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# A Logical Analysis of Existential Dependence and Some Other Ontological Concepts—A Comment to Some Ideas of Eugenia Ginsberg-Blaustein

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**Abstract:** This paper deals with several problems concerning notion of existential dependence and ontological notions of existence, necessity and fusion. Following some ideas of Eugenia Ginsberg-Blaustein, the notions are treated in reference to objects, in relation to the concepts of state of affairs and subject of state of affairs. It provides an axiomatic characterization of these concepts within the framework of a multi-modal propositional logic and then presents a semantic analysis of these concepts. The semantics are a slight modification to the standard relational semantics for normal modal propositional logic.

**Keywords:** existential dependence; object; state of affairs; existence; necessity; fusion

## 1. Introduction

In the paper ‘On the Concepts of Existential Dependence and Independence’ Eugenia Ginsberg-Blaustein, a member of the Philosophical Centre at the University of Lwów and a participant of Twardowski’s seminar in the years 1926–1930, put forward an imaginative definition of existential dependence and existential independence. The paper was published originally in Polish as “W sprawie pojęć samoistności i niesamoistności”, *Księga pamiątkowa Polskiego Towarzystwa Filozoficznego we Lwowie 12. II. 1904–12. II. 1929*, Lwów, 1931 [1]. The English translation was published in *Parts and Moments*, ed. by Barry Smith with Editorial Note by Peter M. Simons [2,3]. The main idea of her approach is that the concepts of existential dependence and independence could be defined by means of the concepts of state of affairs and subject of the state of affairs taken as primitive. According to Ginsberg-Blaustein, ‘object  $Q_1$  is dependent with respect to object  $Q_2$  if, for the existence or occurrence of  $Q_1$ , the occurrence of some state of affairs in which  $Q_2$  is the subject is necessary’ ([4], p. 278). In my view, some other ontological concepts used in reference to objects could be analyzed in terms of state of affairs and subject of the state of affairs as well. The paper provides a tentative formal logical study of ontological concepts of existential dependence, existence, necessity and fusion, used in reference to objects, in relation to the concepts of state of affairs and subject of state of affairs. However, I am not going to give definitions of these ontological concepts. I would like to present an axiomatic characterization instead. Thus, the axioms could be interpreted as implicit definitions of these concepts.

Another logical study of existential dependence provides the paper ‘A Logical Theory of Dependence’, by Kurt Grelling from 1939 [5]. Therefore, Grelling also was an early pioneer in the research.

## 2. Preliminaries and Some Basic Insights

In order to elaborate on this idea, I will study logical interconnections between ontological concepts of existential dependence, existence, necessity and fusion, used in reference to objects, in

relation to the concepts of state of affairs and subject of state of affairs. Thus, I will study contexts like **b** is a subject of **A**, where **A** stands for a state of affairs and **b** stands for an object. Let me adopt an informal notation to express some basic insights. For object **a** and state of affairs **B**, let **aB** mean that **a** is a subject of **B**. For object **a**, let **Exa** mean that **a** exists and let **Neca** mean that **a** is a necessary object. For objects **a** and **b**, let **a\b** mean that object **a** is dependent with respect to object **b** and let **(a◦b)** stand for the fusion of **a** and **b**. The signs  $\sim$ ,  $\&$ ,  $\vee$  and  $\rightarrow$  will then be used, respectively, as symbols for negation, conjunction, disjunction and material implication. In contexts like **Exa**, **Neca**, **a\b** and **(a◦b)** symbols, **a** and **b**, represent objects. In contexts like **aB**, symbol **a** behaves also like one-placed modal operator. Thus, symbols **a**, **b**, **c**, ... are referred to by the term 'object-operators'. It seems intuitively that if state of affairs **B** logically implies state of affairs **C**, and object **a** is a subject of **C**, then **a** is also a subject of **B**. It is quite clear that if object **a** is a subject of state of affairs **B**, and **B** obtains, then object **a** exists:

$$(1) \quad \mathbf{aB} \rightarrow (\mathbf{B} \rightarrow \mathbf{Exa}).$$

Thus, the denial of the existence of **a** implies that no state of affairs **B**, such that object **a** is a subject of state of affairs **B**, obtains:

$$(2) \quad \sim \mathbf{Exa} \rightarrow (\mathbf{aB} \rightarrow \sim \mathbf{B}).$$

On the other hand, object **a** is a subject of the state of affairs that **a** exists:

$$(3) \quad \mathbf{aExa}.$$

Some object could be a contingent being and some object could be a necessary being. Object **a** is a necessary being if and only if **a** is not a contingent being. By necessary object, I understand an ontological generator, using the term of Jerzy Perzanowski [6], or God-like being of the theory. If an object is a contingent object, then there is an object that makes it possible. According to Perzanowski, object **a** is an ontological generator if and only if it makes possible any object. However, the theory does not imply that there are some ontological generators. Maybe ontological realm is not well founded. The theory does not imply that there are some necessary beings. It can be shown, by counter models, that no formula of the form **Neca** is a thesis, as well as no formula of the form  $\sim \mathbf{Neca}$ . If object **a** is a necessary being, then **a** is a subject of any state of affairs.

$$(4) \quad \mathbf{Neca} \rightarrow \mathbf{aB}.$$

The claim that if object **a** is a necessary being, then **a** being a subject of any state of affairs is very controversial. However, the term *necessary being* is understood as *ontological generator* in the sense of Perzanowski, for example, as the God of Spinoza. On the other hand, if object **a** is a subject of tautological state of affairs, then **a** is a necessary being:

$$(5) \quad \mathbf{a1} \rightarrow \mathbf{Neca}.$$

(1) stands for arbitrary chosen tautology of propositional logic. The intuition is that every state of affairs entails every tautology, so if any state of affairs obtains, then tautological state of affairs obtains. Therefore, being a subject of tautological state of affairs is the same as being a subject of any state of affairs. Of course, each necessary object exists:

$$(6) \quad \mathbf{Neca} \rightarrow \mathbf{Exa}.$$

It could remind readers of the conclusion of Thomas Aquinas' Argument from Necessity to the effect that a necessary being exists. However, the implication (6) says only that if **a** is a necessary object, then **a** exists. However, it does not imply that there is an object, which is a necessary being. If object **a** is dependent with respect to object **b**, then for any state of affairs **C**, if **a** is a subject of **C**, then **b** is a subject of **C**:

$$(7) \quad \mathbf{a\b} \rightarrow (\mathbf{aC} \rightarrow \mathbf{bC}).$$

On the other hand, if object **a** is a subject of the state of affairs that object **b** exists, then **b** is dependent with respect to **a**:

$$(8) \quad \mathbf{aExb} \rightarrow \mathbf{b\ a}.$$

Conversely, if object **b** is dependent with respect to **a**, then object **a** is a subject of the state of affairs that object **b** exists:

(9)  $\mathbf{b} \setminus \mathbf{a} \rightarrow \mathbf{a} \text{Ex} \mathbf{b}$ .

