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Some Properties of Bazilevič Functions Involving Srivastava–Tomovski Operator

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[†] Authors dedicate this article to Professor Hari M. Srivastava on the occasion of his 82nd Birthday.

Abstract: We introduce a new class of Bazilevič functions involving the Srivastava–Tomovski generalization of the Mittag-Leffler function. The family of functions introduced here is superordinated by a conic domain, which is impacted by the Janowski function. We obtain coefficient estimates and subordination conditions for starlikeness and Fekete–Szegö functional for functions belonging to the class.

Keywords: analytic function; Srivastava–Tomovski generalization of Mittag-Leffler function; convex function; Bazilevič function; starlike function; Fekete–Szegö problem; differential subordination

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1. Introduction

Researchers have successfully applied the *Mittag-Leffler function and its multi-parameter extensions* to several problems in physics, engineering and other applied sciences. However, the real importance of this function arose from the role it plays in *Fractional Calculus* [1]. The familiar Mittag-Leffler function $E_\alpha(z)$ and its two-parameter version $E_{\alpha,\beta}(z)$ are defined, respectively, by

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)} \quad \text{and} \quad E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)} \quad (z, \alpha, \beta, \in \mathbb{C}, Re(\alpha) > 0),$$

where $z \in \Pi = \{z : | z |< 1\}$, $\mathbb{C}$ denotes the sets of complex numbers and $(x)_n$ denotes the Pochhammer symbol defined by

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = \begin{cases} 1 & if\ n = 0 \\ x(x+1)(x+2)\ \ldots\ (x+n-1) & if\ n \in \mathbb{N}. \end{cases}$$

The Mittag-Leffler function $E_\alpha(z)$ and its two-parameter version $E_{\alpha,\beta}(z)$ were first considered by Gösta Mittag-Leffler in 1903 and A. Wiman in 1905. Refer to Ayub et al. [2], Gorenflo [3], Srivastava [4–6] and Srivastava et al. [7–18] for detailed studies that involve the Mittag-Leffler function.

The Mittag-Leffler function $E_{\alpha,\beta}(z)$ coincides with well-known elementary functions and some special functions. For example,

$$E_{1,2}(z) = \frac{e^z - 1}{z}, \qquad E_{3,1}(z) = \frac{1}{2}\left[e^{z^{1/3}} + 2e^{-\frac{1}{2}z^{1/3}} \cos\left(\frac{\sqrt{3}}{2} z^{1/3}\right)\right].$$

Srivastava et al. [13] considered the following family of the multi-index Mittag-Leffler functions as a kernel of some fractional-calculus operators

$$E^{\gamma,k,\delta,\epsilon}_{(\alpha_j,\beta_j)_m}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{kn}\,(\delta)_{\epsilon n}}{\prod_{j=1}^{m}\Gamma(\alpha_j n+\beta_j)}\frac{z^n}{n!}, \tag{1}$$

$$\left(\alpha_j,\ \beta_j,\ \gamma,\ k,\ \delta,\ \epsilon \in \mathbb{C};\ \mathrm{Re}(\alpha_j)>0, (j=1,\ldots,m);\ \mathrm{Re}\left(\sum_{j=1}^{m}\alpha_j\right) > \mathrm{Re}(k+\epsilon)-1\right).$$

Some Higher Transcendental Functions and Related Mittag-Leffler Functions

The well-known *Meijer G-function* and *Fox's H-function* have almost all elementary and special functions as their special cases. Here we will restrict with a brief overview of the *Fox–Wright function* and *Hurwitz–Lerch type zeta functions* unification with the Mittag-Leffler function and its multi-parameter extensions.

