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An Improved Algorithm for Identification of Dominating Vertex Set in Intuitionistic Fuzzy Graphs

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Abstract: In graph theory, a "dominating vertex set" is a subset of vertices in a graph such that every vertex in the graph is either a member of the subset or adjacent to a member of the subset. In other words, the vertices in the dominating set "dominate" the remaining vertices in the graph. Dominating vertex sets are important in graph theory because they can help us understand and analyze the behavior of a graph. For example, in network analysis, a set of dominant vertices may represent key nodes in a network that can influence the behavior of other nodes. Identifying dominant sets in a graph can also help in optimization problems, as it can help us find the minimum set of vertices that can control the entire graph. Now that there are theories about vagueness, it is important to define parallel ideas in vague structures, such as intuitionistic fuzzy graphs. This paper describes a better way to find dominating vertex sets (DVSs) in intuitive fuzzy graphs (IFGs). Even though there is already an algorithm for finding DVSs in IFGs, it has some problems. For example, it does not take into account the vertex volume, which has a direct effect on how DVSs are calculated. To address these limitations, we propose a new algorithm that can handle large-scale IFGs more efficiently. We show how effective and scalable the method is by comparing it to other methods and applying it to water flow. This work's contributions can be used in many areas, such as social network analysis, transportation planning, and telecommunications.

Keywords: intuitionistic fuzzy set; intuitionistic fuzzy graph; minimal intuitionistic dominating vertex subset; domination set

MSC: 05C72

1. Introduction

In 1736, graph theory started with the famous Konigsberg seven bridges problem and Leonhard Euler's negative answer. It can, therefore, be used to model various aspects of contemporary life. Numerous difficulties from the actual world are modeled mathematically using the graph. When a rule or relationship connects objects, a graph can be used.

The probability theory was regarded as the only theory to address uncertainty based on Aristotle's yes-or-no logic until the middle of the 20th century. A breakthrough in this field of study was made when Zadeh introduced a novel approach for the class of fuzzy sets in 1965 [1]. Fuzzy logic is the term applied afterwards to this new reasoning. Zadeh presented the fuzzy set (FS) notion to convey ambiguity and fuzziness. Since then, numerous fields of study have focused on fuzzy sets. To solve a problem with a spacecraft, Deveci et al. devised a new way to figure out weight coefficients and proposed a combined compromise solution based on interval type-2 fuzzy sets [2]. Another recent application of fuzzy sets can be seen in [3], in which Pythagorean probabilistic hesitant fuzzy sets are utilized to design a fuel supply system modelling approach for electric vehicles. Akram et al. [4] propose

another intriguing application in group decision-making. This technique mainly employs linguistic Pythagorean fuzzy sets. Pamucar et al. discussed a state-of-the-art application of integrated stratified fuzzy rough decision-making in transportation systems [5].

Rosenfeld [6] looked into fuzzy relations in 1975 and, based on fuzzy sets (FSs), made up the theory of fuzzy graphs (FGs). All the vertices and edges are given membership grades in this method. Since this theory came out, many generalized versions of fuzzy graphs have been made and used in many different fields. As a generalization of FGs, Samanta et al. introduced fuzzy threshold graphs [7]. The theory of "fuzzy k-competition graphs and fuzzy p-competition graphs" is explored in further detail in [8]. Samanta et al. [9] also discussed how regular and fully generalized fuzzy graphs are. M-polar fuzzy graphs are a new development in this field of study. Ghorai and Pal [10] talked about how m-polar fuzzy graphs work, how they look, and how to figure out the degrees of the vertices. Additionally, competition graphs for an "interval-valued m-polar fuzzy graph" [11] and psi-morphism on the product of m-polar fuzzy graphs [12] have been studied. FGs have been used successfully in many real-world situations, such as image processing, information retrieval, communication, networking, programming, register assignment, DNA research, determining the molecular structure of chemicals, electrical networks, and flight routines. Recent years have seen a significant increase in applying FGs for modelling communication networks in the digital world (LinkedIn, Google Scholar, WhatsApp, YouTube, and Instagram). Many related domains exist [13]. A FG is a good way to solve combinatorial problems in many areas of computer science and computational intelligence systems [14].

In 1983, Atanassov [15] developed intuitionistic fuzzy sets (IFSs) to generalize FSs. In this development, he included a measure defining the amount of "non-membership" and the "degree of membership". In this approach, "the sum of the membership grade and non-membership grade is less than or equal to one, whereas in fuzzy sets the sum of membership and non-membership grades exactly equals 1". Many variants and generalized models of IFSs have been investigated in the literature. A mathematical method for unifying many features via Maclaurin symmetric mean operator is presented in [16]. Khan et al. [17] used complex T-spherical fuzzy sets to examine how well software packages worked. Lastly, intuitionistic HFS_S [18] and graded soft expert sets [19] have also been explored in a hesitant fuzzy environment and applied in decision analysis.

Returning to the discussion of graphs, Shannon et al. [20] first introduced the parallel concept of IFGs and examined some of their properties. This definition assigns IF "membership and non-membership grades" to vertices and edges. Gulzar et al. proposed complex IFGs and looked into how they could be used in cellular networks [21]. Shao et al. applied IFGs to water supply systems [22]. Some other significant studies include k-polar IFGs and their implementation in "BCK/BCI algebras" [23], "picture fuzzy graphs and its application in social networks" [24], improved interval-valued FGs, interval-valued IFGs, neutrosophic graphs [25], complex IFGs [26], "energy and Laplacian energy of Pythagorean FGs" [27], and "Pythagorean FGs and preference relations" [28].

A dominating vertex set (DVS) in a graph is a set of vertices such that every vertex in the graph is either in the set or adjacent to a vertex in the set. The dominance in graphs dates back to the 1850s, when European chess fans investigated the issue of "dominating queens", as mentioned in [29]. Since the 1960s [30], when they were first studied mathematically, DVSs have been used in many different ways. Numerous problems, such as computer communication networks, social network theory, land surveying, and other related difficulties, can be modelled using dominating sets. For example, in a social network, a DVS might represent a group of influential people who can affect the opinions and behaviors of the entire network. In a transportation network, a DVS could be a group of important places that must be monitored or fixed to ensure the whole network works well. From the viewpoint of graph theory, many related notions have been studied. "Alhevaz et al. discussed the strong equality of perfect Roman and weak Roman domination in trees" in [31].

Thus, determining the minimum dominating sets and the dominance number for graphs, FGs, and IFGs could be helpful. Bozhenyuk et al. [32] suggest finding DVSs in FGs. One of the most important research problems in IFGs is to find DVSs.

The current article includes the study and improvement of the findings of the DVSs presented in [32]. The approach used an intuitionistic degree of domination to find the domination vertex set. After reviewing the algorithm and numerical examples, we discovered some anomalies in the results. In the article, the number of vertices describes how dominant something is. However, this fact needs to be completely ignored in their algorithm to find DVSs for IFGs. Calculations show that their algorithm's DVS does not meet the definition of dominance for IFGs. This lack of authenticity motivated us to improve the algorithm for better results. We show a new way to find the domination vertex set that uses the volume of the vertex instead of the individual intuitionistic degrees.

