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Theoretical Derivation of the Cosmological Constant in the Framework of the Hydrodynamic Model of Quantum Gravity: Can the Quantum Vacuum Singularity Be Overcome?

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Abstract: In the present work, it is shown that the problem of the cosmological constant (CC) is practically the consequence of the inadequacy of general relativity to take into account the quantum property of the space. The equations show that the cosmological constant naturally emerges in the hydrodynamic formulation of quantum gravity and that it does not appear in the classical limit because the quantum energy-impulse tensor gives an equal contribution with opposite sign. The work shows that a very large local value of the CC comes from the space where the mass of a quasi-punctual particle is present but that it can be as small as measured on cosmological scale. The theory shows that the small dependence of the CC from the mean mass density of the universe is due to the null contribution coming from the empty space. This fact gives some hints for the explanation of the conundrum of the cosmic coincidence by making a high CC value of the initial instant of universe compatible with the very small one of the present era.

Keywords: quantum gravity; cosmological constant; dark energy; quantum vacuum energy density; cosmic coincidence conundrum; vacuum catastrophe

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1. Introduction

General relativity is an example of a theory of impressive power and simplicity, but its ambit is purely classical. The CC, on the other hand, is a typical example of a modification originally introduced [1] by hand to help the fit of the physical data [2].

Even if unpleasant, it is the sign that a more general theory (*i.e.* quantum gravity (QG)) is necessary for a complete understanding.

In fact, the quantum version of general relativity is in progress [3–9], and its contribution leads to the emergence of the CC as a correction to the general relativity [10].

Its original role of allowing static homogeneous solutions in general relativity equations in the presence of matter [11] (and then discharged by Einstein himself) turned out to be unnecessary when the expansion of the universe was discovered [12]. Nevertheless, there have been a number of subsequent episodes in which a non-zero CC was considered as an explanation of astronomical observations [13–16].

Meanwhile, particle physicists have realized that the CC can be interpreted as a measure of the energy density of the vacuum [10–18]. This energy density is the sum of a number of contributions, each of magnitude much larger than the upper limits of the CC known today. The question of why the observed cosmological vacuum energy is so small compared to that deriving by the scales of particle physics has become a celebrated puzzle.

By using the hydrodynamic model of quantum gravity [19], the present work shows that the CC naturally appears in the theory. The outcome shows that the energy-impulse tensor density (QEITD) introduces a quantum contribution that leads to a non-zero CC-value only in the place where the mass of a particle is present, and globally to an observationally accessible CC.

The paper is organized as follows: In the first section, the hydrodynamic model of quantum gravity is synthetically resumed, and the emergence of the CC in the gravitational equations is shown. In the second part of the work, the theoretical structure of the CC is analyzed and discussed respect to its large-scale value and time-dependence.

2. The Hydrodynamic Approach to Quantum Gravity

The author has shown [19] that the hydrodynamic model of quantum gravity leads to the equation

$$R_{\nu\mu} - \frac{1}{2} g_{\nu\mu} R_{\alpha}{}^{\alpha} = \frac{8\pi G}{c^4} \left(T_{\nu\mu} + \Lambda g_{\nu\mu} \right), \tag{1}$$

where $T_{\nu\mu}$ is the covariant quantum energy-impulse tensor (QEITD) and where

$$\Lambda = -\frac{8\pi G}{c^2} \frac{m|\psi|^2}{\gamma} \tag{2}$$

is coupled to the quantum equations (defined in Section 2.2).

2.1. The QEITD Derived by the Hydrodynamic Quantum Model

In order to obtain the covariant QEITD, it is enough to calculate its contravariant form in the flat Minkowski space-time.

In this case, the quantum hydrodynamic equations of motion (for scalar uncharged particles), as shown by Guvenis [20], read

$$g^{\mu\nu} \frac{\partial S_{(q,t)}}{\partial q^{\mu}} \frac{\partial S_{(q,t)}}{\partial q^{\nu}} - \hbar^2 \frac{\partial_{\mu} \partial^{\mu} |\psi|}{|\psi|} - m^2 c^2 = 0 \tag{3}$$

and

$$\frac{\partial}{\partial q_{\mu}} \left(|\psi|^2 \frac{\partial S}{\partial q^{\mu}} \right) = \frac{\partial J_{\mu}}{\partial q_{\mu}} = 0, \tag{4}$$

