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A Study of Team Recommended Generalized Assignment Methods

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Abstract: This study considers the team recommendation problem as a generalized assignment problem. Firstly, a formal description of the team recommendation problem is given; secondly, a team-recommended generalized assignment model (TRGAM) is established based on the work ability value of alternative members, the comprehensive work ability value of the team as the core concern index, the importance weight of team tasks and the energy allocation weight of team members as the fusion strategy of the data; thirdly, a solution method for the standard case of TRGAM is designed using the enumeration method and Hungarian algorithms (BEM⊕HM–TRGAMs) as local computational tools; fourthly, the alternative member set refinement methods and standardization measures for TRGAM are given; finally, BEM⊕HM–TRGAMs are analyzed using specific arithmetic examples. The theoretical analysis and experimental results show that TRGAM has good structural features and interpretability and BEM⊕HM–TRGAMs can effectively solve the TRGAM solving problem.

Keywords: team recommendation; comprehensive ability; generalized assignment; enumeration method; Hungarian algorithm

MSC: 90B50; 90B70

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1. Introduction

Team recommendation (TR) is the process of selecting the team members who best meet the overall task requirements among alternative members [1]. Its distinctive features are both divisions of labor and cooperation among members, which is a widespread and realistic problem in many fields such as resource management, new technology development, innovation team cultivation, complex system optimization, etc. [2]. For example, In the process of team formation for large software project development, the selection of team members considers the analysis of market demand, product design, technical expertise, and other aspects of ability, and each member not only can be responsible for one aspect of the work as the main person, but also work on some other aspects of the work as an auxiliary. As TR is a class of nondeterministic polynomial NP-hard problems [1], we do not know a polynomial time algorithm to solve it, and it is difficult to solve this problem with a single traditional planning method (e.g., collaborative filtering, matrix decomposition, etc.).

The essence of TR is an optimization problem. Previous research on the TR problem argued that the relevance of members' skills to task demands should be considered to the maximum possible extent when selecting team members [3,4]. With the deep research of many scholars, they found that relevant skills are not the only determinant of team success, but also that cooperation and communication among members is another core characteristic of TR [5]. The results of previous studies on the performance of TR collaboration (see specifically the literature review in Section 2) can be characterized as follows: (a) some form of statistical laws to determine (estimate) the value of the overall cluster performance metrics; (b) combining intelligent algorithms to determine the recommendation result (this type of research involves random search, the solution result is not guaranteed to be the optimal solution to the problem, and the computational complexity is high); (c) difficulty in reflecting the role value and role orientation of cluster members in the recommended results, and lack of operability; and (d) lack of intuitive interpretability and structural features.

From the above discussions, it is clear that the TR problem and the assignment problem have similar characteristics in terms of member role positioning, both of which select some members to accomplish some tasks, while there are also essential differences as follows: (a) the TR problem should consider the ability of team members to work primarily on tasks and the ability of team members to work on other tasks as an auxiliary member; (b) the existing standard assignment problem cannot describe and solve the TR problem through a simple method. Therefore, it is a great theoretical and application value to construct a TR method that can reflect both the ability of the members and their auxiliary role and reflect both the division of labor and the cooperation. This study considers TR as a generalized assignment problem, and its main contributions are as follows: (a) from the perspective of role positioning, task importance weights and member energy allocation weights are introduced to indicate the importance of different tasks and members' energy allocation for each task, respectively, and to integrate individual ability values into the assignment problem; (b) the team recommended generalized assignment model (abbreviated as TRGAM) is proposed; (c) give a basic guideline for refining alternative members; (d) a solution method based on the enumeration method and the Hungarian algorithm (abbreviated as BEM⊕HM-TRGAMs) is designed for the standard TRGAM; and (e) give the standardization measures for TRGAM.

The study is structured as follows. Section 2 reviews the current research results in terms of both the TR problem and the assignment problem, respectively. Section 3 gives a formal description of the TR problem and the symbolic system used in the subsequent discussion, as well as the construction of a generalized assignment model (TRGAM) for TR from a structural point of view, and an analysis of the properties of TRGAM from a theoretical point of view. Section 4 designs a solution method based on the enumeration method and Hungarian algorithms (BEM⊕HM-TRGAMs) for a particular type of TRGAM (TRGAMs and called standard-type TRGAM). Section 5 gives some standardization measures for TRGAM. Section 6 further analyzes and verifies the effectiveness of BEM⊕HM-TRGAMs and validates it through concrete examples and simulation experiments. Section 7 systematically summarizes the work discussed in this study.

2. Literature Review

2.1. TR Problem

With the rapid development of the internet and information technology, the TR problem has received extensive attention in the academic field. Some scholars have studied the TR problem from different angles of concern by combining relevant theories, and have obtained many research results with certain theoretical and application values. For example, for the problem of forming classroom teams for college students, an online questionnaire was used to collect the influencing factors of team formation that students focused on, and the results showed that students preferred the factor of closeness of the team members [6]. Mutual trust and reliability between teams are key to ensuring smooth teamwork [7]. Higher intimacy among members positively contributes to efficient teamwork, and for this factor, Latorre and Suárez [8] focused on the interpersonal relationships between the members and estimated the compatibility among colleagues based on previous cooperation results as well as individual social skills, significantly improving the expected results of social interactions among team members. Jin et al. [9] focused on the individual ability of team candidates and their ability to maintain interpersonal relationships and proposed a TR model based on cooperation effects. They solved it using a genetic algorithm that helps teams to improve their performance by selecting the right members.

Teamwork performance depends not only on technical ability, but also on the effective interaction between team members. Team communication cost has been the focus of attention when constructing the TR model, and many scholars have made many studies and contributions to the optimization of team communication cost in recent years. As an example, Daş, Altınkaynak, Göçken and Türker [2] constructed a multi-objective planning model by considering both the required skills of the team and the reduction of communication costs. They designed a corresponding heuristic algorithm to ensure that the selected team meets the requirements. Berktaş and Yaman [10] address the problem of building a team that can communicate and collaborates effectively by proposing a novel branching algorithm that ensures minimizing the sum of communication costs by decomposing the original problem into a series of linear problems that need to be solved. Selvarajah et al. [11] proposed a multi-objective algorithm integrating a basic cost function and discussed the importance of variables concerning members' emotions on TR as a unified framework to build effective expert teams. Kalyani Selvarajah et al. [12] argues that the lower the cost of communication, the more collaborative the team will be, and uses the sum of member distances as a communication cost function to have minimal communication cost while covering all the required skills while proposing a knowledge-based evolutionary optimization algorithm to solve this problem. Li et al. [13] developed two greedy algorithms and a heuristic to solve the model by maximizing the ratio of team influence to the cost of communication among members to construct an efficient, low-cost team.

