

Article

Galaxy Rotation Curves in Covariant Hořava-Lifshitz Gravity

Jean Alexandre ^{1,*} and Martyna Kostacinska ²

¹ Department of Physics, King's College London, London WC2R 2LS, UK

² Theoretical Physics, Imperial College London, London SW7 2AZ, UK;

E-Mail: martyna.kostacinska13@imperial.ac.uk

* Author to whom correspondence should be addressed; E-Mail: jean.alexandre@kcl.ac.uk;

Tel.: +44-20-7848-2429 ; Fax: +44-20-7848-2420.

Received: 3 October 2013; in revised form: 28 November 2013 / Accepted: 18 December 2013 /

Published: 23 December 2013

Abstract: Using the multiplicity of solutions for the projectable case of the covariant extension of Hořava-Lifshitz gravity, we show that an appropriate choice for the auxiliary field allows for an effective description of galaxy rotation curves. This description is based on static and spherically symmetric solutions of covariant Hořava-Lifshitz gravity and does not require Dark Matter.

Keywords: modified gravity; covariant Hořava-Lifshitz gravity; static and spherically symmetric solution; galaxy rotation curves

1. Introduction

Although General Relativity (GR) has been accurately checked in the solar system, the small distance behaviour of gravity is still not understood, and several alternatives to GR have been proposed. Amongst these modified theories is Hořava-Lifshitz (HL) gravity [1], which is based on the Lifshitz approach and consists of introducing an anisotropy between space and time: when space is rescaled as $x \rightarrow bx$, time is rescaled as $t \rightarrow b^z t$, where z is an integer. In Lifshitz models, Lorentz invariance must be recovered for $z = 1$, and the situation $z \neq 1$ leads to new renormalizable interactions: the convergence of loop integrals is improved by inclusion of higher orders in space derivatives, without introducing ghosts, since the number of time derivatives remains the minimum. A review of Lifshitz-type quantum field theories in flat space time can be found in [2], and the HL power-counting renormalizable theory of gravity is reviewed in [3–5].

A fundamental problem of the original model of HL gravity is the existence of an additional scalar degree of freedom for the metric, which can be understood as a Goldstone mode arising from breaking of 4-dimensional diffeomorphism [6,7]. A solution to this problem has been proposed in [8], where an auxiliary field A is introduced, such that its “equation of motion” leads to an additional constraint, eliminating the unwanted scalar degree of freedom of the metric. The resulting theory is invariant under a new Abelian gauge symmetry $U_{\Sigma}(1)$ which involves the metric components, the auxiliary field A and an additional auxiliary field ν . This gauge symmetry, together with the 3-dimensional diffeomorphism of HL gravity, can be shown to be equivalent to a 4-dimensional diffeomorphism at the lowest order in a post-Newtonian expansion, showing the equivalence with GR at long distances. For this reason, this modified version of HL gravity is called covariant HL gravity. We note, however, that the long-distance limit is not obviously recovered: it has been shown in [9] that the equivalence principle is not automatically retrieved in the infrared, and that the meaning of the auxiliary field A and its coupling to matter are still open questions.

Because of the anisotropy between space and time, HL gravity is naturally described in terms of the Arnowitt-Deser-Misner (ADM) decomposition of the metric, which expresses a space-time foliation. An important consequence of space-time anisotropy is the possibility of imposing the lapse function N to depend on time only. This situation is called the projectable case and leads to an interesting feature: the solutions of the equations of motion are not unique. Indeed, the Hamiltonian constraint, obtained by variation of the action with respect to N , leads to an integral equation, which does not have a unique solution, as will be seen in the present article. This multiplicity of solutions is independent of the above mentioned new gauge symmetry $U_{\Sigma}(1)$, and the different solutions for A in the projectable case belong to different gauge orbits. On the other hand, in the non-projectable case where N depends on both space and time, the Hamiltonian constraint leads to a differential equation, which has a unique solution, after fixing the constants of integration. The non-projectable case has been studied in [10] for static spherically symmetric solutions of covariant HL gravity.

