it is reasonable to introduce a moderating factor to compress the cyclic stress to an equivalent constant stress.
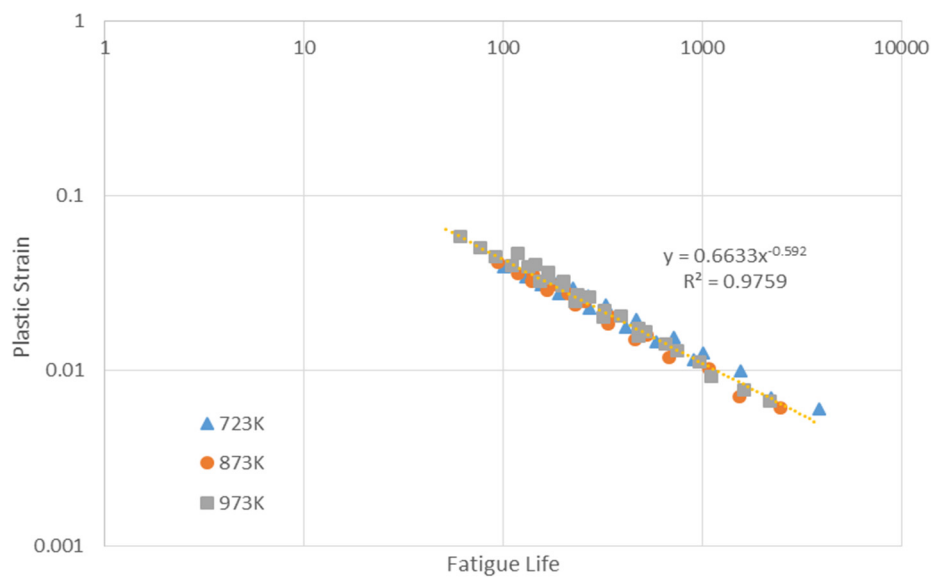
Figure 6. Transformed $\varepsilon_{p,\text{ref}}$–$N$ data of stainless steel 316 using Equation (14) with creep function c (T, t, ε_p) and moderating factor.

5.2. The Heat Treatment

Heat treatment can change fatigue behaviour through hardening or softening. However, stainless steel 316 cannot be hardened by heat treatment, and this is proved by [33], where fatigue life changes slightly between aged condition and annealed condition [33]. This makes the unified creep-fatigue equation more universal for stainless steel 316 under different heat treatments. For example, the creep-fatigue data of aged stainless steel obtained from [33] (T = 839 K and 922 K) and quenched stainless steel 316 obtained from [25] (T = 723 K, 873 K and 973 K) can be collected into one power-law curve $\varepsilon_{p,\text{ref}} = 0.6533N^{-0.588}$ with a quality of fit of R^2 = 0.9827 (Figure 7).
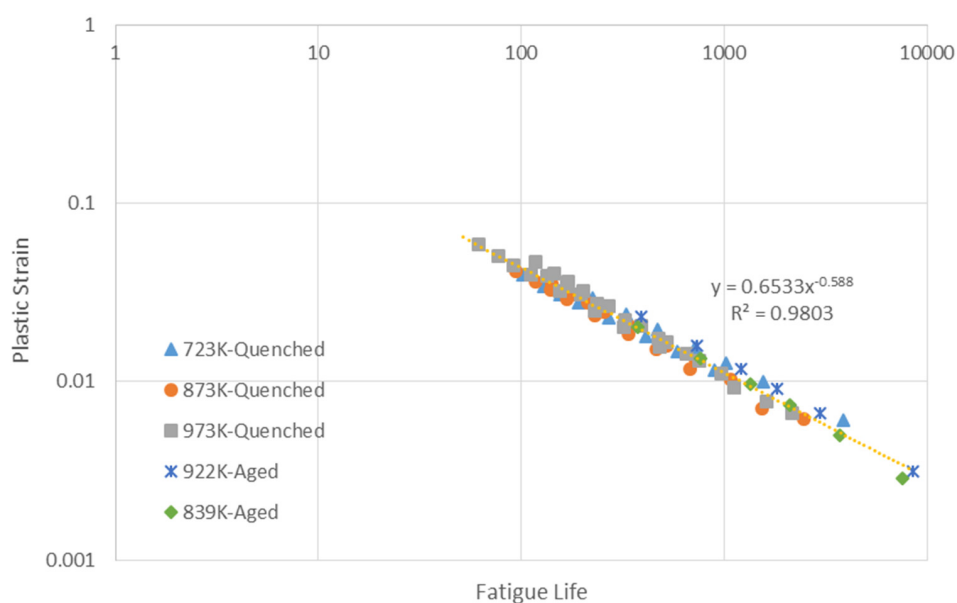
Figure 7. Transformed $\varepsilon_{p,\text{ref}}$–$N$ data of quenched and aged stainless steel 316 using Equation (14) with creep function c (T, t, ε_p).