For $\eta_j \in \mathbb{C}\,(j=1,\ldots,r)$ and $\nu_j \in \mathbb{C}\setminus Z_0^- = \{0,-1,\ldots\}\,(j=1,\ldots,s)$, the *Fox–Wright function* ${}_r\Psi_s$, which is defined by (see ([19], Equation (1.6)), ([20], p. 21) and ([21], p. 19))

$${}_r\Psi_s\left[\begin{matrix}(\eta_1,A_1) & \ldots & (\eta_r,A_r) \\ (\nu_1,B_1) & \ldots & (\nu_s,B_s)\end{matrix}\ ;z\right] = \sum_{n=0}^{\infty}\frac{\prod_{j=1}^{r}\Gamma(\eta_j+A_j n)}{\prod_{j=1}^{s}\Gamma(\nu_j+B_j n)}\frac{z^n}{n!}. \tag{2}$$

where $\mathrm{Re}(A_j) > 0,\ (j=1,\ldots,r)$ and $\mathrm{Re}(B_j) > 0 \in \mathbb{C}\,(j=1,\ldots,s)$ with $1+\mathrm{Re}\left(\sum_{j=1}^{s}B_j-\sum_{j=1}^{r}A_j\right)\geq 0$. Refer to Srivastava ([5], Definition 2) for a detailed discussion on the convergence of the series (2).

Lin and Srivastava [22] introduced and investigated an interesting generalization of the well-known *Hurwitz–Lerch zeta function* $\phi(z,s,a)$ in the following form

$$\phi^{k,\epsilon}_{\eta,\nu}(z,m,a) = \sum_{n=0}^{\infty}\frac{(\eta)_{kn}}{(\nu)_{\epsilon n}}\frac{z^n}{(n+a)^m}.$$

In order to derive a direct relationship with the Fox–Wright function, the function $\phi^{k,\epsilon}_{\eta,\nu}(z,m,a)$ was further generalized to (see Srivastava et al. [23])

$$\phi^{k_1,\ldots,k_r;\epsilon_1,\ldots,\epsilon_s}_{\eta_1,\ldots\eta_r;\nu_1,\ldots,\nu_s}(z,m,a) = \sum_{n=0}^{\infty}\frac{\prod_{j=1}^{r}(\eta_j)_{nk_j}}{n!\prod_{j=1}^{s}(\nu_j)_{n\epsilon_j}}\frac{z^n}{(n+a)^m},$$

where $\eta_j \in \mathbb{C}\,(j=1,\ldots,r)$ and $\nu_j \in \mathbb{C}\setminus Z_0^- = \{0,-1,\ldots\}\,(j=1,\ldots,s)$. Refer to [24–26], for a detailed discussion on the convergence.

When $A_j = 1,\ (j=1,\ldots,r)$ and $B_j=1,\ (j=1,\ldots,s)$ in (2), then

$${}_r\Psi_s\left[\begin{matrix}(\eta_1,1) & \ldots & (\eta_r,1) \\ (\nu_1,1) & \ldots & (\nu_s,1)\end{matrix}\ ;z\right] = {}_rF_s\left[\begin{matrix}\eta_1, & \ldots & \eta_r \\ \nu_1, & \ldots & \nu_s\end{matrix}\ ;z\right] = \sum_{n=0}^{\infty}\frac{(\eta_1)_n\ldots(\eta_r)_n}{(\nu_1)_n\ldots(\nu_s)_n}\frac{z^n}{n!}.$$

The function ${}_rF_s$ is the well-known generalized hypergeometric function (see [27,28]). Similarly, we observe from (1) and (2) that

$$E^{\gamma,k,\delta,\epsilon}_{(\alpha_j,\beta_j)_m}(z) = \frac{1}{\Gamma(\gamma)\Gamma(\delta)}\,{}_2\Psi_m\left[\begin{matrix} & (\gamma,k), & (\delta,\epsilon) \\ (\beta_1,\alpha_1) & \ldots & (\beta_m,\alpha_m)\end{matrix}\ ;z\right].$$