Dependence is a transitive and reflexive relation, thus:

(10)  $(\mathbf{a} \setminus \mathbf{b} \ \& \ \mathbf{b} \setminus \mathbf{c}) \rightarrow \mathbf{a} \setminus \mathbf{c}$ .

and

(11)  $\mathbf{a} \setminus \mathbf{a}$ .

The claim that dependence is a reflexive relation is very controversial. However, this property is quite nice from the technical point of view. By the fusion of objects  $\mathbf{a}$  and  $\mathbf{b}$ , I understand a compound object composed of  $\mathbf{a}$  and  $\mathbf{b}$  as its direct components. The components of a fusion could be understood as its parts, moments, qualities, etc. Therefore, if the fusion of objects  $\mathbf{a}$  and  $\mathbf{b}$  is a subject of state of affairs  $\mathbf{C}$ , then object  $\mathbf{a}$  is a subject of state of affairs  $\mathbf{C}$  and object  $\mathbf{b}$  is a subject of state of affairs  $\mathbf{C}$ :

(12)  $(\mathbf{a} \circ \mathbf{b}) \mathbf{C} \rightarrow (\mathbf{a} \mathbf{C} \ \& \ \mathbf{b} \mathbf{C})$ .

However, the converse implication is not generally valid. There could be a state of affairs  $\mathbf{C}$  such that object  $\mathbf{a}$  is a subject of  $\mathbf{C}$  and object  $\mathbf{b}$  is a subject of  $\mathbf{C}$ , but the fusion of  $\mathbf{a}$  and  $\mathbf{b}$  is not a subject of  $\mathbf{C}$ . If the fusion of  $\mathbf{a}$  and  $\mathbf{b}$  exists, then  $\mathbf{a}$  and  $\mathbf{b}$  also exist:

(13)  $\text{Ex}(\mathbf{a} \circ \mathbf{b}) \rightarrow (\text{Ex} \mathbf{a} \ \& \ \text{Ex} \mathbf{b})$ .

However, it is not converse. It is possible that object  $\mathbf{a}$  exists and object  $\mathbf{b}$  exists, but the compound object composed of  $\mathbf{a}$  and  $\mathbf{b}$  does not exist. Finally, each compound object is dependent with respect to its direct components:

(14)  $(\mathbf{a} \circ \mathbf{b}) \setminus \mathbf{a}$ .

and

(15)  $(\mathbf{a} \circ \mathbf{b}) \setminus \mathbf{b}$ .

Note that the operation of forming the fusion is neither associative nor commutative. A mixture as the fusion of its ingredients and a process as the fusion of its phases provide counterexamples.

In the following two paragraphs, these ideas will be studied in two different ways: a syntactical one in Section 2 and a semantic one in Section 3. The syntactical approach is concerned with a calculus and a theory as a set of all theses. The semantic approach is concerned with models and with truth conditions in a model. By the help of these semantic notions, a set of all valid formulae will be singled out.

### 3. The Theory

Let me introduce a formal language. The alphabet is given by:

- (a) a denumerable set  $P$  of propositional letters. I refer to these as  $p_1, p_2, p_3, \dots$ , etc.,
- (b) the symbols of logical connectives  $\sim$  and  $\&$  for negation and conjunction, respectively,
- (c) a denumerable set  $O$  of object letters. I refer to these as  $a_1, a_2, a_3, \dots$ , etc.,
- (d) the ontological symbols of existential dependence existence  $\text{Ex}$ , necessity  $\text{Nec}$  and fusion  $\circ$ ,
- (e) the auxiliary symbols (and) for parentheses.

The set  $\text{Ob}$  of one-place object-operators is the smallest set  $X$  satisfying the following conditions:

(Ob 1) Each object letter belongs to  $X$ .

(Ob 2) If  $x$  belongs to  $X$  and  $y$  belongs to  $X$ , then  $(x \circ y)$  belongs to  $X$ .

The letters  $\mathbf{a}, \mathbf{b}, \mathbf{c}, \dots$  are used as metalogical variables to range over object-operators. The object operators represent objects.

The set of well-formed sentential formulae is the smallest set  $X$  satisfying the following conditions:

(For 1) Each propositional letter belongs to  $X$ .

(For 2) If  $x$  belongs to  $X$  and  $y$  belongs to  $X$ , then  $(x \ \& \ y), \sim x$  and  $\sim y$  belongs to  $X$ .

- (For 3) If  $\mathbf{a}$  is an object operator, then  $\text{Exa}$  and  $\text{Neca}$  belongs to  $X$ .  
 (For 4) If  $\mathbf{a}$  and  $\mathbf{b}$  are object operators, then  $\mathbf{a} \setminus \mathbf{b}$  belongs to  $X$ .  
 (For 5) If  $x$  belongs to  $X$  and  $\mathbf{a}$  is an object operator, then  $\mathbf{a}x$  belongs to  $X$ .

The letters  $\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$  are used as metalogical variables to range over well-formed sentential formulae. The formulae represent states of affairs.

Other symbols are introduced by following definitions:

- (D1)  $(\mathbf{A} \vee \mathbf{B}) = \sim(\sim\mathbf{A} \ \& \ \sim\mathbf{B})$ ,  
 (D2)  $(\mathbf{A} \rightarrow \mathbf{B}) = \sim(\mathbf{A} \ \& \ \sim\mathbf{B})$ ,  
 (D3)  $(\mathbf{A} \equiv \mathbf{B}) = \sim(\mathbf{A} \ \& \ \sim\mathbf{B}) \ \& \ \sim(\sim\mathbf{A} \ \& \ \mathbf{B})$ .

Symbol **1** stands for arbitrary chosen tautology of propositional logic.