In addition to interpersonal relationships and team costs, many other factors are thought to influence measures of teamwork performance in TR problems, and in response to these issues, several scholars have done the following research. For effective cooperation among team members, some scholars have proposed using alternative members' social networks to build better teams [14]. Wang et al. [15] argued that maximizing team skills and ensuring cost minimization can improve teamwork efficiency. Coordinated team functioning as well as common principles influence teamwork efficiency [16]. Büyükboyaci and Robbett [17] believed that accurate positioning of the members in the team according to their skills promoted positive teamwork. Garousi and Tarhan [18] have found that a mixed group of team members based on their personalities is good teamwork. D'Aniello et al. [19] designed a recommendation model based on knowledge, skills, and attitudes by combining leader recommendation and employee preference. This model was evaluated and analyzed to prove its practicality and reliability. Xiao et al. [20] developed a TR model that integrates individual performance, internal organizational collaboration performance, and external organizational collaboration performance. They gave a solution method based on a non-dominated ranking genetic algorithm. A model is proposed to quantitatively measure collective intelligence for the expert TR problem, which combines two factors, including an expertise score and a trust-based collaboration score. This was solved using a genetic algorithm [21]. Bahargam et al. [22] addressed the influence of team fault lines on TR results. They proposed the idea of differentiating alternative members according to their personal qualities. Then, they divided the same type of members into low-failure teams as a new set of alternative members to ensure the stability of the constructed teams. Shen [23] addressed the multi-objective TR problem by using a multistage decision model for processing and designing a genetic algorithm seeking feasible solutions for all stages to solve the multi-objective multistage human resource allocation problem. Wang and Zhang [24] addressed the problem of lack of consideration of team members' personal opinions in the team formation process and constructed a simulation environment for negotiation by considering the welfare requirements of the leader as well as team members, and the negotiation process is continuously iterated until both parties' interests are satisfied. A synergistic evaluation index system for the knowledge innovation team partner selection problem was given based on the analysis of the synergistic relationships and synergistic effects between partners. A mathematical model for selecting team partners was established, and the corresponding general responsibility assignment software patterns (GRASP) heuristic algorithm was developed [25]. Costa et al. [26] proposed a search method with technical attributes as the focus and nontechnical attributes as supplementary information to improve the quality and highlight the characteristics of software teams. An evaluation index system for competency was established using the comprehensive competency value as the basis for membership selection; a team membership selection method with a questionnaire and statistical analysis as the access to the evaluation matrix and fuzzy comprehensive evaluation as the data synthesis measure was given in the analytic hierarchy process (AHP) framework [27].

2.2. Assignment Problem

The assignment problem is one of the most common decision problems in resource management. Its characteristics and standard form are as follows: m people do m things. Each person can and can only do one thing, and one thing can be done by at most one person. If the benefit of the i-th person completing the j-th thing is c_{ij}, then how to assign tasks to maximize the total benefit of m people completing m thing. The mathematical model is as follows (where, $x_{ij} \in \{0, 1\}$ is the assigned decision variable, $x_{ij} = 0$ denotes that person i does not do the j-th thing and $x_{ij} =$ denotes that person i is responsible for the j-th thing):

$$\begin{aligned} &\max z = \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij} \\ &\text{s.t.} \begin{cases} \sum\limits_{i=1}^{m} x_{ij} = 1,\ j = 1, 2, \cdots, m, \\ \sum\limits_{j=1}^{m} x_{ij} = 1,\ i = 1, 2, \cdots, m, \\ x_{ij} \in \{0,\ 1\},\ i,\ j = 1, 2, \cdots, m. \end{cases} \end{aligned} \tag{1}$$

Because most of the decision problems with assignment characteristics cannot be solved directly using the Hungarian algorithm, many scholars have discussed the extension of the standard assignment model and formed many research results with application value. One of the most representative results is the standardization method based on the change assignment matrix (the specific content is as follows: for the assignment problems with unequal persons and things or special requirements between persons and things, the change assignment matrix is used as the basic strategy to transform the nonstandard assignment problems into standard assignment problems). The second is the result in the generalized assignment method (whose basic feature is that the model structure or solution method is similar to the assignment problem). This model building and solution ideas of the assignment problem try to satisfy the optimal assignment results under different backgrounds, constraints and influencing factors, and have some similarities with the TR problem. For example, for the assignment problem of assigning one agent to each task, Zaozerskaya [28] proposes to maximize the profit of the assignment while satisfying the agent's capacity and workload constraints. For assignment problems such as assigning different teachers to different units of coursework tasks, Zaozerskaya et al. [29] propose a class of integer linear programming models that set the minimum and maximum possible number of academic loads for each teacher and also take into account the interpersonal aspects of teachers in the teaching process. To address the problem of multi-skill training demand allocation with staff uncertainty in the service industry, scholars proposed that the training cost and the expected cost of employee overload should be minimized [30]. Nambiar et al. [31] designed a generalized triple random sequential assignment model that integrates the value of job ability, workload, and worker mobilizability to ensure that the total expected payoff of the assignment is maximized. A mathematical optimization model to maximize the sum of benefits generated by task completion was designed for the optimization of the airport task assignment problem. The actual data issues were solved by applying CPLEX software [32]. The minimum cost flow model is constructed for the shortest time assignment problem, and a fast decision method is proposed for solving the shortest time assignment problem by combining the Duality theorem [33]. In terms of the current research status, the prospection of standardized transformation methods has been perfected. However, the research on generalized assignment is still in the exploratory stage, and how to establish an assignment method that can take into account the interaction between members is a widespread concern in academic and application fields today.

Combining the above two parts, it is easy to see that: (a) TR problems usually require specifying the roles of team members and have similar structural features as assignment problems; and (b) most of the current methods for solving TR are complex intelligent algorithms that do not consider a simple way to handle the role assignment of team members. Therefore, this study considers the TR problem as a generalized assignment problem and uses the ability values of team members and the role assignments of team members as a starting point. Constructing a solution to the TR problem is of important practical significance and is theoretically feasible.

3. TR's Generalized Assignment Model (TRGAM)

3.1. Formal Description of the TR Problem

The selection and teams' recommendation is a widespread decision problem in the real world. The formal description of the problem is as follows: m members are selected from n candidates to form a team, and each team member is responsible for one task for the main and some other tasks for the auxiliary. If the value of each member's ability to complete each task is known, then the method to select team members to maximize the value of the team's overall ability is determined.

For the convenience of the description, we assume that:

- $U = \{1, 2, \cdots, n\}$: candidate members of the team;
- $V = \{1, 2, \cdots, m\}$: tasks of the team;
- $a_{ik} \in [0, \infty)$: the ability value of the k-th task of the i-th alternative candidate member;
- $A = (a_{ik})_{n\times m}$: the ability matrix of the TR problem.