A special case of the multi-index Mittag-Leffler function defined by (1), when $m = 2$ corresponding to the Srivastava–Tomovski generalization of the Mittag-Leffler function [29], is given by

$$E_{\alpha,\beta}^{\gamma,k}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nk} z^n}{\Gamma(\alpha n + \beta) n!}, \quad z,\ \alpha,\ \beta,\ \gamma,\ k \in \mathbb{C}, Re(\alpha) > 0,\ Re(k) > 0,$$
$$= \frac{1}{\Gamma(\gamma)}\, {}_1\Psi_1 \left[\begin{matrix} (\gamma, k) \\ (\beta, \alpha) \end{matrix}; z \right]. \tag{3}$$

In [29], the authors established that $E_{\alpha,\beta}^{\gamma,k}(z)$ defined by (3) is an entire function in the complex z-plane. The function $E_{\alpha,\beta}^{\gamma,k}(z)$ is called the Srivastava–Tomovski generalization of the Mittag-Leffler function. The function $E_{\alpha,\beta}^{\gamma,1}(z)$ is popularly known as Prabhakar function or generalized Mittag-Leffler three-parameter function.

In geometric function theory, several researchers have studied the properties of the Srivastava–Tomovski generalization of the Mittag-Leffler function. The most prominent studies pertaining to the Srivastava–Tomovski generalization were by Aouf and Mostafa [30], Attiya [31], Liu [32] and Tomovski et al. [33].

2. Definitions and Preliminaries

Let $\mathcal{H}(d,n)$ be the class of analytic functions having a series of the form $\varphi(z) = d + d_n z^n + d_{n+1} z^{n+1} + \ldots$.

Let

$$\Lambda_n = \{\varphi \in \mathcal{H},\ \varphi(z) = z + d_{n+1} z^{n+1} + d_{n+2} z^{n+2} + \ldots\}$$

and let $\Lambda = \Lambda_1$. For $\varphi \in \Lambda$, Cang and Liu in [34] introduced an operator using the Srivastava–Tomovski generalization of the Mittag-Leffler function ([29]), which, explicitly for $p = 1$, is given by

$$H_{\alpha,\beta}^{\gamma,k}\varphi(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(\gamma + nk)\Gamma(\alpha + \beta)}{\Gamma(\gamma + k)\Gamma(\alpha n + \beta) n!} d_n z^n. \tag{4}$$

Motivated by [35,36], we now define an operator $\mathfrak{I}_k^m(\alpha,\ \beta,\ \gamma)\varphi(z) : \Lambda \to \Lambda$ is defined by

$$\mathfrak{I}_{k,\lambda}^m(\alpha,\ \beta,\ \gamma)\varphi(z) = z + \sum_{n=2}^{\infty} [1 - \lambda + \lambda n]^m \frac{\Gamma(\gamma + nk)\Gamma(\alpha + \beta)}{\Gamma(\gamma + k)\Gamma(\alpha n + \beta) n!} d_n z^n. \tag{5}$$

Remark 1. *We note that operator $\mathfrak{I}_{k,\lambda}^m(\alpha,\ \beta,\ \gamma)\varphi(z)$ is closely related to the operators studied by Breaz et al. [37], Cang and Liu [34] and Elhaddad et al. [38]. Now here we list some of the special cases:*

1. *$\mathfrak{I}_{1,\lambda}^m(0,\ \beta,\ 1)\varphi(z) = D^m\varphi(z)$, where $D^m\varphi(z)$ is the Al-Oboudi operator (see [39])*
2. *If we let $\alpha = 0$, $k = \gamma = 1$ and $\lambda = 1$ in (5), then $\mathfrak{I}_{k,\lambda}^m(\alpha,\ \beta,\ \gamma)\varphi(z)$ reduces to the well-known Sălăgean operator.*
3. *If we let $m = 0$ and $\lambda = 1$ in (5), then $\mathfrak{I}_{k,\lambda}^m(\alpha,\ \beta,\ \gamma)\varphi(z)$ reduces to the operator defined and studied by Cang & Liu [34].*

Let Υ denote the class of functions having series

$$r(z) = 1 + \sum_{n=1}^{\infty} r_n z^n \quad (z \in \Pi),$$

which satisfies the condition $Re(r(z)) > 0$. We denote by $\mathcal{S}^*(\eta)$, $\mathcal{C}(\eta)$ and $\mathcal{K}(\eta, \tau)$ $(0 \le \eta,\ \tau < 1)$ the familiar subclasses of Λ consisting of functions that are, respectively, starlike of order η, convex of order η and close-to-convex of order η and type τ in Π.