Within the frame of the formal language, a simple calculus can be constructed. Let me construct the calculus on the following axiomatic basis.

The first group of axioms consists of all tautologies of classical propositional logic, with well-formed formulae substituted for the propositional letters.

The second group of axioms is determined by the following schemata:

- (A1)  $(\mathbf{aB} \ \& \ \mathbf{aC}) \rightarrow \mathbf{a(B} \ \vee \ \mathbf{C})$ ,  
 (A2)  $\mathbf{aB} \rightarrow (\mathbf{B} \rightarrow \text{Exa})$ ,  
 (A3)  $\mathbf{aExa}$ ,  
 (A4)  $\mathbf{a1} \rightarrow \text{Neca}$ ,  
 (A5)  $\text{Neca} \rightarrow \mathbf{aB}$ ,  
 (A6)  $\mathbf{a} \setminus \mathbf{b} \rightarrow (\mathbf{aC} \rightarrow \mathbf{bC})$ ,  
 (A7)  $\mathbf{bExa} \rightarrow \mathbf{a} \setminus \mathbf{b}$ ,  
 (A8)  $(\mathbf{a} \circ \mathbf{b})\mathbf{C} \rightarrow (\mathbf{aC} \ \& \ \mathbf{bC})$ .

The rules of inference of the calculus are:

- (R1) Rule of Detachment (*Modus Ponens*),  
 (R2) A Rule of Inverse Monotonicity to the effect that if  $\mathbf{B} \rightarrow \mathbf{C}$  is a provable formula, then  $\mathbf{aC} \rightarrow \mathbf{aB}$  is also a provable formula.

The axiom schemata and the inference rules capture some ontological content discussed in the previous section.

A proof is defined in the standard way as a finite sequence of formulae such that each member either belongs to axioms or is derived from earlier members of the sequence by *Modus Ponens* or the Rule of Inverse Monotonicity. A proof is said to be a proof of the last member in its sequence, and a thesis is a formula of which there is a proof. Among the theses are all formal counterparts of (1)–(15). In fact, formal counterparts of (1), (3), (5), (7), (8) and (12) are axioms. As for the remainder of them, and some other theses, let me state the following theorem.

**Theorem 1.** *The following expressions are thesis schemata of the calculus:*

- (Th1)  $\sim\text{Exa} \rightarrow (\mathbf{aB} \rightarrow \sim\mathbf{B})$ ,  
 (Th2)  $\text{Neca} \rightarrow \mathbf{b} \setminus \mathbf{a}$ ,  
 (Th3)  $\text{Neca} \rightarrow \text{Exa}$ ,  
 (Th4)  $\mathbf{b} \setminus \mathbf{a} \rightarrow \mathbf{aExb}$ ,  
 (Th5)  $(\mathbf{a} \setminus \mathbf{b} \ \& \ \mathbf{b} \setminus \mathbf{c}) \rightarrow \mathbf{a} \setminus \mathbf{c}$ ,  
 (Th6)  $\mathbf{a} \setminus \mathbf{a}$ ,  
 (Th7)  $\text{Ex}(\mathbf{a} \circ \mathbf{b}) \rightarrow (\text{Exa} \ \& \ \text{Exb})$ ,

- (Th8)  $\mathbf{aEx(a \circ b)}$ ,  
 (Th9)  $\mathbf{bEx(a \circ b)}$ ,  
 (Th10)  $(\mathbf{a \circ b}) \setminus \mathbf{a}$ ,  
 (Th11)  $(\mathbf{a \circ b}) \setminus \mathbf{b}$ ,  
 (Th12)  $\mathbf{Nec(a \circ b) \rightarrow (Neca \ \& \ Necb)}$ ,  
 (Th13)  $(\mathbf{a \setminus b \ \& \ b \setminus a}) \rightarrow (\mathbf{Exa \equiv Exb})$ ,  
 (Th14)  $\mathbf{a \setminus b \rightarrow (a \circ c) \setminus b}$ ,  
 (Th15)  $\mathbf{a \setminus (b \circ c) \rightarrow (a \setminus b \ \& \ a \setminus c)}$ .

**Proof.** To prove the theorem, it is sufficient to show that the schemata (Th1)–(Th15) are thesis schemata (where proofs are easy, they are omitted).