The remaining paper is distributed in the following sections: Section 2 briefly discusses the fundamental ideas and notions of FGs and IFGs. Section 3 recalls the DVSs for crisp graphs and IFGs. In the next section, we will look at Bozhenyuk et al.'s [32] algorithm for finding DVSs in IFGs. The mathematical structure of the algorithm and an illustrative example is presented. Section 5 of this article introduces a novel approach to finding DVSs for IFGs. The shortcomings of the existing algorithm are also discussed in this section. The efficiency of the developed approach and comparative remarks are described in Section 6. The section also presents a water flow application of the novel algorithm. In the last section, some concluding comments are given.

2. Preliminaries

This section provides the necessary background information for understanding this article. Specifically, it reviews "fuzzy graphs (FGs) and intuitionistic fuzzy graphs (IFGs)". We recommend referring to [1] for further information on fuzzy sets.

As was already said, the theory of FS continued to grow and be used in many different areas. In fuzzy set theory, several ideas were defined that were similar to those in crisp set theory. Fuzzy relations are one of these notions that are most frequently employed. A crisp relationship uses a characteristic function to display the relationship between the elements. In other words, the elements are either associated or not. In contrast, a fuzzy relation uses a fuzzy MF to find the value of the relationship between the elements. The elements might be related to a certain degree.

Definition 1 ([33])**.** *Let σ be a fuzzy subset of a set X and $\alpha_{\mathcal{N}} : X \times X \to [0,1]$ be a fuzzy MF on $X \times X$. Then, $\alpha_{\mathcal{N}}$ is called a fuzzy relation on X if*

$$\alpha_{\mathcal{N}}(x,y) \leq \sigma(x) \wedge \sigma(y) \text{ for all } x, y \in X.$$

The condition of a fuzzy relation is that the degree of relationship between two elements should not be higher than their grades.

FGs are the main idea of this study. Rosenfeld [6] first developed them in 1975 as an extension of Euler's graph theory. Rosenfeld examined the structure of FGs and explored the relationships between fuzzy sets. He also developed analogies for various ideas in graph theory.

Definition 2 ([6])**.** *The pair $G^* = (V, U)$ represents a simple crisp graph with vertices in V and edges in U. If two vertices a and b are adjacent through an edge, then we write the edge as (a, b). A fuzzy graph $G = (V, \sigma, \alpha)$ correlating to a crisp graph G^* is a triplet containing a non-empty set V of vertices together with a pair of functions $\sigma : V \to [0,1]$ and $\alpha : V \times V \to [0,1]$ such that for all $a, b \in V$,*

$$\alpha(a,b) \leq \min\{\sigma(a), \sigma(b)\}.$$

Here, $\sigma(a)$ denotes the value of membership of vertex a and $\alpha(a, b)$ gives the membership value of the edge (a, b). Thus, a fuzzy vertex set can be characterized by the pair (V, σ).

In fuzzy set theory, several researchers have come up with ideas similar to those in crisp graphs. As a result, various definitions of FGs have emerged. The different definitions come from the fact that some vertices, edges, or both are given fuzzy grades. Some authors in the literature classify these as types of FGs [34]. Kaufmann [35], for example, proposed a definition of FGs that combines fuzzy vertices and edges, while Breshtein et al. [36] developed an approach that treats FGs as bundles of crisp vertices and fuzzy edges. This type of FG is also known as the first kind of FG [36]. Definition 2 gives fuzzy degrees to vertices and edges, while the definition of FGs in [36] assumes that the set of vertices is clear and only gives fuzzy degrees to the edges.

To manage fuzziness and uncertainty, fuzzy set theory became very popular. Atanassov brought attention to the fact that it was a single index theory in 1983, simply explaining the state of membership and taking non-membership into account as its complement $(1 - \alpha_A)$ [37]. As a solution, he came up with IFS, a two-index theory that explains why some elements are members and others are not. Thus, an IFS can represent three states: support, opposition, and neutrality. Hesitancy margins, or hesitancy indices, describe the neutral condition [37].

Definition 3 ([15])**.** *Consider a nonempty set X. An "intuitionistic fuzzy (IF) set" $\mathcal{N}$ in X is represented in the form:*

$$\mathcal{N} = \{\langle e, \, \mu_{\mathcal{N}}(e), \nu_{\mathcal{N}}(e)\rangle | \forall e \in X \, [\mu_{\mathcal{N}}(e) + \nu_{\mathcal{N}}(e) \leq 1]\};$$

where the functions $\mu_{\mathcal{N}}(e), \nu_{\mathcal{N}}(e) : X \rightarrow [0, 1]$ define respectively, the membership grade and the non-membership grade of an object $e \in X$ to the IFS $\mathcal{N}$.

Furthermore, $\pi_{\mathcal{N}}(e) = 1 - \mu_{\mathcal{N}}(e) - \nu_{\mathcal{N}}(e)$ is known as the "intuitionistic fuzzy set index or hesitation margin" of e in $\mathcal{N}$. $\pi_{\mathcal{N}}(e)$ is the indeterminacy grade of e to the IFS $\mathcal{N}$.

Parallel to the concept of fuzzy relations, the IF relation is presented as follows:

Definition 4 ([32])**.** *The "intuitionistic fuzzy relation" R on the set X is an intuitionistic fuzzy set over $X \times X$ of the form:*

$$R = \{\langle (a, b), \, \mu_R(a, b), \, \nu_R(a, b)\rangle | (a, b) \in X \times X \}.$$

Here, μ_R and ν_R represents respectively the IF "membership and non-membership grades" satisfying the following condition

$$(\forall a, b \in X \times X)[\mu_R(a, b) + \nu_R(a, b) \leq 1].$$

A particular member of IFS is called an intuitionistic fuzzy variable. IF relations are used to define membership and non-membership degrees to the edges.

Atanassov et al. [38] defined the following two operators to join two IF variables. These operators define the domination degree for vertices in an IFG.

Definition 5 ([38])**.** *Let $a = (\mu(a), \nu(a))$ and $b = (\mu(b), \nu(b))$ be intuitionistic fuzzy variables. Here, $\mu(a) + \nu(a) \leq 1$, and $\mu(b) + \nu(b) \leq 1$. Then the operators & and $\vee$ are described as below:*

$$a \, \& \, b = (min(\mu(a), \mu(b)), \, max(\nu(a), \nu(b))), \tag{1}$$

$$a \vee b = (max(\mu(a), \mu(b)), \, min(\nu(a), \nu(b))). \tag{2}$$

For comparison of two IF variables, $a < b$ if $\mu(a) < \mu(b)$ and $\nu(a) > \nu(b)$.

