axioms

MDPI

Editorial

New Developments in Geometric Function Theory

Georgia Irina Oros

Department of Mathematics and Computer Science, Faculty of Informatics and Sciences, University of Oradea, RO-410087 Oradea, Romania; georgia_oros_ro@yahoo.co.uk

1. Introduction

This Special Issue aims to highlight the latest developments in the research concerning complex-valued functions from the perspective of geometric function theory. Contributions were sought regarding any aspect of subordination and superordination, different types of operators specific to the research in this field, and special functions involved in univalent function theory with the hope that new approaches would emerge regarding the introduction and study of special classes of univalent functions using operators and the classical theories of differential subordination and superordination, as well as the newer adapted theories of strong differential subordination and superordination and fuzzy differential subordination and superordination. Authors were invited to submit their latest results related to analytic functions in all their variety and also related to their applications in other fields of research. Quantum calculus and its applications related to geometric function theory were also expected to provide interesting outcomes. The presentation of the results obtained by using any other technique that can be applied in the field of complex analysis and its applications was also encouraged.

This Special Issue is devoted especially to complex analysis and was proposed as a means to find new approaches using geometric function theory, to inspire further development in this field.

2. Overview of the Published Papers

The present Special Issue contains 14 papers accepted for publication after a rigorous reviewing process.

In the study [1], Richard D. Carmichael considers vector-valued analytic functions and distributions with values in Banach or Hilbert space. It is proved that certain vector-valued measurable functions generate the analytic functions using the Fourier–Laplace transform, and conversely, measurable functions are generated from the analytic functions, and it is shown that the analytic functions are representable through the generated measurable functions. Certain specific properties are obtained for the analytic functions and measurable functions, and it is proved that, under specified conditions, the analytic functions considered are in fact vector-valued Hardy functions, which immediately result in Cauchy and Poisson integral representations. The existence of boundary values of the analytic functions on the topological boundary is investigated, and problems to consider in future research are suggested. Notably, the author is convinced that future studies can focus on the integral representation, boundary values, and applications of the functions defined in this paper.

In another study [2], Hatun Özlem Güney, Georgia Irina Oros, and Shigeyoshi Owa provide an application of the well-known Sălăgean differential operator for defining a new operator, through which a new class of functions is defined, which has the classes of starlike and convex functions of order α as special cases. The renowned Jack–Miller–Mocanu lemma is applied for obtaining interesting properties for the newly defined class of functions. The new operator defined in this paper can be used to introduce other specific

Citation: Oros, G.I. New Developments in Geometric Function Theory. *Axioms* **2023**, *12*, 59. https://doi.org/10.3390/axioms12010059

Received: 28 December 2022
Accepted: 1 January 2023
Published: 4 January 2023

subclasses of analytic functions, and quantum calculus can be also investigated in future studies.

The research of Gangadharan Murugusundaramoorthy and Teodor Bulboacă presented in reference [3] involves the new subclasses of bi-univalent functions defined in the open-unit disk, which are associated with the Gegenbauer polynomials and satisfy subordination conditions. Coefficient estimates are established for the defined classes, and the remarkable Fekete–Szegő problem is also considered. For particular values of the parameters involved in the definition of the classes, the results obtained in this paper provide new insights into the Yamakawa family of bi-starlike functions associated with the Chebyshev and Legendre polynomials, which are left as an exercise to interested readers.

The authors of reference [4], Alaa H. El-Qadeem and Ibrahim S. Elshazly, study the Hadamard product features of certain subclasses of p-valent meromorphic functions defined in the punctured open-unit disc using the q-difference operator. Convolution properties and coefficient estimates are also established regarding the new subclasses defined in this study. The authors suggest that future researchers focus on the use of these subclasses in studies involving the theories of differential subordination and superordination and also the investigation of the Fekete–Szegő problem.

In the research presented in reference [5], Georgia Irina Oros, Gheorghe Oros, and Ancuța Maria Rus use the confluent hypergeometric function embedded in the theory of strong differential superordinations. The form of the confluent hypergeometric function and that of the previously defined Kummer–Bernardi and Kummer–Libera operators are adopted by considering certain classes of analytic functions depending on an extra parameter previously introduced related to the theory of strong differential subordination and superordination. Strong differential superordinations are investigated, and the best subordinates are given. The applications of the established theoretical results are illustrated through two examples. As potential future studies, the authors suggest the use of the dual notion of strong differential subordination for investigations concerning the confluent hypergeometric function and the two operators used in the present study, which could yield sandwich-type results if combined with the results contained in this paper.