where

$$J_{\mu} = \frac{i\hbar}{2m} (\psi * \frac{\partial \psi}{\partial q^{\mu}} - \psi \frac{\partial \psi *}{\partial q^{\mu}}) \tag{5}$$

is the 4-current, where

$$S = \frac{\hbar}{2i} \ln[\frac{\psi}{\psi *}], \tag{6}$$

and where the connection between the standard quantum notations and the hydrodynamic ones is given by the relation

$$\psi = |\psi| \exp[\frac{iS}{\hbar}]. \tag{7}$$

Equations (3) and (4), describing the (quantum) motion of the mass density $\mu = m|\psi|^2$ (generalized to the non-Euclidean space in Section 2.2), are coupled with Equation (1) through the QEITD [19]

$$T_{\mu}{}^{\nu} = \dot{q}_{\mu} \frac{\partial \boldsymbol{L}}{\partial \dot{q}_{\nu}} - \boldsymbol{L} \delta_{\mu}{}^{\nu} = |\psi|^2 \left(\dot{q}_{\mu} \frac{\partial L}{\partial \dot{q}_{\nu}} - L \delta_{\mu}{}^{\nu} \right), \tag{8}$$

where

$$\boldsymbol{L} = |\psi|_2 L \tag{9}$$

is the Lagrangian density, and L is the hydrodynamic Lagrangian function that defines the quantum hydrodynamic equations of motion as a function of the couple of real variables $|\psi|$ and S [20–23] where the momentum reads

$$p_\mu = (\frac{E}{c}, -p_i) = -\frac{\partial S}{\partial q^\mu}. \tag{10}$$

The quantum hydrodynamic equation of motion (3) that, in the Lagrangian form, reads [19]

$$p_\mu = -\frac{\partial L}{\partial \dot{q}^\mu}. \tag{11}$$

and

$$\dot{p}_\mu = -\frac{\partial L}{\partial q^\mu}, \tag{12}$$

where

$$L = \frac{dS}{dt} = \frac{\partial S}{\partial t} + \frac{\partial S}{\partial q_i}\dot{q}_i = -p_\mu \dot{q}^\mu \tag{13}$$

is coupled to the current conservation Equation (4) where the 4-current J_μ Equation (6), after simple manipulation, reads [19]

$$J_\mu = (c\rho, -J_i) = -|\psi|^2 \frac{p_\mu}{m} = \rho \dot{q}_\mu, \tag{14}$$

where

$$\rho = -\frac{|\psi|^2}{mc^2}\frac{\partial S}{\partial t}. \tag{15}$$

Moreover, by using Equations (14) and (15) it follows that

$$p_\mu = -m\frac{\rho}{|\psi|^2}\dot{q}_\mu = -\frac{1}{c^2}\frac{\partial S}{\partial t}\dot{q}_\mu, \tag{16}$$

and, hence, that

$$\begin{aligned} L &= -p_\mu \dot{q}^\mu = c^2 \left(\tfrac{\partial S}{\partial t}\right)^{-1} p_\mu p^\mu = \rho^{-1} J_\mu p^\mu \\ &= -\tfrac{i\hbar}{2} c^2 \left(\frac{\partial \ln[\frac{\psi}{\psi *}]}{\partial t}\right)^{-1} \frac{\partial \ln[\frac{\psi}{\psi *}]}{\partial q^\mu} \frac{\partial \ln[\frac{\psi}{\psi *}]}{\partial q_{\mu i}} \end{aligned} \tag{17}$$

and

$$\begin{aligned} T_\mu{}^\nu &= |\psi|^2 c^2 \left(\tfrac{\partial S}{\partial t}\right)^{-1} \left(p_\mu p^\nu - p_\alpha p^\alpha \delta_\mu{}^\nu\right) \\ &= |\psi|^2 m^2 c^4 \left(\tfrac{\partial S}{\partial t}\right)^{-1} \left(\tfrac{p_\mu p^\nu}{m^2 c^2} - \left(1 - \tfrac{V_{qu}}{mc^2}\right)\delta_\mu{}^\nu\right) \\ &= m|\psi|^2 c^2 \left(\tfrac{1}{mc^2}\tfrac{\partial S}{\partial t}\right)^{-1} \left(u_\mu u^\nu - \left(1 - \tfrac{V_{qu}}{mc^2}\right)\delta_\mu{}^\nu\right) \\ &= m|\psi|^2 c^2 \left(\frac{\frac{\hbar}{2im^2c^2}\partial \ln[\frac{\psi}{\psi *}]}{\partial t}\right)^{-1} \left(\begin{array}{c} \left(\frac{\hbar}{2mc}\right)^2 \frac{\partial \ln[\frac{\psi}{\psi *}]}{\partial q^\mu} \frac{\partial \ln[\frac{\psi}{\psi *}]}{\partial q_\nu} \\ + \left(1 - \frac{V_{qu}}{mc^2}\right)\delta_\mu{}^\nu \end{array}\right) \end{aligned} \tag{18}$$

