Galaxies **2015**, *3*, 103-112; doi:10.3390/galaxies3020103

OPEN ACCESS

galaxies

ISSN 2075-4434

www.mdpi.com/journal/galaxies

Article

Quantum Corrected Non-Thermal Radiation Spectrum from the Tunnelling Mechanism

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Academic Editor: Lorenzo Iorio

Received: 9 February 2015 / Accepted: 28 May 2015 / Published: 12 June 2015

Abstract: The tunnelling mechanism is today considered a popular and widely used method in describing Hawking radiation. However, in relation to black hole (BH) emission, this mechanism is mostly used to obtain the Hawking temperature by comparing the probability of emission of an outgoing particle with the Boltzmann factor. On the other hand, Banerjee and Majhi reformulated the tunnelling framework deriving a black body spectrum through the density matrix for the outgoing modes for both the Bose-Einstein distribution and the Fermi-Dirac distribution. In contrast, Parikh and Wilczek introduced a correction term performing an exact calculation of the action for a tunnelling spherically symmetric particle and, as a result, the probability of emission of an outgoing particle corresponds to a non-strictly thermal radiation spectrum. Recently, one of us (C. Corda) introduced a BH effective state and was able to obtain a non-strictly black body spectrum from the tunnelling mechanism corresponding to the probability of emission of an outgoing particle found by Parikh and Wilczek. The present work introduces the quantum corrected effective temperature and the corresponding quantum corrected effective metric is written using Hawking's periodicity arguments. Thus, we obtain further corrections to the non-strictly thermal BH radiation spectrum as the final distributions take into account both the BH dynamical geometry during the emission of the particle and the quantum corrections to the semiclassical Hawking temperature.

Keywords: quantum tunnelling, quantum corrected effective temperature, black hole information puzzle

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1. Introduction

Considering Hawking radiation [1] in the tunnelling approach, [2–11] particle creation mechanism caused by the vacuum fluctuations near the BH horizon works as follows. A virtual particle pair is created just inside the horizon and the virtual particle with positive energy can tunnel out the BH horizon as a real particle. Otherwise, the virtual particle pair is created just outside the horizon and the negative energy particle can tunnel inwards. Thus, for both the possibilities, the particle with negative energy is absorbed by the BH and as a result the mass of the BH decreases. The flow of positive energy particles towards infinity is considered as Hawking radiation. Earlier, this approach was limited to obtain only the Hawking temperature through a comparison of the probability of emission of an outgoing particle with the Boltzmann factor rather than the actual radiation spectrum with the correspondent distributions. This problem was formally addressed by Banerjee and Majhi [7]. By a novel formulation of the tunnelling formalism, they were able to directly reproduce the black body spectrum for either bosons or fermions from a BH with standard Hawking temperature. However, considering contributions beyond semiclassical approximation in the tunnelling process, Parikh and Wilczek [2,3] found a probability of emission compatible with a non-thermal spectrum of the radiation from BH. This non precisely thermal character of the spectrum is important to resolve the information loss paradox of BH evaporation [12] because arguments that information is lost during hole's evaporation partially rely on the assumption of strict thermal behavior of the radiation spectrum [3,12]. Interesting approaches to resolve the BH information puzzle have been recently proposed in [13,14]

The basic difference between the works [2,3] and the work [7] is consideration or non-consideration of the energy conservation. As a result, there will be a dynamical [2,3] or static [7] BH geometry. In fact, due to conservation of energy, in [2,3] the BH horizon contracts during the radiation process which deviates from the perfect black body spectrum. This non-thermal spectrum has profound implications for realizing the underlying quantum gravity theory. In the language of the tunnelling mechanism, a trajectory in imaginary or complex time joins two separated classical turning points [3]. The key point is that the forbidden region traversed by the emitting particle has a *finite* size [3] from $r = r_{initial}$ to $r = r_{final}$ ($r_{initial}$ is the radius of the horizon of the BH initially and r_{final} is the radius of the horizon of the BH after particle emission). This finite size implies a discrete nature of the tunnelling mechanism, which is characterized by the physical state before the emission of the particle and that after the emission of the particle. As a result, the radiation spectrum is also discrete [14,15]. Consequently, particle emission can be interpreted like a quantum transition of frequency ω between the two discrete states [14,15]. It is the particle itself which generates a tunnel through the horizon [3,14,15] having finite size. In thermal spectrum, the tunnelling points have zero separation, so there is no clear trajectory because there is no barrier [3,14,15].

