**Abstract:** For disk galaxies (spirals and irregulars), the inner circular-velocity gradient $d_R V_0$ (inner steepness of the rotation curve) correlates with the central surface brightness $\Sigma_{*,0}$ with a slope of $\sim 0.5$. This implies that the central dynamical mass density scales almost linearly with the central baryonic density. Here I show that this empirical relation is consistent with a simple model where the central baryonic fraction $f_{\text{bar},0}$ is fixed to 1 (no dark matter) and the observed scatter is due to differences in the baryonic mass-to-light ratio $M_{\text{bar}}/L_R$ (ranging from 1 to 3 in the $R$-band) and in the characteristic thickness of the central stellar component $\Delta z$ (ranging from 100 to 500 pc). Models with lower baryonic fractions are possible, although they require some fine-tuning in the values of $M_{\text{bar}}/L_R$ and $\Delta z$. Regardless of the actual value of $f_{\text{bar},0}$, the fact that different types of galaxies do not show strong variations in $f_{\text{bar},0}$ is surprising, and may represent a challenge for models of galaxy formation in a $\Lambda$ Cold Dark Matter ($\Lambda$CDM) cosmology.

**Keywords:** dark matter; galaxies: formation; galaxies: evolution

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**1. Introduction**

Galaxies are known to follow tight scaling relations linking their observed baryonic content to their dynamical properties. Pressure-supported systems (ellipticals, bulges, and dwarf spheroidals) follow the Faber–Jackson relation [1,2], which links the total luminosity of the system (proxy for the baryonic mass) to its mean velocity dispersion (proxy for the dynamical mass). Rotation-supported systems (lenticulars, spirals, and dwarf irregulars) follow the baryonic Tully–Fisher relation (BTFR) [3], which links the total baryonic mass $M_{\text{bar}}$ (gas plus stars) to the asymptotic velocity along the flat part of the rotation curve...
While $V_{\text{flat}}$ is related to the total dynamical mass of the galaxy, the inner steepness of the rotation curve provides information on the central dynamical mass density, including both baryons and dark matter (DM). Early studies [7–10] pointed out that, in disk galaxies, the shape of the luminosity profile and the shape of the rotation curve are related, suggesting a close link between the distribution of baryons and the distribution of the dynamical mass (luminous and/or dark). This has been confirmed by several subsequent studies [11–16], which have substantially increased the number of galaxies with high-quality rotation curves, spanning the Hubble sequence from lenticulars to irregulars (Irrs). The observational evidence is concisely summarized by the so-called “Renzo’s rule” [17]: for any feature in the luminosity profile of a galaxy there is a corresponding feature in the rotation curve, and vice versa.

In [18] we measured the inner circular-velocity gradient $d R V_0$ for a sample of 52 galaxies with high-quality rotation curves, ranging from bulge-dominated galaxies (S0 to Sb) to disk-dominated ones (Sc to Irrs). $d R V_0$ is defined as $dV/dR$ for $R \to 0$, thus measuring the inner steepness of the galaxy rotation curve. We found that $d R V_0$ correlates with the central surface brightness $\Sigma_{*0}$ over more than two orders of magnitude in $d R V_0$ and four orders of magnitude in $\Sigma_{*0}$ (see Figure 1). This is a scaling relation for rotation-supported systems, which is analogous to the BTFR for the innermost regions of disk galaxies. These two empirical laws imply that, to a first approximation, the baryonic properties of a galaxy can predict the shape of the rotation curve, and vice versa. For example, the rotation curves of late-type galaxies can be roughly described by a nearly solid-body rise in the inner regions ($d R V_0$) and the flattening in the outer parts ($V_{\text{flat}}$) (e.g., [15,19]). Thus, if the central surface brightness and the total baryonic mass of a galaxy are known, one can approximately predict the shape of its rotation curve using the $d R V_0 - \Sigma_{*0}$ and the $V_{\text{flat}} - M_{\text{bar}}$ relations. The predictive power of baryons is an empirical fact, independent of any underlying theory. This is quite surprising in a DM-dominated Universe.

In this paper I build a toy model that naturally reproduces the slope, normalization, and scatter of the $d R V_0 - \Sigma_{*0}$ relation. I also discuss the general implications that this relation poses to the baryonic fraction in the innermost regions of disk galaxies.