where $u_\mu = \frac{\gamma}{c}\dot{q}_\mu$ and where

$$\frac{\partial S}{\partial q^\mu}\frac{\partial S}{\partial q_\mu} = p_\mu p^\mu = \left(\frac{E^2}{c^2} - p^2\right) = m^2 c^2 \left(1 - \frac{V_{qu}}{mc^2}\right), \tag{19}$$

has been used in [19], where the quantum potential reads $V_{qu} = -\frac{\hbar^2}{m}\frac{\partial_\mu \partial^\mu |\psi|}{|\psi|}$. As shown by Bialiniki-Birula *et al.* and by the author himself [19–23], it is noteworthy to observe that, due to the biunique relation between the quantum hydrodynamic approach (with the quantization bond) and the standard quantum equations [21–24], Equations (3) and (4) are equivalent to the Klein-Gordon equation (KGE) [19–23]

$$g^{\mu\nu}\partial_\nu\partial_\mu\psi = -\frac{m^2c^2}{\hbar^2}\psi, \tag{20}$$

and, hence, the QEITD (18) makes the quantum gravity Equation (1) independent by the hydrodynamic approach used to derive it.

2.2. The Quantum Gravity System of Equations

Now we have all the equations of the system that define the quantum gravitational evolution.

The Einstein Equation (1) is coupled to the KGE that (for scalar uncharged particles) in a non-Euclidean space-time reads

$$\partial^\mu\psi_{;\mu} = \frac{1}{\sqrt{-g}}\partial^\mu\sqrt{-g}\left(g^{\mu\nu}\partial_\nu\psi\right) = -\frac{m^2c^2}{\hbar^2}\psi, \tag{21}$$

with the covariant form of the QEITD (8) $T_{\mu\nu} = T_\mu{}^\alpha g_{\alpha\nu}$ that includes the quantum contribution, given by the quantum potential, that reads

$$V_{qu} = -\frac{\hbar^2}{m}\frac{1}{|\psi|\sqrt{-g}}\partial^\mu\sqrt{-g}\left(g^{\mu\nu}\partial_\nu|\psi|\right). \tag{22}$$

3. The Cosmological Constant

In this section, we derive the cosmological constant Λ by requiring that the classical Einstein equation and the minimum action principle (MAP) are recovered in the classical limit.

By considering, for the sake of simplicity, states of matter (or antimatter), without mixed superposition of them, we obtain the identity [19]

$$-\left(\frac{1}{c^2}\frac{\partial S}{\partial t}\right) = \frac{E}{c^2} = \pm m\gamma\sqrt{1-\frac{V_{qu}}{mc^2}}, \tag{23}$$

where, according to the Dirac interpretation, the negative energy states apply to the antimatter. By using Equation (23), the QEITD (18) reads

$$\begin{aligned} T_\mu{}^\nu &= |\psi|^2c^2\left(\tfrac{\partial S}{\partial t}\right)^{-1}\left(p_\mu p^\nu - p_\alpha p^\alpha\delta_\mu{}^\nu\right) \\ &= |\psi|^2\left(\tfrac{c^2}{\gamma^2}\left(\tfrac{1}{c^2}\tfrac{\partial S}{\partial t}\right)u_\mu u^\nu - m^2c^2\left(\tfrac{1}{c^2}\tfrac{\partial S}{\partial t}\right)^{-1}\left(1-\tfrac{V_{qu}}{mc^2}\right)\delta_\mu{}^\nu\right) \\ &= -(\pm)\,\tfrac{m|\psi_\pm|^2c^2}{\gamma}\left(\sqrt{1-\tfrac{V_{qu}}{mc^2}}\right)\left(u_\mu u^\nu - \delta_\mu{}^\nu\right), \end{aligned} \tag{24}$$

where $|\psi_+|^2$ and $|\psi_-|^2$ are the matter and antimatter particle density, respectively, so that we can explicitly write the hydrodynamic motion equation [19], which reads