Recently, Breaz et al. [40] defined and studied the following function

$$\Gamma(Q, R;\ p;\ \sigma;\Psi) = \frac{[(1+Q)p + \sigma(R-Q)]\Psi(z) + [(1-Q)p - \sigma(R-Q)]}{[(R+1)\Psi(z) + (1-R)]}, \tag{6}$$

where $\Psi(z) \in \Upsilon$ and has a series representation of the form

$$\Psi(z) = 1 + L_1 z + L_2 z^2 + \cdots. \tag{7}$$

A detailed geometric interpretation of $\Gamma(Q, R;\ p;\ \sigma;\Psi)$ was discussed by Karthikeyan et al. in [41]. The function $\Gamma(Q, R;\ p;\ \sigma;\Psi)$ was mainly motivated by the study of Noor and Malik [42] and Srivastava et al. [43–49].

By making use of the function $\Gamma(Q, R;\ p;\ \sigma;\Psi)$, we now define the following.

Definition 1. *For $0 \le \delta < \infty$, a function $\varphi(z) \in \Lambda$ is said to be in $\mathcal{B}^m_\lambda(\alpha, \beta, \gamma;\ \delta;\ Q, R;\ \Psi)$ if and only if for all $\mathfrak{I}^m_{k,\lambda}(\alpha,\ \beta,\ \gamma)\chi \in \mathcal{S}^*(0)$ it satisfies the condition*

$$\frac{z\left[\mathfrak{I}^m_{k,\lambda}(\alpha,\ \beta,\ \gamma)\varphi(z)\right]'}{[\mathfrak{I}^m_{k,\lambda}(\alpha,\ \beta,\ \gamma)\varphi(z)]^{1-\delta}\,[\mathfrak{I}^m_{k,\lambda}(\alpha,\ \beta,\ \gamma)\chi(z)]^{\delta}} \prec \Gamma(Q, R;\ 1;\ \sigma;\Psi), \quad (z \in \Pi) \tag{8}$$

where $\mathfrak{I}^m_{k,\lambda}(\alpha,\ \beta,\ \gamma)\chi(z) \neq 0$ for all $z \in \Pi$.

If we let $m = \sigma = \alpha = 0$, $k = 1$, $Q = 1$, $R = -1$ and $\chi \in \mathcal{S}^*(1 + z/1 - z)$, then $\mathcal{B}^m_\lambda(\alpha, \beta, \gamma;\ Q, R;\ \Psi)$ reduces to the class

$$\mathcal{B}(\delta;\ \Psi) = \left\{\varphi \in \Lambda : \frac{z\varphi'(z)}{[\varphi(z)]^{1-\delta}\,[\chi(z)]^{\delta}} \prec \Psi(z), \quad \chi \in \mathcal{S}^*(0)\right\}.$$

The function $\mathcal{B}(\delta;\ \Psi)$ was studied by Goyal and Goswami in [50] but with φ and χ belonging to Λ_n.

If we let $\chi(z) = z$, $\sigma = 0$, $\Psi(z) = \frac{1+z}{1-z}$, $Q = 1 - 2\eta$ and $R = -1$ we get the class $\mathcal{B}(\delta,\ \eta)$ which satisfies the condition $\mathrm{Re}\,\dfrac{z^{1-\delta}\left[\mathfrak{I}^m_{k,\lambda}(\alpha,\ \beta,\ \gamma)\varphi(z)\right]'}{\left[\mathfrak{I}^m_{k,\lambda}(\alpha,\ \beta,\ \gamma)\varphi(z)\right]^{1-\delta}} > \eta$, $z \in \Pi$, where $\varphi \in \Lambda$. Further, on letting $m = \sigma = \alpha = 0$ and $k = 1$ in $\mathcal{B}(\delta,\ \eta)$, it reduces to the well-known class $\mathcal{B}(\delta)$ (see [51]), which satisfies the condition $\mathrm{Re}\,\dfrac{z^{1-\delta}\varphi'(z)}{(\varphi(z))^{1-\delta}} > 0$, $z \in \Pi$, where $\varphi \in \Lambda$. For recent developments pertaining to the study of Bazilevič functions, refer to [52,53].