Although the unified creep-fatigue equation shows a good universality for stainless steel 316 under different heat treatments, the potential limitation for this equation still cannot be ignored. For example, this strong influence of heat treatment on fatigue life is shown on Inconel 718, and the aged condition can provide better fatigue strength than annealed condition [33]. Therefore, if the unified creep-fatigue equation is imposed on Inconel 718, the coefficients obtained at one specified heat treatment condition cannot be used to predict the fatigue life of this material under other heat treatments. This is an opportunity for future development of the formulation.

5.3. Reliability

The proposed new equations were validated against existing experimental data in the literature. The equations provide an excellent fit to data ($r^2 = 0.97$ or better). This demonstrates that the equations provide the desired level of fidelity to the original experimental data.

The validation only can show these selected data follow the unified creep-fatigue equation, but not all data. Hence it is important to consider the degree of reliability. In this section, the strain-form unified creep-fatigue equation was used to explore the reliability.

The fatigue life, temperature and strain rate are defined as random variables, and then 100 random creep-fatigue data points (*plastic strain* vs. *fatigue life*) were derived from [25] and the Coffin-Manson equation. These random data then were transformed into a reference condition through Equation (16). The transformation of 10 sets of random data (1000 data points) shows that these creep-fatigue data can collapse into almost one straight line at log-log scale (two of them are shown in Figure 8) with the quality of fit $R^2 = 0.975$–0.985. This transformation based on random data has further verified the unified equation, $\varepsilon_p = C_0 c\,(T,\ t, \varepsilon_p)\,N^{-\beta_0}$ and the form of creep function $c\,(T, t, \varepsilon_p)$.
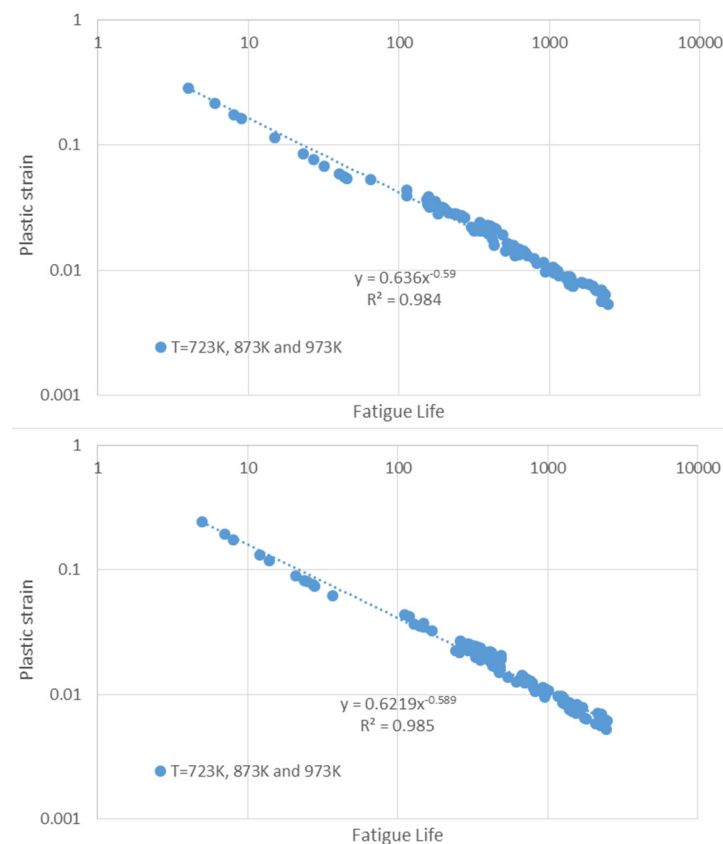
Figure 8. Transformed $\varepsilon_{p,\text{ref}}$–$N$ data of stainless steel 316 using Equation (14) with creep function $c\,(T, t, \varepsilon_p)$ and random variables. Data shown are for two of ten sets, as evidence of a consistent transformation process.