$$\sqrt{1-\frac{V_{qu}}{mc^2}}\frac{du_\mu}{ds} = -u_\mu\frac{d}{ds}\left(\sqrt{1-\frac{V_{qu}}{mc^2}}\right) + \frac{\partial}{\partial q^\mu}\left(\sqrt{1-\frac{V_{qu}}{mc^2}}\right) = \pm\frac{\gamma}{mc^2}\frac{\partial \mathrm{T}_\mu{}^\nu}{\partial q^\nu}, \tag{25}$$

where the quantum energy-impulse tensor $\mathrm{T}_\mu{}^\nu = \frac{T_\mu{}^\nu}{|\psi|^2}$ [19] reads

$$\mathrm{T}_\mu{}^\nu = \frac{T_\mu{}^\nu}{|\psi_\pm|^2} = (\pm) - \frac{mc^2}{\gamma}\sqrt{1-\frac{V_{qu}}{mc^2}}\left(u_\mu u^\nu - \delta_\mu{}^\nu\right). \tag{26}$$

Moreover, since, from the mechanical point of view,

$$\frac{\partial \Lambda_{(\dot{q},t)} \delta_\mu{}^\nu}{\partial q^\nu} = 0, \tag{27}$$

it follows that all QEITD tensors of the form

$$T_\nu{}^\mu \equiv T_\nu{}^\mu + \Lambda_{(\dot{q},t)} \delta_\nu{}^\mu \tag{28}$$

are able to lead to the same motion equation (in Euclidean space). Nevertheless, from the point of view of general relativity, since, in non-Euclidean space, the motion Equation (25)

$$\begin{aligned} &\tfrac{du_\mu}{ds} - \tfrac{1}{2}\tfrac{\partial g_{\lambda\kappa}}{\partial q^\mu} u^\lambda u^\kappa \\ &= -u_\mu \tfrac{d}{ds}\left(ln\sqrt{1-\tfrac{V_{qu}}{mc^2}}\right) + \tfrac{\partial}{\partial q^\mu}\left(ln\sqrt{1-\tfrac{V_{qu}}{mc^2}}\right) \end{aligned} \tag{29}$$

depends by the metric tensor $g_{\lambda\kappa}$, which, by Equation (1), is a function of $\Lambda_{(\dot{q},t)}$, only one form (that minimizes the action) of $\Lambda_{(\dot{q},\ t)}$ is possiblein Equation (28) .

In order to determine $\Lambda_{(\dot{q},t)}$, we require that the Einstein equation (which minimizes the action) is recovered by Equation (1) in the classical limit (*i.e.* $\hbar \to 0, V_{qu} \to 0$).

By imposing this condition on Equation (1), the explicit Expression (2) is obtained for the CC, which leads to the quantum gravitational equation

$$R_{\nu\mu} - \frac{1}{2} g_{\nu\mu} R_\alpha{}^\alpha + \frac{8\pi G}{c^2}\frac{\mu}{\gamma} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\nu\mu}. \tag{30}$$

Moreover, if we write the QEITD $T_{\nu\mu}$ as

$$T_{\nu\mu} = \left(\frac{1}{\gamma mc^2}\frac{\partial S}{\partial t}\right)^{-1}\left(T_{\nu\mu\ \mathrm{CL}} - \frac{\mu c^2}{\gamma}\left(1 - \frac{V_{qu}}{mc^2}\right) g_{\mu\nu}\right), \tag{31}$$

where

$$T_{\nu\mu\ \mathrm{CL}} = \frac{\mu c^2}{\gamma} u_\mu u_\nu \tag{32}$$

is the classical part of the QEITD (*i.e.* for dust matter [21]), Equation (30) reads

$$R_{\nu\mu} - \frac{1}{2} g_{\nu\mu} R_\alpha{}^\alpha - \delta\Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}\left(\frac{1}{\gamma mc^2}\frac{\partial S}{\partial t}\right)^{-1} T_{\nu\mu\ \mathrm{CL}}, \tag{33}$$

where $\delta\Lambda$, the overall CC, reads

$$\delta\Lambda = \left[1 + \left(\frac{1}{\gamma mc^2}\frac{\partial S}{\partial t}\right)^{-1}\left(1 - \frac{V_{qu}}{mc^2}\right)\right]\Lambda = \left[1 - \sqrt{1 - \frac{V_{qu}}{mc^2}}\right]\Lambda. \tag{34}$$

In the classical limit (when $V_{qu} = 0$, $\sqrt{1-\frac{V_{qu}}{mc^2}} = 1$), $\delta\Lambda$ is equal to zero, and the general relativity equation is recovered without the cosmological term.