In this article we use the freedom of choice of the auxiliary field A in the projectable case to study the possibility of fitting galaxy rotations curves without the need for Dark Matter. One can argue that the ambiguity in the choice of A leaves its physical interpretation unclear. This may be understood as one of the incomplete aspects of covariant HL gravity, and one could conjecture the existence of an additional symmetry which would restrict the set of solutions for the auxiliary field A , but this still remains to be studied. We note that the multiplicity of solutions, arising from a Hamiltonian constraint expressed in terms of an integral over space, has been discussed in [11] as an alternative to Dark Matter, in the context of the original HL gravity. Fits to galaxy rotation curves have been studied in the framework of the original HL gravity in [12,13], where a detailed comparison with experimental data is presented. Also, an interesting analogy between HL gravity for $z = 0$ and Modified Newtonian Dynamics theories has been described in [14].

In Section 2 we review the static and spherically symmetric solutions of covariant HL gravity, and in Section 3 we show how galaxy rotation curves can be described by the covariant extension of HL gravity. Although the curves we use were derived from Dark Matter models, we use them as experimental data fits. It is interesting to note that we obtain exact solutions for the auxiliary field A . Another exact solution in the Lifshitz context has been derived in [15], where the exact effective potential for a Liouville scalar

theory, renormalizable in 3 + 1 dimensions with anisotropic scaling $z = 3$, proves to be an exponential, as in the usual 1 + 1 dimensional relativistic Liouville theory. Finally, we conclude by discussing the possibility to describe the solar system with our solution.

2. Static and Spherically Symmetric Solutions of Covariant HL Gravity

The static and spherically symmetric solutions of covariant HL gravity were derived in [25,26], and we review here the aspects relevant to our present study.

2.1. Action

The ADM metric we consider is:

$$ds^2 = -c^2 N^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt) \quad (1)$$

where N and N^i are the lapse and shift functions respectively, and g_{ij} is the three-dimensional space metric. We consider the anisotropic scaling with $z = 3$, for which the different operators, allowed for the model to be power-counting renormalizable, have the mass dimension of 6, at most. We note that in this case the mass dimension of the speed of light is $[c] = z - 1 = 2$, and we will keep c in the different expression for the sake of clarity.

The covariant Hořava-Lifshitz action, in the absence of cosmological constant, is:

$$S = \int dt d^3x \sqrt{g} (N [K_{ij} K^{ij} - K^2 - V + \nu \Theta^{ij} (2K_{ij} + \nabla_i \nabla_j \nu)] - AR^{(3)}) \quad (2)$$

where g is the determinant of the three-dimensional metric g_{ij} , with curvature scalar $R^{(3)}$. A is an auxiliary field (without kinetic term) of mass dimension $z + 1 = 4$ and ν is an auxiliary field of mass dimension $z - 2 = 1$. In the above expression, the extrinsic curvature is:

$$K_{ij} = \frac{1}{2N} \{ \dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \}, \quad i, j = 1, 2, 3 \quad (3)$$

where a dot denotes a time derivative, and,

$$\Theta^{ij} = R^{(3)ij} - \frac{1}{2} R^{(3)} g^{ij} \quad (4)$$

where where $R^{(3)ij}$ is the Ricci tensor corresponding to the three-dimensional space metric g_{ij} . Note that a more general kinetic term $K_{ij} K^{ij} - \lambda K^2$ can be considered, with λ a generic parameter. The latter should be equal to 1 if one wishes to recover GR in the infrared, but in the framework of HL gravity, no symmetry imposes this parameter to be equal to 1. Constraints on λ resulting from observations of Type Ia Supernovae, Baryon Acoustic Oscillations, CMB and the requirement of Big Bang Nucleosynthesis all point towards a value near the GR parameter $\lambda = 1$ [16], while it has also been shown that covariant HL gravity with $\lambda \neq 1$ leads to inconsistencies, if one compares the predictions of the model with solar system tests [17]. We therefore focus here on the situation where $\lambda = 1$ only. Independently of these tests, the mathematical consistency of covariant HL gravity for $\lambda \neq 1$ has been studied in [18], and the corresponding Hamiltonian structure in [19]. We stress again here that the purpose of this covariant

extension of HL gravity is to eliminate the unwanted additional scalar graviton from the spectrum. As shown in [8], no singularity arises in the limit $\lambda \rightarrow 1$, and the strong coupling of the scalar graviton does not exist anymore, as it is the case for the original HL gravity. Besides [6,7], one can find detailed analysis of the strong coupling problem in [20–22], and also related discussions in [23] on a extension of the original HL gravity [24].