- (Th1)  $\sim \mathbf{Exa} \rightarrow (\mathbf{aB} \rightarrow \sim \mathbf{B})$ : from (A2) by propositional logic.  
 (Th2)  $\mathbf{Neca} \rightarrow \mathbf{b \setminus a}$ : By (A5),  $\mathbf{Neca} \rightarrow \mathbf{aB}$  is a thesis schema. Then,  $\mathbf{Neca} \rightarrow \mathbf{aExb}$  is a thesis schema, and by (A7) and propositional logic,  $\mathbf{Neca} \rightarrow \mathbf{b \setminus a}$  is a thesis schema.  
 (Th3)  $\mathbf{Neca} \rightarrow \mathbf{Exa}$ : By (A2),  $\mathbf{aB} \rightarrow (\mathbf{B} \rightarrow \mathbf{Exa})$  is a thesis schema. Then, by (Th2) and propositional logic,  $\mathbf{Neca} \rightarrow (\mathbf{B} \rightarrow \mathbf{Exa})$  is a thesis schema and  $\mathbf{Neca} \rightarrow (\mathbf{1} \rightarrow \mathbf{Exa})$  is also a thesis schema ( $\mathbf{1}$  stands for arbitrary chosen tautology of propositional logic). Thus, by propositional logic,  $\mathbf{Neca} \rightarrow \mathbf{Exa}$  is a thesis schema.  
 (Th4)  $\mathbf{b \setminus a} \rightarrow \mathbf{aExb}$ : By (A6),  $\mathbf{b \setminus a} \rightarrow (\mathbf{bExb} \rightarrow \mathbf{aExb})$  is a thesis schema. However, by (A3),  $\mathbf{bExb}$  is a thesis schema, then, by propositional logic,  $\mathbf{b \setminus a} \rightarrow \mathbf{aExb}$  is a thesis schema.  
 (Th5)  $(\mathbf{a \setminus b \ \& \ b \setminus c}) \rightarrow \mathbf{a \setminus c}$ : By (A6) and propositional logic,  $(\mathbf{a \setminus b \ \& \ b \setminus c}) \rightarrow (\mathbf{aD} \rightarrow \mathbf{cD})$  is a thesis schema and therefore  $(\mathbf{a \setminus b \ \& \ b \setminus c}) \rightarrow (\mathbf{aExa} \rightarrow \mathbf{cExa})$  is a thesis schema. However, by (A3),  $\mathbf{aExa}$  is a thesis schema and  $(\mathbf{a \setminus b \ \& \ b \setminus c}) \rightarrow \mathbf{cExa}$  is a thesis schema. Hence, by (A7) and propositional logic,  $(\mathbf{a \setminus b \ \& \ b \setminus c}) \rightarrow \mathbf{a \setminus c}$  is a thesis schema.  
 (Th6)  $\mathbf{a \setminus a}$ : from (A3) and (A7) by propositional logic.  
 (Th7)  $\mathbf{Ex(a \circ b) \rightarrow (Exa \ \& \ Exb)}$ : By (A2),  $\mathbf{aEx(a \circ b) \rightarrow (Ex(a \circ b) \rightarrow Exa)}$  and  $\mathbf{bEx(a \circ b) \rightarrow (Ex(a \circ b) \rightarrow Exb)}$  are thesis schemata. However, by (A8),  $(\mathbf{a \circ b})\mathbf{Ex(a \circ b) \rightarrow aEx(a \circ b)}$  and  $(\mathbf{a \circ b})\mathbf{Ex(a \circ b) \rightarrow bEx(a \circ b)}$  are thesis schemata. Hence, by propositional logic,  $(\mathbf{a \circ b})\mathbf{Ex(a \circ b) \rightarrow (Ex(a \circ b) \rightarrow Exa)}$  and  $(\mathbf{a \circ b})\mathbf{Ex(a \circ b) \rightarrow (Ex(a \circ b) \rightarrow Exb)}$  are thesis schemata and, by (A3) and propositional logic,  $\mathbf{Ex(a \circ b) \rightarrow (Exa \ \& \ Exb)}$  is a thesis schema.  
 (Th8)  $\mathbf{aEx(a \circ b)}$ : By (Th7),  $\mathbf{Ex(a \circ b) \rightarrow Exa}$  is a thesis schema. Hence, by (R2),  $\mathbf{aExa} \rightarrow \mathbf{aEx(a \circ b)}$  is a thesis schema. However, by (A3),  $\mathbf{aExa}$  is a thesis schema and, by (R1),  $\mathbf{aEx(a \circ b)}$  is a thesis schema.  
 (Th9)  $\mathbf{bEx(a \circ b)}$ : in the same way as for (Th8).  
 (Th10)  $(\mathbf{a \circ b}) \setminus \mathbf{a}$ : By (Th8),  $\mathbf{aEx(a \circ b)}$  is a thesis schema. Hence, by (A7),  $(\mathbf{a \circ b}) \setminus \mathbf{a}$  is a thesis schema.  
 (Th11)  $(\mathbf{a \circ b}) \setminus \mathbf{b}$ : in the same way as for (Th10).  
 (Th12)  $\mathbf{Nec(a \circ b) \rightarrow (Neca \ \& \ Necb)}$ : By (A4),  $\mathbf{a1} \rightarrow \mathbf{Neca}$  and  $\mathbf{b1} \rightarrow \mathbf{Necb}$  are thesis schemata. However, by (A8),  $(\mathbf{a \circ b})\mathbf{C} \rightarrow \mathbf{aC}$  and  $(\mathbf{a \circ b})\mathbf{C} \rightarrow \mathbf{bC}$  are thesis schemata and, by (A5) and propositional logic,  $\mathbf{Nec(a \circ b) \rightarrow aC}$  and  $\mathbf{Nec(a \circ b) \rightarrow bC}$  are thesis schemata. Thus, by propositional logic,  $\mathbf{Nec(a \circ b) \rightarrow a1}$  and  $\mathbf{Nec(a \circ b) \rightarrow b1}$  are thesis schemata. Hence, by (A4),  $\mathbf{Nec(a \circ b) \rightarrow Neca}$  and  $\mathbf{Nec(a \circ b) \rightarrow Necb}$  are thesis schemata; therefore, by propositional logic,  $\mathbf{Nec(a \circ b) \rightarrow (Neca \ \& \ Necb)}$  is a thesis schema.  
 (Th13)  $(\mathbf{a \setminus b \ \& \ b \setminus a}) \rightarrow (\mathbf{Exa \equiv Exb})$ : By (A6),  $\mathbf{a \setminus b} \rightarrow (\mathbf{aExa} \rightarrow \mathbf{bExa})$  is a thesis schema and, by (A3) and propositional logic,  $\mathbf{a \setminus b} \rightarrow \mathbf{bExa}$  is a thesis schema. By (A2),  $\mathbf{bExa} \rightarrow (\mathbf{Exa} \rightarrow \mathbf{Exb})$  is a thesis schema. Hence, by propositional logic,  $\mathbf{a \setminus b} \rightarrow (\mathbf{Exa} \rightarrow \mathbf{Exb})$  is a thesis schema. On the other hand, by (A6),  $\mathbf{b \setminus a} \rightarrow (\mathbf{bExb} \rightarrow \mathbf{aExb})$  is a thesis schema and, by (A3) and propositional logic,  $\mathbf{b \setminus a} \rightarrow \mathbf{aExb}$  is a thesis schema. By (A2),  $\mathbf{aExb} \rightarrow (\mathbf{Exb} \rightarrow \mathbf{Exa})$  is a thesis schema. Hence, by propositional logic,  $\mathbf{b \setminus a} \rightarrow (\mathbf{Exb} \rightarrow \mathbf{Exa})$  is a thesis schema. Thus, by propositional logic,  $(\mathbf{a \setminus b \ \& \ b \setminus a}) \rightarrow (\mathbf{Exa \equiv Exb})$  is a thesis schema.

- (Th14)  $a \setminus b \rightarrow (a \circ c) \setminus b$ : from (Th10) and (Th5) by propositional logic.  
 (Th15)  $a \setminus (b \circ c) \rightarrow (a \setminus b \ \& \ a \setminus c)$ : By (A6),  $a \setminus (b \circ c) \rightarrow (aC \rightarrow (b \circ c)C)$  is a thesis schema. Hence, by (A8),  $a \setminus (b \circ c) \rightarrow (aC \rightarrow (bC \ \& \ cC))$  is a thesis schema. Therefore,  $a \setminus (b \circ c) \rightarrow (aExa \rightarrow (bExa \ \& \ cExa))$  is also a thesis schema and, by (A3),  $a \setminus (b \circ c) \rightarrow (bExa \ \& \ cExa)$  is a thesis schema. Thus, by (A7) and propositional logic,  $a \setminus (b \circ c) \rightarrow (a \setminus b \ \& \ a \setminus c)$  is a thesis schema.

