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A Non-Polynomial Gravity Formulation for Loop Quantum Cosmology Bounce

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Abstract: Recently the so-called mimetic gravity approach has been used to obtain corrections to the Friedmann equation of General Relativity similar to the ones present in loop quantum cosmology. In this paper, we propose an alternative way to derive this modified Friedmann equation via the so-called non-polynomial gravity approach, which consists of adding geometric non-polynomial higher derivative terms to Hilbert–Einstein action, which are nonetheless polynomials and lead to a second-order differential equation in Friedmann–Lemaître–Robertson–Walker space-times. Our explicit action turns out to be a realization of the Helling proposal of effective action with an infinite number of terms. The model is also investigated in the presence of a non-vanishing cosmological constant, and a new exact bounce solution is found and studied.

Keywords: cosmological bounces; non-polynomial gravity; effective action; second-order equations of motion; metric formalism; cosmological constant; perfect fluid; exact regular solutions

1. Introduction

Because they indicate a breakdown of the predictivity of the theory under consideration, it is believed that singularities are not part of nature. It is also well known that General Relativity (GR) plus ordinary matter admits solutions for the space-time metric which are singular. For our purposes, a space-time metric will be singular if there exists ill-defined curvature invariants at some points. Simple and familiar examples are the static Schwarzschild metric and the GR solution with ordinary matter or radiation in a Friedmann–Lemaître–Robertson–Walker (FLRW) space-time. Of course, this is not the most general definition of singular space-time (for example, see [1] and references therein), but it will be sufficient for us.

In this paper, we will only be interested in working within flat FLRW space-times. One possible approach to face this singularity issue may consist of considering high-energy corrections to GR action in order to cure this divergence. Different attempts have been proposed to deal with these divergences, essentially in three directions: either by taking into account quantum gravity corrections, by considering new kinds of matter, or by modifying GR (which is usually equivalent to the introduction of new fields). As a consequence, the resulting modified Friedmann equations may contain regular solutions—for example, bounces. The number of papers dealing with cosmological bounces is quite huge. A partial list of papers is [2–14]. Some recent progress in the general features of the problem has also been presented in [1,15–25].

Within the specific approaches of string theory and loop quantum cosmology (LQC), models with no singularities in their cosmological sector have been proposed; for example, see [26–29] for LQC and [30,31] for string theory.

With regard to the second proposal, Helling—[32] and independently Date and Sengupta [33]—suggested a modification of GR Lagrangian which gives the same correction to the Friedmann equation as LQC, with therefore the same bounce. This approach was intended to be an effective action formulation of the loop quantization procedure of FLRW space-times. Helling showed that a formulation in terms of an infinite sum of curvature invariants is possible, but it was not possible to write it explicitly.

More recently, working within the so-called "mimetic approach", Chamseddine and Mukhanov [34,35] (see also the similar construction in [36]) followed this idea, and in two papers [37,38] made use of a non-polynomial function of the mimetic field in a simple manner, and were able to reproduce the LQC result. Note that within the mimetic approach, but including in the action a suitable potential for the mimetic field, it is possible to show the existence of cosmological bounces. Examples are provided in [35,39]. Furthermore, the bounce mimetic approach has recently been generalized in [40]. Other recent papers on bounce loop cosmology are [41,42], while mimetic modified gravity is discussed in [43].

Furthermore, it should be stressed that when dealing with modified gravity models on FLRW space-times, in general other singularities may arise (e.g., [44] and references therein).

In this paper, we propose the implementation of the Helling construction by finding an explicit Lagrangian built only from the metric field and which leads to the LQC corrections. This Lagrangian is constructed via the so-called non-polynomial gravities (NPGs) [45]. The NPG approach is intended to mimic a specific sector of a fundamental (i.e., background-independent) effective theory, in which only gravitational metric corrections with no additional derivatives are present. In this way, invariants built making use of non-polynomial terms in the metric become polynomials in the FLRW sector, becoming candidates to build an effective action there.

This paper is organized as follows. In Section 2, after expliciting the construction of the scalars, the action is written and the associated equations of motion are derived. In Section 3, in the presence of a cosmological constant and a perfect fluid, the exact solutions of the model are found and discussed. The paper ends with Section 4, in which a discussion of our results is presented.

2. Action and Equations of Motion

To begin with, let us consider a flat Friedmann–Lemaître–Robertson–Walker metric (FLRW) $\bar{g}$ defined by the following space-time interval:

$$ds^2 = -N(t)dt^2 + a(t)^2 d\vec{x}^2. \tag{1}$$

Here $N(t)$ is an arbitrary function which implements the time reparametrization invariance.