2. A Toy Model for the $d R V_0 - \Sigma_{*0}$ Relation

For a general 3D distribution of mass, the circular velocity $V$ of a test particle orbiting at radius $R$ is given, to a first approximation, by:

$$ \frac{V^2}{R} = \frac{\alpha G M_{\text{dyn}}}{R^2} = \frac{4}{3} \pi \alpha G \bar{\rho}_{\text{dyn}} R $$

(1)

where $G$ is Newton’s constant, $M_{\text{dyn}}$ is the dynamical mass within $R$, $\bar{\rho}_{\text{dyn}} = M_{\text{dyn}}/\frac{4}{3} \pi R^3$ is the mean dynamical mass density within $R$, and $\alpha$ is a factor of the order of unity that depends on the 3D distribution of mass. For a spherical mass distribution, the Newton’s theorem gives $\alpha = 1$ (e.g., [20]). For a thin exponential disk with scale length $R_d$, $\alpha$ varies with radius (cf. [21]): $\alpha \simeq 1$ at $R = R_d$ and monotonically decreases for $R \lesssim R_d$ ($\alpha \simeq 0.75$ at $R = 0.5 R_d$; $\alpha \simeq 0.5$ at $R = 0.25 R_d$). The surface brightness profiles of disk galaxies often deviate from a pure exponential in the inner regions: if the luminosity profile shows an inner “flattening”, as it is often observed in dwarf galaxies (e.g., [22]), the value of $\alpha$ near the center decreases with respect to an exponential disk. For these reasons, I consider that $\alpha$ can vary between 0.5 and 1 in the inner parts of different galaxies ($R \lesssim 0.5 R_d$).
Figure 1. The $d_R V_0 - \Sigma_{*,0}$ scaling relation. Galaxies are coded by the value of the maximum velocity $V_{\text{max}}$ observed along the rotation curve. Galaxies with $\Sigma_{*,0} \gtrsim 10^3 \, L_\odot \, \text{pc}^{-2}$ are typically dominated by a bulge component in the inner parts and have more uncertain values of $d_R V_0$ (see [18], for details). The yellow band shows a model where the central baryonic fraction is constant for each galaxy ((a): $f_{\text{bar},0} = 1$; (b): $f_{\text{bar},0} = 0.17$); the width of the band considers that the value of $M_{\text{bar}}/L_R$ can vary from 1 to 3, $\Delta z$ from 100 to 500 pc, and $\alpha$ from 0.5 to 1. The dashed line indicates an “intermediate” model with $M_{\text{bar}}/L_R = 2$, $\Delta z = 300$ pc, and $\alpha = 0.75$.

If we assume that the dynamical mass density converges to a finite value $\rho_{\text{dyn},0}$ towards the center (as is reasonable for actual galaxies), in the limit $R \to 0$ we have:

$$ \frac{dV}{dR} \simeq \frac{V}{R} = \sqrt{\frac{4}{3} \pi \alpha G \rho_{\text{dyn},0}} = \sqrt{\frac{4}{3} \pi \alpha G \rho_{\text{bar},0}} f_{\text{bar},0} \tag{2} $$

where $\rho_{\text{bar},0}$ is the central baryonic mass density and $f_{\text{bar},0} = \rho_{\text{bar},0}/\rho_{\text{dyn},0}$ is the central baryonic fraction. Note that $f_{\text{bar},0}$ may strongly differ from the “cosmic” baryonic fraction (=0.17) given by the Cosmic Microwave Background [23] and observed in galaxy clusters [24]. In a $\Lambda$ Cold Dark Matter ($\Lambda$CDM) cosmology, $f_{\text{bar},0}$ is determined by the complex formation and evolution history of the object, involving galaxy mergers, gas inflows, star formation, stellar and Active Galactic Nuclei (AGN) feedback, etc. Thus, we expect $f_{\text{bar},0}$ to vary from galaxy to galaxy, possibly from 0.17 (the cosmic value) up to 1 (baryon dominance). Moreover, in a given galaxy, the baryonic fraction $f_{\text{bar}}$ can vary with radius up to a factor of 10 (see, e.g., [25]) due to the relative contributions of baryons and DM to each point of the rotation curve. The baryonic fractions deduced from Equation (2) are formal extrapolations for $R \to 0$, but in practice they are representative of the innermost galaxy regions accessible by the available rotation curves (typically for $R \lesssim 0.5 R_d$, see [18] for details).
Observationally, we measure either $\mu_0$ (in units of mag arcsec$^{-2}$) or $\Sigma_{*,0}$ (in units of $L_\odot$ pc$^{-2}$). The latter is given by:

$$\Sigma_{*,0} = \int_{-\infty}^{\infty} \frac{\rho_{\text{bar},0}}{M_{\text{bar}}/L} dz \simeq \frac{\rho_{\text{bar},0}}{M_{\text{bar}}/L} 2\Delta z$$

where $\Delta z$ is the characteristic thickness of the stellar component in the central regions (either a disk, a bulge, a bar/pseudo-bulge, or a nuclear star cluster) and $M_{\text{bar}}/L$ is the baryonic mass-to-light ratio, including molecules and other undetected baryons (the atomic gas density is generally $<10$ $M_\odot$ pc$^{-2}$ in the inner regions and, thus, negligible to a first approximation). Therefore, we expect the following relation:

$$d_R V_0 = \sqrt{\frac{2}{3} \pi \alpha G M_{\text{bar}}/L} \frac{\Delta z}{f_{\text{bar},0}} \sqrt{\Sigma_{*,0}}$$