Let me draw your attention to (Th8), (Th9) and to the last four schemata (Th12)–(Th15), which capture some ontological intuition about objects. Thesis schemata (Th8) and (Th9) say that objects  $a$  and  $b$  are both subjects of the state of affairs that the fusion of  $a$  and  $b$  exists. According to (Th12), if the fusion of  $a$  and  $b$  is a necessary object, then  $a$  is a necessary object and  $b$  is a necessary object. Due to (Th13), if  $a$  is dependent with respect to  $b$ , and  $b$  is dependent with respect to  $a$ , then  $a$  exist if and only if  $b$  exist. Schema (Th14) says that if  $a$  is dependent with respect to  $b$ , then a fusion of  $a$  and  $c$  is also dependent with respect to  $b$ , and schema (Th15) says that if  $a$  is dependent with respect to the fusion of  $b$  and  $c$ , then  $a$  is dependent with respect to  $b$  and  $a$  is dependent with respect to  $c$ .

For any set of formulae  $X$ , I shall say that  $A$  is *deducible* from  $X$  if and only if there are formulas  $B_1, B_2, \dots, B_n$  belonging to  $X$  such that the formula  $(B_1 \ \& \ B_2 \ \& \ \dots \ \& \ B_n) \rightarrow A$  is a thesis. A set of formulae  $X$  is called *consistent* if and only if there is no formula  $A$ , such that  $A$  and  $\sim A$  are both deducible from  $X$ . Otherwise,  $X$  is called *inconsistent*. The definition implies that if any set of formulae  $X$  is inconsistent, then some finite subset of  $X$  is also inconsistent, and that if any set of formulae  $X$  is consistent and a formula  $A$  is not deducible from  $X$ , then the set  $X \cup \{\sim A\}$  is also consistent. By the theory, I mean the class of all theses. Thus, the theory is the smallest set containing all axioms and closed with respect to *Modus Ponens* and the Rule of Inverse Monotonicity. Note that if the theory is inconsistent, then all sets of formulae are inconsistent. Fortunately, the following theorem holds.

**Theorem 2.** *The theory is consistent.*

**Proof.** *To prove that the theory is consistent, let me take advantage of a standard language of classical propositional logic. The alphabet is given by a denumerable set  $Q$  of propositional letters, I refer to these as  $q_1, q_2, q_3, \dots$ , etc., the symbols of logical connectives  $\sim$  and  $\&$  for negation and conjunction, respectively, and parentheses (and). The set of well-formed sentential formulae is defined inductively in the standard way. The symbols  $\vee, \rightarrow$  and  $\equiv$  are introduced by definitions in the standard way. Let me define the mapping  $T$ , from the language of the theory to the language of classical propositional calculus, as follows:*

- (T1)  $T(p_n) = q_{2n}$ ,
- (T2)  $T(a_n) = q_{2n+1}$ ,
- (T3)  $T(a \circ b) = T(a) \ \& \ T(b)$ ,
- (T4)  $T(\sim A) = \sim T(A)$ ,
- (T5)  $T(A \ \& \ B) = T(A) \ \& \ T(B)$ ,
- (T5)  $T(Exa) = T(a)$ ,
- (T6)  $T(Neca) = T(a)$ ,
- (T7)  $T(a \setminus b) = T(a) \rightarrow T(b)$ ,
- (T8)  $T(aB) = T(B) \rightarrow T(a)$ .

Thus, for every formula  $A$ , there is a unique formula  $T(A)$  in the language of classical propositional logic. Let me call it the PC-transform of  $A$ . It is easy to show that the PC-transform of every thesis of the theory is a tautology of classical propositional calculus. It follows that for every formula  $A$ ,  $A$  and  $\sim A$  are not both theses, for if they were,  $T(A)$  and  $\sim T(A)$  would both be tautologies of classical propositional calculus, which is impossible. Thus, by definition of the theory, for every formula  $A$ ,  $A$  and  $\sim A$  are not both deducible from the theory.

A set of formulae is *complete* if and only if, for any formula  $A$ , either  $A$  belongs to  $X$  or  $\sim A$  belongs to  $X$ . A set of formulae that is both consistent and complete is called a *maximal consistent* set of formulae.

The theorem known as the Lindenbaum’s Lemma to the effect that any consistent set of formulae is a subset of a maximal consistent set of formulae holds for the theory. Note that if  $X$  is a maximal consistent set of formulae, and  $A$  is deducible from  $X$ , then  $A$  belongs to  $X$ . It is easy to show that for every maximal consistent set of formulae  $X$  and for every formulae  $A$  and  $B$ ,  $\sim A$  belongs to  $X$  if and only if  $A$  does not belong to  $X$ , and  $(A \ \& \ B)$  belongs to  $X$  if and only if  $A$  belongs to  $X$  and  $B$  belongs to  $X$ .

#### 4. Semantics

As was said, the object-operators behave like modalities, so that they can be handled using possible-world semantics. Thus, the semantics for the theory is a slight modification to the standard relational semantics for normal modal propositional logic. Possible worlds are also referred to by the term ‘possibilities’, ‘stand points’ or simply ‘points’. The idea is roughly as follows. Usually, for any given object, there are possibilities that are relevant to this object, and there are possibilities that are not. Thus, some possible worlds involve this object and some of them do not. Moreover, the class of possibilities that are relevant for an object can vary from different standpoints. Thus, for any given object, the set of possible worlds, which involve this object from the standpoint of given possible world, can be singled out. Then, the object operators are to be interpreted as binary relations on the set of possible worlds. The intuition behind this modeling is that each object  $a$  is determined by the binary relation  $R_a$ , which correlates a possible world  $w$  with possible worlds, which involve object  $a$  at the possible world  $w$ . The symbol of fusion is to be interpreted as a binary operation  $\bullet$  defined on the set of binary relations assigned to object operators.