The topic of introducing new subclasses of bi-starlike and bi-convex functions of a complex order associated with the Erdély–Kober-type integral operator in the open-unit disc is considered by Alhanouf Alburaikan, Gangadharan Murugusundaramoorthy, and Sheza M. El-Deeb [6]. The estimates of initial coefficients are given, and Fekete–Szegő inequalities are investigated for the functions in those classes. Several consequences of the results are also highlighted as examples.

For the study presented in reference [7], Feras Yousef, Ala Amourah, Basem Aref Frasin, and Teodor Bulboacă again consider certain new subclasses of bi-univalent functions by exploiting the zero-truncated Poisson distribution probabilities and involving Gegenbauer polynomials and the concept of subordination. Coefficient-related problems are investigated, and the Fekete–Szegő functional problem is solved for those classes. The authors suggest that the results offered in this paper would lead to other different new results involving Legendre and Chebyshev polynomials.

Considering the importance of the logarithmic coefficients, in reference [8], Sevtap Sümer Eker, Bilal Şeker, Bilal Çekiç, and Mugur Acu obtain the sharp bounds for the second Hankel determinant concerning the logarithmic coefficients of strongly starlike functions and strongly convex functions. The results presented here could inspire further studies that focus on other subclasses of univalent functions and obtain the boundaries for higher-order Hankel determinants.

New results on the radius of uniform convexity of two kinds of normalization of the Bessel function J_ν in the case of $\nu \in (-2, -1)$ are presented by Luminiţa-Ioana Cotîrlă, Pál Aurel Kupán, and Róbert Szász in reference [9]. This study provides alternative proof regarding the radius of convexity of order alpha. The authors also provide alternative proof regarding the radius of convexity of order alpha and derive an interesting correlation between convexity and uniform convexity.

The research presented by Richard D. Carmichael in reference [10] is connected to the results obtained in reference [1]. A boundary value result concerning vector-valued tempered distributions as the boundary values of vector-valued analytic functions is given under the general norm growth on the analytic function, which is equivalent to the growth of Tillmann. The second goal of this paper was to obtain a Cauchy integral representation of the analytic functions by using the generally known structure of the spectral function and the structure of the tempered distributional boundary value. The analytic function used to obtain the boundary value was equated to the product of a polynomial and the constructed Cauchy integral. This paper concerns theoretical mathematics; however, the considered topics find applications in mathematical physics and the field of mathematics involving physical problems.

New results are obtained concerning fuzzy differential subordination theory and are highlighted by Alina Alb Lupaș [11]. A previously introduced operator defined by applying the Riemann–Liouville fractional integral to the convex combination of well-known Ruscheweyh and Sălăgean differential operators is used for defining a new fuzzy subclass. The convex property of this class is proved, and certain fuzzy differential subordinations involving the functions from this class and the operator mentioned earlier are obtained. The best fuzzy dominants are given for the considered fuzzy differential subordinations in theorems, and interesting corollaries emerge when specific functions with remarkable geometric properties are used as the best fuzzy dominants. Inspired by the research presented here, researchers can apply the operator used in this paper in future studies for the introduction of other subclasses of analytic functions. The dual theory of fuzzy differential superordination can also be used for obtaining similar results involving the operator and the class defined in this paper.

Using beta-negative binomial distribution series and Laguerre polynomials, Isra Al-Shbeil, Abbas Kareem Wanas, Afis Saliu, and Adriana Cătaş [12] investigate a new family of normalized holomorphic and bi-univalent functions associated with Ozaki close-to-convex functions. They provide estimates on the initial Taylor–Maclaurin coefficients and discuss Fekete–Szegő type inequality for the functions in this family in the special case of generalized Laguerre polynomials.

A symmetric–convex differential formula of normalized Airy functions in the open-unit disk is developed by Samir B. Hadid and Rabha W. Ibrahim in reference [13]. The equation is taken into account as a differential operator in the development of a class of normalized analytic functions. Two-dimensional wave propagation in the earth–ionosphere wave path using k-symbol Airy functions is used for the investigation. It is shown that the standard wave-mode working formula may be determined by orthogonality considerations without the use of intricate justifications of the complex plane.