5.4. The Initial Proposal of Creep-Fatigue-Equation-Based Temperature Modified Coffin-Manson Equation

Frequency modified Coffin-Manson equation can be expressed as Equation (36) [34]:

$$\Delta \varepsilon_p = C_0 \left[N_f \nu^{k-1} \right]^m \tag{36}$$

where ν is the frequency, and k and m are the constant determined by experiments. Comparing this equation with the conventional Coffin-Manson equation (Equation (2)), the $N_f \nu^{k-1}$ can be defined as the frequency modified fatigue life. Similarly, comparing the unified creep-fatigue equations (Equations (14) and (17)) with the conventional Coffin-Manson equation (Equation (2)), $\left[c\left(T,\ t,\ \varepsilon_p\right) \right]^{-1/\beta_0} N$ and $\left[c\left(T,t\right) \right]^{-1/\beta_0 b(T,t)} N$ can be defined as the temperature-frequency modified fatigue life (creep-fatigue life). If we get rid of frequency effect from the unified creep-fatigue equations, the creep-fatigue life ($N_{f-\text{creep}+\text{fatigue}}$) can be transformed to temperature modified fatigue life ($N_{f-\text{temp}}$) through Equation (37), then the temperature modified Coffin-Manson equation could be developed through this transformation.

$$N_{f-\text{temp}} = N_{f-\text{creep}+\text{fatigue}} \cdot \nu^{k-1} \tag{37}$$

Because c_1 function in the strain-form unified creep-fatigue equation related to time-temperature relation, it is difficult to remove the frequency effect from this function. Therefore, the power-law unified creep-fatigue equation is used to develop the temperature modified Coffin-Manson equation (Equation (38)) through removing the frequency-related items and transforming creep-fatigue life to temperature modified fatigue life.

$$\varepsilon_p = C_0 \left[1 - c_1 \left(T - T_{\text{ref}} \right) \right] N_{f-\text{temp}}^{-\beta_0 \left[1 - b_1 \left(T - T_{\text{ref}} \right) \right]} \tag{38}$$

Based on the creep-fatigue data [25], the coefficients of this equation are evaluated (shown in Table 1). Then, using the coefficient shown in Table 1, the generated raw fatigue data (ε_p–N) obtained from [25] (T = 723 K, 873 K and 973 K) are transformed to the reference condition ($\varepsilon_{p,\text{ref}}$–N) through Equation (16). The transformed data at "723 K and 873 K", and "873 K and 973 K" are plotted in Figures 9 and 10 respectively.

Table 1. The coefficients of Equation (38).