Similar to fuzzy graph definitions, various IFG definitions exist in the literature. Shannon et al. defined an IFG as a collection of an IF set of vertices and edges [20]. In this definition of IFG, IF degrees are assigned to both the vertices and edges under certain conditions for membership and non-membership degrees. For our research, we will use the following definition:

Definition 6 ([32]). *An IFG $\widetilde{G} = (V, U)$ is said to be first kind if V is crisp set of vertices and U is an intuitionistic set of edges. Here, $U = \langle V \times V, \mu, \nu \rangle$ is an intuitionistic fuzzy relation that satisfies the following condition*

$$(\forall x, y \in V)[0 \leq \mu(x,y) + \nu(x,y) \leq 1].$$

3. Dominating Vertex Set (DVS)

In this section, we will discuss what DVSs are and how they work for crisp, fuzzy, and IFG graphs. The problem dates back to the 1950s [39], but the scope of study greatly increased in the middle of the 1970s.

In 1977, Allan et al. [40] proposed a DVS based on an independent vertex set.

Definition 7 ([40]). *The subgraph of a graph G that is created by all of the vertices around a vertex v is known as the neighbourhood of vertex v denoted by $N(v)$.*

Definition 8 ([40]). *Consider $G = (V, E)$ be a crisp graph where V represents the collection of vertices and E represents the collection of edges. A set $D \subseteq V$ is a DVS of G if $\forall v \in V \backslash D$, $N(v) \cap D \neq \varnothing$.*

The collection of red vertices in Figure 1 represents the DVS for the given graph.
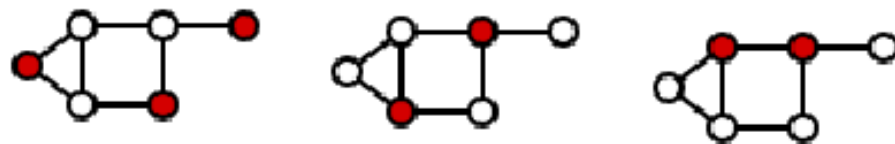
Figure 1. DVS for crisp graph.

In other words, we can say every vertex not in D is adjacent to at least one member of D.

DVS for IFG

After the advent of FGs, Bershtein et al. [36] defined the theory of fuzzy DVS based on Maghouts' method. Following finding DVSs for fuzzy graphs in [36], Bozhenyuk et al. [32] developed a way to obtain DVSs on IFGs.

Suppose $\widetilde{G} = (V, U)$ be an IFGs of the first kind. Let $p(x, y) = (\mu(x, y), \nu(x, y))$ be an IF variable that defines the grades of adjacency and the grades of non-adjacency from vertex x to vertex y. Suppose X be an arbitrary subset of the vertices of the set. For each vertex $y \in V \backslash X$, we represent the volume:

$$p_X(y) = \bigvee_{x \in X} p(x, y). \tag{3}$$

Definition 9 ([32]). *The set X is called an intuitionistic DVS for vertex y with the intuitionistic degree of domination $p_X(y)$.*

The above definition gives a degree of domination for each element. Extending this concept, the DVS for a graph is defined as follows:

Definition 10 ([32]). *The set X is an intuitionistic DVS for graph $\widetilde{G} = (V, U)$ with the intuitionistic degree of domination:*

$$\beta(X) = \underset{y \in V \setminus X}{\&} p_X(y) = \underset{y \in V \setminus X}{\&} \bigvee_{x \in X} p(x, y). \tag{4}$$

Example 1. *Consider the IFG in Figure 2. Let $X = \{x_3, x_4\}$. Then*

$$
\begin{aligned}
\beta(X) \quad &= p_X(x_1) \,\&\, p_X(x_2) \\
&= [(0.3, 0.4) \bigvee (0.5, 0.5)] \& [(0.4, 0.4) \bigvee (0, 1)] \\
&= (0.5, 0.4) \& (0.4, 0.4) = (0.4, 0.4)
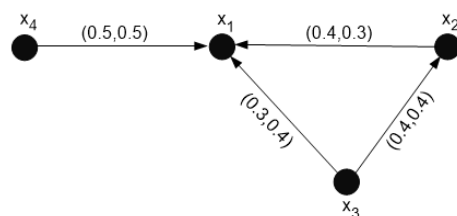\end{aligned}
$$

Figure 2. Example of an IFG.

Definition 11 ([32]). *An IFS*

$$\widetilde{D} = \{\langle \beta_1^0/1 \rangle, \langle \beta_2^0/2 \rangle, \dots, \langle \beta_n^0/n \rangle\} \tag{5}$$

is known as domination set of graphs $\widetilde{G}$, where $\left\langle \beta_j^0/j \right\rangle$ denotes the minimal DVS with j vertices having degree of domination β_j^0. We shall call β_j^0 a degree of DVS of order j.

4. Existing Algorithm for Finding DVS for an IF Graph

Bozhenyuk et al. [32] introduced a method for computing a DVS for an IF graph, extending the fuzzy graph method. However, some important steps should have been taken into account during the process of generalization, which raises questions about the validity of this method. In this section, we describe the technique and highlight its limitations.

Let X_β be an intuitionistic DVS of the IFG $\widetilde{G} = (V, U)$ with the degree of domination $\beta = (\mu_\beta, \nu_\beta)$.

In X_β, for each arbitrary vertex $x_i \in V$ any one of the following conditions must hold

- $x_i \in X_\beta$;
- If $x_i \notin X_\beta$ then there exists a vertex x_j so that it corresponds to set X_β while the vertex x_j is adjacent to vertex x_i with the degree $(\mu(x_j, x_i), \nu(x_j, x_i)) \geq \beta$.

Consequently, the accompanying assertion is true:

$$(\forall x_i \in V)[x_i \in X_\beta \ \vee \ (\exists x_j \in X_\beta \,|\, \mu(x_j, x_i) \geq \mu_\beta \& \nu(x_j, x_i) \leq \nu_\beta)]. \tag{6}$$

Each vertex $x_i \in V$ is allocated a Boolean variable p_i with the value 1 if $x_i \in X_\beta$ and 0 otherwise.

Assigning the intuitionistic variable ξ_{ji} for the proposition $(\mu(x_j, x_i), \nu(x_j, x_i)) \geq (\mu_\beta, \nu_\beta)$ the statement in (6) can be written in logical proposition as below:

$$\Phi_D = \underset{i=1,n}{\&} (p_i \vee \underset{j=1,n}{\bigvee} (p_j \& \xi_{ji})) \tag{7}$$

Here, $n = |V|$. Supposing $\xi_{ii} = (1,0)$ and considering the following equality

$$p_i \vee \bigvee_j (p_j \& \xi_{ji})) = \bigvee_j (p_j \& \xi_{ji}));$$

(7) is re-written as

$$\Phi_D = \underset{i=1,n}{\&} (\underset{j=1,n}{\bigvee} (p_j \& \xi_{ji})) \tag{8}$$

To find Φ_D for any IF graph, the following propositional laws are used.

$$x \vee x\&y = x \; ; x\&y \vee x\&\bar{y} = x;$$
$$(\xi_1 \geq \xi_2) \rightarrow (\xi_1 \& x \vee \xi_2 \& x \& y = \xi \& x) \tag{9}$$

All the minimum intuitionistic DVSs are defined which correspond to the disjunctive members as

$$\Phi_D = \bigvee_{i=1,i} (p_{1i} \& p_{2i} \& \dots \& p_{ki} \& \beta_i). \tag{10}$$

This yields the DVS for the graph $\widetilde{G}$.