Throughout this paper, we let

$$\aleph(1;\ Q, R;\ \sigma;\ \Psi; z) = \frac{[(1+Q) + \sigma(R-Q)]\Psi(z) + [(1-Q) - \sigma(R-Q)]}{[(R+1)\Psi(z) + (1-R)]}. \tag{9}$$

From ([40], Theorem 2), with

$$w(z) = \frac{1}{2}\rho_1 z + \frac{1}{2}\left(\rho_2 - \frac{1}{2}\rho_1^2\right)z^2 + \frac{1}{2}\left(\rho_3 - \rho_1\rho_2 + \frac{1}{4}\rho_1^3\right)z^3 + \cdots,\ z \in \Pi,$$

we can get

$$\aleph(1;\ Q, R;\ \sigma; w(z)) = 1 + \frac{L_1\rho_1(Q-R)(1-\sigma)}{4}z + \frac{(Q-R)(1-\sigma)L_1}{4}\left[\rho_2 - \rho_1^2\left(\frac{(R+1)L_1 + 2\left(1 - \frac{L_2}{L_1}\right)}{4}\right)\right]z^2 + \ldots. \tag{10}$$

Now we will state some results, which we will be using to establish the coefficient inequalities.

Lemma 1 ([54]). *Let* $\rho(z) = 1 + \sum_{n=1}^{\infty} \rho_1 z^n$ *be analytic in the unit disc satisfying* $\operatorname{Re}\rho(z) > 0$. *Then, for each complex number* ϑ*, we have*

$$|\rho_2 - \vartheta\rho_1^2| \leq 2 \max\{1, |2\vartheta - 1|\},$$

the result is sharp for functions given by

$$\rho(z) = \frac{1+z^2}{1-z^2}, \qquad \rho(z) = \frac{1+z}{1-z}.$$

Lemma 2 ([55]). *If* $R(z) = z + \sum_{n=2}^{\infty} c_n z^n \in \mathcal{S}^*$*, then* $|c_n| \leq n$*. Further, for each complex number* ϑ *we have* $| c_3 - \vartheta c_2^2 | \leq \max(1, | 3 - 4\vartheta |)$ *and the result is sharp for the Koebe functon*

$$r(z) = \frac{z}{(1-z)^2} \quad if \quad \left|\rho - \frac{3}{4}\right| \geq \frac{1}{4}$$

and for

$$r^{\frac{1}{2}}\left(z^2\right) = \frac{z}{1-z^2} \quad if \quad | \rho - \frac{3}{4} | \leq \frac{1}{4}.$$

3. Fekete–Szegö Inequalities for the Class $\mathcal{B}_{\lambda}^{m}(\alpha, \beta, \gamma; \delta; Q, R; \Psi)$

In this section, we obtain the solution to the Fekete–Szegö problem for functions in class $\mathcal{B}_{\lambda}^{m}(\alpha, \beta, \gamma; \delta; Q, R; \Psi)$.