The generation of each alternative team T contains the selection of several alternative members, each of them is mainly responsible for one task and for some tasks for the auxiliary members, where the member t_k can be arranged in the order of the tasks for which they are mainly responsible. At the same time, since each task has a member who is mainly responsible for and a member who is auxiliary responsible for, in order to understand more directly the combination of team overall ability values, F_1 and F_2 are introduced to denote the team's main responsibility ability values and auxiliary work ability values, respectively. Since the TR problem has similar structural features as the assignment problem, the most direct and effective solution method is to build the assignment matrix $C(T, A)$. Synthesizing the above description, we assume that:

- $T = (t_1, t_2, \cdots, t_m)$: a team that the t_k-th candidate member for the main of the responsible with the k-th task ($k = 1, 2, \cdots, m$,$t_1, t_2, \cdots, t_m$ is not identical to each other, that is, $(t_1, t_2, \cdots, t_m)$ can be considered as an optional arrangement of $1, 2, \cdots, n$ with a ability of m),
- $F_1(t_1, t_2, \cdots, t_m)$: the comprehensive work ability value of the t_k-th candidate member as the main responsibility to complete the k-th task (called the main work ability value of team $(t_1, t_2, \cdots, t_m)$),
- $F_2(t_1, t_2, \cdots, t_m)$: the best comprehensive ability value of each member of team $(t_1, t_2, \cdots, t_m)$ as auxiliary members for various tasks (called the best auxiliary work ability value of team $(t_1, t_2, \cdots, t_m)$),
- $C(T, A) = \left(c_{ij}(t_1, t_2, \cdots, t_m)\right)_{m\times m}$: the $m \times m$ matrix formed by the t_k-th row of A as the k-th row (referred to as the ability matrix of team $T = (t_1, t_2, \cdots, t_m)$).

Since the work ability value of team T consists of two components, the main work ability value of T and the optimal auxiliary work ability value, the TR problem can thus be formally described according to the above notational convention as ($\mathcal{P}(n, m)$ denotes the entire selection arrangement of $1, 2, \cdots, n$ that the ability is m):

$$\max F(t_1, t_2, \cdots, t_m) = F_1(t_1, t_2, \cdots, t_m) + F_2(t_1, t_2, \cdots, t_m)$$
$$\text{s.t.} \begin{cases} (t_1, t_2, \cdots, t_m) \in \mathcal{P}(n, m), \\ \text{Other requirements for members as primary and secondary.} \end{cases} \tag{2}$$

Because the mutually supportive role of each task and the cooperative utility of each member in the TR problem are difficult to be known precisely, thus, the construction of a TR method that takes into account both the cooperative utility of the overall members and the mutually supportive role of the whole task is a matter that cannot be precisely achieved. (2) is only a formal description. In order to obtain a TR method with some guidance and operability, the discussion in this paper follows the following hypothesis:

Hypothesis 1 (H1). *Appropriate increase of auxiliary members under the same task has a positive contribution to the completion of the task (considering the effect of marginal effects, the number of people under the same task should not be too large).*

Hypothesis 2 (H2). *Ignoring the possibility of a jump in ability values when auxiliary members join, the combination of ability values satisfies some sense of linear accumulation.*

3.2. TRGAM Model Proposed

As we can learn from the aforementioned discussion, TR is an optimization problem with similar characteristics and essential differences from the assignment problem, considering both the main and auxiliary roles of each member, and one of the core issues is the measure of the overall team ability value. If the competency values of team members working on different tasks as main and auxiliary are taken as local work competency values, then with the premise that the combined competence value is linearly additive, the combined competence value of the team $(t_1, t_2, \cdots, t_m)$ can be formally described as follows (see Section 3.3, Theorem 1 for specific calculations):

$$\max F(t_1, t_2, \cdots, t_m) = F_1(t_1, t_2, \cdots, t_m) + F_2(t_1, t_2, \cdots, t_m)$$
$$\text{s.t.} \begin{cases} (t_1, t_2, \cdots, t_m) \in \mathcal{P}(n, m), \\ \sum\limits_{i=1, j\neq k}^{m} x_{ik} = r_k,\ k = 1, 2, \cdots, m, \\ \sum\limits_{k=1, k\neq i}^{m} x_{ik} = s_i,\ i = 1, 2, \cdots, m, \\ x_{ik} \in \{0, 1\},\ i, k = 1, 2, \cdots, m. \end{cases} \tag{3}$$

where: (a) $x_{ik} \in \{0, 1\}$ is a variable describing whether member t_i is engaged in the k-th task as an auxiliary member, $x_{ik} = 0$ means t_i is not engaged in the k-th task as an auxiliary member, $x_{ik} = 1$ means t_i is engaged in the k-th task as an auxiliary member; (b) $r_k \in \{0, 1, 2, \cdots, m-1\}$ denotes the number of persons engaged in the k-th task as an auxiliary member, $s_i \in \{0, 1, 2, \cdots, m-1\}$ denotes the number of other tasks engaged in as an auxiliary member, and to ensure that each task is assigned to the required number of main and auxiliary members and to avoid unnecessary waste of personnel.

It is easy to see that the determination of $F_2(t_1, t_2, \cdots, t_m)$ is the core work of (3), its essence can be expressed as follows (where b_{ik} is satisfied: when $i = k, b_{ii} = 0$; when $i \neq k, b_{ik}$ is the value of the t_i-th member's ability to perform the k-th task as an auxiliary member):

$$F_2(t_1, t_2, \cdots, t_m) = \max \sum_{i=1}^{m}\sum_{k=1}^{m} b_{ik} \cdot x_{ik}$$
$$\text{s.t.} \begin{cases} \sum\limits_{i=1}^{m} x_{ik} = r_k,\ k = 1, 2, \cdots, m, \\ \sum\limits_{k=1}^{m} x_{ik} = s_i,\ i = 1, 2, \cdots, m, \\ x_{ij} \in \{0, 1\},\ i, k = 1, 2, \cdots, m, \end{cases} \tag{4}$$

Additionally, when $r_1 = r_2 = \cdots = r_m = s_1 = s_2 = \cdots = s_m = 1$, (4) is the standard assignment problem with $(b_{ik})_{m\times m}$ as the assignment matrix.

The determination of $F_2(t_1, t_2, \cdots, t_m)$ is the core work of (3), and its content as well as its form have similar characteristics to the standard assignment model (1). Thus, in the following discussion, (3) is called the generalized assignment model of TR (abbreviated as

TRGAM), call (3) at time $r_1 = r_2 = \cdots = r_m = s_1 = s_2 = \cdots = s_m = 1$ the standard form of TRGAM (abbreviated as TRGAMs).