Remarkably, a least-square fit to the data-points in Figure 1 returns a slope of $\sim 0.5$ (equivalent to $-0.2$ when the central surface brightness is expressed in units of mag arcsec$^{-2}$ instead of $L_\odot$ pc$^{-2}$, see [18,26]). The actual value of the slope remains uncertain due to several effects in the determination of $d_R V_0$ and $\Sigma_{*,0}$, but it can be constrained between $\sim 0.4$ and $\sim 0.6$. Despite these uncertainties, it is interesting to check whether the zero point and the observed scatter along the relation are consistent with typical values of $\alpha$, $\Delta z$, $M_{\text{bar}}/L_R$, and $f_{\text{bar},0}$. In both panels of Figure 1, the yellow band is calculated assuming that $f_{\text{bar},0}$ is constant for each galaxy ($f_{\text{bar},0} = 1$ in the left panel, $f_{\text{bar},0} = 0.17$ in the right one), while $\alpha$ varies from 0.5 to 1, $\Delta z$ varies from 100 to 500 pc, and $M_{\text{bar}}/L_R$ varies from 1 to 3. The dashed line shows an “intermediate” model with $\alpha = 0.75$, $\Delta z = 300$ pc, and $M_{\text{bar}}/L_R = 2$. The width of the band is dominated by the variation in $\Delta z$ (factor $\sqrt{5}$) followed by the variations in $M_{\text{bar}}/L_R$ (factor $\sqrt{3}$) and $\alpha$ (factor $\sqrt{2}$). Note that the objects in the sample cover a wide range in total mass ($20 \lesssim V_{\text{max}} \lesssim 300$ km s$^{-1}$, as indicated by the different symbols in Figure 1), and span the entire Hubble sequence going from bulge-dominated galaxies (S0 to Sb), typically characterized by old stellar populations in the central regions, to disk-dominated galaxies (Sc to Irr), characterized by young stellar populations. Considering possible differences in their star formation history, molecular content, metallicity, internal extinction, and initial mass function, a variation in $M_{\text{bar}}/L_R$ by a factor of 3 is a rather conservative choice (cf. [27–29]).

In the left panel of Figure 1, I consider $f_{\text{bar},0} = 1$ (no DM). The agreement with the observations is striking. The scatter along the relation can be fully explained by variations in the 3D distribution of baryons ($\Delta z$ and $\alpha$) and stellar populations ($M_{\text{bar}}/L_R$), without any need of DM in the innermost galaxy regions (typically within $\sim 0.5 R_\odot$). In the right panel, I consider the opposite, extreme case where $f_{\text{bar},0}$ is fixed to 0.17 (the cosmic value). Clearly, this low baryonic fraction cannot reproduce the observed relation, unless one significantly increases $\Delta z$ and/or decreases $M_{\text{bar}}/L_R$. The characteristic thickness of the central stellar component should be increased up to values of $\sim 3$ kpc, which appear quite unrealistic. The baryonic mass-to-light ratio, instead, should be decreased down to $\sim 0.1$–$0.2$, which is inconsistent with standard stellar population synthesis models (e.g., [27–29]). Models with intermediate values of $f_{\text{bar},0}$ are possible, provided that $M_{\text{bar}}/L_R$ and/or $\Delta z$ are properly fine-tuned. For example, a plausible model is obtained by fixing $f_{\text{bar},0} \simeq 0.5$ ($\sim 3$ times the cosmic value) and varying $M_{\text{bar}}/L_R$ between 0.5 and 1.5, $\Delta z$ between 100 and 500 pc, and $\alpha$ between 0.5 and 1. Such a model cannot be distinguished...
from the one in Figure 1 (left panel) due to the degeneracies between $M_{\text{bar}}/L_{R}$, $\Delta z$, and $f_{\text{bar},0}$. Surface photometry in infrared (IR) bands ($K$-band or 3.6 $\mu$m) may improve the situation and help to break these degeneracies, given that one expects a smaller scatter in the values of $M_{\text{bar}}/L$ (e.g., [4,27]), which may translate into a smaller scatter around the $d_R V_0 - \Sigma_{*,0}$ relation. It would also be useful to investigate whether the residuals around the central relation correlate with some galaxy properties. For example, if baryons dominate the central galaxy regions ($f_{\text{bar},0} \simeq 1$), one may expect the residuals to correlate with the central colors (tracing $M_{\text{bar}}/L_{R}$) and/or the central morphology (roughly tracing $\alpha$ and $\Delta z$). These issues will be addressed in future studies.