The potential term V contains up to six spatial derivatives of the metric g_{ij} :

$$V = -c^2 R^{(3)} - \alpha_1 (R^{(3)})^2 - \alpha_2 R^{(3)ij} R_{ij}^{(3)} - \beta_1 (R^{(3)})^3 - \beta_2 R^{(3)} R^{(3)ij} R_{ij}^{(3)} - \beta_3 R_j^{(3)i} R_k^{(3)j} R_i^{(3)k} - \beta_4 R^{(3)} \nabla^2 R^{(3)} - \beta_5 \nabla^i R_{jk}^{(3)} \nabla_i R^{(3)jk} \quad (5)$$

where the mass dimensions of the different couplings are:

$$[\alpha_1] = [\alpha_2] = z - 1 = 2, \quad [\beta_1] = [\beta_2] = [\beta_3] = [\beta_4] = [\beta_5] = z - 3 = 0 \quad (6)$$

We consider the most general, static, spherically symmetric metric, of the form:

$$ds^2 = -c^2 N^2 dt^2 + \frac{1}{f(r)} (dr + n(r) dt)^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (7)$$

where $n(r) = N^r(r)$ is the radial component of shift function, and $N_r = n(r)/f(r)$ since $g_{rr} = 1/f(r)$. Note that n has mass dimension $z - 1 = 2$.

2.2. Constraints and Equations of Motion

The variation of the action (Equation (2)) with respect to the different degrees of freedom leads to the following constraints, or equations of motion, where a prime denotes a derivative with respect to the radial coordinate r .

- The variation with respect to A gives $R^{(3)} = 0$, or equivalently:

$$r f' + f - 1 = 0 \quad (8)$$

which imposes the function $f(r)$ in the metric (7) to be:

$$f(r) = 1 - \frac{2B}{r} \quad (9)$$

where B is a constant of integration.

- The variation with respect to ν gives:

$$\Theta^{ij} \nabla_i \nabla_j \nu + \Theta^{ij} K_{ij} = 0 \quad (10)$$

In what follows we will assume the Gauge fixing of $\nu = 0$, then the above constraint gives $\Theta^{ij} K_{ij} = 0$, which is satisfied for spherically symmetric solutions.

- The variation with respect to N gives the Hamiltonian constraint:

$$\int_0^\infty dr \frac{r^2}{\sqrt{f(r)}} (K_{ij} K^{ij} - K^2 + V) = 0 \quad (11)$$

which is an space-integral equation since N depends on time only.

- The variation with respect to n gives:

$$f'(r)n(r) = 0 \quad (12)$$

such that either f is a constant ($B = 0$) or $n = 0$.

- The variation with respect to f gives:

$$A' + \frac{A}{2r} \left(1 - \frac{1}{f}\right) + 4 \frac{fn(\sqrt{rn})'}{\sqrt{r}} = \left\{ \frac{r}{4f} - \frac{\sqrt{f}}{2r} \sum_{n=0}^3 (-1)^n \frac{d^n}{dr^n} \left(\frac{r^2}{\sqrt{f}} \frac{\partial}{\partial f^{(n)}} \right) \right\} V \quad (13)$$

where the time has been rescaled such that $N = 1$.

2.3. Solutions

The different solutions of the previous set of equations were derived in [25,26], and here we shortly review the different cases.

- $n = 0$: the solution of the differential Equation (13) gives short-distance corrections to the Schwarzschild solution, details of which can be found in [25,26];
- $n \neq 0$ and $A = 0$: leads to the Schwarzschild solution in the Painleve-Gullstrand coordinates ($n \propto r^{-1/2}$), and the Hamiltonian constraint is automatically satisfied;
- $n \neq 0$ and $A \neq 0$: corresponds to the situation with multiple solutions for A and n , on which this article focuses. Since $B = 0$, we have $f = 1$ and $V = 0$, such that the Equations (11) and (13) give respectively:

$$\begin{aligned} \int_0^\infty dr (rn^2)' &= 0 \\ rA' + 2(rn^2)' &= 0 \end{aligned} \quad (14)$$

with solutions:

$$n^2(r) = \frac{C}{r} - \frac{1}{2}A(r) + \frac{1}{2r} \int_0^r d\rho A(\rho) \quad (15)$$

where C is a constant of integration and the auxiliary field A satisfies:

$$\int_0^\infty dr r A'(r) = 0 \quad (16)$$

The flexibility in the choice of solution for A will help describe galaxy rotation curves, as explained in the next section.