Temperature Regimes	C_0	c_1	β_0	b_1	k
723 K–873 K	1.997	0.002955	0.62375	−0.000309	723 K: 0.728 873 K: 0.758
873 K–973 K	2.452	0.002668	0.80713	0.00088	873 K: 0.758 973 K: 0.873
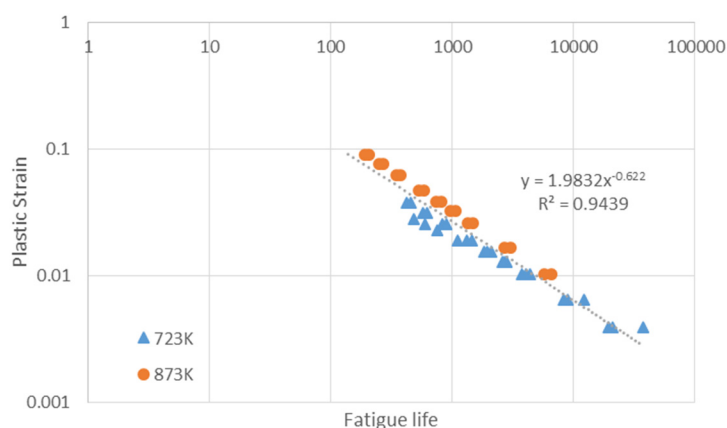
Figure 9. Transformed $\varepsilon_{p,\text{ref}}$–N data (below 873 K) of stainless steel 316 using Equation (17).
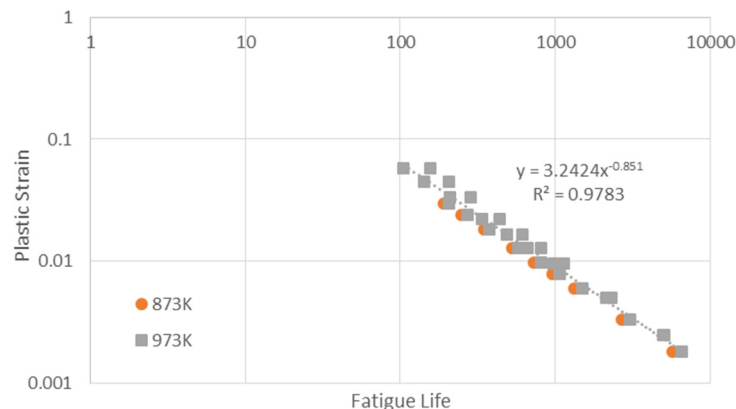
Figure 10. Transformed $\varepsilon_{p,\text{ref}}$–$N$ data (above 873 K) of stainless steel 316 using Equation (17).

Figures 9 and 10 show that the creep-fatigue data below 873 K and above 873 K can be collapsed into power-law curves of $\varepsilon_{p,\text{ref}} = 1.9832N^{-0.622}$ with the quality of fit as $R^2 = 0.9439$ and $\varepsilon_{p,\text{ref}} = 3.2424N^{-0.851}$ with the quality of fit as $R^2 = 0.9783$ respectively. The errors between fatigue life from experiments and creep-fatigue equation for these two temperature regimes are 0.814 and 0.294. This transformation has verified the creep-fatigue-equation-based temperature modified Coffin-Manson equation.

5.5. Application and Future Research

The results have provided a derivation of the equations and the methods for determining the coefficients. While this may appear mathematically complex, it is a necessary feature of the work as it provides reproducibility and assists other researchers in applying and adapting the work.

The implications for practitioners, e.g., nuclear industry, are given in Sections 4.3.3 and 4.4.3, using the two forms of the unified creep-fatigue equation. The equations are provided for stainless steel 316 and encapsulate the data for multiple temperatures, fatigue life, and plastic strain. In application, the equation might be used to determine fatigue life for given conditions (plastic strain, temperature and cycle time), or to determine acceptable plastic strain for a given life and other conditions.