Algorithm 1. Minimization of expression (7)

Input: An IFG $\widetilde{G} = (V, U)$ with intuitionistic DVS $X_\beta; \beta = (\mu_\beta, \nu_\beta)$ being degree of domination.
Output: Minimized expression (7).
1　**Begin**
2　　　**Construct** the expression: $\Phi_D = \underset{i=1,n}{\&} (p_i \vee \underset{j=1,n}{\vee} (p_j \& \xi_{ji}))$.// ξ_{ji} represents
3　$(\mu(x_j, x_i), \nu(x_j, x_i)) \geq (\mu_\beta, \nu_\beta)$ and $p_i = 1$ if $x_i \in X_\beta$ and 0 otherwise
4　　　**Construct** the expression: $\Phi_D = \underset{i=1,n}{\&} (\underset{j=1,n}{\vee} (p_j \& \xi_{ji}))$.
5　　　**Compute** the expression: $\Phi_D = \bigvee_{i=1,i} (p_{1i} \& p_{2i} \& \dots \& p_{ki} \& \beta_i)$.//According to the rules in (9)
6　**end**

Example 2. *Consider the IFG in Figure 2. The adjacency matrix of the graph is*

$$R_x = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ \left[\begin{array}{cccc} (1,0) & (0,1) & (0,1) & (0,1) \\ (0.4,0.3) & (1,0) & (0,1) & (0,1) \\ (0.3,0.4) & (0.4,0.4) & (1,0) & (0,1) \\ (0.5,0.5) & (0,1) & (0,1) & (1,0) \end{array}\right] \end{array}$$

The corresponding expression of this matrix is represented as follows:

$$\varphi_D = [(1,0)p_1 \vee (0.4, 0.3)p_2 \vee (0.3,0.4)p_3 \vee (0.5,0.5)p_4]\&[(1,0)p_2 \vee (0.4,0.4)p_3]\& [(1,0)p_3]\&[(1,0)p_4]$$

Multiplying expressions in the first two brackets and the expressions in last two brackets, we obtain

$$\varphi_D = [(1,0)p_1p_2 \quad \vee (0.4,\ 0.3)p_2 \vee (0.3,0.4)p_2p_3 \vee (0.5,0.5)p_2p_4 \vee (0.4,0.4)p_1p_3 \vee (0.4,0.4)p_2p_3 \vee (0.3,0.4)p_3$$
$$\vee (0.4,0.5)p_3p_4] \& [(1,0)p_3p_4]$$

Further simplification yields

$$\varphi_D = \big[(1,0)p_1p_2p_3p_4 \vee (0.4,0.3)p_2p_3p_4 \vee (0.5,0.5)p_2p_3p_4 \vee (0.4,0.4)p_1p_3p_4 \vee (0.3,\ 0.4)p_3p_4 \vee (0.4,0.5)p_3p_4\big]$$

and finally,

$$\varphi_D = \big[(0.4,0.4)p_3p_4 \vee (0.5,0.3)p_2p_3p_4 \vee (1,0)p_1p_2p_3p_4\big].$$

5. Modified Algorithm for Finding DVS

The volume of vertices impacts intuitionistic DVSs, as discussed in Section 3 of [32]. However, this aspect should have been addressed when developing the algorithm for calculating DVS, as described in the same article. This casts serious doubt on its authenticity.

This section is dedicated to designing a novel algorithm to derive all intuitionistic DVSs. The defined method, based on the volume of vertices, is a variant of the method given in [32]. It is also based on Maghout's method for defining all minimal fuzzy DVSs for fuzzy graphs.

Consider that set X_β is an intuitionistic DVS of the graph $\widetilde{G}$ with the degree of domination $\beta = (\mu_\beta, \nu_\beta)$. Then, for an arbitrary vertex $x_i \in V$, one of the subsequent requirements must hold.

(a) $x_i \in X_\beta$;

(b) $x_i \notin X_\beta$,

In the second case, the following algorithmic steps will be applied:

Step 1: To find DVSs of degree 1 (if they exist), find x_j, such that

$$p(x_j, x_r) \neq (0,1) \text{ for all } x_r \in V - \{x_j\}$$

Then,

$$\beta_1^0 = \overset{\vee}{_j} \beta_{1j};$$

where $\beta_{1j} = \overset{\&}{_{x_r \in V - \{x_j\}}} p(x_j, x_r)$.

Here, β_1^0 denotes the degree of domination of DVS of order 1.

Step 2: For deriving DVSs of order 2 or more, there exists $X_\beta^* \subseteq X_\beta$, such that

$$p_{X_\beta^*}(x_i) \geq \beta.$$

The following line of reasoning is sound from a logical standpoint:

$$(\forall x_i \in V)\Big[x_i \in X_\beta \vee \Big(\exists x_j,\ x_k \in X_\beta \mid \mu(x_i,\ x_j) \geq \mu_\beta \bigwedge \nu(x_i,\ x_k) \leq \nu_\beta\Big].\qquad (11)$$

For every vertex $x_i \in V$, we define a Boolean variable p_i that takes the value 1, if $x_i \in X_\beta$ and 0 if $x \notin X_\beta$.

We define the intuitionistic variable $\zeta_{jik} = (\mu_\beta, \nu_\beta)$ for the proposition $\mu(x_i, x_j) \geq \mu_\beta \bigwedge \nu(x_i, x_k) \leq \nu_\beta$.

Passing from the quantifier, we obtain

$$\varphi_D = \underset{i=1,n}{\&} \Big(p_i \vee \bigvee_{j=1,n} \big(p_j \& \zeta_{jik}\big)\Big).$$

Here, $n = |V|$. Supposing $\xi_{ii} = (1,0)$ and considering the following equality

$$p_i \vee \bigvee_j \big(p_j \& \zeta_{jik}\big) = \bigvee_j \big(p_j \& \zeta_{jik}\big),$$

we write

$$\varphi_D = \underset{i=1,n}{\&} \left(\bigvee_{j=1,n} \left(p_j \,\&\, \zeta_{jik} \right) \right). \tag{12}$$

We transform Equation (12) using the following laws.

$$p \,\vee\, p\&q = p \,;\, p\&q \,\vee\, p\&\overline{q} = p;$$

$$(\zeta_1 \geq \zeta_2) \rightarrow (\zeta_1\& \, p \vee \zeta_2\&p\&q = \zeta\&p).$$

The disjunctive members of all minimal intuitionistic DVSs are defined as below.

$$\varphi_D = \bigvee_{i=1,i} \left(p_{1i} \,\&\, p_{2i} \,\&\, \ldots \,\&\, p_{ki} \,\&\, \beta_i \right) \tag{13}$$

Based on φ_D, DVS for an IFG $\widetilde{G}$ is defined.