We want to build an effective action that reproduces some quantum geometry corrections. For this reason, we are interested in scalars that are built from a particular geometric property of FLRW space-times; namely, that the following projector:

$$\tau^{\alpha}_{\beta} = \delta^{\alpha}_{t}\delta^{t}_{\beta} = \mathrm{diag}(1,0,0,0), \tag{2}$$

is actually a true tensor, and that the quantity $\sqrt{N}\delta^{t}_{\alpha}$ is a true vector in FLRW. In order to see why, we can provide explicit tensorial forms to these objects. Consider the following vector and tensor:

$$V_{\alpha} := \frac{\partial_{\alpha} R}{\sqrt{-\partial_{\sigma} R \partial^{\sigma} R}} \quad \text{and} \quad V_{\alpha\beta} := V_{\alpha} V_{\beta}\,. \tag{3}$$

For the considered metric, these geometric tensors are of order-0; that is, they do not depend on the derivatives of the metric (here, the scale factor). Indeed, denoting the restrictions of the tensors (3) on (1) by $V\rfloor = V\big|_{g=\bar{g}}$, etc., one can see that they are indeed order-0 tensors with the claimed geometrical interpretation:

$$V_{\alpha}\rfloor = -\sqrt{N}\delta^{t}_{\alpha} \quad \text{and} \quad V_{\alpha\beta}\rfloor = -\tau_{\alpha\beta} \tag{4}$$

This property follows from the fact that for any scalar Q, $\partial_\alpha Q$ has only one component when evaluated on (1). We have chosen $Q = R$ here for simplicity. It is exactly the same type of property that the Weyl and Cotton tensors have in spherical symmetry [45–47], except that here the property is quite trivial. See [48,49] for more details in the case of spherical symmetry.

In all classes of space-times that share this property (e.g., spherical symmetric space-times or Bianchi type I), one can build scalars from these tensors that will be second-order in these classes (in particular in (1)), but higher-order otherwise. The two second-order invariants we shall be interested in are

$$K := \frac{1}{9}\left(\nabla^\alpha \nabla^\beta V_{\alpha\beta} - V_\alpha \nabla^\alpha \nabla^\beta V_\beta\right) \quad \text{and} \quad \Omega := \frac{R}{6} - 2K\,. \tag{5}$$

With these invariants, we may construct gravity models denoted NPG in [45], and so we will use this name here. Furthermore, these scalars are chosen so that their restrictions to (1) are:

$$K\Big| = \frac{H^2}{N} \quad \text{and} \quad \Omega\Big| = \frac{\dot{H}}{N} - \frac{H\,\dot{N}}{2N^2}\,, \tag{6}$$

because, together with the Ricci scalar, they form a basis of order-2 scalars in FLRW space-times, and setting $N(t) = 1$, they are actually the simplest ones. Here, H is the Hubble parameter, and $\dot{H} = \frac{dH}{dt}$. Note that working in flat FLRW space-times, there exists other invariants which have similar properties (e.g., [46,50]), but the ones we have chosen are also relevant in spherically symmetric space-times [48].

We recall that, in principle, it is possible to reproduce the loop quantum cosmology modification of the Friedmann equation—and therefore the bounce that replaces the big bang—via higher-order corrections to Einstein–Hilbert action [32,33]. These corrections must lead to second-order equations of motion (as shown by Helling), and so are truly geometrical corrections in the sense that unlike a generic modified gravity model, they do not involve additional fields with no direct geometrical meaning compared to the metric, or compared to the scalar field responsible for the local rescaling invariance in some models of conformal gravity, for example.

In the paper [32], it was also shown that it is possible to write such corrections as an infinite series of polynomials of contractions of Ricci tensors, even though it was not possible to write this effective action explicitly. In our approach, making use of the two scalars (5) defined above, a possible way to achieve this task is to start with the following action:

$$\mathcal{I} = \int d^4x \sqrt{-g}\left(\frac{R - 2\Lambda + \left[\mathscr{L}^{\infty}_{NPG}\right]}{2\kappa} + \mathscr{L}_m\right), \tag{7}$$

where $\kappa = 8\pi$, with the Newton constant $G = 1$, Λ is the cosmological constant, $\mathscr{L}_m$ is the Lagrangian density of matter, and

$$\mathscr{L}^{\infty}_{NPG} = -2\Omega + \frac{4\Omega}{S}\left(1 - \sqrt{1 - S}\right). \tag{8}$$

Here we have introduced the dimensionless scalar $S = \frac{3}{2\pi\rho_c}K$, with ρ_c playing the role of critical density, which in our approach is a *free* dimensional parameter.