As far as the vacuum is concerned, which is characterizd by the values $\psi = |\psi| = 0$, $V_{qu} = 0$, it follows that

$$\delta\Lambda = \left[1 - \sqrt{1 - \frac{V_{qu}}{mc^2}}\right]\Lambda = 0. \tag{35}$$

4. The Cosmological Constant in Presence of Matter

If, in the vacuum, the overall CC $\delta\Lambda$ is null, in the regions of space where the matter is present and localized in particles (*i.e.* $\sqrt{1-\frac{V_{qu}}{mc^2}} \neq 1$), the exact cancellation of the CC does not happen. Since the quantum potential is larger wherethe mass concentration is higher, the CC becomes relevant in places where the matter is localized in quasi-punctual particles.

The maximal contribution from the cosmological term comes from those regions of space where the quantum potential is of order of the mass energy (as happens for the Plank mass black holes [19]). In this case, we can set $V_{qu} = (1-\varepsilon)\, mc^2$ with $\varepsilon << 1$ so that the overall CC reads

$$\begin{aligned} \delta\Lambda = \; & \Lambda(1-\gamma^{-1}\sqrt{1-\frac{V_{qu}}{mc^2}}) = \frac{8\pi\; Gm|\psi_+|^2}{\gamma\; c^2}(1-\gamma^{-1}\sqrt{1-\frac{V_{qu}}{mc^2}}) \\ = \; & \frac{8\pi\; m|\psi_+|^2}{\gamma\; c^2}(1-\frac{\varepsilon}{\gamma}) \end{aligned} \quad (36)$$

which, since the contribution from low velocity matter is dominant (e.g., for neutrinos whose speed is close to that of light, it follows that $\gamma \to \infty$ and $\Lambda \to 0$), leads to

$$\delta\Lambda \approx \frac{8\pi G\; m|\psi_+|^2}{c^2}(1-\varepsilon) \quad (37)$$

Since the system of Equations (1)–(4) and (18) only considers the gravitational interaction, the only possible localized mass distribution is the black hole (BH) whose smallest mass has the value of the Planck mass [19].

In this case, for a "particle" of a Planck mass localized in the sphere of its gravitational radius $r_g = \frac{2Gm}{c^2} \approx 0.8 \times 10^{-35} m$ [19], we have $|\psi_+|_2 \approx 0.5 \times 10^{105} m^{-3}$ and $V_{qu} = mc^2$ [19] (so that $\varepsilon = 0$), that , introduced in Equation (37), gives the local value of

$$\delta\Lambda \approx \frac{8\pi G\; m|\psi_+|^2}{c^2} \cong 4.9 \times 10^{70}. \quad (38)$$

On the other hand, the mass density is practically null outside the BH radius so that

$$\psi_{+(x)} = 0, \quad (39)$$

as well as

$$V_{qu} = 0, \quad (40)$$

so that

$$\delta\Lambda = 0. \quad (41)$$

Thence, if we consider the mean value of $\delta\Lambda$ (which gives rise to the cosmological effect)

$$< \delta\Lambda > \approx \frac{8\pi G\Omega}{c^2}, \quad (42)$$

where Ω represents the mean mass density of the nowadays universe [24], whose estimated value is

$$\Omega \approx 9.66 \times 10^{-27} kg/m^3, \quad (43)$$

it follows that the ratio $\frac{<\delta\Lambda>}{\delta\Lambda}$ for a BH of Planck mass (given that the universe mean number of black holes with the Planck mass, per cubic meter, is $\frac{\Omega}{m_P} \approx 2 \times 10^{-7}\; m^{-3}$) reads

$$\frac{< \delta\Lambda >}{\delta\Lambda} = \frac{\Omega_{BH}}{m_P}\frac{1}{|\psi_+|^2} \approx 2.5 \times 10^{-113}, \quad (44)$$

which shows an enormous difference between the local values of the CC respect to its mean on cosmological scale.

Given that the mass density of universe is $2 \times 10^{-7}\ m^{-3}$ BH of Planck mass per cubic meter, which own a density of $|\psi_+|^2 \approx 0.5 \times 10^{105}\ m^{-3}$ in a volume $\frac{4}{3}\ \pi {r_g}^3 \approx 2 \times 10^{-105}\ m$, we obtain $\Omega \approx m_P \times 2 \times 10^{-7}\ m^{-3} =< m_P|\psi_+|^2 >$.