3. Galaxy Rotation Curves

In what follows, we consider each star in the spiral arms of the galaxy as a test particle, moving on a circular trajectory under the influence of the potential $\phi(r)$, generated by the centre of the galaxy, which is assumed to be static and spherically symmetric. Our approach is to start from a stellar velocity

distribution and derive the corresponding expression for the auxiliary field A , in the above situation with $n \neq 0$ and $A \neq 0$, where multiple solutions for A are allowed. We note the following few points:

(1) We consider vacuum solutions of covariant HL gravity, for which one cannot in principle describe the region in the centre of the galaxy. Nevertheless, as we will see in case of the following velocity profiles, a consistent solution for A can be found for $0 \leq r \leq R$, where R is a typical radial length describing the galaxy. To be consistent with the usual studies of galaxy rotations, we will choose here $R = r_{200}$, corresponding to the virial radius of the galaxy. The latter is defined as the distance from the centre of the galaxy, where the density ρ_{200} is 200 times the critical density ρ_c of the Universe:

$$\rho_{200} = 200\rho_c = 200 \frac{3H^2}{8\pi G} \quad (17)$$

where H is the Hubble constant;

(2) For $r > r_{200}$, we assume the solution $A = 0$, which leads to the usual Schwarzschild solution outside the galaxy. Indeed, far from the galaxy, one expects to see the Newtonian potential. This solution implies that the Hamiltonian constraint is expressed in terms of an integral over the finite range $[0, r_{200}]$ of radial coordinate r , which we can impose to vanish by fixing the constants of integration;

(3) For $r \leq r_{200}$, since the shift $A \rightarrow A + \text{constant}$ does not have a physical implication, one can expect to find a consistent solution for A which is continuous at $r = r_{200}$, *i.e.*, $A(r_{200}) = 0$. On the other hand, one cannot expect the auxiliary field to be differentiable at $r = r_{200}$, but this is not necessary for the consistency of covariant HL gravity: the second derivative of A does not appear in any equation of motion or constraint, so the first derivative can be discontinuous.

3.1. Gravitational Potential

On a circular trajectory, the relation between the speed v of a star and the Newtonian potential ϕ is:

$$\frac{v^2}{r} = \frac{d\phi}{dr} \quad (18)$$

where ϕ is obtained from the g_{00} component of the low energy effective theory:

$$-c^2 + n^2 = -c^2(1 + 2\phi) \quad (19)$$

and is thus given by:

$$\phi(r) = -\frac{n^2(r)}{2c^2} \quad (20)$$

We note here that, in the situation where the second auxiliary field ν does not vanish, the Newtonian potential is instead given by:

$$\phi(r) = -\frac{(n(r) - \nabla_r \nu(r))^2}{2c^2} \quad (21)$$

and ϕ is independent of the $U_\Sigma(1)$ gauge choice for ν and A . Also, ϕ is dimensionless, so that the speed v we use in this article is also dimensionless, which corresponds to a usual definition of speed, for isotropic space-time. The corresponding ‘‘Lifshitz velocity’’ is cv , with mass dimension $z - 1 = 2$.

From the relations (15) and (18), the speed v and the auxiliary field A are then related by:

$$\frac{rA'(r)}{4c^2} = \frac{d}{dr} \left(r \int^r d\rho \frac{v^2(\rho)}{\rho} \right) = v^2(r) + \int^r d\rho \frac{v^2(\rho)}{\rho} \quad (22)$$

and we have to check that the Hamiltonian constraint (16) is satisfied, in order to show the consistency of the approach.

We also note from Equations (15) and (18) that the solution $A = 0$ gives,

$$v(r) = \sqrt{\frac{C}{2c^2r}} \quad (23)$$

which is expected from Newtonian mechanics outside the galaxy, for $r > r_{200}$. If one fixes the constant of integration C to the value:

$$C = 2c^2r_{200}v_{200}^2 \quad (24)$$

where $v_{200} = v(r_{200})$, one obtains a continuous speed at $r = r_{200}$. Finally a different choice of the auxiliary field ν would modify the field A in such a way that physical results would not be changed.