2. Basic Equations for Tunnelling Approach to Radiation Spectrum

In Planck units ($G = c = k_B = \hbar = \frac{1}{4\pi\epsilon_0} = 1$), the strictly thermal tunnelling probability is given by [1–3]

$$\Gamma \sim \exp\left(-\frac{\omega}{T_H}\right), \tag{1}$$

where $T_H = \frac{1}{8\pi M}$ is the Hawking temperature and ω is the energy-frequency of the emitted radiation. However, considering contributions beyond semiclassical approximation and taking into account the conservation of energy, Parikh and Wilczek reformulate the tunnelling probability as [2,3]

$$\Gamma \sim \exp\left[-\frac{\omega}{T_H}\left(1-\frac{\omega}{2M}\right)\right]. \tag{2}$$

This non-thermal spectrum enables the introduction of an intriguing way to consider the BH dynamical geometry through the *BH effective state.* In fact, one introduces the *effective temperature* as [14–17]

$$T_E(\omega) \equiv \frac{2M}{2M-\omega}T_H = \frac{1}{4\pi(2M-\omega)}, \tag{3}$$

which permits to rewrite the probability of emission (2) in Boltzmann-Hawking form as [14–17]

$$\Gamma \sim \exp[-\beta_E(\omega)\omega] = \exp(-\frac{\omega}{T_E(\omega)}), \tag{4}$$

where the effective Boltzmann factor takes the form [14–17]

$$\beta_E(\omega) \equiv \frac{1}{T_E(\omega)}. \tag{5}$$

One interpretes the effective temperature as the temperature of a black body emiting the same total amount of radiation [14–17]. Hence Hawking temperature is replaced by the effective temperature in the expression for the probability of emission. It should be noted that this notion of effective temperature has already been introduced in the literature for the Schwarzschild BH [16,17], for the Kerr BH [18] and for the Reissner-Nordstrom BH [19]. Further, the ratio $\frac{T_E(\omega)}{T_H} = \frac{2M}{2M-\omega}$ characterize the deviation of the radiation spectrum of a BH from the strictly thermal feature [14–17]. Also, the tunnelling approach of Parikh and Wilczek shows the probability of emission of Hawking quanta (see Equation (2)) is non-thermal in nature (*i.e.*, BH does not emit like a perfect black body). Moreover, due to perfect black body character of Bose-Einstein and Fermi-Dirac distributions, it is natural to have deviations from these distributions in case of the above effective temperature. Thus in analogy to BH, the effective temperature of a body (say star) can be defined as the temperature of a black body that would emit the same total amount of electromagnetic radiation [14,20]. So, one can consider this effective temperature and the bolometric luminosity as the two fundamental physical parameters to identify a star on the Hertzsprung-Russel diagram. It is worthy to mention here that both the above two physical parameters however depend on the chemical composition of the star [14–17,20].

Further, in analogy with the effective temperature, one can define the *effective mass* and the *effective horizon* radius as [14–17]

$$M_E = M - \frac{\omega}{2} \qquad and \qquad r_E = 2M_E = 2M - \omega. \tag{6}$$

Note that these effective quantities are nothing but the average value of the corresponding quantities before (initial) and after (final) the particle emission (*i.e.*, $M_i = M$, $M_f = M - \omega$; $r_i = 2M_i$ and $r_f = 2M_f$). Accordingly, T_E is the inverse of the average value of the inverses of the initial and final Hawking temperatures [14–17]. Hence, there is a discrete character (in time) of the Hawking temperature. Thus, the effective temperature may be interpreted as the Hawking temperature *during* the emission of the particle [14–17].