From the aforementioned analysis, TRGAM has the characteristics of a generalized assignment model that reflects the value of team members as a main and auxiliary role. These characteristics indicate that TRGAM has good structural characteristics and interpretability and is a mathematical model with a certain guiding significance. However, the solution of TRGAM becomes an NP-hard problem, and identifying a suitable method to construct a solution method with certain general significance is another key issue in solving the TR problem.

3.3. Properties of TRGAM

This part focuses on the fundamental properties of TRGAM and provides some theoretical basis for the design of subsequent solution methods.

Since the implementation of any project, in reality, is subject to time constraints, the actual work ability value of team members will inevitably depend on the share of energy invested by the members. If this dependence is expressed as a proportional function of the work ability value and the share of energy input, then the objective function in the combined ability value (3) can be expressed as Theorem 1 below.

Theorem 1. *Let $A = (a_{ik})_{n\times m}$ be the ability matrix of the TR problem, and $T = (t_1,\ t_2,\ \cdots,\ t_m)$ denote the team whose t_k-th alternative member is mainly responsible for the k-th task. $w_k \in (0,\ 1)$ denotes the important weight of the k-th task (satisfies $\sum_{k=1}^{m} w_k = 1$), $\beta_1(s_k)$, $\beta_2(s_k) \in [0,\ 1]$ denotes the value of energy allocation weights of t_k for the k-th task as a main member and other s_k tasks as an auxiliary member (satisfies $\beta_1(s_k) + s_k \cdot \beta_2(s_k) = 1$). Then, the best overall ability value of the team$T = (t_1, t_2, , t_m)$ in (3) can be expressed as:*

$$\begin{aligned} \max F(t_1,t_2,\ldots,t_m) &= F_1(t_1,t_2,\ldots,t_m) + F_2(t_1,t_2,\ldots,t_m) \\ &= \sum_{k=1}^{m} w_k \left[\beta_1(s_k)\cdot c_{kk}(t_1,t_2,\ldots,t_m) + \max \sum_{i=1,i\neq k}^{m} \beta_2(s_i)\cdot c_{ik}(t_1,t_2,\ldots,t_m)\cdot x_{ik} \right] \\ &= \sum_{k=1}^{m} w_k\cdot\beta_1(s_k)\cdot c_{kk}(t_1,t_2,\ldots,t_m) + \max \sum_{k=1}^{m}\sum_{i=1,i\neq k}^{m} w_k\cdot\beta_2(s_i)\cdot c_{ik}(t_1,t_2,\cdot,t_m)\ldots x_{ik} \end{aligned} \tag{5}$$

where: (a) $s_k = 0 \Leftrightarrow \beta_1(s_k) = 1$, the intuitive meaning of $\beta_1(s_k) = 1$ is that all the energy of t_k is devoted to the k-th task and no other task is performed for the auxiliary; the intuitive meaning of $\beta_1(s_k) < 1$ is that the team member can devote some energy to other tasks while working on the k-th task; (b) $\beta_1(s_k) + s_k \cdot \beta_2(s_k) = 1$ is a balanced distribution of energy to other tasks; (3) since taking some other tasks as an auxiliary member into consideration often helps one to work on the task as a main member. Thus, in the actual construction of TRGAM, one can choose the case of $s_k \neq 0$. To ensure the proper implementation of the main task as well as the basic energy input for the auxiliary task, $\beta_1(s_k)$ should not be very small (in general, $\beta_1(s_k) \in [0.5,\ 1]$) and s_k not be very large (in general, $s_k \in \{1,\ 2,\ 3\}$).

Note: for the ability matrix $A = (a_{ik})_{n\times m} = (P_1,\ P_2,\ \cdots,P_m)$ of the TR problem and the importance weight vector of the task $W = (w_1,\ w_2,\ \cdots,\ w_m)$, if we record a $\widetilde{A} = (w_1\cdot P_1,\ w_2\cdot P_2,\ \cdots,\ w_m\cdot P_m) \triangleq (\widetilde{a}_{ik})_{n\times m}$ (call it the effect ability matrix of the TR problem), then (5) can be simplified as (where $\widetilde{A} = (w_1\cdot P_1,\ w_2\cdot P_2,\ \cdots,\ w_m\cdot P_m) \triangleq (\widetilde{a}_{ik})_{n\times m}$ denotes the matrix of order $m\times m$ consisting of the i-th row of the t_i-th row of $\widetilde{A} i = 1,\ 2, \cdots,\ m$):

$$\begin{aligned} F(t_1,t_2,\ldots,t_m) &= \sum_{k=1}^{m} \beta_1(s_k)\cdot\widetilde{c}_{kk}(t_1,t_2,\ldots,t_m) + \max\sum_{k=1}^{m}\sum_{i=1,i\neq k}^{m} \beta_2(s_i)\cdot\widetilde{c}_{ik}(t_1,t_2,\ldots,t_m)\cdot x_{ik} \\ &\triangleq \widetilde{F}_1(t_1,t_2,\ldots,t_m) + \widetilde{F}_2(t_1,t_2,\ldots,t_m) \end{aligned} \tag{6}$$

This indicates that if the ability matrix in the above discussion is replaced by the effect ability matrix, then the task importance weights in TRGAM can be considered all equal to

1. In the following discussion, we consider A as the effect ability matrix, which simplifies the form of the model and reduces the computational effort of the solution process.

Theorem 2. *Let us denote the set of alternative members of TR as* $U = \{1, 2, \cdots, n\}$, *denote the set* $V = \{1, 2, \cdots, m\}$ *of tasks of TR, and denote* $A = (a_{ik})_{n\times m}$ *the ability matrix of U with respect to V. For task k, if we write* $u_{1k}, u_{2k}, \cdots, u_{nk}$ *to denote the result of sorting* $a_{1k}, a_{2k}, \cdots, a_{nk}$ *according to the largest to the smallest,* $U_k = \{i | i \in \{1, 2, \cdots, n\} \text{ and } u_{ik} \geq u_{mk}\}$, $U_k^{(s_k)} = \{i | i \in \{1, 2, \cdots, n\} \text{ and } u_{ik} \geq u_{s_k k}\}$; *then, for TRGAM, we have the following conclusion (where,* $|B|$ *denotes the number of elements in the finite set B).*

(a) *The set with* $\cup_{k=1}^{m} U_k$ *as an alternative member has the same optimal solution as the set with U as an alternative member.*
(b) *When* $\left|\cup_{k=1}^{m} U_k^{(s_k)}\right| \geq m$, *the set with* $\cup_{k=1}^{m} U_k^{(s_k)}$ *as an alternative member has the same optimal solution as the set with U as an alternative member.*