Some comments are in order. This contribution—which modifies the GR term—is similar to the Born–Infeld Lagrangian for non-linear electrodynamics and does not follow from first principles. On the other hand, it may be interpreted as

$$\mathscr{L}^{\infty}_{NPG} = -4\sum_{i=0}^{\infty}(-1)^{i+1}\binom{1/2}{1+i}\,S^i\,\Omega\,, \tag{9}$$

where $\binom{n}{m}$ is the generalized binomial coefficient defined by $\binom{n}{m} := \frac{\Gamma(n+1)}{\Gamma(m+1)\Gamma(n-m+1)}$. The brackets in Equation (7) are used to emphasize that within this kind of minisuperspace approach, one can only hope to find the desired scalar up to scalars that vanish or are boundary terms (at least) in the class of space-times in which the reconstruction is done—in our case, in flat FLRW (1). For example, one could add scalars involving the Weyl tensor or background-dependent boundary terms (e.g., those of [46]) in the action without modifying the dynamics of (7) for FLRW space-time. Therefore, $\mathscr{L}^{\infty}_{NPG}$ is only a particular NPG representative of an infinite class of scalars (including polynomial ones like in [32]) with equivalent contributions to the equations of motion in (1).

Furthermore, for this specific space-time, the additional term, despite its non-polynomiality, may be considered in (9) as an infinite sum of polynomials in the metric, and therefore (7) constitutes a suitable effective action, whose coupling constants are fixed in order to reproduce the LQC modification of Friedmann equations. Note also that the $i = 0$ term of the sum (9)—namely $2\,\Omega$—is equivalent to the Ricci scalar in FLRW, since they differ from each other by a total derivative. Moreover, in its present form, the correction $\mathscr{L}^{\infty}_{NPG}$ seems of higher order, but it is in fact equivalent—up to (background-dependent) boundary terms—to the correction of [32,33]. Indeed, they differ from each other by a total derivative:

$$\left(R - 2\Omega + \tfrac{4\Omega}{S}\left(1-\sqrt{1-S}\right)\right)\Big\rfloor = 8\pi\rho_c\left(1-\sqrt{1-S}-\sqrt{S}\arcsin(\sqrt{S})\right)\Big\rfloor + \tfrac{4}{\sqrt{-g}}\sqrt{\tfrac{2\pi\rho_c}{3}}\,\dot{B}\Big\rfloor, \tag{10}$$

with

$$B = \frac{\sqrt{-g}}{\sqrt{N}}\left(\csc^{-1}\left(\frac{1}{\sqrt{S}}\right) - \frac{1-S-\sqrt{1-S}}{\sqrt{S}}\right). \tag{11}$$

Note that the GR contribution is cancelled in both cases, because $\sqrt{-g}\left(R-2\Omega\right)\rfloor = \frac{d}{dt}\left(\frac{4\,a^3 H}{\sqrt{N}}\right)$, and what is left is only a non-polynomial effective action and an effective cosmological constant $8\,\pi\rho_c$ in the first-order form of the right-hand-side. Therefore, in FLRW and up to boundary terms, the sequence (9) of polynomial curvature scalars is the only one that gives the LQC modification of Friedmann equation, as we will see now.

Making use of a minisuperspace approach (Weyl method), from ansatz (1) and action (7) we can derive the Euler–Lagrange (EL) equations of motion by making the variation with respect to Lagrangian coordinates $N(t)$ and $a(t)$. The Principle of symmetric criticality applied to the isometry group of an homogeneous and isotropic universe assures that the reverse process (the right one) will give the same results [51,52].

We also assume that the matter is a perfect fluid, with equation of state $p = w\rho$, ρ and p being the density and the pressure. Making the variation with respect to $N(t)$, one gets the Friedmann equation, and by setting $N(t) = 1$ after the variation, one has

$$4\pi\rho_c\left(1-\sqrt{1-\frac{3H^2}{2\pi\rho_c}}\right) = 8\pi\rho + \Lambda\,. \tag{12}$$

As a first check, when $\frac{H^2}{\rho_c} \ll 1$, one recovers the Friedmann equation of GR.

Defining $\bar{\rho} := \frac{\Lambda}{8\pi} + \rho$, one gets the standard form of the LQC-corrected Friedmann equation:

$$H^2 = \frac{8\pi\bar{\rho}}{3}\left(1-\frac{\bar{\rho}}{\rho_c}\right). \tag{13}$$

Making the variation with respect to $a(t)$, one gets the other Friedmann equation, which contains the acceleration. For our purposes, we do not need it, since it can be derived from (12) and the energy conservation equation

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0\,, \tag{14}$$

as a consequence of the diffeomorphism invariance of our invariant action. Thus, only two of these three equations are independent, and one may use only Equations (12) and (14).