Following [15] one can use Hawking's periodicity argument [15,21,22] to obtain the *effective Schwarzschild line element*

$$ds_E^2 = -(1 - \frac{2M_E}{r})dt^2 + \frac{dr^2}{1 - \frac{2M_E}{r}} + r^2(\sin^2\theta d\varphi^2 + d\theta^2), \tag{7}$$

which takes into account the BH *dynamical* geometry during the emission of the particle.

Recently, one of us (C. Corda) introduced the above discussed BH effective state [14–17] and was able to obtain a non-strictly black body spectrum from the tunnelling mechanism corresponding to the probability of emission of an outgoing particle found by Parikh and Wilczek [15]. The final non-strictly thermal distributions which take into account the BH dynamical geometry are [14,15]

$$\begin{array}{c} < n >_{boson} = \frac{1}{\exp[4\pi(2M-\omega)\omega]-1} \\ < n >_{fermion} = \frac{1}{\exp[4\pi(2M-\omega)\omega]+1}. \end{array} \tag{8}$$

3. Quantum Corrections

Now, we further modify the effective temperature by incorporating the quantum corrections to the semiclassical Hawking temperature discussed in [4]. As a result, the quantum physics of BHs will be further modified. Banerjee and Majhi [4] have formulated the quantum corrected Hawking temperature using the Hamilton-Jacobi method [23] beyond semiclassical approximation. According to them [4], the quantum corrected Hawking temperature (termed as *modified Hawking temperature*) is given by

$$T_H^{(m)} = \left[1 + \Sigma_i \frac{\beta_i}{M^{2i}}\right]^{-1} T_H, \tag{9}$$

where the β_i are dimensionless constant parameters. However, if these parameters are chosen as powers of a single parameter α, then in compact form [4]

$$T_H^{(m)} = \left(1 - \frac{\alpha}{M^2}\right) T_H. \tag{10}$$

This modified Hawking temperature is very similar in form to the temperature correction in the context of one-loop back reaction effects [24,25] in the spacetime with α related to the trace anomaly [26]. Further, using conformal field theory, if one considers one-loop quantum correction to the surface gravity for Schwarzschild BH then α has the expression [4]

$$\alpha = -\frac{1}{360\pi}\left(-N_0 - \frac{7}{4}N_{\frac{1}{2}} + 13N_1 + \frac{233}{4}N_{\frac{3}{2}} - 212N_2\right), \tag{11}$$

where N_s denotes the number of field with spin s. Also considering two-loop back reaction effects in the spacetime, the quantum corrected Hawking temperature becomes [4]

$$T_H^{(m)} = \left[1 - \frac{\alpha}{M^2} - \frac{\gamma}{M^4}\right] T_H, \tag{12}$$

where second loop contributions are related to the dimensionless parameter γ. Thus, it is possible to incorporate higher loop quantum corrections by proper choices of the β_i. It should be noted that these correction terms dominate at large distances [27].

Using the above mentioned modified Hawking temperature, the modified form of the Boltzmann factor is

$$\beta^{(m)} = \frac{1}{T_H^{(m)}} = \frac{1}{T_H\left(1 - \frac{\alpha}{M^2} - \frac{\gamma}{M^4}\right)} = \frac{\beta_H}{\left(1 - \frac{\alpha}{M^2} - \frac{\gamma}{M^4}\right)}. \tag{13}$$

Thus, the (quantum corrected) modified BH mass has the expression

$$M^{(m)} = \frac{M}{\left(1 - \frac{\alpha}{M^2} - \frac{\gamma}{M^4}\right)}. \tag{14}$$