3.2. Navarro, Frenk and White Profile

This profile originates from Cold Dark Matter halo models and is characterised by the ‘‘cusp’’ shape of the density distribution. Its validity has been tested on variety of observational results, including Low Surface Brightness galaxies in [27] and spherical galaxies and clusters in [28]. For our purposes, we use the circular velocity profile derived from the mass density profile in [29], based on the assumption of a Dark Matter halo and ignoring the contribution of baryons,

$$v(r) = v_0 \sqrt{\frac{r_{200}}{r} \ln \left(1 + \frac{ar}{r_{200}} \right) - \frac{a}{1 + ar/r_{200}}} \quad (25)$$

where a is the concentration parameter and:

$$v_0 = \frac{v_{200}}{\sqrt{\ln(1+a) - a/(1+a)}} \quad (26)$$

where v_{200} , the virial velocity, is the circular velocity at the virial radius. Some typical values [30] are in Table 1:

Table 1. Parameters for the Navarro, Frenk and White profile.

Galaxy	a	v_{200} (km/s)	r_{200} (kpc)
NGC 2403	10.9 ± 0.6	106.1 ± 1.9	88.4 ± 2.6
NGC 3198	11.2 ± 0.43	104.0 ± 0.7	86.6 ± 1.6
NGC 3521	14.0 ± 12.6	122.5 ± 20.4	102.0 ± 18

With the profile (Equation (25)), Equation (22) leads to:

$$\frac{rA'(r)}{4c^2v_0^2} = \phi_1 - \frac{a}{1 + ar/r_{200}} \quad (27)$$

where ϕ_1 is a constant of integration, and therefore:

$$\frac{A(r)}{4c^2v_0^2} = (\phi_1 - a) \ln \left(\frac{r}{r_{200}} \right) + a \ln \left(1 + a \frac{r}{r_{200}} \right) + A_1 \quad (28)$$

where A_1 is a constant of integration. Since we consider the solution $A(r) = 0$ for $r > r_{200}$ and impose $A(r)$ to be continuous at $r = r_{200}$, then $A_1 = -a \ln(1 + a)$ and:

$$\begin{aligned} \frac{A(r)}{4c^2v_0^2} &= (\phi_1 - a) \ln\left(\frac{r}{r_{200}}\right) + a \ln\left(\frac{1 + ar/r_{200}}{1 + a}\right) \quad \text{if } 0 \leq r \leq r_{200} \\ A(r) &= 0 \quad \text{if } r > r_{200} \end{aligned} \quad (29)$$

Finally, the Hamiltonian constraint:

$$0 = \int_0^\infty dr r A'(r) = 4c^2v_0^2 \int_0^{r_{200}} dr \left(\phi_1 - \frac{a}{1 + ar/r_{200}} \right) \quad (30)$$

is satisfied if:

$$\phi_1 = \ln(1 + a) \quad (31)$$

We note that the solution (Equation (29)) is consistent for any relevant values of the parameters a, v_0, r_{200} and therefore allows the description of a whole range of galaxies.

3.3. Pseudoisothermal Profile

The pseudoisothermal mass density profile assumes an existence of a “cored” dark matter halo component in the galaxy [31], with mass density of an approximately constant value in the central region of the galaxy, for $r \leq R_c$. Empirically motivated, the model is often contrasted with the aforementioned Navarro, Frenk and White (NFW) “cuspy” profile, while evidence shows that it provides better fit of the galaxy rotation velocities [27]. The velocity profile is:

$$v(r) = v_0 \sqrt{1 - \frac{R_c}{r} \arctan\left(\frac{r}{R_c}\right)} \quad (32)$$

where v_0 is given by:

$$v_0 = \sqrt{4\pi G \rho_0 R_c^2} \quad (33)$$

and ρ_0 , the central density, is the density within R_c . Typical values [30] are in Table 2:

Table 2. Parameters for the Pseudoisothermal profile.

Galaxy	R_c (kpc)	ρ_0 ($10^{-3} \mathbf{M}_\odot \mathbf{pc}^{-3}$)	r_{200} (kpc)
NGC 2403	2.51 ± 0.32	59.1 ± 14.3	88.4 ± 2.6
NGC 2841	1.36 ± 0.75	674.8 ± 736.4	159.0 ± 3.7
NGC 3621	5.88 ± 0.32	13.0 ± 1.1	106.6 ± 4.0

We note that in the case of NCG 2403, both the NFW and the present profile can be shown to produce fits of comparably good quality.