3. Exact Solutions for General Equation of State Parameter W and Cosmological Constant Λ

We recall the equation of state for the perfect fluid $p = w\rho$. Then, introducing for the sake of simplicity $\tilde{\rho} = 8\pi\rho$ and $\mu = \frac{1}{8\pi\rho_c}$, one has

$$\begin{aligned} 3H^2 &= (\tilde{\rho} + \Lambda) - \mu(\tilde{\rho} + \Lambda)^2, \\ \frac{d\tilde{\rho}}{\tilde{\rho}} &= -3(1+w)Hdt. \end{aligned} \tag{15}$$

First we note that without solving the differential equation, it is possible to show that a bounce solution is present; that is there exists $a_* > 0$ such that $H_* = 0$ and $\dot{H}_* > 0$. In fact, the first equation on the bounce $H_* = 0$ gives the condition $1 - \mu\Lambda = \mu\tilde{\rho_*}$; that is, $\mu\Lambda < 1$, which is therefore a necessary condition.

We now derive the exact solution. Inserting the first equation into the second one leads to:

$$\frac{dX}{(X - \Lambda)\sqrt{X - \mu X^2}} = \pm\sqrt{3}(1+w)dt\,, \tag{16}$$

where $X = \rho + \Lambda$. Thus,

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{\Lambda}\sqrt{1-X\mu}}{\sqrt{X}\sqrt{1-\Lambda\mu}}\right)}{\sqrt{\Lambda}\sqrt{1-\Lambda\mu}} = \pm\sqrt{3}(1+w)t + c\,, \tag{17}$$

where c is the integration constant. In the following, we may put $c = 0$ without any problem. Solving in X and thus in $\tilde{\rho}$ gives:

$$\tilde{\rho}(t) = -\frac{2\Lambda(-1+\Lambda\mu)}{-1 + 2\Lambda\mu + \cosh\left(\left(\pm\sqrt{3}t(1+w)\right)\sqrt{\Lambda}\sqrt{1-\Lambda\mu}\right)}. \tag{18}$$

The second equation of (15) admits the usual well-known solution $a = a_0\tilde{\rho}^{\frac{-1}{3(1+w)}}$. As a consequence, one has

$$a(t) = a_0\left(\frac{-1 + 2\Lambda\mu + \cosh\left(\left(\sqrt{3}(1+w)t\right)\sqrt{\Lambda}\sqrt{1-\Lambda\mu}\right)}{2\Lambda(1-\Lambda\mu)}\right)^{\frac{1}{3(1+w)}}. \tag{19}$$

Here, we recover the condition $1 - \mu\Lambda > 0$, $\Lambda > 0$. Given this solution, one can check that the scalar $\partial_\sigma R \partial^\sigma R$ is not vanishing everywhere, and the scalars (5) are indeed well-defined.

As a further check of the solution, we can study the two limits $\mu \to 0$ and $\Lambda \to 0$. First, the GR limit, namely

$$\lim_{\mu\to 0} \tilde{\rho}(t) = \Lambda \operatorname{csch}^2\Big(\frac{1}{2}\sqrt{\Lambda}(\sqrt{3}t(1+w))\Big),$$
$$\lim_{\mu\to 0} a(t) = a_0\Big(\frac{1-\cosh(\sqrt{\Lambda}(\sqrt{3}t(1+w)))}{2\Lambda}\Big)^{\frac{1}{3(1+w)}}. \tag{20}$$

This is the solution of GR with non-vanishing cosmological constant, and one recovers the Big Bang solution at $t = 0$.

In the other limit, one has

$$\lim_{\Lambda\to 0} \tilde{\rho}(t) = \frac{4}{\left(\sqrt{3}t(1+w)\right)^2 + 4\mu},$$
$$\lim_{\Lambda\to 0} a(t) = a_0\Big(\mu + \frac{1}{4}(\sqrt{3}t(1+w))\Big)^{\frac{1}{3(1+w)}}. \tag{21}$$

and one recovers the original LQG bounce solution in the absence of cosmological constant.