In case of emitted radiation from the BH, the modified Hawking temperature (with quantum correction) becomes

$$T_H^{(m)} = \frac{1}{8\pi M^{(m)}}. \tag{15}$$

As a result, following [21], one can again use Hawking's periodicity argument [15,21,22] to obtain the modified Schwarzschild like line element, which takes the form [14,15]

$$(ds_m)^2 = -(1 - \frac{2M^{(m)}}{r})dt^2 + \frac{dr^2}{1 - \frac{2M^{(m)}}{r}} + r^2(\sin^2\theta d\varphi^2 + d\theta^2) \tag{16}$$

with modified surface gravity

$$\kappa^{(m)} = \frac{1}{4M^{(m)}} = \frac{1}{2r^{(m)}} = \frac{\left(1 - \frac{\alpha}{M^2} - \frac{\gamma}{M^4}\right)}{4M}. \tag{17}$$

Equation (16) enables the replacement $M \to M^{(m)}$ and $T_H \to T_H^{(m)}$ in Equations (3), (5) and (6). In other words, one can define the (quantum corrected) *modified effective temperature*

$$T_E^m(\omega) \equiv \frac{2M^{(m)}}{2M^{(m)} - \omega} T_H^{(m)} = \frac{1}{4\pi(2M^{(m)} - \omega)}, \tag{18}$$

the (quantum corrected) *modified effective Boltzmann factor*

$$\beta_E^{(m)}(\omega) \equiv \frac{1}{T_E^m(\omega)} \tag{19}$$

and the (quantum corrected) *modified effective mass* and *effective horizon* radius

$$M_E^{(m)} = M^{(m)} - \frac{\omega}{2} \quad \textit{and} \quad r_E^{(m)} = 2M_E^{(m)} = 2M^{(m)} - \omega. \tag{20}$$

A clarification is needed concerning the definition (18) (Communication with the 5-th referee.). Equations (14)–(17) give the quantum corrections using Hamilton-Jacobi method beyond semiclassical approximation. Here we considered the contributions of the non-thermal spectrum by reformulation of tunnelling probability choosing Equation (16) as the modified Schwarzschild line element. It should be noted that a full calculation involving the action of a particle on the BH spacetime also leads to this result. So Equation (3) now becomes Equation (18). Following [15,21,22], one uses again Hawking's periodicity argument. Then, the euclidean form of the metric will be given by [28]

$$\left[ds_E^{(m)}\right]^2 = x^2 \left[\frac{d\tau}{4M^{(m)}\left(1-\frac{\omega}{2M^{(m)}}\right)}\right]^2 + \left(\frac{r}{r_E^{(m)}}\right)^2 dx^2 + r^2(\sin^2\theta d\varphi^2 + d\theta^2), \tag{21}$$

which is regular at $x = 0$ and $r = r_E^{(m)}$. τ is treated as an angular variable with period $\beta_E^{(m)}(\omega)$ [15,21,22]. Replacing the quantity $\sum_i \beta_i \frac{\hbar^i}{M^{2i}}$ in [21] with the quantity $-\frac{\omega}{2M^{(m)}}$, if one follows step by step the detailed analysis in [21] at the end one easily gets the (quantum corrected) *modified effective Schwarzschild line element*

$$\left[ds_E^{(m)}\right]^2 = -(1-\frac{2M_E^{(m)}}{r})dt^2 + \frac{dr^2}{1-\frac{2M_E^{(m)}}{r}} + r^2(\sin^2\theta d\varphi^2 + d\theta^2). \tag{22}$$

One also easily shows that $r_E^{(m)}$ in Equation (21) is the same as in Equation (20). Thus, the line element (22) takes into account both the BH dynamical geometry during the emission of the particle and the quantum corrections to the semiclassical Hawking temperature.