Formally, I shall introduce the notion of a model. A model  $M$  is to consist of a non-empty set of possible worlds  $W$ , an infinite sequence  $P_1, P_2, P_3, \dots$  of subsets of  $W$ , let me abbreviate it as  $P_i$ , a set of binary relations on  $W$ , let me abbreviate it as  $R$ , a binary operation  $\bullet$  defined on  $R$ , and an infinite sequence  $R_1, R_2, R_3, \dots$ , of binary relations from  $R$ , let me abbreviate it as  $R_i$ . Thus, I define an ontological model as a structure  $M \leq \langle W, P_i, R, \bullet, R_i \rangle$  satisfying the following additional conditions:

- (C1) for any relations  $R$  and  $S$  belonging to  $R$ , and for any  $w$  and  $v$  belonging to  $W$ , if  $vR\bullet Sw$ , then  $vRw$  and  $vSw$ ;
- (C2) for any relation  $R$  belonging to  $R$ , and for any  $w$  and  $v$  belonging to  $W$ , if  $vRv$  then  $vRw$ ;
- (C3) for any relations  $R$  and  $S$  belonging to  $R$ , and for any  $w$  and  $v$  belonging to  $W$ , if  $vRv$  implies that  $vSw$ , then  $vRw$  implies that  $vSw$ .

Condition (C1) reflects the conviction that a possible world, which involves a fusion, also involves the direct components of the fusion. Conditions (C2) and (C3) are, respectively, semantic counterparts of the axiom schemata (A3) and (A7), and they are necessary for the completeness result.

Given the definition of a model, I shall state the following theorem.

**Theorem 3.** *There are structures that are models.*

**Proof.** Let  $W^*$  be the set of maximal consistent sets of formulae. Due to theorem 2,  $W^*$  is a non-empty set. Let  $P_i^*$  be the infinite sequence of subsets of  $W^*$ , such that for each natural number  $k$ ,  $P_k^*$  is the set of maximal consistent sets of formulae containing propositional letter  $p_k$ . For each object operator  $a$ , let  $R_a$  be the binary relation on  $W^*$ , such that for any  $v$  and  $w$  belonging to  $W^*$ ,  $v R_a w$  if and only if  $\{\sim C: aC \in w\} \not\subseteq v$ . Let  $R^*$  be the set of binary relations on  $W^*$  that contains, for every object operator  $a$ , the relation  $R_a$  and no other relations. Let  $\bullet^*$  be the binary operation on  $R^*$ , such that for any  $R_a$  and  $R_b$  belonging to  $W^*$ ,  $R_a \bullet^* R_b = R_{(a \circ b)}$ . Let  $R_i^*$  be the infinite sequence of binary relations on  $W^*$ , such that for each natural number  $k$ , for any  $v$  and  $w$  belonging to  $W^*$ ,  $v R_k^* w$  if and only if  $\{\sim C: a_k C \in w\} \not\subseteq v$ . In order to show that the structure  $M^* \leq \langle W^*, P_i^*, R^*, \bullet^*, R_i^* \rangle$  is a model, it is sufficient to prove that the structure satisfies the conditions (C1), (C2) and (C3).

To prove (C1), assume that it is not the case that  $vR_a w$  or it is not the case that  $vR_b w$ . Thus,  $\{\sim C:aC \in w\} \subseteq v$  or  $\{\sim C:bC \in w\} \subseteq v$ . Now suppose  $(a \circ b)C$  belongs to  $w$ . Then, by (A8),  $aC$  and  $bC$  also belong to  $w$ , and by the assumption,  $\sim C$  belongs to  $v$ . Hence,  $\{\sim C:(a \circ b)C \in w\} \subseteq v$  and it is not the case that  $vR_a \bullet^* S_b w$ .

To prove (C2), assume that it is not the case that  $vR_a w$ . Thus,  $\{\sim C:aC \in w\} \subseteq v$  and, by (A3),  $\sim Exa$  belongs to  $v$ . Hence, by (A2), any formula depicted by schema  $(aC \rightarrow \sim C)$  also belongs to  $v$ . Thus, for any formula  $aC$ , if  $aC$  belongs to  $v$ , then  $\sim C$  belongs to  $v$ . Hence,  $\{\sim C:aC \in v\} \subseteq v$  and it is not the case that  $vR_a v$ .

To prove (C3), assume that for any  $v$ ,  $vR_a v$  implies that  $vR_b w$ . Thus, for any  $v$ , if  $\{\sim C:bC \in w\} \subseteq v$ , then  $\{\sim C:aC \in v\} \subseteq v$ . However, due to (A3), for any  $v$ , if  $\{\sim C:aC \in v\} \subseteq v$ , then  $\sim Exa$  belongs to  $v$ . Thus, for any  $v$ , if  $\{\sim C:bC \in w\} \subseteq v$ , then  $\sim Exa$  belongs to  $v$ . Hence, no maximal consistent set of formulae includes the set  $\{\sim C:bC \in w\} \cup \{Exa\}$ , and therefore this set is inconsistent. Thus, there is a finite subset of this set  $\{\sim C_1, \sim C_2, \sim C_3, \dots, \sim C_k, Exa\}$ , which is inconsistent, and therefore formula  $Exa \rightarrow (C_1 \vee C_2 \vee C_3 \vee \dots \vee C_k)$  is a thesis. Hence,  $Exa \rightarrow (C_1 \vee C_2 \vee C_3 \vee \dots \vee C_k)$  belongs to  $w$  and, by (A1), (R1) and (R2),  $(bC_1 \ \& \ bC_2 \ \& \ bC_3 \ \& \ \dots \ \& \ bC_k) \rightarrow bExa$  also belongs to  $w$ . However, formulae  $bC_1, bC_2, bC_3, \dots$ , and  $bC_k$  belong to  $w$ , and by (R1),  $bExa$  also belongs to  $w$ . Thus, by (A7),  $a \setminus b$  belongs to  $w$  and, by (A6), any formula depicted by schema  $(aC \rightarrow bC)$  also belongs to  $w$ . Hence,  $\{\sim C:aC \in w\} \subseteq \{\sim C:bC \in w\}$ , and, therefore, for any  $v$ , if  $\{\sim C:bC \in w\} \subseteq v$ , then  $\{\sim C:aC \in w\} \subseteq v$ . Thus,  $vR_a w$  implies that  $vR_b w$ .