With the profile (Equation (32)), Equation (22) leads to:

$$\frac{rA'(r)}{4c^2v_0^2} = \phi_2 + \frac{1}{2} \ln\left(1 + \frac{r^2}{R_c^2}\right) \quad (34)$$

where ϕ_2 is a constant of integration. The same steps as those describe for the previous profile lead to the solution:

$$\begin{aligned} \frac{A(r)}{4c^2v_0^2} &= \phi_2 \ln\left(\frac{r}{r_{200}}\right) + \frac{1}{2} \int_0^{r/R_c} \frac{dt}{t} \ln(1+t^2) + A_2 \quad \text{if } 0 \leq r \leq r_{200} \\ A(r) &= 0 \quad \text{if } r > r_{200} \end{aligned} \quad (35)$$

where the constants of integration are:

$$A_2 = -\frac{1}{2} \int_0^{r_{200}/R_c} \frac{dt}{t} \ln(1+t^2) \quad (36)$$

for the continuity of $A(r)$ at $r = r_{200}$, and:

$$\phi_2 = -\frac{1}{2} \ln\left(1 + \frac{r_{200}^2}{R_c^2}\right) + 1 - \frac{R_c}{r_{200}} \arctan\left(\frac{r_{200}}{R_c}\right) \quad (37)$$

for the Hamiltonian constraint to be satisfied.

3.4. Vacuum Energy Contribution of the Auxiliary Field

Since the present model does not require Dark Matter to explain galaxy rotation curves, the resulting mass content of the Universe might therefore be reduced. But the static configuration of A actually results in an effective vacuum energy. This can be seen from the original action (Equation (2)), after integrating by parts the term $-\sqrt{g}AR^{(3)}$, which involve second-order space derivatives of the metric. This leads to the following contribution to the Hamiltonian:

$$M \propto \frac{1}{c} \int d^3x \sqrt{g} g_{ij} \nabla^i \nabla^j A = \frac{4\pi}{c} \int_0^{r_{200}} r^2 dr \partial_r^2 A \quad (38)$$

where the factor $1/c$ arises from the absence of integration over time, and restores the correct dimension.

In the case of the Navarro, Frenk and White profile, the energy (Equation (38)) reads:

$$M_1 = 16\pi cv_0^2 r_{200} \left[\ln(1+a) - \frac{a}{1+a} \right] \quad (39)$$

which is always positive, for any value of the parameter $a \geq 0$.

For the Pseudoisothermal case, we obtain from Equation (38):

$$M_2 = 16\pi cv_0^2 r_{200} \left[1 - \frac{R_c}{r_{200}} \arctan\left(\frac{r_{200}}{R_c}\right) \right] \quad (40)$$

which is also always positive, for any value of the ratio r_{200}/R_c .

The auxilliary field A has therefore a cosmological role, by contributing to the vacuum energy of the Universe, consistently with its observed acceleration.

4. Conclusions

The multiplicity of solutions in covariant HL gravity has been used to describe galaxy rotation curves, without the need for Dark Matter. This multiplicity is a consequence of the breaking of 4-dimensional diffeomorphism, in the projectable case, where the lapse function is imposed to depend on time only. We considered two specific velocity profiles and showed in each case that a corresponding consistent solution can be derived for the auxiliary field A . Clearly, similar results can be expected with other profiles. A more detailed analysis would consist in taking into account the finite star density at the centre of the galaxy, as well as its rotation.

One should note that the solutions found, for the auxiliary field A , depend on parameters which cannot appear in the Lagrangian in a universal way. These parameters arise as constants of integrations, as a result of the integral form of the Hamiltonian constraint, and they have to be fitted to each galaxy which is studied. This is a bit similar to the Schwarzschild metric in General Relativity, involving a mass which is a constant of integration and depends on the star/black hole under consideration.