Theorem 1. *If* $\varphi(z) \in \mathcal{B}_{\lambda}^{m}(\alpha, \beta, \gamma; \delta; Q, R; \Psi)$*, then we have*

$$|d_2| \leq \frac{(Q-R)(1-\sigma)|L_1|}{2|M_2|(1+\delta)} + \frac{2\delta}{(\delta+1)}, \tag{11}$$

and

$$\begin{aligned} |d_3| \leq \frac{(Q-R)(1-\sigma)|L_1|}{2|M_3|(\delta+2)} \max\{1, |2\Gamma_1 - 1|\} + \frac{\delta}{(\delta+2)} \max\{1, |3 - 4\Gamma_2|\} \\ + \frac{(Q-R)\delta(1-\sigma)|L_1 M_2|}{|M_3|(\delta+1)(\delta+2)}, \end{aligned} \tag{12}$$

where Γ_1 *and* Γ_2 *are given by*

$$\Gamma_1 = \frac{1}{4}\left[L_1(R+1) + 2\left(1 - \frac{L_2}{L_1}\right) - \frac{(Q-R)(1-\sigma)L_1(\delta^2+\delta-2)}{2(1+\delta)^2}\right]$$

$$\Gamma_2 = \frac{M_2^2}{2M_3}\left[(\delta+1) + \frac{\delta(\delta^2+\delta-2)}{(1+\delta)^2} + \frac{2\delta}{(\delta+1)}\right].$$

Further, for all $\vartheta \in \mathbb{C}$ *we have*

$$\begin{aligned} \left|d_3 - \vartheta d_2^2\right| \leq \frac{(Q-R)(1-\sigma)|L_1|}{2|M_3|(\delta+2)} \max\{1, |2\mathcal{H}_1 - 1|\} \\ + \frac{2(Q-R)\delta(1-\sigma)|L_1 M_2|}{(\delta+1)}\left|\frac{1}{2M_3(\delta+2)} - \frac{\vartheta}{M_2(1+\delta)}\right| + \frac{2\delta}{(\delta+2)} \max\{1, |3 - 4\mathcal{H}_2|\}, \end{aligned} \tag{13}$$

where $\mathcal{H}_1$ *and* $\mathcal{H}_2$ *are given by*

$$\mathcal{H}_1 = \frac{1}{4}\left[L_1(R+1) + 2\left(1 - \frac{L_2}{L_1}\right) + \left\{\frac{\vartheta M_3(\delta+2)}{M_2^2} - \frac{(\delta^2+\delta-2)}{2}\right\}\frac{(Q-R)(1-\sigma)L_1}{(1+\delta)^2}\right]$$

$$\mathcal{H}_2 = \frac{M_2^2}{2M_3}\left[(\delta+1) - \frac{2\vartheta\delta(\delta+2)M_3}{M_2^2(\delta+1)^2} + \frac{\delta(\delta^2+\delta-2)}{(1+\delta)^2} + \frac{2\delta}{(\delta+1)}\right].$$

The inequality is sharp for each $\vartheta \in \mathbb{C}$.

Proof. By the definition of $\mathcal{B}_\lambda^m(\alpha, \beta, \gamma; \delta; Q, R; \Psi)$, we have

$$\frac{z\left[\mathfrak{J}_{k,\lambda}^m(\alpha,\, \beta,\, \gamma)\varphi(z)\right]'}{[\mathfrak{J}_{k,\lambda}^m(\alpha,\, \beta,\, \gamma)\varphi(z)]^{1-\delta}\,[\mathfrak{J}_{k,\lambda}^m(\alpha,\, \beta,\, \gamma)\chi(z)]^{\delta}} = \aleph(1;\, Q, R;\, \sigma; w(z)), \tag{14}$$

where $\aleph(1;\, Q, R;\, \sigma; w(z))$ is defined as in (9). The left-hand side of (14) is given by

$$\begin{aligned}&\frac{z\left[\mathfrak{J}_{k,\lambda}^m(\alpha,\, \beta,\, \gamma)\varphi(z)\right]'}{[\mathfrak{J}_{k,\lambda}^m(\alpha,\, \beta,\, \gamma)\varphi(z)]^{1-\delta}\,[\mathfrak{J}_{k,\lambda}^m(\alpha,\, \beta,\, \gamma)\chi(z)]^{\delta}}\\ &= 1 + [d_2(1+\delta) - \delta c_2]M_2\, z + \left[M_3(\delta+2)\left(d_3 - \frac{1}{2M_3(\delta+2)}d_2^2M_2^2(\delta^2+\delta-2)\right)\right.\\ &\qquad \left. - d_2M_2^2\delta c_2 + M_3\delta\left(c_3 - \frac{M_2^2}{2M_3}c_2^2(\delta+1)\right)\right]z^2 + \cdots .\end{aligned} \tag{15}$$