Now we study our exact solution with respect to the coordinate time t. We have already shown the existence of the bounce. In particular, for small t , one has

$$a(t\to 0) = a_0\left(\frac{\mu}{1-\mu\Lambda}\right)^{1/3(1+w)}\left(1 + \frac{(1-\mu\Lambda)(1+w)}{4\mu}t^2 + ...\right). \tag{22}$$

We see that the minimal value is $a(0) = a_0\left(\frac{\mu}{1-\mu\Lambda}\right)^{1/3(1+w)}$, corresponding to the bounce. Moreover, already Equation (19) shows that there $a(t)$ is never vanishing: indeed, the hyperbolic cosine is always greater than 1, so $\cosh x - 1 \geq 0$; and since μ and Λ are both positive, the scale factor is always positive and never vanishing.

The other interesting limit is the one for very large t . Since we have already taken the cosmological constant into account, we take $w > -1$. We remind that $\cosh x \to e^{|x|}$, for $x \to \pm\infty$, and one has

$$a(t\to\infty) = \frac{a_0}{(2\Lambda(1-\mu\Lambda))^{1/3(1+w)}}\left(2\mu\Lambda - 1 + \exp\left(\sqrt{3\Lambda(1-\mu\Lambda)}(1+w)t\right)\right)^{1/3(1+w)}, \tag{23}$$

the exponential becomes dominant, corresponding to an accelerating universe. Thus, our solution may represent dark energy (DE), with a chosen suitable scale, and for large t [53,54].

We conclude this Section by discussing the limits μ and large Λ. We have seen that the product $\mu\Lambda$ must be $\mu\Lambda < 1$. This is not a problem for the DE issue, because $\mu = \frac{1}{8\pi\rho_c}$ mimics a quantum correction and thus it can be taken small, because ρ_c is very large, and for DE Λ is small. The situation is different with Λ which is not small (as during inflation), and in this case our solution may not be interesting.

Finally, concerning the scalars used in the construction, given their non-polynomial forms, one could wonder if they are regular at the bounce, like polynomial scalars. One can check that given the solution (19), their behaviours are:

$$\lim_{t\to 0} \partial_\sigma R \partial^\sigma R = \lim_{t\to 0} K = 0,$$
$$\lim_{t\to 0} \nabla^\alpha\nabla^\beta V_{\alpha\beta} = \lim_{t\to 0} V_\alpha \nabla^\alpha\nabla^\beta V_\beta = 3\lim_{t\to 0}\Omega = \frac{-3(1+w)(-1+\Lambda\mu)}{2\mu}; \tag{24}$$

that is, no problem when $\mu \neq 0$.

4. Discussion

In this paper, we have found an explicit covariant Lagrangian formulation of the loop quantum cosmology tree level correction [55] to Friedmann equation in terms of an infinite sum of non-polynomial gravity corrections to Einstein–Hilbert action. We have seen that they constitute a suitable effective action in FLRW because despite their non-polynomiality for general metric fields, their contributions evaluated on FLRW space-times are in fact polynomial. Furthermore, we have found the exact solution of the model in the presence of a positive cosmological constant and a perfect fluid with state parameter w, and we have seen that it represents a bounce that replaces the big bang singularity of GR.

Our result implements quite simply the argument of [32,33], consisting of interpreting the LQC corrections as purely geometrical corrections to GR. In fact, the corrections lead to second-order equations of motion in their FLRW sector, and therefore do not involve additional degrees of freedom with no direct geometrical interpretations—just like the Ricci and Gauss–Bonnet scalars that lead to second-order equations of motion for general metric. In some sense, this might be expected since LQC is a quantum geometry theory, and this implies that within a suitable limit, it could be written as an effective action with initial term the Einstein–Hilbert one, plus high-energy corrections with a direct geometrical interpretation.

For this effective action, such new degrees of freedom would be necessarily present, since the Lovelock theorem [56–58] prevents corrections from being found, involving only the metric field, and with associated second-order differential equations for any metric field (excluding NPG). The effective equations of motion would therefore be higher-order ones for general metric field. This means that the theory would involve additional fields. If such an effective action formulation exists, it must therefore involve new degrees of freedom—at least in some specific backgrounds. In this sense, the NPG approach should be thought of as a convenient and simple way to explore high-energy corrections to the classical degrees of freedom only, in some specific backgrounds, in order to find effective (possibly non-singular) space-time solutions. The case of static spherically symmetric space-times—namely, the case of regular black hole—is investigated in [48].

In our opinion, the mimetic approach followed in [37,38,40] might also be an interesting step to understand a would-be modified gravity formulation of LQG semi-classical corrections, because—at least in a cosmological context—the additional field ϕ of this theory has a direct geometrical meaning $\phi = t$ due to the presence of a Lagrange multiplier. This mimetic approach was also used in [34,39,59–63], and there might be a very large class of theories that have the same property to convert additional fields without a clear geometrical meaning into ones which are related to geometry via Lagrange multipliers.