5. The Cosmological Constant of Particles with Mass Smaller than the Planck One

To evaluate the local value of the CC for a localized mass m (not due to gravitational force), we have to include into the gravitational problem the other interactions of the nature (in fact, Equation (3) is just for an uncharged scalar particle).

To introduce an external potential, in order to have the localization of a mass distribution, we have to consider at least the electromagnetic (em) force.

In this case (for Euclidean space, for instance), the KGE

$$g_{\mu\nu}\left(\partial^\nu - \frac{e}{i\hbar}A^\nu\right)\left(\partial^\mu - \frac{e}{i\hbar}A^\mu\right)\varphi + \frac{m^2c^2}{\hbar^2}\varphi = 0 \tag{45}$$

or the Dirac equation

$$\left(i\hbar\gamma^\mu\left(\partial_\mu + \frac{ie}{\hbar}A_\mu\right) + mc\right)\ \Psi = \frac{\partial L_{int}}{\partial\psi} \tag{46}$$

(where $\frac{\partial L_{int}}{\partial\phi} = 0$ for the em field) are coupled to the Maxwell one

$$F^{\mu\nu}{}_{;\nu} = -4\pi\ J^\mu \tag{47}$$

for the photon [25]. If we want to include the "strong" force of fermions (e.g., due to a scalar meson), we must add the field equation of the meson (like the Maxwell one for the photon),

$$g_{\mu\nu}\partial^\nu\partial^\mu\phi + \frac{m^2c^2}{\hbar^2}\phi - j_{(x)} = \frac{\partial L_{int}}{\partial\phi} \tag{48}$$

(where, for instance, $j_{(x)} = -\frac{\partial V_{(\phi)}}{\partial\phi}$, where $V_{(\phi)}$ is the "self-interacting" field that, in renormalizable quartic interaction, reads $V_{(\phi)} = \lambda\phi^4$ [26]) and the fermion-meson interaction term

$$L_{int} = -g\overline{\psi}\phi\psi \tag{49}$$

All these equations are coupled each other and, through the QEITD, with the gravitational one that defines the metric tensor and, thence, the covariant derivatives. However, the definition of the QEITD through the hydrodynamic representation of quantum mechanics is still not available for the strong and weak force. Thence, to evaluate the order of magnitude of the CC due to mass localization, in the appendix, the CC is calculated for localized particles in quantum wells.

Moreover, if the localized mass is much smaller than that of the Planck mass, we can approximately substitute the covariant derivatives in Equations (45)–(49) with the normal derivatives.

By using this approximation, we do not need to solve the coupled gravitational system of motion equations, but the local overall CC can be evaluated by using the Euclidean limit of quantum equations. From Equation (44) and from the outputs given in the appendix (A.11–A.12), we can see that the mean value $\frac{8\pi G\ \Omega}{c^2}$ of the CC can differ by many orders of magnitude by the local one $\frac{8\pi G\ m|\psi_+|^2}{c^2}$. On the other hand, the huge local value of the CC in the place where the mass of a particle is concentrated (e.g., in a volume whose radius is of the order of the Compton length) can lead to a cosmological value that agrees in order of magnitude with that astronomically measured as a consequence of the dilution in a large vacuum space (see Appendix).

6. Connection with the Quantum Field Theory

As for the quantum mechanics, the quantum field approach also has its own hydrodynamic representation that, as shown by Bhom and colleagues [27], makes use of the "super-quantum potential."

More recently, Koide and Kodama [28] showed that the hydrodynamic quantum field model can also be obtained by means of the stochastic variational method.

If in the "mechanical" approach we have in the vacuum $|\psi|_2 = 0$ in the quantum field approach, assigned the probability distribution (PD) $\Psi(\psi(x,t))$, even if the mean value $\overline{\psi}$ in the vacuum is equal to zero, the variance $\overline{\psi\psi * - \overline{\psi}\overline{\psi *}} = \overline{|\psi|^2} \neq 0$ is not null given that, generally speaking, the PD $\Psi(\psi, x, t)$ is not a delta-function. This leads to the description of the vacuum as a sea of virtual particles and antiparticles (with zero mean real particles) but with an intrinsic zero-point energy density.