From (15) and (10), the coefficients of z and z^2 are given by

$$d_2 = \frac{(Q-R)(1-\sigma)L_1\rho_1}{4M_2(1+\delta)} + \frac{\delta}{(\delta+1)}c_2 \tag{16}$$

and

$$\begin{aligned}d_3 = {} & \frac{(Q-R)(1-\sigma)L_1}{4M_3(\delta+2)}\left[\rho_2 - \frac{\rho_1^2}{4}\left(L_1(R+1) + 2\left(1 - \frac{L_2}{L_1}\right)\right.\right.\\ &\left.\left. - \frac{(Q-R)(1-\sigma)L_1(\delta^2+\delta-2)}{2(1+\delta)^2}\right)\right] + \frac{c_2(Q-R)\delta(1-\sigma)L_1\rho_1M_2}{4M_3(\delta+1)(\delta+2)}\\ & - \frac{\delta}{(\delta+2)}\left\{c_3 - \frac{M_2^2}{2M_3}\left[(\delta+1) + \frac{\delta(\delta^2+\delta-2)}{(1+\delta)^2} + \frac{2\delta}{(\delta+1)}\right]c_2^2\right\}.\end{aligned} \tag{17}$$

Using $|\rho_n| \le 2\,(n \ge 1)$ in (16), we can obtain (11). Using (17) together with Lemma 1, we have

$$\begin{aligned}|d_3| \le {} & \frac{(Q-R)(1-\sigma)|L_1|}{4|M_3|(\delta+2)}\left|\rho_2 - \frac{\rho_1^2}{4}\left(L_1(R+1) + 2\left(1 - \frac{L_2}{L_1}\right)\right.\right.\\ &\left.\left. - \frac{(Q-R)(1-\sigma)L_1(\delta^2+\delta-2)}{2(1+\delta)^2}\right)\right|\\ & + \frac{\delta}{(\delta+2)}\left|c_3 - \frac{M_2^2}{2M_3}\left[(\delta+1) + \frac{\delta(\delta^2+\delta-2)}{(1+\delta)^2} + \frac{2\delta}{(\delta+1)}\right]c_2^2\right|\\ & + \frac{|c_2|(Q-R)\delta(1-\sigma)|L_1\rho_1M_2|}{4|M_3|(\delta+1)(\delta+2)}.\end{aligned}$$

Hence the proof of (1).
To establish (1), we consider

$$
\begin{aligned}
\left|d_3-\vartheta d_2^2\right| = \Bigg| &\frac{(Q-R)(1-\sigma)L_1}{4M_3(\delta+2)}\left[\rho_2-\frac{\rho_1^2}{4}\left(L_1(R+1)+2\left(1-\frac{L_2}{L_1}\right)\right.\right.\\
&\left.\left.+\left\{\frac{\vartheta M_3(\delta+2)}{M_2^2}-\frac{(\delta^2+\delta-2)}{2}\right\}\frac{(Q-R)(1-\sigma)L_1}{(1+\delta)^2}\right)\right]\\
&+\frac{c_2(Q-R)\delta(1-\sigma)L_1\rho_1M_2}{2(\delta+1)}\left\{\frac{1}{2M_3(\delta+2)}-\frac{\vartheta}{M_2(1+\delta)}\right\}-\frac{\delta}{(\delta+2)}\Bigg\{c_3\\
&-\frac{M_2^2}{2M_3}\left[(\delta+1)-\frac{2\vartheta\delta(\delta+2)M_3}{M_2^2(\delta+1)^2}+\frac{\delta(\delta^2+\delta-2)}{(1+\delta)^2}+\frac{2\delta}{(\delta+1)}\right]c_2^2\Bigg\}\Bigg|.
\end{aligned}
\tag{18}
$$