6. Conclusions

The following major results were obtained:

(1) The strain-form unified creep-fatigue equation: $\varepsilon_p = C_0 c\,(T,\ t,\ \varepsilon_p)\,N^{-\beta_0}$ and power-law form: $\varepsilon_p = C_0 c\,(T,t)\,N^{-\beta_0 b(T,t)}$ have been verified on stainless steel 316, and the methods of extracting the fatigue and the creep coefficients by limited creep-fatigue tests and creep rupture tests have been presented. The equations are efficient as they represent a whole range of conditions, in contrast to some other formulations which have to be calibrated separately for each set of environmental conditions.

(2) A moderating factor is introduced into the unified creep-fatigue equation to compress the amplitude of cyclic stress, and leads to a more reasonable result than the situation without moderating factor.

(3) The reliability of this unified creep-fatigue equation is verified through random variables.

(4) The creep-fatigue equation-based, temperature-modified Coffin-Manson equation is proposed, and this equation is validated through transforming raw data to the reference condition.

(5) However, there is a potential limitation. For the materials whose fatigue behaviour is strongly influenced by heat treatment, the coefficients obtained from one specific material condition cannot be used to predict the fatigue life of the same material at material conditions. This is identified as an area for future development of the theory.

Author Contributions: The work was conducted by D.L. and supervised by D.J.P. and E.W. The initial form of the unified creep-fatigue equation was proposed by E.W., and the method of extracting the coefficients was

then developed by D.L. and E.W. The discussion on the moderating factor, heat treatment and reliability was developed by D.L. and D.J.P. The review of creep-fatigue models in the nuclear industry and the evaluation and validation of the unified creep-fatigue equation for stainless steel 316 were conducted by D.L. The initial form of the creep-fatigue equation-based, temperature-modified Coffin-Manson equation was proposed by D.L.

Conflicts of Interest: The authors declare no conflict of interest. The research was conducted without personal financial benefit from any funding body, and no such body influenced the execution of the work.

Appendix A

A1. The Coefficients in the Strain-Form Unified Creep-Fatigue Equation

At pure creep-rupture condition, plastic strain $\varepsilon_p = 0$, then c function can be presented as:

$$c\left(T, t, \sigma\right) = 1 - c_1\left(\sigma\right)\left(T_R - T_{\text{ref}}\right) - c_2\left(\sigma\right) log\left(t_R / t_{\text{ref}}\right) = 0 \tag{A1}$$

where T_R is rupture temperature and t_R is rupture time. According to Manson-Haferd Parameter (Equation (8)), the creep occurs at the point of convergence. At this point, $T_R = T_a = T_{\text{ref}}$ and $t_R = t_a$, then Equation (A1) can be deduced to Equation (A2):

$$c_2 = \frac{1}{\log\left(t_a / t_{\text{ref}}\right)} \tag{A2}$$

This equation shows that c_2 is independent of stress/strain, and it can be obtained through creep rupture test.

In addition, creep rupture also occurs at the reference cyclic time, where $t = t_{\text{ref}}$, then Equation (A1) can be deduced to Equation (A3):

$$c_1\left(\sigma\right) = \frac{1}{T_R - T_{\text{ref}}} \tag{A3}$$

The Manson-Haferd Parameter shows the gradients of T vs. $log t$ curves at different stresses, and keeps constant at one specific stress. The Manson–Haferd Parameter can be regarded as a function of stress $P_{MH}\left(\sigma\right)$. When the stress is converted into plastic strain through Ramberg-Osgood relation, the T vs. $log t$ curves at different strains become nonlinear, and the Manson-Haferd Parameter is presented as the gradient of tangents at one specific strain and temperature. In this case, the Manson-Haferd Parameter can be shown as a function of strain and temperature $P_{MH}\left(\varepsilon_p, T\right)$.