It is interesting to note that the solutions (Equations (29) and (35)) actually do not only describe galaxy rotation curves, but also the usual Schwarzschild solutions for the Solar System, by taking the limit $r_{200} \rightarrow 0$ such that $A = 0$ for all r . In this sense these solutions provide an effective unified description of gravitational effects, from the solar system scale to the galaxy scale, where local Lorentz violation appears gradually, as the observational length increases from the size of the solar system to the size of the galaxy.

We finally note that, although HL gravity was initially developed to modify ultraviolet behaviour of gravity, our present work uses another consequence of local Lorentz symmetry violation, which is the existence of a non-unique solution to the Hamiltonian constraint. Since it has been shown that canonical Hořava-Lifshitz gravity provides a description of Cosmology which is consistent with constraints from observational data [32], it would be interesting to see if the non-unique time-dependent solutions of the covariant version of the theory could also allow for an effective description of Cosmology, including an alternative to Dark Energy.

Acknowledgments

The authors would like to thank the two anonymous reviewers for their insightful comments and the thorough discussion that helped to improve the paper.

Conflicts of Interest

The authors declare no conflict of interest.

References

1. Hořava, P. Quantum gravity at a Lifshitz point. *Phys. Rev. D* **2009**, *79*, doi:10.1103/PhysRevD.79.084008.
2. Alexandre, J. Lifshitz-type quantum field theories in particle physics. *Int. J. Mod. Phys. A* **2011**, *26*, 4523, doi:10.1142/S0217751X11054656.

3. Padilla, A. The good, the bad and the ugly of Hořava gravity. *J. Phys. Conf. Ser.* **2010**, *259*, 012033, doi:10.1088/1742-6596/259/1/012033.
4. Mukohyama, S. Hořava-Lifshitz cosmology: A review. *Class. Quantum Gravity* **2010**, *27*, 223101, doi:10.1088/0264-9381/27/22/223101.
5. Sotiriou, T.P. Hořava-Lifshitz gravity: A status report. *J. Phys. Conf. Ser.* **2011**, *283*, 012034, doi:10.1088/1742-6596/283/1/012034.
6. Sotiriou, T.P.; Visser, M.; Weinfurter, S. Quantum gravity without Lorentz invariance. *J. High Energy Phys.* **2009**, *2009*, 033, doi:10.1088/1126-6708/2009/10/033.
7. Charmousis, C.; Niz, G.; Padilla, A.; Saffin, P.M. Strong coupling in Hořava gravity. *J. High Energy Phys.* **2009**, *2009*, 070, doi:10.1088/1126-6708/2009/08/070.
8. Hořava, P.; Melby-Thompson, C.M. General covariance in quantum gravity at a Lifshitz point. *Phys. Rev. D* **2010**, *82*, 064027, doi:10.1103/PhysRevD.82.064027.
9. Abdalla, E.; da Silva, A.M. On the motion of particles in covariant Hořava-Lifshitz gravity and the meaning of the A-field. *Phys. Lett. B* **2012**, *707*, 311–314.
10. Lin, K.; Wang, A. Static post-Newtonian limits in non-projectable Hořava-Lifshitz gravity with an extra U(1) symmetry. *ArXiv E-Prints*, **2013**, [arXiv:1212.6794](https://arxiv.org/abs/1212.6794).
11. Mukohyama, S. Dark matter as integration constant in Hořava-Lifshitz gravity. *Phys. Rev. D* **2009**, *80*, 064005, doi:10.1103/PhysRevD.80.064005.
12. Cardone, V.F.; Radicella, N.; Ruggiero, M.L.; Capone, M. The Milky Way rotation curve in Hořava-Lifshitz theory. *ArXiv E-Prints*, **2010**, [arXiv:1003.2144](https://arxiv.org/abs/1003.2144).
13. Cardone, V.F.; Capone, M.; Radicella, N.; Ruggiero, M.L. Spiral galaxies rotation curves in the Hořava-Lifshitz gravity theory. *ArXiv E-Prints*, **2012**, [arXiv:1202.2233](https://arxiv.org/abs/1202.2233).
14. Romero, J.M.; Bernal-Jaquez, R.; Gonzalez-Gaxiola, O. Is it possible to relate MOND with Hořava Gravity? *Mod. Phys. Lett. A* **2010**, *25*, 2501, doi:10.1142/S0217732310033396.
15. Alexandre, J.; Farakos, K.; Tsapalis, A. Liouville-Lifshitz theory in 3 + 1 dimensions. *Phys. Rev. D* **2010**, *81*, 105029, doi:10.1103/PhysRevD.81.105029.
16. Dutta, S.; Saridakis, E.N. Overall observational constraints on the running parameter λ of Hořava-Lifshitz gravity. *J. Cosmol. Astropart. Phys.* **2010**, *2010*, 013, doi:10.1088/1475-7516/2010/05/013.
17. Lin, K.; Mukohyama, S.; Wang, A. Solar system tests and interpretation of gauge field and Newtonian prepotential in general covariant Hořava-Lifshitz gravity. *Phys. Rev. D* **2012**, *86*, 104024, doi:10.1103/PhysRevD.86.104024.
18. Da Silva, A.M. An alternative approach for general covariant Hořava-Lifshitz gravity and matter coupling. *Class. Quantum Gravity* **2011**, *28*, 055011, doi:10.1088/0264-9381/28/5/055011.
19. Klusoň, J. Hamiltonian analysis of nonrelativistic covariant restricted-foliation-preserving diffeomorphism invariant Hořava-Lifshitz gravity. *Phys. Rev. D* **2011**, *83*, 044049, doi:10.1103/PhysRevD.83.044049.
20. Blas, D.; Pujolas, O.; Sibiryakov, S. On the extra mode and inconsistency of Hořava gravity. *J. High Energy Phys.* **2009**, *2009*, 029, doi:10.1088/1126-6708/2009/10/029.
21. Bogdanos, C.; Saridakis, E.N. Perturbative instabilities in Hořava gravity. *Class. Quantum Gravity* **2010**, *27*, 075005, doi:10.1088/0264-9381/27/7/075005.