Starting from the standard Schwarzschild line element, *i.e.*, [7,15]

$$ds^2 = -(1-\frac{2M}{r})dt^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2(\sin^2\theta d\varphi^2 + d\theta^2), \tag{23}$$

the analysis in [7] permitted to write down the (normalized) physical states of the system for bosons and fermions as [7]

$$\begin{aligned} |\Psi>_{boson} &= (1-\exp(-8\pi M\omega))^{\frac{1}{2}} \textstyle\sum_n \exp(-4\pi n M\omega)\, |n_{out}^{(L)}> \otimes |n_{out}^{(R)}> \\ |\Psi>_{fermion} &= (1+\exp(-8\pi M\omega))^{-\frac{1}{2}} \textstyle\sum_n \exp(-4\pi n M\omega)\, |n_{out}^{(L)}> \otimes |n_{out}^{(R)}> . \end{aligned} \tag{24}$$

Hereafter we focus the analysis only on bosons. In fact, for fermions the analysis is identical [7]. The density matrix operator of the system is [7]

$$\begin{aligned} &\hat{\rho}_{boson} \equiv \Psi>_{boson} < \Psi|_{boson} \\ &= (1-\exp(-8\pi M\omega)) \textstyle\sum_{n,m} \exp[-4\pi(n+m)M\omega]\, |n_{out}^{(L)}> \otimes |n_{out}^{(R)}> < m_{out}^{(R)}| \otimes < m_{out}^{(L)}|. \end{aligned} \tag{25}$$

If one traces out the ingoing modes, the density matrix for the outgoing (right) modes reads [7]

$$\hat{\rho}_{boson}^{(R)} = (1-\exp(-8\pi M\omega)) \sum_n \exp(-8\pi n M\omega)\, |n_{out}^{(R)}> < n_{out}^{(R)}|. \tag{26}$$

This implies that the average number of particles detected at infinity is [7]

$$< n >_{boson} = tr\left[\hat{n}\hat{\rho}^{(R)}_{boson}\right] = \frac{1}{\exp\left(8\pi M\omega\right) - 1}, \tag{27}$$

where the trace has been taken over all the eigenstates and the final result has been obtained through a bit of algebra, see [7] for details. The result of Equation (27) is the well known Bose-Einstein distribution. A similar analysis works also for fermions [7], and one easily gets the well known Fermi-Dirac distribution

$$< n >_{fermion} = \frac{1}{\exp\left(8\pi M\omega\right) + 1}, \tag{28}$$

Both the distributions correspond to a black body spectrum with the Hawking temperature $T_H = \frac{1}{8\pi M}$. On the other hand, if one follows step by step the analysis in [7], but starting from the (quantum corrected) modified effective Schwarzschild line element (22) at the end obtains the correct physical states for boson and fermions as

$$\begin{aligned} |\Psi >_{boson} &= \left(1 - \exp\left(-8\pi M_E^{(m)}\omega\right)\right)^{\frac{1}{2}} \sum_n \exp\left(-4\pi n M_E^{(m)}\omega\right) |n^{(L)}_{out} > \otimes |n^{(R)}_{out} > \\ |\Psi >_{fermion} &= \left(1 + \exp\left(-8\pi M_E^{(m)}\omega\right)\right)^{-\frac{1}{2}} \sum_n \exp\left(-4\pi n M_E^{(m)}\omega\right) |n^{(L)}_{out} > \otimes |n^{(R)}_{out} > \end{aligned} \tag{29}$$

and the correct distributions as

$$\begin{aligned} < n >_{boson} &= \frac{1}{\exp\left(8\pi M_E^{(m)}\omega\right) - 1} = \frac{1}{\exp\left[4\pi\left(2M^{(m)} - \omega\right)\omega\right] - 1} = \frac{1}{\exp\left[4\pi\left(2\frac{M}{\left(1 - \frac{\alpha}{M^2} - \frac{\gamma}{M^4}\right)} - \omega\right)\omega\right] - 1} \\ < n >_{fermion} &= \frac{1}{\exp\left(8\pi M_E^{(m)}\omega\right) + 1} = \frac{1}{\exp\left[4\pi\left(2M^{(m)} - \omega\right)\omega\right] + 1} = \frac{1}{\exp\left[4\pi\left(2\frac{M}{\left(1 - \frac{\alpha}{M^2} - \frac{\gamma}{M^4}\right)} - \omega\right)\omega\right] + 1}, \end{aligned} \tag{30}$$

which are not thermal because they take into account both the BH dynamical geometry during the emission of the particle and the quantum corrections to the semiclassical Hawking temperature. We note that setting $\alpha = \gamma = 0$ in Equation (30) we find the results in [15], *i.e.*, Equation (8). In fact, in [15] only the BH dynamical geometry was taken into account. Here, we further improved the analysis by taking into account also the quantum corrections to the semiclassical Hawking temperature.