Thence, since for the quantum vacuum $\overline{|\psi|^2} \neq 0$, if we make the identification $\Lambda_{vacuum} \equiv \overline{\Lambda}$, it follows that

$$\Lambda_{vacuum} \equiv \overline{\Lambda} = -\frac{8\pi G}{c^2}\frac{m\overline{|\psi|^2}}{\gamma} \neq 0 \tag{50}$$

and, hence,

$$< \Lambda_{vacuum} > \equiv < \overline{\Lambda} > \neq 0 \tag{51}$$

Nevertheless, by Equation (35) for the vacuum (*i.e.* $V_{qu} = 0$), it follows that

$$< \delta\Lambda > = < \left[1 - \sqrt{1 - \frac{V_{qu}}{mc^2}}\right]\overline{\Lambda} > = [1-1] < \overline{\Lambda} > = 0 \tag{52}$$

so that, again, the overall CC $< \delta\Lambda >$ has a null contribution from the zero-point vacuum fluctuations.

In order to establish an analytical link between the absence of the quantum vacuum singularity of the hydrodynamic model and the quantum vacuum catastrophe within the standard QED and QCD, the present work envisages that the hydrodynamic model of the quantum field [27] in non-Euclidean space-time has to be investigated.

7. Critical Overview

The derivation of the QG equations show that CC $\Lambda_{(\dot{q},\, t)}$ warrants that, in the classical limit, the Einstein equation (which satisfies the MAP) is recovered.

Nevertheless, the recovery of the classical gravity equation is not exact so as to fully derive the QG by the MAP.

The application of the MAP to the hydrodynamic equation poses some questions given that, among the hydrodynamic solutions of Equation (25), only those that own the "irrotational" property [19,21] (which warrants the quantization condition) describe the quantum mechanics. This fact generates an interplay between the MAP and the quantization postulate.

The full evaluation of such problem goes beyond the purpose of the present paper and has been left to be investigated in a future work.

Furthermore, it must be observed that, in the term

$$< m|\psi_+|^2 > \approx \frac{c^2}{8\pi G} < \delta\Lambda >, \tag{53}$$

represents the mean mass density of the nowaday universe based on some implicit assumptions that deserve a comment. $m|\psi_+|^2$ represents the total matter density if we attribute the classical meaning to it. In fact, the term $|\psi_+|^2$ has quantum properties with a contribution from the quantum superposition of states that can be disregarded just in the "macroscopic" classic universe, where the quantum decoherence takes place [29–31].

For the initial instant of the universe, especially before of the inflation, when both the gravity and the quantum laws were all contemporaneously effective and strongly coupled, the value of $<|\psi_+|_2>$ had to be very different from the one it owns today since the mass of the particles was concentrated in a very small volume with an overall quantum coherence.

Finally, it is worth mentioning that, if the low curvature limit of Equation (1) in the classical case (*i.e.* $\hbar = 0 \;\; V_{qu} = 0$) leads to the Newtonian gravity, the quantum corrections (for $V_{qu} \neq 0$) in the Galilean limit of Equation (1) [32] can furnish a excellent experimental confirmation of the present model.

8. Conclusions

In the present work it has been shown that the CC originates by the quantum property of the vacuum where the particles are quasi-punctual and their mass is not smoothly distributed all around the space.

The equations show that the cosmological constant emerges in the hydrodynamic formulation of quantum gravity in order to obtain the Einstein equation in the classical limit that satisfies the MAP.

This work shows that the CC does not appear in the classical limit since it is deleted by an equal contribution, with opposite sign, coming from the quantum energy-impulse tensor.

This paper also shows that the contribution to the CC of the vacuum in the absence of matter is null as a consequence of the subtraction of an equal amount coming from the QEITD.

The outputs of the theory show that the overall CC can own a very large local value only in the place where the mass distribution of a quasi-punctual particle is present and that its effect can be as small as measured on cosmological scale. The theory shows that the CC owns a time dependence coming from the state of the universe. This fact makes possible a high CC value, at the initial instant of universe, compatibly with the very small one of the nowaday universe.

Appendix

The local cosmological constant value of a particle in a quantum well.