It is useful to note the following results: $n_1 = \left|\cup_{k=1}^{m} U_k\right|$, $U^* = \cup_{k=1}^{m} U_k = \{1, 2, \cdots, n_1\}$. Then, using the construction process of U^*, we know that: $U^* \subset U$ and $n_1 \geq m$, $|U_k| \geq m$, $k = 1, 2, \cdots, m$. For any $i_0 \in \{n_1 + 1, n_1 + 2, \cdots, n\}$ and $k \in \{1, 2, \cdots, m\}$, there is $\min\{a_{1k}, a_{2k}, \cdots, a_{n_1 k}\} > a_{i_0 k}$. The following proves that for any $\{1, 2, \cdots, n\}$ of the chosen arrangement $T^{(1)} = \left(t_1^{(1)}, t_1^{(1)}, \cdots, t_m^{(1)}\right)$, there exists a chosen arrangement $T^{(2)} = \left(t_1^{(2)}, t_1^{(2)}, \cdots, t_m^{(2)}\right)$ of $\{1, 2, \cdots, n_1\}$ such that $F\left(t_1^{(2)}, t_1^{(2)}, \cdots, t_m^{(2)}\right) > F\left(t_1^{(1)}, t_1^{(1)}, \cdots, t_m^{(1)}\right)$, the basic idea is to replace the alternative members belonging to $\{n_1 + 1, n_1 + 2, \cdots, n\}$ in $T^{(1)} = \left(t_1^{(1)}, t_1^{(1)}, \cdots, t_m^{(1)}\right)$ with the alternative members in $\{1, 2, \cdots, n_1\}$ one by one. In the following, only the case where $T^{(1)} = \left(t_1^{(1)}, t_1^{(1)}, \cdots, t_m^{(1)}\right)$ contains one alternative member in $\{n_1 + 1, n_1 + 2, \cdots, n\}$ is described, and for the case where $T^{(1)}$ contains more than one alternative member in $\{n_1 + 1, n_1 + 2, \cdots, n\}$, the conclusion is known to hold using recursive methods.

The conclusion clearly holds when $n_1 = n$. Consider the following case $n_1 < n$ and assume that $T^{(1)} = (1, 2, \cdots, m-1, n)$. Using $m \leq n_1 < n$, we know that $m \in \{1, 2, \cdots, n_1\}$. Thus, if n in $T^{(1)} = (1, 2, \cdots, m-1, n)$ is replaced by m, then $T^{(1)} = (1, 2, \cdots, m-1, m)$ is a chosen arrangement of $\{1, 2, \cdots, n_1\}$. Since only the m-th member in $T^{(1)} = (1, 2, \cdots, m-1, n)$ and $T^{(1)} = (1, 2, \cdots, m-1, m)$ differs and the m-th member has the ability value $a_{mk} > a_{nk}$, $k = 1, 2, \cdots, m$, it follows that with the structural system of $F(t_1, t_2, \cdots, t_m)$, we can know that $F(1, 2, \cdots, m-1, m) > F(1, 2, \cdots, m-1, n)$.

(a) can be proved by a process exactly like (b) and will not be repeated here.

Since the number of alternative members is an important factor affecting the computation time of TRGAM, reducing the number of alternative members can significantly reduce the computation time of TRGAM, and thus Theorem 2 can be used as a theoretical basis for refining the set of alternative members of TRGAM. However, it is worth noting that (a) using (a) in Theorem 2 often fails to obtain a smaller set of alternative members because $\left|\cup_{k=1}^{m} U_k\right|$ is larger; (b) using (b) in Theorem 2 often fails to obtain a smaller set of alternative members because $\left|\cup_{k=1}^{m} U_k^{(s_k)}\right| \geq m$ is not satisfied. For this problem, we can use the strategy of increasing s_k by 1 each time until $\left|\cup_{k=1}^{m} U_k^{(s_k)}\right| \geq m$ (where $\hat{s}_k = \min\{s_k + d, m\}$, $U_k^{(\hat{s}_k)} = \{i | i \in \{1, 2, \cdots, n\} \text{ and } u_{ik} \geq u_{\hat{s}_k k}\}$) to obtain the smaller alternative membership set.

4. Solving TRGAMs Based on Enumeration Method and Hungarian Algorithm (BEM⊕HM-TRGAMs)

Combining the discussions in the previous sections, it can be seen that the computational workload of solving TRGAMs mainly comes from the following two aspects: (1) considering the workload of various teams; (2) the workload of determining the optimal auxiliary work ability of teams. Since Theorem 2 involves the idea of the refinement of people can effectively reduce the computational complexity of the enumeration method, the Hungarian algorithm can solve the standard assignment problem and the optimal auxiliary work ability of teams in TRGAMs can be reduced to a standard assignment problem. This part uses the enumeration method as the search strategy for excellent teams and the Hungarian algorithm as the team optimal auxiliary work ability determination method. A solution algorithm for TRGAMs (BEM⊕HM-TRGAMs) is designed.

Integrating the discussion in the previous sections, we can implement BEM⊕HM-TRGAMs by following the steps below:

Step 1: Data refinement (including determining the effect ability matrix; refining the set of alternative members using Theorem 2).

Step 2: Run BEM⊕HM–TRGAMs to give the recommended results (including team members, combined work ability values of the team, main work ability values of the team, auxiliary work ability values of the team, tasks performed by each team member as main and auxiliary).

5. Standardized Solving Measures for TRGAM

The comprehensive discussion in Sections 3–5 shows that: (a) BEM⊕HM–TRGAMs can be used to solve TRGAMs based on the size of the alternative member set; (b) the difference between TRGAM and TRGAMs lies in the $s_i \neq 1$ or $r_k \neq 1$, and the optimization process mainly involves the problem of calculating the optimal auxiliary work ability values of team members. As $F_2(t_1, t_2, \cdots, t_m)$ in TRGAMs is determined by the Hungarian algorithm of the standard assignment problem, i.e., (using the notation convention of Section 3, where: when $i = k$, $b_{ii} = 0$; when $i \neq k$, $b_{ik} = \beta_2(s_i) \cdot c_{ik}(t_1, t_2, \cdots, t_m)$),

$$F_2(t_1, t_2, \cdots, t_m) = \max \sum_{i=1}^{m} \sum_{k=1}^{m} b_{ik} \cdot x_{ik}$$
$$\text{s.t.} \begin{cases} \sum\limits_{i=1}^{m} x_{ik} = 1,\ k = 1, 2, \cdots, m, \\ \sum\limits_{k=1}^{m} x_{ik} = 1,\ i = 1, 2, \cdots, m, \\ x_{ij} \in \{0, 1\},\ i, k = 1, 2, \cdots, m, \end{cases} \tag{7}$$

The intuitive meaning of $s_i \neq 1$ or $r_k \neq 1$ is a change in the energy allocation weights of team members, which is essentially a change in the assignment matrix. Therefore, $s_i \neq 1$ or $r_k \neq 1$ can be understood as a nonstandard form of assignment problem using $B = (b_{ik})_{m \times m}$ as the base assignment matrix, which can be transformed into a standard form of assignment problem by equivalent deformation. The following are some basic cases of standardization for $s_i \in \{0, 2\}$ and $r_k \in \{0, 2\}$ (with s_1 and r_1 as an example). For the general case of $s_i \neq 1$ or $r_k \neq 1$, the normalization operation can be implemented similarly to these basic cases.