| **Algorithm 2.** Minimization of expression (11) |
|---|
| **Input:** An IFG $\widetilde{G} = (V, U)$ with intuitionistic DVS X_β; $\beta = \left(\mu_\beta, \, \nu_\beta \right)$ being degree of domination. |
| **Output:** Minimized expression (11). |
| 1   **Begin** |
| 2   **Construct** $\beta_1^0 = \bigvee_j \underset{x_r \in V - \{x_j\}}{\&} p(x_j, x_r).//$ calculated for all x_j such that $p\left(x_j, x_r\right) \neq (0,1)$ for all $x_r \in V - \left\{x_j\right\}$ |
| 2   **Construct** the expression: $\Phi_D = \underset{i=1,n}{\&}(p_i \vee \underset{j=1,n}{\bigvee} \left(p_j \,\&\, \xi_{jik} \right)).//$ ξ_{jik} represents |
| 3 $\mu\left(x_i, \, x_j\right) \geq \mu_\beta \wedge \nu(x_i, \, x_k) \leq \nu_\beta$ and $p_i = 1$ if $x_i \in X_\beta$ and 0 otherwise |
| 4   **Construct** the expression: $\Phi_D = \underset{i=1,n}{\&}(\underset{j=1,n}{\bigvee} \left(p_j \,\&\, \xi_{jik} \right)).$ |
| 5   **Compute** the expression: $\Phi_D = \bigvee_{i=1,i}(p_{1i}\&p_{2i}\& \ldots \&p_{ki}\&\beta_i).//$ According to the rules in (9) |
| 6   **end** |

Example 3. *Consider the IFG in Figure 3.*
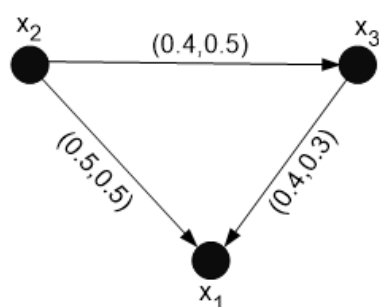
Figure 3. Example of an IFG.

The adjacency matrix for this graph can be expressed as below.

$$R_x = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} x_1 \\ \left[\begin{array}{ccc} (1,0) & (0,1) & (0,1) \\ (0.5,0.5) & (1,0) & (0.4,0.5) \\ (0.4,0.3) & (0,1) & (1,0) \end{array} \right] \end{array} \begin{array}{c} x_2 \qquad x_3 \end{array}$$

Step 1:
The set of all vertices is $X = \{x_1, x_2, x_3\}$.

Here, x_2 and x_3 are two vertices whose $p(x_2, x_r) \neq (0,1)$, such that $x_r \in X - \{x_2\}$ and $p(x_3, x_j) \neq (0,1)$, such that $x_j \in X - \{x_3\}$.

Now

$$X - x_2 = \{x_1, x_3\}$$

$$p_{x_2}(x_1) = (0.5, 0.5)$$

$$p_{x_2}(x_3) = (0.4, 0.5)$$

Hence,

$$\beta_1(x_2) = \beta_{12} = \{(0.5, 0.5) \& (0.4, 0.5)\} = (0.4, 0.5)$$

Again

$$X - x_3 = \{x_1, x_2\}$$

$$p_{x_3}(x_1) = (0.4, 0.3)$$

Hence,

$$\beta_1(x_3) = \beta_{13} = (0.4, 0.3)$$

So,

$$\beta_1^0 = \{(0.4, 0.5) \vee (0.4, 0.3)\} = (0.4, 0.3)$$

Step 2:

The corresponding expression of this matrix is represented as follows:

$$\Psi_D = [(1,0)p_1 \vee (0.5, \ 0.3)p_2 p_3] \& [(1,0)p_2] \& [(0.4, 0.5)p_2 \vee (1,0)p_3]$$

Simplifying expression 1 and multiply with expression 2

$$\Psi_D = [(1,0)p_1 p_2 \vee (0.5, 0.3)p_2 p_3] \ \& \ [(0.4, 0.5)p_2 \vee (1,0)p_3]$$

Simplifying expressions

$$\Psi_D = [(0.4, 0.5)p_1 p_2 \vee (1,0)p_1 p_2 p_3 \vee (0.4, 0.5)p_2 p_3 \vee (0.5, 0.3)p_2 p_3]$$

After simplification, we will obtain

$$\Psi_D = \left[\underline{(0.5, 0.3)p_2 p_3} \vee (0.4, 0.5)p_1 p_2 \vee (1,0)p_1 p_2 p_3 \right]$$

6. Comparative Analysis

For the sake of comparison between Algorithms 1 and 2, we consider a water flow network. The details have been outlined below.

Example 4. *Consider a city with two water utilities (x_1 and x_2) that provide water to the communities of x_3, x_4, and x_5. The water flow network is illustrated in Figure 4, where each edge represents the flow of water from the source node to the destination node. Each edge is given an IF membership or non-membership degree, which is the fraction of water flow capacity (out of 1), to figure out how much water can flow through it and how much water is lost because of leaks and drying up in the pipes. Our goal is to determine the optimal collective flow capacity from the source nodes (x_1 and x_2) while minimizing the anticipated water loss across the entire region.*
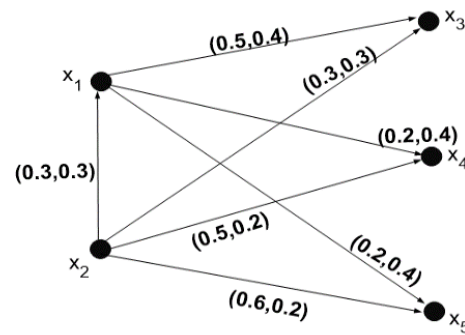
Figure 4. Intuitionistic fuzzy graph for water flow network.

DVSs will be calculated to find the required numbers. We first apply Algorithm 1 to this problem for obtaining domination set of graphs $\widetilde{G}$ shown in Figure 4.

6.1. Solution of Water Flow Network through Algorithm 1

The adjacency matrix of the graph $\widetilde{G}$ is as below.

$$R_x = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} x_1 \\ \left[\begin{array}{c} (1,0) \\ (0.3,0.3) \\ (0,1) \\ (0,1) \\ (0,1) \end{array} \right. \begin{array}{c} x_2 \\ (0,1) \\ (1,0) \\ (0,1) \\ (0,1) \\ (0,1) \end{array} \begin{array}{c} x_3 \\ (0.5,0.4) \\ (0.3,0.3) \\ (1,0) \\ (0,1) \\ (0,1) \end{array} \begin{array}{c} x_4 \\ (0.2,0.4) \\ (0.5.0.2) \\ (0,1) \\ (1,0) \\ (0,1) \end{array} \begin{array}{c} x_5 \\ (0,1) \\ (0.6,0.2) \\ (0,1) \\ (0,1) \\ (1,0) \end{array} \left. \begin{array}{c} \\ \\ \\ \\ \end{array} \right]$$

Using Equation (9) this matrix is represented as follows.

$$\varphi_D = [(1,0)p_1 \quad \vee (0.3,\ 0.3)p_2] \& [(1,0)p_2] \& [(0.5,0.4)p_1 \vee (0.3,0.3)p_2 \vee (1,0)p_3] \& [(0.2,0.4)p_1 \vee (0.5,0.2)p_2$$
$$\vee (1,0)p_4] \& [(0.6,0.2)p_2 \vee (1,0)p_5]$$