For a particle in a quantum well of infinite height whose side is equal to $a = \sqrt{3}\frac{\hbar}{2mc}$ (*i.e.* of the order of the Compton length so that the modulus of its momentum in the fundamental state is equal to mc), the relativistic wave function

$$\psi_{\pm(x)} = \frac{1}{a^{3/2}}\exp[\pm i\frac{\boldsymbol{p}}{\hbar}x]\exp[\pm i\frac{E}{\hbar}t] \tag{A.1}$$

where

$$E^2 = \boldsymbol{p}^2c^2 + m^2c^4 \tag{A.2}$$

undergoes the quantum restriction $\psi_{(x=\pm\frac{a}{2})} = 0$ to the edge of the well

$$\boldsymbol{a} = (\pm\frac{a}{2}, \pm\frac{a}{2}, \pm\frac{a}{2}), \tag{A.3}$$

which, for $-\frac{a}{2} \leqslant x_i \leqslant \frac{a}{2}$, leads to the wave function

$$\psi_{n_\pm(x)} = \frac{\psi_{+(x)} + \psi_{-(x)}}{\sqrt{2}} = \frac{1}{\sqrt{2}a^{3/2}}\cos[\pm i\frac{\boldsymbol{p_n}}{\hbar}x]\exp[\pm i\frac{E_n}{\hbar}t] \tag{A.4}$$

with

$$\boldsymbol{p_n} = \frac{(2n+1)}{\sqrt{3}}mc(1,1,1); \tag{A.5}$$

hence, the quantum potential value for the fundamental state reads

$$V_{qu} = -\frac{\hbar^2}{m}\frac{\partial_\mu \partial^\mu |\psi|}{|\psi|} = \frac{\hbar^2}{m}\sum_i \left(\frac{p_i}{\hbar}\right)^2 = mc^2. \tag{A.6}$$

On the other hand, outside the potential well, we have that

$$\psi_{\pm(x)} = 0, \tag{A.7}$$

which leads to

$$V_{qu} = 0. \tag{A.8}$$

Therefore, inside the volume where the particle is localized, the net CC reads

$$\delta\Lambda \approx \frac{8\pi G\, m|\psi_+|^2}{c^2}, \tag{A.9}$$

while, outside,

$$\delta\Lambda = 0. \tag{A.10}$$

Thence, as a function of mass particle and potential well wideness, the local ratio $\frac{<\delta\Lambda>}{\delta\Lambda}$ reads:

(i) Particle of a proton mass and potential well of side $a \approx 10^{-15}m$.

In this case, we have $|\psi_+|_2 \approx 10^{45}m^{-3}$, and, since the mean mass density of he universe is about 5.5 hydrogen atoms per m^3 (including the dark matter and the dark energy) leading to a value $\frac{\Omega}{m_+} \approx 5.5\ m^{-\ 3}$, it follows that

$$\frac{<\delta\Lambda>}{\delta\Lambda} = \frac{\Omega}{m_+}\frac{1}{|\psi_+|^2} \approx 5.5 \times 10^{-45}. \tag{A.11}$$

Given that $\frac{\Omega}{m_+} \approx 5.5\ m^{-\ 3}$ protons per cubic meter with a proton density $|\psi_+|_2 \approx 10^{45}m^{-3}$ in a volume $a^3 = 10^{-45}$, it follows that $<|\psi_+|_2> \approx 5.5m^{-3}$ and, hence, that $\Omega \approx m_+ 5.5\ m^{-3} = < m_+|\psi_+\ |_2>$.

(ii) Particle of electron mass and potential well of side $a \approx 1.2 \times 10^{-13}m$.

In this case, we have $|\psi_{e-}|_2 \approx 0.7 \times 10^{39}m^{-3}$. On the other hand, given that, if the electron mass is about $10^{-\ 3}$ times of that of the proton , it follows that the universe mean number of electrons per cubic meter is equal to $\frac{\Omega}{m_{e-}} \approx 5.5 \times 10^3\ m^{-3}$, it follows that

$$\frac{<\delta\Lambda>}{\delta\Lambda} = \frac{\Omega}{m_{e-}}\frac{1}{|\psi_{e-}|^2} \approx 0.7 \times 10^{-35}. \tag{A.12}$$

Given the universe matter density of $\frac{\Omega}{m_{e-}} \approx 5.5 \times 10^3\ m^{-3}$ electrons per cubic meter, with an electron density $|\psi_{+e}|_2 \approx 0.7 \times 10^{39}m^{-3}$ in a volume $a^3 = 10^{-39}$, it follows that $<|\psi_{+e}|_2> \approx 3.85 \times 10^3 m^{-3}$ and, hence, that the universe mass density $\Omega \approx 6.43 \times 10^{-27}\ Kgm^{-3}$ that is still satisfactory close to the cosmologically observed value.

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