3. Discussion & Conclusions

I investigated the implications that the $d_R V_0 - \Sigma_{*,0}$ relation poses to the central baryonic fraction $f_{\text{bar},0}$ of disk galaxies (spirals and irregulars). Surprisingly, I found that the observed relation is consistent with a model where $f_{\text{bar},0}$ is fixed to 1 (no DM) and the scatter is entirely given by variations in the baryonic mass-to-light ratio $M_{\text{bar}}/L_{R}$ (from 1 to 3 in the $R$-band), in the characteristic thickness of the central stellar component $\Delta z$ (from 100 to 500 pc), and in the 3D shape of the gravitational potential (parametrized by $\alpha \simeq 0.5$ to 1). Models with very low values of $f_{\text{bar},0}$ (such as the “cosmic” value of 0.17) are very unlikely, since they would require values of $M_{\text{bar}}/L_{R}$ and $\Delta z$ that are in tension with our current knowledge on stellar populations and galaxy structure. Models with intermediate values of $f_{\text{bar},0}$ ($\simeq$0.5) are possible, provided that the parameters $M_{\text{bar}}/L_{R}$ and $\Delta z$ are properly fine-tuned.

If $f_{\text{bar},0} \simeq 1$ and, thus, baryons dominate the innermost regions of disk galaxies ($R \lesssim 0.5 R_d$), even in low-luminosity and low-surface-brightness ones, the whole controversy about cuspy versus cored DM density profiles is undermined (see, e.g., [17]). Cored DM profiles may be allowed if their characteristic central surface density is relatively low ($<100$ M$_\odot$ pc$^{-2}$), whereas cuspy DM profiles would generally be disfavored as they should have low concentrations that are unexpected in a $\Lambda$CDM cosmology (e.g., [30]). Broadly speaking, a dominant baryonic component in the innermost galaxy regions would leave “little room” for a central DM cusp. If $f_{\text{bar}} < 1$, both cored and cuspy DM profiles may be allowed, but one should explain why, for galaxies of very different masses and rotation velocities, the central baryonic density scales almost linearly with the central dynamical mass density due to the dominant DM halo.

Regardless of the actual value of $f_{\text{bar},0}$, it is surprising that models with fixed $f_{\text{bar},0}$ can naturally reproduce the $d_R V_0 - \Sigma_{*,0}$ relation. In a $\Lambda$CDM cosmology, the central baryonic fractions of galaxies are the result of complex baryonic physics (including mergers, gas inflows, star formation, stellar and AGN feedback, etc.), which likely depends on the specific properties of the galaxy (total mass, angular momentum, environment, etc.). It is puzzling, therefore, that the data can be accurately described using a fixed value of $f_{\text{bar},0}$ for very different types of galaxies, ranging from dwarf irregulars with $V_{\text{max}} \simeq 20–30$ km s$^{-1}$ to bulge-dominated spirals with $V_{\text{max}} \simeq 200–300$ km s$^{-1}$. A large variation in $f_{\text{bar},0}$ from galaxy to galaxy would introduce further scatter along the relation that is not observed. Intriguingly, the situation is very different for the BTFR. The zero point and slope of the BTFR imply that the global baryonic fraction in the disk $f_d$ is smaller than the cosmic value by $\sim$1–2 orders of magnitude and systematically decreases with $V_{\text{fast}}$ [24,31]. In other words, galaxies must progressively lose more and more baryons during their formation with decreasing mass (as suggested by the BTFR),
but the fraction of baryons in the inner regions should remain higher than the cosmic value and almost constant in any type of galaxy (as suggested by the $dV_0 - \Sigma_{*0}$ relation). This may be a challenge for models of galaxy formation in a ΛCDM cosmology. I stress, however, that the current observations constrain the slope of the $dV_0 - \Sigma_{*0}$ relation between ~0.4 and ~0.6 [18]: values slightly lower/higher than 0.5 may point to systematic variations of $f_{\text{bar},0}$ with $\Sigma_{*0}$. Future observational studies, therefore, should aim at obtaining a better calibration of this scaling law.

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Conflicts of Interest

The author declares no conflict of interest.

References


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