It completes the proof that the structure  $M^* \leq W^*, P_i^*, R^*, \bullet^*, R_i^* >$  is a model. I shall call this structure the *canonical model*.

For any object-operator  $a$ , there is a unique binary relation  $R_a$ , which corresponds to  $a$  in a model  $M$ . For each natural number  $k$ ,  $R_k$  correspond to  $a_k$  and  $R_a \bullet R_b$  correspond to  $(a \circ b)$ . For any possible world  $w$ , let  $[w]^{Ra}$  be the set of possible worlds which involve object  $a$  at the possible world  $w$ . Thus,  $[w]^{Ra} = \{v:vR_a w\}$ . I shall call it the *range of object a* at the possible world  $w$ . The range of object  $a$  at the possible world  $w$  contains the possibilities that are relevant to object  $a$  according to the standpoint  $w$ .

In terms of possible world in a model I state, the truth conditions for formulae according to their forms. I write  $w \models^M A$  to mean that formula  $A$  is true at the possible world  $w$  in model  $M$ . The truth conditions are as follows:

1.  $w \models^M p_k$  if and only if  $w$  belongs to  $P_k$ , for  $k = 1, 2, 3, \dots$
2.  $w \models^M \sim A$  if and only if not  $w \models^M A$ .
3.  $w \models^M (A \ \& \ B)$  if and only if both  $w \models^M A$  and  $w \models^M B$ .
4.  $w \models^M aB$  if and only if for any possible world  $v$  if  $v \models^M B$  then  $v \in [w]^{Ra}$ .
5.  $w \models^M Exa$  if and only if  $w \in [w]^{Ra}$ .
6.  $w \models^M Neca$  if and only if  $[w]^{Ra} = W$ .
7.  $w \models^M a \setminus b$  if and only if  $[w]^{Ra} \subseteq [w]^{Rb}$ .

Clause (1) states what true value of each propositional letter is to be at each possible world. It reflects the stipulation that in a model  $M$ , a propositional letter  $p_k$  is true at a possible world  $w$ , just in the case  $w$  is a member of the set  $P_k$ . Clauses (2) and (3) are simply repeats of the usual propositional truth clauses. Due to definitions (D1)–(D4), they yield the classical truth tables for standard propositional connectives. Clause (4) formulates the interpretation of being a subject of a state of affairs:  $aB$  is true at possible world  $w$  if and only if  $B$  is true only at possible worlds that involve object  $a$  at possible world  $w$ , or an object is a subject of a state of affairs at a possible world  $w$  if and only if the state of affairs obtained only in possible worlds that belong to the range of object  $a$  at the possible world  $w$ . Clauses (5), (6) and (7) reflect some ideas about ontological concepts of existence, necessity and existential dependence. According to (5), object  $a$  exists at point  $w$  if and only if from standpoint  $w$ ,  $w$  itself involves object  $a$ . Clause (6) states that object  $a$  is a necessary being at point  $w$  if and only if the class of possibilities that are relevant to object  $a$  according to standpoint  $w$  is universal. The content of (7) is that object  $a$  is dependent with respect to object  $b$  at point  $w$  if and only if the

class of possibilities that are relevant to object **a** according to standpoint *w* is included in the class of possibilities that are relevant to object **b** according to standpoint *w*.

For any given formula **A**, let  $|A|^M$  be the set of possible worlds that verify formula **A** in model *M*. Thus,  $|A|^M = \{v:v \models^M A\}$ . I shall call it the *propositional content of formula A* in model *M*. The propositional content of a formula in a model could be interpreted as the state of affairs, which corresponds to the formula in the model. Let me reformulate the clause (4) in terms of the range of an object and propositional content of a formula.

$$(4^*) w \models^M aB \text{ if and only if } |B|^M \subseteq [w]^{Ra}.$$

According to (4\*), object **a** is a subject of state of affairs **B** at point *w* if and only if state of affairs **B** is included in the range of object **a** at possible world *w*.

A formula true at every possible world in a model *M* is said to be *valid in the model M*, a formula valid in every model is said to be *valid*. I write  $\models^M A$  to mean that formula **A** is valid in model *M*, and  $\models A$  to mean that formula **A** is a valid formula.

### 5. Completeness Theorem

In the previous two sections, the formulae were studied in two different ways, a syntactical one in section 2 and a semantic one in section 3. A fruitful blend of the two approaches results in the following completeness theorem.

**Theorem 4.** *A formula is a thesis of the theory if and only if it is a valid formula.*

**Proof.** Before turning to the proof proper, note that the lemma to the effect that for any formula **A** and any  $w \in W^*$ ,  $w \models^{M^*} A$  if and only if  $A \in w$  holds for the canonical model. I shall call it the *fundamental lemma*.

The proof of the lemma is of course by induction on the construction of formulae. The definition of the canonical ontological model assures that for any propositional letter  $p_k$ , and for any  $w \in W^*$ ,  $w \models^{M^*} p_k$  if and only if  $p_k \in w$ . In the case of propositional connectives  $\sim$  and  $\&$  you rely on the maximal consistency of each *w*, to assure you that  $A \in w$  if and only if it is not the case that  $\sim A \in w$  and that  $(A \& B) \in w$  if and only if  $A \in w$  and  $B \in w$ .

In the case of one-place object operator **a**, the induction hypothesis is that for any  $w \in W^*$ ,  $w \models^{M^*} aA$  if and only if  $A \in w$ . Now, suppose  $aA$  belongs to *w*. Hence, if it is not the case that  $vR_a w$ , then  $\sim C:aC \in w \subseteq v$  and therefore  $A \notin v$ . Thus, by the induction hypothesis, if it is not the case that  $vR_a w$ , then  $v \not\models^{M^*} A$ . Hence, if  $v \models^{M^*} A$ , then  $vR_a w$  and therefore  $w \models^{M^*} aA$ . Next, suppose that  $w \not\models^{M^*} aA$ . Thus, if it is not the case that  $vR_a w$ , then  $v \not\models^{M^*} A$  and, by the definition of the canonical model and by the induction hypothesis, if  $\{\sim C:aC \in w\} \subseteq v$ , then  $A \notin v$ . Hence, no maximal consistent set of formulae includes the set  $\{\sim C:aC \in w\} \cup \{A\}$ , and therefore this set is inconsistent. Thus, there is a finite subset of this set  $\{\sim B_1, \sim B_2, \sim B_3, \dots, \sim B_k, A\}$ , which is inconsistent, and therefore formula  $A \rightarrow (B_1 \vee B_2 \vee B_3 \vee \dots \vee B_k)$  is a thesis. Hence,  $A \rightarrow (B_1 \vee B_2 \vee B_3 \vee \dots \vee B_k)$  belongs to *w* and, by (A1), (R1) and (R2),  $(aB_1 \& aB_2 \& aB_3 \& \dots \& aB_k) \rightarrow aA$  also belongs to *w*. However formulae  $aB_1, aB_2, aB_3, \dots,$  and  $aB_k$  belong to *w*, and, by (R1),  $aA$  also belongs to *w*.