Case 1. $s_1 = 0$, $s_2 = s_3 = \cdots = s_m = 1$.

The intuitive meaning of this scenario is that t_1 does not engage in any task for the auxiliary member (i.e., member t_1's energy will be fully devoted to the task he or she is main engaged in, while there must be one task without a supporting member), and the energy allocation weights of each member satisfy $\beta_1(s_1) = 1$, $\beta_2(s_1) = 0$, $\beta_i(s_2) = \beta_i(s_3) = \cdots = \beta_i(s_m), i \in \{1, 2\}$.

Let$B^* = (b^*_{ik})_{m \times m}$denote the matrix generated by replacing all elements of the first row of$B = (b_{ik})_{m \times m}$with 0 and leaving the rest of the elements unchanged, i.e.,

$$b^*_{ik} = \begin{cases} 0, & \text{if } i = 1, \\ b_{ik}, & \text{if } i \neq 1, \end{cases} \tag{8}$$

Then, determining the value of $F_2(t_1, t_2, \cdots, t_m)$ in TRGAM translates into a standard assignment problem with $B^* = \left(b^*_{ik}\right)_{m \times m}$ as the assignment matrix, and $x_{1k} = 1$ in its optimal assignment scheme $(x_{ik})_{m \times m}$ indicates that the k-th task has no auxiliary members.

Case 2. $s_1 = 2, s_2 = s_3 = \cdots = s_m = 1$.

The intuition of this situation is that t_1 will be engaged in two tasks as an auxiliary member (i.e., member t_1's energy will be invested in one task that he or she is main engaged in and two tasks that he or she is auxiliary to according to some principle), and the energy allocation weights of each member satisfy $\beta_1(s_1) + 2\beta_2(s_1) = 1$, $\beta_i(s_2) = \beta_i(s_3) = \cdots = \beta_i(s_m), i \in \{1, 2\}$.

Let $B^* = (b^*_{ik})_{(m+1)\times(m+1)}$ denote the matrix generated by $B = (b_{ik})_{m\times m}$ in the following steps: (a) adjust the elements $b_{12}, b_{13}, \cdots, b_{1m}$ of the first row of $B = (b_{ik})_{m\times m} to b'_{1k} = \beta_2(s_2) \cdot \widetilde{c}_{ik}(t_1, t_2, \cdots, t_m)$, $k \in \{2, 3, \cdots, m\}$ and leave the rest of the elements unchanged (the new matrix generated is denoted as $B' = (b'_{ik})_{m\times m}$); (b) the maximum value of the elements of each row of B' is used as the (m + 1)-th element of that row to generate the matrix B'' of order $m \times (m+1)$; and (c) add the first row of B'' as the (m + 1)-th row to generate the matrix B^* of order $(m+1) \times (m+1)$, i.e.,

$$b^*_{ik} = \begin{cases} b'_{ik}, & \text{if } i, k \in \{1, 2, \cdots, m\}, \\ \max\limits_{1\le j\le m} b'_{ij}, & \text{if } k = m+1, i \in \{1, 2, \cdots, m\}, \\ b^*_{1k}, & \text{if } i = m+1, k \in \{1, 2, \cdots, m, m+1\}, \end{cases} \tag{9}$$

Then, determine the value of $F_2(t_1, t_2, \cdots, t_m)$ in TRGAM to be transformed into a standard assignment problem with $B^* = (b^*_{ik})_{(m+1)\times(m+1)}$ as the assignment matrix, and the optimal assignment scheme $(x_{ik})_{(m+1)\times(m+1)}$ in which $x_{ik} = 1$ indicates that the i-th person is responsible for the k-th thing (where: (a) the 1st person and the (m + 1)-th person each denote the member t_1 in team $(t_1, t_2, \cdots, t_m)$; (b) wheni $\in \{2, 3, \cdots, m\}$, the i-th individual denotes the member t_i in team $(t_1, t_2, \cdots, t_m)$; (c) when $k \in \{1, 2, \cdots, m\}$, the k-th thing denotes the k-th task; and (d) When $k = m+1$, the (m + 1)-th thing is the k_0-th task that satisfies $b^*_{i(m+1)} = \max\{b_{i1}, b_{i2}, \cdots, b_{im}\} = b_{ik_0}$).

Case 3. $r_1 = 0, r_2 = r_3 = \cdots = r_m = 1$.

The intuition for this scenario is that the first task does not require an auxiliary member (i.e., no team member performs the first task as an auxiliary). Since $(t_1, t_2, \cdots, t_m)$ is the optimization variable of TRGAM, where t_1 can be any alternative member, it can be agreed that t_1 does not perform the first task as an auxiliary member (i.e.,$s_1 = 0, s_2 = s_3 = \cdots = s_m = 1$) during the optimization process, and the energy allocation weights of each member in team $(t_1, t_2, \cdots, t_m)$ satisfy $\beta_1(s_1) = 1$, $\beta_2(s_1) = 0$, $\beta_i(s_2) = \beta_i(s_3) = \cdots = \beta_i(s_m), i \in \{1, 2\}$.

Let $B^* = (b^*_{ik})_{m\times m}$ denote the matrix generated by replacing the elements of both the first row and the first column of $B = (b_{ik})_{m\times m}$ with 0 and leaving the remaining elements unchanged, i.e.,

$$b^*_{ik} = \begin{cases} 0, & \text{if } i = 1 \text{ or } k = 1, \\ b_{ik}, & \text{if } i \neq 1 \text{ and } k \neq 1, \end{cases} \tag{10}$$

Then, determining the value of $F_2(t_1, t_2, \cdots, t_m)$ in TRGAM can be transformed into a standard assignment problem with $B^* = (b^*_{ik})_{m\times m}$ as the assignment matrix.

Case 4. $r_1 = 2, r_2 = r_3 = \cdots = r_m = 1$.

The intuition of this scenario is that the first task requires two team members other than t_1 to assist in (i.e., a member has to assist in two tasks as an auxiliary member). Similar to the analysis in Case 3, it can be agreed that t_1 will perform two tasks (i.e., $s_1 = 2, s_2 = s_3 = \cdots = s_m = 1$), and the energy allocation weights of each member in team $(t_1, t_2, \cdots, t_m)$ satisfy $\beta_1(s_1) + 2\beta_2(s_1) = 1$, $\beta_i(s_2) = \beta_i(s_3) = \cdots = \beta_i(s_m), i \in \{1, 2\}$.