In order to continue this work, one could try to generalize the construction for more general cosmological models (e.g., Bianchi I). In this special case, the construction of (3) is preserved because once again $\partial_\alpha R$ has only one component, and therefore the NPG approach can be used in a simple way without first searching for new order-0 tensors. However, in this way or even using mimetic gravity, one faces the issue raised earlier that an infinite number of scalars that do not contribute to FLRW dynamics (like those involving the Weyl tensor, boundary and vanishing terms that are such only in FLRW, etc.) could be present. However, a straight and naive generalization of action (7) in the context of NPG could nonetheless lead to interesting results.

We conclude by observing that it might be interesting within this NPG approach to see numerically if a bounce can be obtained at the lowest order of corrections. With regard to the action (7), we used an infinite sum $\mathscr{L}^{\infty}_{NPG} = \sum_{i=0}^{\infty} \alpha_i \, S^i \, \Omega$, and we had the necessity to reconstruct all the constants α_i given the result of LQC. If a bounce can already be obtained for a truncation at order $2(m+1)$ using the corrections $\mathscr{L}^{m}_{NPG} = \sum_{i=0}^{m} \alpha_i \, S^i \, \Omega$, it might be possible to constrain the constant α_i for $i > 2(m+1)$ in order to preserve the bounce. This could be an interesting way to single out a class of possible corrections without the need to reconstruct the whole sequence of constants.

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Abbreviations

The following abbreviations are used in this manuscript:

NPG	Non-polynomial gravity
LQC	Loop quantum cosmology
FLRW	Friedmann–Lemaître–Robertson–Walker space-time
EL	Euler–Lagrange
LQG	Loop quantum gravity

References

1. Lilley, M.; Peter, P. Bouncing alternatives to inflation. *C. R. Phys.* **2015**, *16*, 1038.
2. Khoury, J.; Ovrut, B.A.; Steinhardt, P.J.; Turok, N. The Ekpyrotic universe: Colliding branes and the origin of the hot big bang. *Phys. Rev. D* **2001**, *64*, 123522
3. Tsujikawa, S.; Brandenberger, R.; Finelli, F. On the construction of nonsingular pre—Big bang and ekpyrotic cosmologies and the resulting density perturbations. *Phys. Rev. D* **2002**, *66*, 083513.
4. Piao, Y.-S.; Feng, B.; Zhang, X.-M. Suppressing CMB quadrupole with a bounce from contracting phase to inflation. *Phys. Rev. D* **2004**, *69*, 103520.
5. Guo, Z.-K.; Piao, Y.-S. Obtaining the CMB anomalies with a bounce from the contracting phase to inflation. *Phys. Rev. D* **2013**, *88*, 063539.
6. Novello, M.; Bergliaffa, S.E.P. Bouncing Cosmologies. *Phys. Rept.* **2008**, *463*, 127.
7. Brandenberger, R. Matter Bounce in Horava-Lifshitz Cosmology. *Phys. Rev. D* **2009**, *80*, 043516.
8. Easson, D.A.; Sawicki, I.; Vikman, A. G-Bounce. *J. Cosmol. Astropart. Phys.* **2011**, *1111*, 021.
9. Bamba, K.; Makarenko, A.N.; Myagky, A.N.; Nojiri, S.; Odintsov, S.D. Bounce cosmology from $F(R)$ gravity and $F(R)$ bigravity. *J. Cosmol. Astropart. Phys.* **2014**, *1*, 8.
10. Xue, B.; Garfinkle, D.; Pretorius, F.; Steinhardt, P.J. Nonperturbative analysis of the evolution of cosmological perturbations through a nonsingular bounce. *Phys. Rev. D* **2013**, *88*, 083509.
11. Myrzakulov, R.; Sebastiani, L. Bounce solutions in viscous fluid cosmology. *Astrophys. Space Sci.* **2014**, *352*, 281.
12. Boisseau, B.; Giacomini, H.; Polarski, D.; Starobinsky, A.A. Bouncing Universes in Scalar-Tensor Gravity Models admitting Negative Potentials. *J. Cosmol. Astropart. Phys.* **2015**, *7*, 002.
13. Gielen, S.; Turok, N. Perfect Quantum Cosmological Bounce. *Phys. Rev. Lett.* **2016**, *117*, 021301.
14. Ijjas, A.; Steinhardt, P.J. Fully stable cosmological solutions with a non-singular classical bounce. *Phys. Lett. B* **2017**, *764*, 289.
15. Qiu, T.; Wang, Y.T. G-Bounce Inflation: Towards Nonsingular Inflation Cosmology with Galileon Field. *J. High Energy Phys.* **2015**, *2015*, 130.
16. Dzierzak, P.; Malkiewicz, P.; Piechocki, W. Turning Big Bang into Big Bounce: I. Classical Dynamics. *Phys. Rev. D* **2009**, *80*, 104001.
17. Malkiewicz, P.; Piechocki, W. Foamy structure of spacetime. *arXiv* **2009**, arXiv:0907.4647.
18. Malkiewicz, P.; Piechocki, W. Turning big bang into big bounce: Quantum dynamics. *arXiv* **2010**, arXiv:0908.4029.
19. Nojiri, S.; Odintsov, S.D.; Oikonomou, V.K. Modified Gravity Theories on a Nutshell: Inflation, Bounce and Late-time Evolution. *Phys. Rept.* **2017**, *692*, 1.
20. Odintsov, S.D.; Oikonomou, V.K. Big-Bounce with Finite-time Singularity: The $F(R)$ Gravity Description. *Int. J. Mod. Phys. D* **2017**, *26*, 1750085.
21. Odintsov, S.D.; Oikonomou, V.K. Bouncing cosmology with future singularity from modified gravity. *Phys. Rev. D* **2015**, *92*, 024016.