4. Concluding Remarks

The present work deals with the quantum correction of non-thermal radiation spectrum in the framework of tunnelling mechanism. Staring from the Schwarzschild BH, at first the quantum corrections are considered. As a result, the Hawking temperature and Schwarzschild mass are modified (see Equations (14) and (15)). So one obtains the modified Schwarzschild line element (see Equation (16)). Then we consider the non-thermal radiation spectrum of this modified Schwarzschild BH by the reformulation of the tunnelling probability. The resulting quantum corrected effective Schwarzschild metric is rewritten using Hawking's periodicity arguments. Also we have shown the correct distributions of bosons and fermions using the above quantum corrections to the semiclassical Hawking temperature. Thus due to quantum correction at the semiclassical level, BH parameters (and its radiation spectrum), namely, its mass, temperature, surface gravity, and Boltzmann factors are modified and as a result, we have quantum corrected effective Schwarzschild metric. Moreover, the one-loop correction which comes from interaction between graviton and particles of various species (characterized) in Equation (11) occurred at the horizon. Hence, the quantum effects lead to a redefinition of surface gravity and other parameters. However, it should be noted that the BH's gravitational potential may not only be characterized by this modified mass far away from the horizon. Therefore, the modified metric (in Equation (16) or Equation (22)) can only be trusted for its near-horizon geometry, but nowhere else—and the effective metric for arbitrary distance could be elaborated in a perturbative way [29,30] and, moreover, the potential is not Coloumb-like in general.

Acknowledgments

The authors Subenoy Chakraborty and Subhajit Saha are thankful to IUCAA, Pune, India for their warm hospitality and research facilities as the work has been done there during a visit. Also Subenoy Chakraborty acknowledges the UGC-DRS Programme in the Department of Mathematics, Jadavpur University. The author Subhajit Saha is thankful to UGC-BSR Programme of Jadavpur University for awarding research fellowship. The authors thank the unknown referees for important advices which permitted to improve this work.

Author Contributions

All authors contributed equally to this paper. All authors have read and approved the final manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

References

1. Hawking, S.W. Particle creation by black holes. *Commun. Math. Phys.* **1975**, *43*, 199–220.
2. Parikh, M.K.; Wilczek, F. Hawking Radiation as Tunneling. *Phys. Rev. Lett.* **2000**, *85*, 5042.
3. Parikh, M.K. A Secret Tunnel Through The Horizon. *Gen. Rel. Grav.* **2004**, *36*, 2419–2422.
4. Banerjee, R.; Majhi, B.R. Quantum tunneling beyond semiclassical approximation. *J. High Energ. Phys.* **2008**, *2008*, 095.
5. Angheben, M.; Nadalini, M.; Vanzo, L.; Zerbini, S. Hawking radiation as tunneling for extremal and rotating black holes. *J. High Energ. Phys.* **2005**, *2005*, 014.
6. Arzano, M.; Medved, A.J.M.; Vagenas, E.C. Hawking radiation as tunneling through the quantum horizon. *J. High Energ. Phys.* **2005**, *2005*, 037.
7. Banerjee, R.; Majhi, B.R. Hawking black body spectrum from tunneling mechanism. *Phys. Lett. B* **2009**, *675*, 243.
8. Jiang, Q.Q.; Wu, S.Q.; Cai, X. Hawking radiation as tunneling from the Kerr and Kerr-Newman black holes. *Phys. Rev. D* **2006**, *73*, 064003.
9. Jiang, Q.Q.; Wu, S.Q.; Cai, X. Erratum-ibid, Publisher's Note: Hawking radiation as tunneling from the Kerr and Kerr-Newman black holes [Phys. Rev. D 73, 064003 (2006)]. *Phys. Rev. D* **2006**, *73*, 069902.