Let $B^* = (b^*_{ik})_{(m+1)\times(m+1)}$ denote the matrix generated by $B = (b_{ik})_{m\times m}$ in the following steps: (a) adjust the element $b_{12}, b_{13}, \cdots, b_{1m}$ in the first row of $B = (b_{ik})_{m\times m}$ to $b'_{1k} = \beta_2(s_2) \cdot \widetilde{c}_{ik}(t_1, t_2, \cdots, t_m)$, $k \in \{2, 3, \cdots, m\}$. The rest of the elements remain unchanged (the new matrix generated is denoted as $B' = (b'_{ik})_{m\times m}$). (b) Add the first column of B' as the (m + 1)-th column *to generate the matrix* $B'' = (b''_{ik})_{m\times m}$ *of order* $m \times (m+1)$. *(c) Add the first row of* B'' *to the (m + 1)-th row to generate the matrix* B^* *of order* $(m+1) \times (m+1)$, *i.e.,*

$$b^*_{ik} = \begin{cases} b'_{ik}, & \text{if } i, k \in \{1, 2, \cdots, m\}, \\ b'_{i1}, & \text{if } k = m+1,\, i \in \{1, 2, \cdots, m\}, \\ b^*_{ik}, & \text{if } i = m+1,\, k \in \{1, 2, \cdots, m, m+1\} \end{cases} \tag{11}$$

Then, determining the value of $F_2(t_1, t_2, \cdots, t_m)$ *in TRGAM translates into a standard assignment problem with* $B^* = (b^*_{ik})_{(m+1)\times(m+1)}$ *as the assignment matrix, and* $x_{ik} = 1$ *in its optimal assignment scheme* $(x_{ik})_{(m+1)\times(m+1)}$ *indicates that the i-th person is responsible for the k-th thing (where: both the first thing and the (m + 1)-th thing represent the first task; and both the first person and the (m + 1)-th person represent the member* t_1 *of the team* $(t_1, t_2, \cdots, t_m)$*).*

Case 5. *The k-th task requires two members as the main responsibility, and each task requires one auxiliary member.*

The intuition of this scenario is that the number of members who choose to build a team is m + 1; therefore, there must be a task that requires two members to work as main members. Furthermore, there must be a member who does not work on any task as an auxiliary member. The solution for the main task assignment can be implemented as in case 2, and the solution for the auxiliary task assignment can be implemented as in case 3.

6. Application Case Study of BEM⊕HM–TRGAMs

In this section, BEM⊕HM-TRGAMs will be used to consider a selection problem for a software company.

Problem Description: In response to the growing demand for online learning among college students, H Software plans to develop a new learning software to meet the changing market needs and improves the company's overall competitiveness. To ensure the smooth implementation of the learning software project and the effective operation as well as maintenance after it is put into the market, the company divides the project into five phases according to the software development process: requirement analysis (a_1), product design (a_2), product development (a_3), product testing (a_4), and product maintenance (a_5). Among them, stage a_1 involves the analysis of the market and the demand characteristics of the user group among college students, the overall planning of the project and the feasibility analysis of the product; stage a_2 involves the structural design of each module of the project, the design of the program structure, and the design of error handling during the operation of the product; stage a_3 involves the related professional skills, the report of the stage results and the anticipation of the next stage of work; stage a_4 involves the development of test plans, test protocols, test analysis, and test reports; stage a_5 involves the design and production of user operation manuals and the online collection of user feedback, and the analysis and modification of the feedback.

Because the project has good market demand and social benefits, the company leadership decided to select five employees from the 37 employees of the technical development department to form a team to complete the development of the learning software, the selection principles are as follows: (a) each team member can and can only be the main member for one task; (b) because a_1 and a_2 involve a comprehensive grasp of the market, users and software design, are related to the overall direction of the team's next work. Two team members are assigned to assist in a_1 and a_2, respectively, and one team member is assigned to assist in each of the remaining tasks; (c) the main and auxiliary members are not the same for each task; and (d) each team member is assigned to assist in one or two tasks. Because of each task's specific characteristics, the company combined its previous work experience as well as work performance and conducted a quantitative assessment of the work ability of 37 employees regarding each task through various means such as mutual evaluation by employees and collective evaluation by experts (the assessment details are shown in Table 1), and the results are shown in Table 2.

Table 1. Work ability evaluation rules.

Task	Assessment Content	Individual Scoring Range	Total Scoring Range for Each Task
Requirement analysis (a_1)	GRASP of market and user needs	(0,25)	(0,50)
	Overall planning of the project and feasibility analysis	(0,25)	
Product design (a_2)	Design the structure of each module of the project	(0,25)	(0,50)
	Program structure design and error handling design	(0,25)	
Product development (a_3)	Expertise (mainstream languages such as MATLAB, R)	(0,25)	(0,50)
	Report on the results of the stage and the next stage of anticipation	(0,25)	
Product testing (a_4)	Develop test plans and implement them	(0,25)	(0,50)
	Aggregate test analysis reports and opinions on results	(0,25)	
Product maintenance (a_5)	Design and production of user operation manuals	(0,25)	(0,50)
	Collect user feedback and analyze and modify it	(0,25)	

Table 2. Competency scores of the 37 alternative members.

Members	a_1	a_2	a_3	a_4	a_5
1	34	32	37	32	31
2	33	35	38	28	34
3	34	31	29	31	31
4	28	29	32	29	27
5	35	34	30	41	35
6	36	35	36	40	33
7	28	29	24	36	28
8	29	36	32	33	29
9	24	29	24	25	26
10	33	32	28	41	31
11	21	17	19	19	19
12	27	25	25	24	22
13	44	36	36	37	38
14	29	36	36	34	34
15	21	25	24	22	28
16	23	22	20	24	19
17	37	33	30	37	35
18	32	39	40	31	37
19	21	27	23	29	27
20	31	27	25	32	31
21	38	30	41	35	35
22	29	24	29	22	28
23	28	22	21	29	25
24	21	30	24	32	30
25	34	31	30	41	35
26	34	41	40	35	40
27	31	24	28	29	24
28	24	21	18	29	23
29	37	29	27	41	33
30	30	39	33	37	37
31	28	21	22	25	22
32	20	17	16	21	20
33	38	34	33	42	38
34	22	28	22	29	28
35	41	34	34	38	36
36	33	31	30	33	28
37	33	24	30	30	30

The trial determined the best team for the project based on the selection requirements and the ability scores of the 37 alternative members.

It is easy to see that the team selection problem can be described by TRGAM. Following the structural features of the TRGAM, the optimal team for this problem is identified in four steps as follows.

Step 1. Parameter setting: (i) importance weights $W = (0.3, 0.3, 0.2, 0.1, 0.1)$ for task $(a_1, a_2, a_3, a_4, a_5)$; (ii) the energy allocation weights $\beta_1 = 0.7$ for each team member working on the main task; (iii) the sum of the energy allocation weights for the auxiliary engagement task is 0.3, and the energy allocation energy weights for the auxiliary engagement two tasks are both 0.15.