Multiplying expressions in parentheses 1, 2, 3, and 4, we obtain

$$\varphi_D = [(1,0)p_1 p_2 \quad \vee (0.3,\ 0.3)p_2] \& [(0.2,0.4)p_1 \vee (0.5,0.4)p_1 p_2 \vee (0.5,0.4)p_1 p_4 \vee (0.2,0.4)p_1 p_2 \vee (0.3,0.3)p_2$$
$$\vee (0.3,0.3)p_2 p_4 \vee (0.2,0.4)p_1 p_3 \vee (0.5,0.2)p_2 p_3 \vee (1,0)p_3 p_4] \& [(0.6,0.2)p_2 \vee (1,0)p_5].$$

Multiplying expressions in parenthesis 1 and 2

$$\varphi_D = [(0.2,0.4)p_1 p_2 \quad \vee (0.5,0.4)p_1 p_2 \vee (0.5,0.4)p_1 p_2 p_4 \vee (0.3,0.3)p_1 p_2 \vee (0.5,0.2)p_1 p_2 p_3 \vee (1,0)p_1 p_2 p_3 p_4$$
$$\vee (0.2,0.4)p_1 p_2 \vee (0.3,0.4)p_1 p_2 \vee (0.3,0.4)p_1 p_2 p_4 \vee (0.3,0.3)p_2 \vee (0.3,0.3)p_2 p_3$$
$$\vee (0.3,0.3)p_2 p_3 p_4] \& [(0.6,0.2)p_2 \vee (1,0)p_5]$$

Simplifying expression 1 and multiplying with expression 2

$$\varphi_D = [(0.5,0.4)p_1 p_2 \quad \vee (0.5,0.4)p_1 p_2 p_4 \vee (0.5,0.2)p_1 p_2 p_3 \vee (0.6,0.2)p_1 p_2 p_3 p_4 \vee (0.3,0.3)p_2 \vee (0.5,0.4)p_1 p_2 p_5$$
$$\vee (0.5,0.4)p_1 p_2 p_4 p_5 \vee (0.5,0.2)p_1 p_2 p_3 p_5 \vee (1,0)p_1 p_2 p_3 p_4 p_5 \vee (0.3,0.3)p_2 p_5]$$

After simplifying

$$\varphi_D = (0.3,0.3)p_2 \quad \vee (0.5,0.4)p_1 p_2 \vee (0.5,0.2)p_1 p_2 p_3 \vee (0.5,0.4)p_1 p_2 p_4 p_5 \vee (0.6,\ 0.2)p_1 p_2 p_3 p_4$$
$$\vee (1,0)p_1 p_2 p_3 p_4 p_5$$

From the last equation, the graph $\widetilde{G}$ has the following domination set.

$$\widetilde{D} = \{ \langle (0.3,0.3)/1 \rangle, \langle (0.5,0.4)/2 \rangle, \langle (0.5,0.2)/3 \rangle, \langle (0.6,0.2)/4 \rangle, \langle (1,0)/5 \rangle \} \tag{14}$$

6.2. Some Observations

Deeply studying Algorithm 1 reveals certain limitations. Figure 4 is a digraph adapted from [32] (Figure 1 in [32]). The DVS calculated for this IFG in Example 3 in [32] is as below

$$\widetilde{D} = \{\langle (0.3, 0.3) / 1 \rangle, \ \langle (0.5, 0.3) / 2 \rangle, \ \langle (0.5, 0.3) / 3 \rangle, \ \langle (0.6, 0.2) / 4 \rangle, \ \langle (1, 0) / 5 \rangle \}.$$

However, applying the algorithm presented in the same paper to the above example yields a different DVS (12) than the aforementioned $\widetilde{D}$.

The subset $X \subseteq V$ is known as a minimal intuitionistic DVS with degree $\beta(X)$, if the condition $\beta(X') < \beta(X)$ is true for any subset $X' \subseteq X$ [32].

However, Algorithm 1 presented in the same article does not satisfy this definition of minimality.

- For the IFG in Figure 4, taking

$$X = \{x_2\} \subseteq \{x_1, \ x_2\} = Y;$$

the degrees of dominations are $\beta(X) = (0.3, 0.3)$ and $\beta(Y) = (0.5, 0.4)$.
However

$$\beta(X) = (0.3, 0.3) \nless (0.5, 0.4) = \beta(Y)$$

As

$$0.3 < 0.5$$

However

$$0.3 \ngtr 0.4.$$

Thus, their algorithm does not give minimal DVS as was claimed.

In some instances, we cannot identify the unique minimal DVS of a given order. The reason lies in the inability of the algorithm to find the supremum of IFVs. Take the graph in Figure 5, for instance.
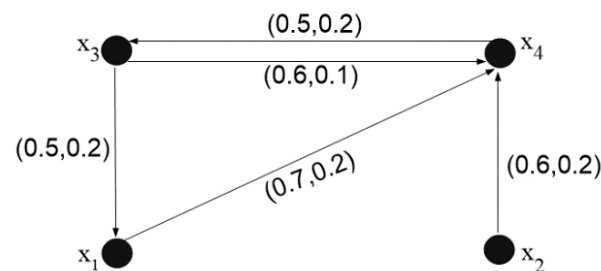

Figure 5. An intuitionistic fuzzy graph.

The corresponding adjacency matrix of the graph in Figure 5 is given below.

$$R_x = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ \left[\begin{array}{cccc} (1,0) & (0,1) & (0,1) & (0.7,0.2) \\ (0,1) & (1,0) & (0,1) & (0.6,0.2) \\ (0.5,0.2) & (0,1) & (1,0) & (0.5,0.2) \\ (0,1) & (0,1) & (0.6,0.1) & (1,0) \end{array} \right] \end{array}$$

Applying Algorithm 1, we obtain

$$\varphi_D = [(0.5, 0.2)p_2p_3 \vee (0.7, 0.2)p_1p_2p_3 \vee (0.5, 0.2)p_2p_3p_4 \vee (0.6, 0.1) \vee p_1p_2p_4 \vee (1, 0)p_1p_2p_3p_4].$$

In this expression we are unable to find the supremum for order 3 DVS. Thus, we cannot derive the minimal DVS of order 3.

6.3. Solution of Water Flow Network through Algorithm 2

For comparison, Algorithm 2 will be applied on the water flow problem in Example 4.

Step 1: The set of all vertices is $X = \{x_1, x_2, x_3, x_4, x_5\}$. Here x_1 and x_2 are two vertices whose $p(x_1, x_r) \neq (0, 1)$ s.t $x_r \in X - \{x_1\}$ and $p(x_2, x_j) \neq (0, 1)$ s.t $x_j \in X - \{x_2\}$.