To prove that  $w \models^{M^*} Exa$  if and only if  $Exa \in w$ , at first suppose that  $w \not\models^{M^*} Exa$ . Hence, it is not the case that  $w R_a w$  and, by the definition of  $M^*$ ,  $\{\sim C:aC \in w\} \subseteq w$ . Thus, any formula depicted by schema  $aC \rightarrow \sim C$  belongs to *w*. However, by (A3),  $aExa$  belongs to *w*, and therefore  $\sim Exa$  also belongs to *w*. Hence,  $Exa \notin w$ . Next, suppose  $Exa \notin w$ . Thus,  $\sim Exa \in w$  and, by (Th1), any formula depicted by schema  $aC \rightarrow \sim C$  belongs to *w*. Hence,  $\{\sim C:aC \in w\} \subseteq w$  and, by the definition of  $M^*$ , it is not the case that  $w R_a w$ . Therefore,  $w \not\models^{M^*} Exa$ .

To prove that  $w \models^{M^*} Neca$  if and only if  $Neca \in w$ , at first suppose  $w \not\models^{M^*} Neca$ . Hence,  $[w]^{Ra} \neq W^*$ , and therefore there is a maximal consistent set of formulae *v*, such that  $\{\sim C:aC \in w\} \subseteq v$ . Thus,  $\{\sim C:aC \in w\}$  is a consistent set, and therefore any formula depicted by schema  $aC \rightarrow \sim a\sim C$  belongs to *w*. In particular,  $aExa \rightarrow \sim a\sim Exa$  belongs to *w*. Hence, by (A3) and (R1),  $\sim a\sim Exa$  belongs to *w* and, by (A5),  $Neca$  does not belong to *w*. Next, suppose  $Neca$  does not belong to *w*. Hence, by (A4), any

formula depicted by schema **a1** does not belong to  $w$ . Therefore,  $\{\sim C:aC \in w\}$  is a consistent set. Thus, there is a maximal consistent set of formulae  $v$ , such that  $\{\sim C:aC \in w\} \subseteq v$ , and, by the definition of  $M^*$ ,  $[w]^{Ra} \neq W^*$ . Therefore,  $w \not\models^{M^*} \text{Neca}$ .

To prove that  $w \models^{M^*} a \setminus b$  if and only if  $a \setminus b \in w$ , at first suppose  $w \models^{M^*} a \setminus b$ . Hence,  $[w]^{Ra} \subseteq [w]^{Rb}$  and, by the definition of  $M^*$ , for any  $v$ , if  $\{\sim C:bC \in w\} \subseteq v$  then  $\{\sim C:aC \in w\} \subseteq v$ . However, due to (A3), **aExa** belongs to  $w$ , and therefore for any  $v$ , if  $\{\sim C:bC \in w\} \subseteq v$ , then **Exa**  $\notin v$ . Hence, no maximal consistent set of formulae includes the set  $\{\sim C:aC \in w\} \cup \{\text{Exa}\}$ , and, therefore, this set is inconsistent. Thus, there is a finite subset of this set  $\{\sim B_1, \sim B_2, \sim B_3, \dots, \sim B_k, \text{Exa}\}$  that is inconsistent, and therefore formula **Exa**  $\rightarrow (B_1 \vee B_2 \vee B_3 \vee \dots \vee B_k)$  is a thesis. Hence, **Exa**  $\rightarrow (B_1 \vee B_2 \vee B_3 \vee \dots \vee B_k)$  belongs to  $w$  and, by (A1), (R1) and (R2), **(bB<sub>1</sub> & bB<sub>2</sub> & bB<sub>3</sub> & ... & bB<sub>k</sub>)**  $\rightarrow$  **bExa** also belongs to  $w$ . However, formulae **bB<sub>1</sub>**, **bB<sub>2</sub>**, **bB<sub>3</sub>**, ..., and **bB<sub>k</sub>** belong to  $w$ , and by (R1), **bExa** also belongs to  $w$ . However, by (A4), any formula depicted by schema **bExa**  $\rightarrow$  **a**  $\setminus$  **b** belongs to  $w$  and therefore  $a \setminus b \in w$ .

Next, suppose  $a \setminus b$  belongs to  $w$ . Hence, by (A6), any formula depicted by schema **aC**  $\rightarrow$  **bC** belongs to  $w$ , and therefore  $\{\sim C:aC \in w\} \subseteq \{\sim C:bC \in w\}$ . Thus, for any  $v$ , if  $\{\sim C:bC \in w\} \subseteq v$  then  $\{\sim C:aC \in w\} \subseteq v$ . Therefore,  $[w]^{Ra} \subseteq [w]^{Rb}$  and  $w \models^{M^*} a \setminus b$ . It completes the proof of the fundamental lemma. I can now get back to the proper proof of Theorem 4.

In order to show that, a formula is a thesis if and only if it is a valid formula, it is sufficient to prove that any thesis is a valid formula and that any formula that is not a thesis is not valid. The proof of the first implication requires the establishment of the ontological validity of all axioms and the demonstration that the rules of inference (R1) and (R2) preserve validity. It could be easily done. To prove the converse implication, suppose a formula **A** is not a thesis. Then,  $\{\sim A\}$  is a consistent set. Thus, there is a maximal consistent set of formulae  $v$ , such that  $\{\sim A\} \subseteq v$ . Hence, by the definition of  $M^*$ , for some  $w \in W^*$ ,  $\sim A$  belongs to  $w$ , and **A** does not belong to  $w$ . Therefore, by the fundamental lemma, for some  $w \in W^*$ ,  $w \not\models^{M^*} A$ , and so **A** is not a valid formula.

Thus, the syntactical and the semantic approach depict the same class of logically true formulae.

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