Step 2. Revise the competency scoring matrix of the alternative members into a utility competency matrix as shown in Table 3 below.

Table 3. Utility ability values for the 37 alternative members.

Members	a_1	a_2	a_3	a_4	a_5
1	10.2	9.6	7.4	3.2	3.1
2	9.9	10.5	7.6	2.8	3.4
3	10.2	9.3	5.8	3.1	3.1
4	8.4	8.7	6.4	2.9	2.7
5	10.5	10.2	6	4.1	3.5
6	10.8	10.5	7.2	4	3.3
7	8.4	8.7	4.8	3.6	2.8
8	8.7	10.8	6.4	3.3	2.9
9	7.2	8.7	4.8	2.5	2.6
10	9.9	9.6	5.6	4.1	3.1
11	6.3	5.1	3.8	1.9	1.9
12	8.1	7.5	5	2.4	2.2
13	13.2	10.8	7.2	3.7	3.8
14	8.7	10.8	7.2	3.4	3.4
15	6.3	7.5	4.8	2.2	2.8
16	6.9	6.6	4	2.4	1.9
17	11.1	9.9	6	3.7	3.5
18	9.6	11.7	8	3.1	3.7
19	6.3	8.1	4.6	2.9	2.7
20	9.3	8.1	5	3.2	3.1
21	11.4	9	8.2	3.5	3.5
22	8.7	7.2	5.8	2.2	2.8
23	8.4	6.6	4.2	2.9	2.5
24	6.3	9	4.8	3.2	3
25	10.2	9.3	6	4.1	3.5
26	10.2	12.3	8	3.5	4
27	9.3	7.2	5.6	2.9	2.4
28	7.2	6.3	3.6	2.9	2.3
29	11.1	8.7	5.4	4.1	3.3
30	9	11.7	6.6	3.7	3.7
31	8.4	6.3	4.4	2.5	2.2
32	6	5.1	3.2	2.1	2
33	11.4	10.2	6.6	4.2	3.8
34	6.6	8.4	4.4	2.9	2.8
35	12.3	10.2	6.8	3.8	3.6
36	9.9	9.3	6	3.3	2.8
37	9.9	7.2	6	3	3

Step 3. Refine the set of alternative members according to a) Theorem 2 as shown in Table 4: (i) according to the utility ability values under each task corresponding to each of the 37 alternative members in Table 3, a utility ability matrix of 37×5 can be obtained; (ii) find the top five members with the largest utility ability values under each column (i.e., under each task), and extract the elements of the corresponding whole row where the found values are located to form a refined data set.

Table 4. Utility ability values of alternative members after refinement.

Members	a_1	a_2	a_3	a_4	a_5
1	10.2	9.6	7.4	3.2	3.1
2	9.9	10.5	7.6	2.8	3.4
5	10.5	10.2	6	4.1	3.5
8	8.7	10.8	6.4	3.3	2.9
10	9.9	9.6	5.6	4.1	3.1
13	13.2	10.8	7.2	3.7	3.8
14	8.7	10.8	7.2	3.4	3.4
17	11.1	9.9	6	3.7	3.5
18	9.6	11.7	8	3.1	3.7
21	11.4	9	8.2	3.5	3.5
25	10.2	9.3	6	4.1	3.5
26	10.2	12.3	8	3.5	4
29	11.1	8.7	5.4	4.1	3.3
30	9	11.7	6.6	3.7	3.7
33	11.4	10.2	6.6	4.2	3.8
35	12.3	10.2	6.8	3.8	3.6

Step 4. Run BEM⊕HM–TRGAMs.

Since 16 alternative members are left after refinement, it is not very computationally intensive to find the optimal team by enumeration method. Thus, it is appropriate to use BEM⊕HM–TRGAMs for the solution. The following solution tests are performed using BEM⊕HM-TRGAMs based on the refined set of alternative members and the corresponding utility ability matrix (The test environment was a computer with a 3.30 GHz Inter(R) Core(TM) i5-4590 processor, 4 GB of RAM, and MATLAB 2016a as the auxiliary computing software). The results are shown in Table 5.

Table 5. Task arrangement and performance of team $T = (13, 26, 21, 33, 18)$.

	13	**26**	**21**	**33**	**18**	**Overall Ability Value**
For the main engaged in the task	a_1	a_2	a_3	a_4	a_5	29.12
For the auxiliary engaged in the task	a_4 and a_2	a_3	a_1	a_5 and a_1	a_2	13.785
$\max F(T)$	—	—	—	—	—	42.905

The above solution process further demonstrates the effectiveness of BEM⊕HM–TRGAMs, and since task weights w and energy allocation coefficients β have subjective factors in the selection process, the selection of w and β should consider the conditions related to the actual problem. These two variables, which represent the importance of the task and the energy input of the members, respectively, have an impact on the calculation of the ability value of the optimal team, and theoretically changing w and β may lead to different allocation results. The time computational complexity of the algorithm is $O(V * E)$ (where V is the number of members and E is the edge between members). It is also shown that BEM⊕HM–TRGAMs is suitable for cases where the dataset is not very large. Since team formation problems with the above case characteristics are widely available, and the team size and number of tasks in them are not too large, the BEM⊕HM–TRGAMs have wide practicality. This suggests that the core problem of TR can be reduced to the problem of determining the work ability value of each member regarding each task based on BEM⊕HM–TRGAMs. In real life, if the team formed requires high ability, then it is necessary to include an expert mathematical modeling team to make the recommendation more accurate and effective, and if the team formed does not require a high level of ability, the inclusion of an expert mathematical modeling team is not necessary.

7. Conclusions

Based on the systematic analysis of the essential characteristics of TR, this study addresses the problem that existing TR methods cannot effectively solve the complementarity problem among members. The generalized allocation model (TRGAM) of TR is established with the value of the comprehensive ability of the team as the measurement index of team selection, and the task importance and ability distribution of members as the entry points. Further, around the TRGAM solution problem, the basic properties of TRGAM are discussed, a basic strategy for refining the set of alternative members is given, a solution method based on the enumeration method and Hungarian algorithm for the standard case of TRGAMs is designed, and given the standardized measures for TRGAM. Finally, the feasibility and effectiveness of BEM$\oplus$HM-TRGAMs are verified by combining specific arithmetic examples, and it is also proved that the algorithm is suitable for cases where the dataset is not very large. The theoretical analysis and example calculations show that TRGAM has good structural characteristics and interpretability, and is a TR model with certain guiding significance, which enriches the theory and methods of generalized assignment problems to a certain extent, and has a wide application prospect in many fields such as resource management and data decision making, and has certain practical significance.

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