Now,

$$X - x_1 = \{x_2, x_3, x_4, x_5\}$$

$$p_{x_1}(x_3) = (0.5, 0.4)$$

$$p_{x_1}(x_4) = (0.2, 0.4)$$

Hence,

$$\beta_1(x_1) = \beta_{11} = \{(0.5, 0.4)\&(0.2, 0.4)\} = (0.2, 0.4)$$

$$X - x_2 = \{x_1, x_3, x_4, x_5\}$$

$$p_{x_2}(x_1) = (0.3, 0.3)$$

$$p_{x_2}(x_3) = (0.3, 0.3)$$

$$p_{x_2}(x_4) = (0.5, 0.2)$$

$$p_{x_2}(x_5) = (0.6, 0.2)$$

$$\beta_1(x_2) = \beta_{12} = \{(0.3, 0.3)\&(0.3, 0.3)\&(0.5, 0.2)\&(0.6, 0.2)\}$$

$$\beta_1(x_2) = \beta_{12} = (0.3, 0.3)$$

So,

$$\beta_1^0 = \{(0.2, 0.4) \vee (0.3, 0.3)\} = (0.3, 0.3)$$

Step 2: Using the adjacency matrix for the graph in Figure 4 (calculated in Section 6.1), φ_D will be as below

$$\varphi_D = [(1, 0)p_1 \quad \vee (0.3, 0.3)p_2]\&[(1, 0)p_2]\& [(0.5, 0.3)p_1p_2 \vee (1, 0)p_3]\&[(1, 0)p_4 \vee (0.5, 0.2)p_1p_2]\&[(0.6, 0.2)p_2 \\ \vee (1, 0)p_5].$$

Multiplying expressions 1, 2 and 3, 4, we obtain

$$\varphi_D = [(1, 0)p_1p_2 \quad \vee (0.3, 0.3)p_2]\&[(0.5, 0.3)p_1p_2p_4 \vee (0.5, 0.3)p_1p_2 \vee (1, 0)p_3p_4 \vee (0.5, 0.2)p_1p_2p_3]\& [(0.6, 0.2)p_2 \\ \vee (1, 0)p_5]$$

Simplifying expression 2 and multiplying with 1

$$\varphi_D = [(0.5, 0.3)p_1p_2 \quad \vee (1, 0)p_1p_2p_3p_4 \vee (0.5, 0.2)p_1p_2p_3 \vee (0.3, 0.3)p_1p_2 \vee (0.3, 0.3)p_2p_3p_4 \\ \vee (0.3, 0.3)p_1p_2p_3]\& [(0.6, 0.2)p_2 \vee (1, 0)p_5]$$

Simplifying expression 1 and multiply with expression 2

$$\varphi_D = [(0.5, 0.3)p_1p_2 \quad \vee (0.6, 0.2)p_1p_2p_3p_4 \vee (0.5, 0.2)p_1p_2p_3 \vee (0.3, 0.3)p_2p_3p_4 \vee (0.5, 0.3)p_1p_2p_5 \\ \vee (1, 0)p_1p_2p_3p_4p_5 \vee (0.5, 0.2)p_1p_2p_3p_5 \vee (0.3, 0.3)p_2p_3p_4p_5]$$

After simplification, we will obtain

$$\varphi_D = \left[(0.5, 0.3)p_1p_2 \vee (0.5, 0.2)p_1p_2p_3 \vee (0.6, 0.2)p_1p_2p_3p_4 \vee (1, 0)p_1p_2p_3p_4p_5\right]$$

$$\widetilde{D} = \{\langle(0.5, 0.3)/2\rangle, \langle(0.5, 0.2)/3\rangle, \langle(0.6, 0.2)/4\rangle, \langle(1, 0)/5\rangle\}. \tag{15}$$

φ_D above reveals that the DVS $\{x_1, x_2\}$ has a membership value of 0.5, which shows the collective flow capacity of x_1 and x_2, and the non-membership is 0.3, representing the average water loss from the two utilities is 0.3.

Hence, the above calculation shows that the novel approach generates minimal DVS. This algorithm is more dependable, as it is based on the volume of vertices instead of their intuitionistic degrees.

7. Conclusions

Many concepts were transformed as the paradigm in set theory shifted from crispness to vagueness. The study of fuzzy graphs and IFGs is currently a hot issue, along with studies in graph theory. DVSs are a frequently explored concept. The literature outlines a method for locating IFG DVSs. The current strategy, however, has some serious drawbacks. The most important mistake is not considering the number of vertices, which directly affects how DVS is calculated for IFGs. Additionally, it is occasionally impossible to find the minimum DVS for IFGs. These factors diminish the applicability and authenticity of the approach.

The current paper attempts to redefine the methodology to address its flaws. The comparative analysis demonstrates its superiority to the state-of-the-art method. Applying this algorithm to the ideal amount of water flow also shows how useful and flexible it is. However, more research needs to be performed on figuring out DVS for IFGs where some membership values still need to be included.

Author Contributions: Conceptualization, N.N., T.S. (Tanzeela Shaheen), L.J. and T.S. (Tapan Senapati); Methodology, L.J.; Validation, T.S. (Tanzeela Shaheen) and T.S. (Tapan Senapati); Formal analysis, N.N. and T.S. (Tanzeela Shaheen); Investigation, L.J. and T.S. (Tapan Senapati); Data curation, N.N.; Writing—original draft, T.S. (Tanzeela Shaheen); Writing—review & editing, L.J.; Supervision, T.S. (Tapan Senapati). All authors have read and agreed to the published version of the manuscript.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

List of abbreviations used in the paper are listed in below. The plural of a notion X will be represented by Xs. For example, fuzzy sets will be abbreviated as FSs.

Abbreviation	Meaning
IF	Intuitionistic fuzzy
IFS	Intuitionistic fuzzy set
IFG	Intuitionistic fuzzy graph
DVS	Dominating vertex set
FS	Fuzzy set
FG	Fuzzy graph
MF	Membership function

References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Deveci, M.; Pamucar, D.; Gokasar, I.; Tavana, M. Spacecraft tracking control and synchronization: An assessment of conventional, unconventional, and combined methods. *Adv. Space Res.* 2022; *available online*.
3. Qahtan, S.; Alsattar, H.A.; Zaidan, A.A.; Deveci, M.; Pamucar, D.; Ding, W. A novel fuel supply system modelling approach for electric vehicles under Pythagorean probabilistic hesitant fuzzy sets. *Inf. Sci.* **2023**, *622*, 1014–1032. [CrossRef]
4. Akram, M.; Ramzan, N.; Deveci, M. Linguistic Pythagorean fuzzy CRITIC-EDAS method for multiple-attribute group decision analysis. *Eng. Appl. Artif. Intell.* **2023**, *119*, 105777. [CrossRef]

5.   Pamucar, D.; Gokasar, I.; Torkayesh, A.E.; Deveci, M.; Martínez, L.; Wu, Q. Prioritization of unmanned aerial vehicles in transportation systems using the integrated stratified fuzzy rough decision-making approach with the hamacher operator. *Inf. Sci.* **2023**, *622*, 374–404. [CrossRef]

6.   Rosenfeld, A. Fuzzy graphs. In *Fuzzy Sets and Their Applications*; Zadeh, L.A., Fu, K.S., Shimura, M., Eds.; Academic Press: New York, NY, USA, 1975; pp. 77–95.