22. Odintsov, S.D.; Oikonomou, V.K. Matter Bounce Loop Quantum Cosmology from $F(R)$ Gravity. *Phys. Rev. D* **2014**, *90*, 124083.
23. Cai, Y.; Wan, Y.; Li, H.G.; Qiu, T.; Piao, Y.S. The Effective Field Theory of nonsingular cosmology. *J. High Energy Phys.* **2017**, *77*, 090.
24. Cai, Y.; Li, H.G.; Qiu, T.; Piao, Y.S. The Effective Field Theory of nonsingular cosmology: II. *Eur. Phys. J. C* **2017**, *77*, 369.
25. Cai, Y.; Piao, Y.S. A covariant Lagrangian for stable nonsingular bounce. *arXiv* **2017**, arXiv:1705.03401.
26. Bojowald, M. Absence of singularity in loop quantum cosmology. *Phys. Rev. Lett.* **2001**, 86, 5227.
27. Bojowald, M. Loop quantum cosmology. *Living Rev. Rel.* **2005**, *8*, 11.
28. Ashtekar, A.; Pawlowski, T.; Singh, P. Quantum nature of the big bang: Improved dynamics. *Phys. Rev. D* **2006**, *74*, 084003.
29. Ashtekar, A.; Corichi, A.; Singh, P. On the robustness of key features of loop quantum cosmology. *Phys. Rev. D* **2008**, 77, 024046.
30. Helling, R.C.; Policastro, G. String quantization: Fock vs. LQG representations. *arXiv* **2004**, arXiv:hep-th/0409182.
31. Helling, R.C. A lesson from the lqg string: Diffeomorphism covariance is enough. *AIP Conf. Proc.* **2009**, *1196*, 154–160.
32. Helling, R.C. Higher curvature counter terms cause the bounce in loop cosmology. *arXiv* **2009**, arXiv:0912.3011.
33. Date, G.; Sengupta, S. Effective Actions from Loop Quantum Cosmology: Correspondence with Higher Curvature Gravity. *Class. Quant. Grav.* **2009**, *26*, 105002.
34. Chamseddine, A.H.; Mukhanov, V. Mimetic Dark Matter. *J. High Energy Phys.* **2013**, *135*, 1311.
35. Chamseddine, A.H.; Mukhanov, V.; Vikman, A. Cosmology with Mimetic Matter. *J. Cosmol. Astropart. Phys.* **2014**, *2014*, 017.
36. Lim, E.A.; Sawicki, I.; Vikman, A. Dust of Dark Energy. *J. Cosmol. Astropart. Phys.* **2010**, *2016*, 012.
37. Chamseddine, A.H.; Mukhanov, V. Resolving Cosmological Singularities. *J. Cosmol. Astropart. Phys.* **2017**, *2017*, 009.
38. Chasmeddine, A.H.; Mukhanov, V. Nonsingular black hole. *Eur. Phys. J. C* **2017**, *77*, 183.
39. Y. Rabochaya and S. Zerbini, A note on a mimetic scalar–tensor cosmological model. *Eur. Phys. J. C* **2016**, *76*, 85.
40. Liu, H.; Noui, K.; Wilson-Ewing, E.; Langlois, D. Effective loop quantum cosmology as a higher-derivative scalar-tensor theory. *arXiv* **2017**, arXiv:1703.10812.
41. Zhu, T.; Wang, A.; Kirsten, K.; Cleaver, G.; Sheng, Q. Universal features of quantum bounce in loop quantum cosmology. *Phys. Lett. B* **2017**, *773*, 196.
42. Zhu, T.; Wang, A.; Kirsten, K.; Cleaver, G.; Sheng, Q. Pre-inflationary universe in loop quantum cosmology. *arXiv* **2017**, arXiv:1705.07544.
43. Odintsov, S.D.; Oikonomou, V.K. Accelerating cosmologies and the phase structure of $F(R)$ gravity with Lagrange multiplier constraints: A mimetic approach. *Phys. Rev. D* **2016**, *93*, 023517.
44. Nojiri, S.; Odintsov, S.D.; Tsujikawa, S. Properties of singularities in (phantom) dark energy universe. *Phys. Rev. D* **2005**, *71*, 063004.
45. Deser, S.; Sarioglu, O.; Tekin, B. Spherically symmetric solutions of Einstein + non-polynomial gravities. *Gen. Rel. Grav.* **2008**, *40*, 1–7.
46. Colléaux, A.; Zerbini, S. Modified gravity models admitting second order equations of motion. *Entropy* **2015**, *17*, 6643–6662.
47. Bellini, E.; di Criscienzo, R.; Sebastiani, L.; Zerbini, S. Black Hole Entropy for Two Higher Derivative Theories of Gravity. *Entropy* **2010**, *12*, 2186–2198.
48. Colléaux, A. Rational regular black holes in non-polynomial gravity. **2017**, in preparation.
49. Deser, S.; Ryzhov, A.V. Curvature invariants of static spherically symmetric geometries. *Class. Quant. Grav.* **2005**, *22*, 3315–3324.
50. Gao, C. Generalized modified gravity with the second-order acceleration equation. *Phys. Rev. D* **2012**, 86, 103512.
51. Palais, R.S. The principle of symmetric criticality. *Commun. Math. Phys.* **1979**, *69*, 19–30.
52. Torre, C.G. Symmetric Criticality in Classical Field Theory. *AIP Conf. Proc.* **2011**, *1360*, 63–74.