22. Koyama, K.; Arroja, F. Pathological behaviour of the scalar graviton in Hořava-Lifshitz gravity. *J. High Energy Phys.* **2010**, *2010*, 061, doi:10.1007/JHEP03(2010)061.
23. Blas, D.; Pujolas, O.; Sibiryakov, S. Comment on “Strong coupling in extended Hořava-Lifshitz gravity”. *Phys. Lett. B* **2010**, *688*, 350–355.
24. Papazoglou, A.; Sotiriou, T.P. Strong coupling in extended Hořava-Lifshitz gravity. *Phys. Lett. B* **2010**, *685*, 197–200.
25. Alexandre, J.; Pasipoularides, P. Spherically symmetric solutions in covariant Hořava-Lifshitz gravity. *Phys. Rev. D* **2011**, *83*, 084030, doi:10.1103/PhysRevD.83.084030.
26. Greenwald, J.; Satheeshkumar, V.H.; Wang, A. Black holes, compact objects and solar system tests in non-relativistic general covariant theory of gravity. *J. Cosmol. Astropart. Phys.* **2010**, *2010*, 007, doi:10.1088/1475-7516/2010/12/007.
27. De Naray, R.K.; McGaugh, S.S.; de Blok, W.J.G. Mass models for low surface brightness galaxies with high resolution optical velocity fields. *Astrophys. J.* **2008**, *676*, 920, doi:10.1086/527543.
28. Lokas, E.L.; Mamon, G.A. Properties of spherical galaxies and clusters with an nfw density profile. *Mon. Not. R. Astron. Soc.* **2001**, *321*, 155–166.
29. Navarro, J.F.; Frenk, C.S.; White, S.D.M. The structure of cold dark matter halos. *Astrophys. J.* **1996**, *462*, 563–575.
30. De Blok, W.J.G.; Walter, F.; Brinks, E.; Trachternach, C.; Oh, S.-H.; Kennicutt, R.C., Jr. High-resolution rotation curves and galaxy mass models from THINGS. *Astron. J.* **2008**, *136*, 2648, doi:10.1088/0004-6256/136/6/2648.
31. Lynden-Bell, D. Statistical mechanics of violent relaxation in stellar systems. *Mon. Not. R. Astron. Soc.* **1967**, *136*, 101–121.
32. Dutta, S.; Saridakis, E.N. Observational constraints on Hořava-Lifshitz cosmology. *J. Cosmol. Astropart. Phys.* **2010**, *2010*, 013, doi:10.1088/1475-7516/2010/01/013.

© 2013 by the authors; licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution license (<http://creativecommons.org/licenses/by/3.0/>).