7.   Samanta, S.; Pal, M. Fuzzy threshold graphs. *CIIT Int. J. Fuzzy Syst.* **2011**, *3*, 360–364.

8.   Samanta, S.; Pal, M. Fuzzy k-competition graphs and p-competition fuzzy graphs. *Fuzzy Inf. Eng.* **2013**, *5*, 191–204. [CrossRef]

9.   Samanta, S.; Sarkar, B.; Shin, D.; Pal, M. Completeness and regularity of generalized fuzzy graphs. *SpringerPlus* **2016**, *5*, 1–4. [CrossRef]

10.   Ghorai, G.; Pal, M. On degrees of m-polar fuzzy graph with application. *J. Uncertain Syst.* **2017**, *11*, 294–305.

11.   Mahapatra, T.; Ghorai, G.; Pal, M. Competition graphs under interval-valued m-polar fuzzy environment and its application. *Comput. Appl. Math.* **2022**, *41*, 1–35. [CrossRef]

12.   Ghorai, G.; Pal, M. Results of m-polar fuzzy graphs with application. *J. Uncertain Syst.* **2018**, *12*, 47–55.

13.   Pal, M.; Samanta, S.; Ghorai, G. *Modern Trends in Fuzzy Graph Theory*; Springer: Berlin/Heidelberg, Germany, 2020; pp. 7–93.

14.   Koam, A.N.; Akram, M.; Liu, P. Decision-making analysis based on fuzzy graph structures. *Math. Probl. Eng.* **2020**, *2020*, 6846257. [CrossRef]

15.   Atanassov, K.T. *Intuitionistic Fuzzy Sets*; Studies in Fuzziness and Soft Computing; Physica: Heidelberg, Germany, 1999; pp. 1–137.

16.   Mahmood, T.; Ali, Z. Analysis of Maclaurin Symmetric Mean Operators for Managing Complex Interval-valued q-Rung Orthopair Fuzzy Setting and Their Applications. *J. Comput. Cogn. Eng.* **2022**, 1–18. [CrossRef]

17.   Khan, R.; Ullah, K.; Pamucar, D.; Bari, M. Performance measure using a multi-attribute decision making approach based on Complex T-spherical fuzzy power aggregation operators. *J. Comput. Cogn. Eng.* **2022**, *1*, 138–146.

18.   Mahmood, T.; Ali, W.; Ali, Z.; Chinram, R. Power aggregation operators and similarity measures based on improved intuitionistic hesitant fuzzy sets and their applications to multiple attribute decision making. *Comput. Model. Eng. Sci.* **2021**, *126*, 1165–1187. [CrossRef]

19.   Qayyum, A.; Shaheen, T. Graded Soft Expert Set as a Generalization of Hesitant Fuzzy Set. *J. Intell. Syst.* **2020**, *29*, 223–236. [CrossRef]

20.   Shannon, A.; Atanassov, K. A first step to a theory of the intuitionistic fuzzy graphs. In Proceedings of the First Workshop on Fuzzy Based Expert Systems, Sofia, Bulgaria, 28–30 September 1994; pp. 59–61.

21.   Gulzar, M.; Mateen, M.H.; Alghazzawi, D.; Kausar, N. A novel applications of complex intuitionistic fuzzy sets in group theory. *IEEE Access* **2020**, *8*, 196075–196085. [CrossRef]

22.   Shao, Z.; Kosari, S.; Rashmanlou, H.; Shoaib, M. New concepts in intuitionistic fuzzy graph with application in water supplier systems. *Mathematics* **2020**, *8*, 1241. [CrossRef]

23.   Kang, K.T.; Song, S.Z.; Jun, Y.B. Multipolar intuitionistic fuzzy set with finite degree and its application in BCK/BCI-algebras. *Mathematics* **2020**, *8*, 177. [CrossRef]

24.   Zuo, C.; Pal, A.; Dey, A. New concepts of picture fuzzy graphs with application. *Mathematics* **2019**, *7*, 470. [CrossRef]

25.   Jan, N.; Ullah, K.; Mahmood, T.; Garg, H.; Davvaz, B.; Saeid, A.B.; Broumi, S. Some root level modifications in interval valued fuzzy graphs and their generalizations including neutrosophic graphs. *Mathematics* **2019**, *7*, 72. [CrossRef]

26.   Yaqoob, N.; Gulistan, M.; Kadry, S.; Wahab, H.A. Complex intuitionistic fuzzy graphs with application in cellular network provider companies. *Mathematics* **2019**, *7*, 35. [CrossRef]

27.   Akram, M.; Naz, S. Energy of Pythagorean fuzzy graphs with applications. *Mathematics* **2018**, *6*, 136. [CrossRef]

28.   Naz, S.; Ashraf, S.; Akram, M. A novel approach to decision-making with Pythagorean fuzzy information. *Mathematics* **2018**, *6*, 95. [CrossRef]

29.   Snyder, K.; c-Dominating Sets for Families of Graphs. Metamorphosis 15 January 2019. Available online: https://metamorphosis. coplac.org/index.php/metamorphosis/article/view/237 (accessed on 14 January 2023).

30.   Pitcher, T. A more general property than domination for sets of probability measures. *Pac. J. Math.* **1965**, *15*, 597–611. [CrossRef]

31.   Alhevaz, A.; Darkooti, M.; Rahbani, H.; Shang, Y. Strong equality of perfect Roman and weak Roman domination in trees. *Mathematics* **2019**, *7*, 997. [CrossRef]

32.   Bozhenyuk, A.; Belyakov, S.; Knyazeva, M.; Rozenberg, I. On computing domination set in intuitionistic fuzzy graph. *Int. J. Comput. Intell. Syst.* **2021**, *14*, 617–624. [CrossRef]

33.   Zadeh, L.A.; Klir, G.J.; Yuan, B. *Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems: Selected Papers*; World Scientific: Singapore, 1996; Volume 6.

34.   Blue, M.; Bush, B.; Puckett, J. Unified approach to fuzzy graph problems. *Fuzzy Sets Syst.* **2002**, *125*, 355–368. [CrossRef]

35.   Kaufmann, A. *Introduction à la Théorie des Sous-Ensembles Flous: Éléments Théoriques de Base*; Masson: Issy Les Moulineaux, France, 1977.

36.   Bershtein, L.S.; Bozhenyuk, A.V. Fuzzy graphs and fuzzy hypergraphs. In *Encyclopedia of Artificial Intelligence*; IGI Global: Hershey, PA, USA, 2009; pp. 704–709.

37.   Li, D.F. *Decision and Game Theory in Management with Intuitionistic Fuzzy Sets*; Springer: Berlin/Heidelberg, Germany, 2014.

38.   Atanassov, K.; Gargov, G. Elements of intuitionistic fuzzy logic. *Part I. Fuzzy Sets Syst.* **1998**, *95*, 39–52. [CrossRef]

39. Haynes, T.W.; Hedetniemi, S.; Slater, P. *Fundamentals of Domination in Graphs*; CRC Press: Boca Raton, FL, USA, 2013.
40. Allan, R.B.; Laskar, R. On domination and independent domination numbers of a graph. *Discret. Math.* **1978**, *23*, 73–76. [CrossRef]