Invoking the Manson–Haferd Parameter (Equation (8)) and Equations (A2) and (A3) can be reduced to Equation (A5):

$$c_1\left(\sigma\right) = -\frac{c_2}{P_{MH}\left(\sigma\right)} = -\frac{c_2}{P_{MH}\left(\varepsilon_p, T\right)} \tag{A4}$$

Then, the c function can be defined through Equations (A2) and (A4):

$$c\left(T, t, \varepsilon_p\right) = 1 + \frac{T - T_{\text{ref}}}{P_{MH}\left(\varepsilon_p, T\right)\Delta\log\left(t_a / t_{\text{ref}}\right)} - \frac{\log\left(t / t_{\text{ref}}\right)}{\log\left(t_a / t_{\text{ref}}\right)} \tag{A5}$$

If plastic strain is expressed as C'_0 when $N = 1$ and $t = t_{\text{ref}}$, Equation (14) can be presented as:

$$C_0\left[1 - c_1\left(C'_0\right)\left(T - T_{\text{ref}}\right)\right] = \varepsilon_p\left(T, t_{\text{ref}},\ N = 1\right) = C'_0 \tag{A6}$$

Because C'_0 can be obtained from a single fatigue test performed at an arbitrary temperature and reference cyclic time, C_0 can be solved numerically. Meanwhile, β_0 also can be evaluated from this single fatigue test.

A2. The Coefficients in the Power-Law Unified Creep-Fatigue Equation

In Equation (18), the coefficient c_2 can be evaluated through Equation (A2).

When $N = 1$, Equation (17) can be represented as:

$$\varepsilon_p \left(T, t, \ N = 1 \right) = C_0 \left[1 - c_1 \left(T - T_{\text{ref}} \right) - c_2 \log \left(t / t_{\text{ref}} \right) \right] \tag{A7}$$

The coefficient C_0 can be extracted through performing fatigue tests at one arbitrary temperature (T_1) at t_{ref} and one arbitrary cyclic time (t_1). Substituting these fatigue data into Equation (A7) gives Equations (A8) and (A9), then C_0 can be solved.

$$\varepsilon_p \left(T, t_{\text{ref}}, \ N = 1 \right) = C_0 \left[1 - c_1 \left(T_1 - T_{\text{ref}} \right) \right] \tag{A8}$$

$$\varepsilon_p \left(T, t_1, \ N = 1 \right) = C_0 \left[1 - c_1 \left(T - T_{\text{ref}} \right) - c_2 \log \left(t_1 / t_{\text{ref}} \right) \right] \tag{A9}$$

The same fatigue experiments performed for the magnitude of C_0 can be used to extract $\beta_0 b_2$ through Equations (A10) and (A11)

$$\beta_0 b \left(T_1, \ t_{\text{ref}} \right) = \beta_0 \left[1 - b_1 \left(T_1 - T_{\text{ref}} \right) \right] \tag{A10}$$

$$\beta_0 b \left(T_1, \ t_1 \right) = \beta_0 \left[1 - b_1 \left(T_1 - T_{\text{ref}} \right) - b_2 \log \left(\ t_1 / t_{\text{ref}} \right) \right] \tag{A11}$$

Next, another set of fatigue testes at another arbitrary temperature (T_2) at t_{ref} is performed to obtain coefficient c_1. When $N = 1$ and $t = t_{\text{ref}}$, Equation (17) can be represented as:

$$\varepsilon_p \left(T, t_{\text{ref}}, \ N = 1 \right) = C_0 \left[1 - c_1 \left(T_2 - T_{\text{ref}} \right) \right] \tag{A12}$$

The combination of Equation (A8) and (A12) gives $-C_0 c_1$, then c_1 can be solved.

In addition, when $T = T_2$ and $t = t_{\text{ref}}$, the exponent of the power-law unified creep-fatigue equation can be represented as:

$$\beta_0 b \left(T, \ t \right) = \beta_0 \left[1 - b_1 \left(T_2 - T_{\text{ref}} \right) \right] \tag{A13}$$

The combination of Equations (A10) and (A13) gives β_0 and b_1. Then β_0 can be used to solve b_2.

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