10. Kerner, R.; Mann, R.B. Tunnelling, temperature, and Taub-NUT black holes. *Phys. Rev. D* **2006**, *73*, 104010.
11. Vanzo, L.; Acquaviva, G.; di Criscienzo, R. Tunnelling Methods and Hawking's radiation: Achievements and prospects. *Class. Quant. Grav.* **2011**, *28*, 183001.
12. Hawking, S.W. Breakdown of predictability in gravitational collapse. *Phys. Rev. D* **1976**, *14*, 2460.
13. Zhang, B.; Cai, Q.-Y.; Zhan, M.S.; You, L. Information conservation is fundamental: Recovering the lost information in Hawking radiation. *Int. J. Mod. Phys. D* **2013**, *22*, 1341014.
14. Corda, C. Time-dependent Schrodinger equation for black hole evaporation: No information loss. *Ann. Phys.* **2015**, *353*, 71–82.
15. Corda, C. Non-strictly black body spectrum from the tunnelling mechanism. *Ann. Phys.* **2013**, *337*, 49–54.
16. Corda, C. Effective temperature for black holes. *J. High Energ. Phys.* **2011**, *1108*, 101.
17. Corda, C. Effective temperature, Hawking radiation and quasinormal modes. *Int. J. Mod. Phys. D* **2012**, *21*, 1242023 (Awarded Honorable mention in the Gravity Research Foundation Essay Competition).
18. Corda, C.; Hendi, S.H.; Katebi, R.; Schmidt, N.O. Effective state, Hawking radiation and quasi-normal modes for Kerr black holes. *J. High Energ. Phys.* **2013**, *1306*, 008.
19. Corda, C.; Hendi, S.H.; Katebi, R.; Schmidt, N.O. Hawking radiation-quasi-normal modes correspondence and effective states for nonextremal Reissner-Nordstrom black holes. *Adv. High Energy Phys.* 2014, 527874.
20. Roy, A.E.; Clarke, D. *Astronomy: Principles and Practice*, 4th ed.; Institute of Physics Publishing: London, UK, 2003.
21. Banerjee, R.; Majhi, B.R. Quantum Tunneling and Trace Anomaly. *Phys. Lett. B* **2009**, *674*, 218.
22. Hawking, S.W. The Path Integral Approach to Quantum Gravity. In *General Relativity: An Einstein Centenary Survey*; Hawking, S.W., Israel, W., Eds.; Cambridge University Press: Cambridge, UK, 1979.
23. Srinivasan, K.; Padmanabhan, T. Particle production and complex path analysis. *Phys. Rev. D* **1999**, *60*, 024007.
24. York, J.H., Jr. Black hole in thermal equilibrium with a scalar field: The back-reaction. *Phys. Rev. D* **1985**, *31*, 775.
25. Lousto, C.O.; Sanchez, N.G. Back reaction effects in black hole spacetimes. *Phys. Lett. B* **1988**, *212*, 411.
26. Fursaev, D.V. Temperature and entropy of a quantum black hole and conformal anomaly. *Phys. Rev. D* **1995**, *51*, 5352.
27. Chakraborty, S.; Saha, S. Quantum tunnelling for Hawking radiation from both static and dynamic black holes. *Adv. High Energy Phys.* **2014**,*2014*, 168487.
28. Hawking, S.W. Information loss in black holes. *Phys. Rev. D* **2005**, *72*, 084013.
29. Donoghue, J.F. General relativity as an effective field theory: The leading quantum corrections. *Phys. Rev. D* **1994**, *50*, 3874.

30. Bjerrum-Bohr, N.E.J.; Donoghue, J.F.; Holstein, B.R. Erratum: Quantum corrections to the Schwarzschild and Kerr metrics [Phys. Rev. D 68, 084005 (2003)]. *Phys. Rev. D* **2005**, *68*, 069904.