The applications of fractional differential operators in the field of geometric function theory are obtained by Mohammad Faisal Khan, Shahid Khan, Saqib Hussain, Maslina Darus, and Khaled Matarneh in reference [14]. The fractional differential operator and the Mittag–Leffler functions are combined to formulate and arrange a new operator of fractional calculus. A new class of normalized analytic functions is introduced using the newly defined fractional operator, and some of its interesting geometric properties are discussed in the open-unit disk. The authors suggest that the operator introduced here can be utilized to define other classes of analytic functions or to generalize other types of differential operators.

3. Conclusions

The 14 papers published as part of this Special Issue entitled "New Developments in Geometric Function Theory" concern a broad range of subjects. Researchers interested in different aspects of geometric function theory and its related topics would find interesting insights and inspiring results, leading to increased reference to these contributions and the propagation of this Special Issue to a large audience.

Acknowledgments: The guest editor of this Special Issue would like to thank all the authors who decided to submit their works and have contributed to the success of this Special Issue, as well as

all the reviewers for their time, constructive remarks, and help in maintaining high standards for the published materials. Special thanks are also given to the editors of *Axioms* and especially to the Managing Editor of this Special Issue, Alex Zhang.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Carmichael, R.D. Generalized Vector-Valued Hardy Functions. *Axioms* **2022**, *11*, 39. [CrossRef]
2. Güney, H.Ö.; Oros, G.I.; Owa, S. An Application of Sălăgean Operator Concerning Starlike Functions. *Axioms* **2022**, *11*, 50. [CrossRef]
3. Murugusundaramoorthy, G.; Bulboacă, T. Subclasses of Yamakawa-Type Bi-Starlike Functions Associated with Gegenbauer Polynomials. *Axioms* **2022**, *11*, 92. [CrossRef]
4. El-Qadeem, A.H.; Elshazly, I.S. Hadamard Product Properties for Certain Subclasses of *p*-Valent Meromorphic Functions. *Axioms* **2022**, *11*, 172. [CrossRef]
5. Oros, G.I.; Oros, G.; Rus, A.M. Applications of Confluent Hypergeometric Function in Strong Superordination Theory. *Axioms* **2022**, *11*, 209. [CrossRef]
6. Alburaikan, A.; Murugusundaramoorthy, G.; El-Deeb, S.M. Certain Subclasses of Bi-Starlike Function of Complex Order Defined by Erdély–Kober-Type Integral Operator. *Axioms* **2022**, *11*, 237. [CrossRef]
7. Yousef, F.; Amourah, A.; Frasin, B.A.; Bulboacă, T. An Avant-Garde Construction for Subclasses of Analytic Bi-Univalent Functions. *Axioms* **2022**, *11*, 267. [CrossRef]
8. Sümer Eker, S.; Şeker, B.; Çekiç, B.; Acu, M. Sharp Bounds for the Second Hankel Determinant of Logarithmic Coefficients for Strongly Starlike and Strongly Convex Functions. *Axioms* **2022**, *11*, 369. [CrossRef]
9. Cotîrlă, L.-I.; Kupán, P.A.; Szász, R. New Results about Radius of Convexity and Uniform Convexity of Bessel Functions. *Axioms* **2022**, *11*, 380. [CrossRef]
10. Carmichael, R.D. Cauchy Integral and Boundary Value for Vector-Valued Tempered Distributions. *Axioms* **2022**, *11*, 392. [CrossRef]
11. Alb Lupaş, A. On Special Fuzzy Differential Subordinations Obtained for Riemann–Liouville Fractional Integral of Ruscheweyh and Sălăgean Operators. *Axioms* **2022**, *11*, 428. [CrossRef]
12. Al-Shbeil, I.; Wanas, A.K.; Saliu, A.; Cătaş, A. Applications of Beta Negative Binomial Distribution and Laguerre Polynomials on Ozaki Bi-Close-to-Convex Functions. *Axioms* **2022**, *11*, 451. [CrossRef]
13. Hadid, S.B.; Ibrahim, R.W. Geometric Study of 2D-Wave Equations in View of K-Symbol Airy Functions. *Axioms* **2022**, *11*, 590. [CrossRef]
14. Khan, M.F.; Khan, S.; Hussain, S.; Darus, M.; Matarneh, K. Certain New Class of Analytic Functions Defined by Using a Fractional Derivative and Mittag-Leffler Functions. *Axioms* **2022**, *11*, 655. [CrossRef]