53. Riess, A.G.; Filippenko, A.V.; Challis, P.; Clochiatti, A.; Diercks, A.; Garnavich, P.M.; Gilliland, R.L.; Hogan, C.J.; Jha, S.; Kirchner, R.P. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astrophys. J.* **1998**, *116*, 1009–1038.
54. Perlmutter, S.; Adering, G.; Goldhaberm G.; Knop, R.A.; Nugent, P.; Castro, P.G.; Deustua, S.; Fabbro, S.; Goobar, A.; Groom, D.E.; et al. Measurements of Ω and Λ from 42 high-redshift supernovae. *Astrophys. J.* **1999**, *517*, 565–586.
55. Bojowald, M. Consistent Loop Quantum Cosmology. *Class. Quant. Grav.* **2009**, *26*, 075020.
56. Lovelock, D. Divergence-free tensorial concomitants. *Aequat. Math.* **1970**, *4*, 127–138.
57. Lovelock, D. The Einstein tensor and its generalizations. *J. Math. Phys.* **1971**, *12*, 498–501.
58. Lovelock, D. The four dimensionality of space and the Einstein tensor. *J. Math. Phys.* **1972**, *13*, 874–876.
59. Hammer, K.; Vikman, A. Many Faces of Mimetic Gravity. *arXiv* **2015**, arXiv:1512.09118.
60. Sebastiani, L.; Vagnozzi, S.; Myrzakulov, R. Mimetic gravity: A review of recent developments and applications to cosmology and astrophysics. *Adv. High Energy Phys.* **2017**, *2017*, 3156915.
61. Cognola, G.; Myrzakulov, R.; Sebastiani, L.; Vagnozzi, S.; Zerbini, S. Covariant Horava-like and mimetic Horndeski gravity: Cosmological solutions and perturbations. *Class. Quant. Grav.* **2016**, *33*, 225014.
62. Myrzakulov, R.; Sebastiani, L.; Vagnozzi, S.; Zerbini, S. Static spherically symmetric solutions in mimetic gravity: rotation curves & wormholes. *Class. Quant. Grav.* **2016**, *33*, 125005.
63. Capozziello, S.; Matsumoto, J.; Nojiri, S.; Odintsov, S.D. Dark energy from modified gravity with Lagrange multipliers. *Phys. Lett. B* **2010**, *693*, 198–208.