Applying Lemmas 1 and 2 in (18), we can establish inequality (1). □

We denote by $\mathcal{S}(\eta,\tau)$ $(0\le\eta<1<\tau)$ (see [56]) the class of functions $\varphi\in\Lambda$ satisfying the inequality

$$
\eta<\operatorname{Re}\frac{z\varphi'(z)}{\varphi(z)}<\tau,\ z\in\Pi. \tag{19}
$$

Corollary 1 ([57], Theorem 5)**.** *Let $0\le\eta<1<\tau$ and let the function $\varphi\in\mathcal{S}(\eta,\tau)$. Then, for all $\vartheta\in\mathbb{C}$ we have*

$$
\begin{aligned}
\left|d_3-\vartheta d_2^2\right| &\le \frac{\tau-\eta}{\pi}\sin\frac{\pi(1-\eta)}{\tau-\eta}\\
&\cdot\max\left\{1;\left|\frac{1}{2}+(1-2\vartheta)\frac{\tau-\eta}{\pi}i+\left(\frac{1}{2}-(1-2\vartheta)\frac{\tau-\eta}{\pi}i\right)e^{2\pi i\frac{1-\eta}{\tau-\eta}}\right|\right\}.
\end{aligned}
$$

Proof. From [56], (19) can be rewritten in the form

$$
\frac{z\varphi'(z)}{\varphi(z)}\prec 1+\frac{\eta-\tau}{\pi}i\log\left(\frac{1-e^{2\pi i((1-\eta)/(\eta-\tau))}z}{1-z}\right)=\mathcal{T}(z).
$$

Further, it is known that $\operatorname{Re}\{\mathcal{T}(z)\}>0$ $(z\in\Pi)$ and has a series of the form

$$
\mathcal{T}(z)=1+\sum_{n=1}^{\infty}L_nz^n,\ z\in\Pi,
$$

where $L_n=\dfrac{\eta-\tau}{n\pi}i\left(1-e^{2n\pi i((1-\eta)/(\eta-\tau))}\right)$, $n\ge1$. Substituting the values of $m=\sigma=\delta=\alpha=0$, $k=1$, $Q=1$, $R=-1$, L_1 and L_2 in Theorem 1 we obtain assertion of our theorem. □

If we take $m=\sigma=\delta=\alpha=0$, $k=1$, $Q=1$ and $R=-1$ in Theorem 1, then we have the following corollary.

Corollary 2 ([58])**.** *Suppose $\varphi(z)=z+d_2z^2+d_3z^3+\cdots\in\mathcal{S}^*(\Psi)$ $(z\in\Pi)$. Then*

$$
\left|d_3-\vartheta d_2^2\right|\le\frac{L_1}{2}\max\left\{1;\left|L_1+\frac{L_2}{L_1}-2\vartheta L_1\right|\right\}\quad(\vartheta\in\mathbb{C}).
$$

Equality is attained if

$$
\varphi(z)=\begin{cases} z\exp\int_0^z[\Psi(t)-1]\frac{1}{t}dt, & \text{if } \left|L_1+\frac{L_2}{L_1}-2\vartheta L_1\right|\ge1\\ z\exp\int_0^z\left[\Psi(t^2)-1\right]\frac{1}{t}dt, & \text{if } \left|L_1+\frac{L_2}{L_1}-2\vartheta L_1\right|\le1.\end{cases}
$$

4. Subordination Results

In general, we note that $\Gamma(Q, R; 1; \sigma; \Psi)$ need not be convex univalent in Π. However, the function $\Gamma(Q, R; 1; \sigma; \Psi)$ is convex depending on the choice of $\Psi(z)$ (see [41,59]).

Lemma 3. *Let ℓ be convex in Π, with $\ell(0) = d$, $\nu \neq 0$ and $\mathrm{Re}\,\nu \geq 0$. If $r \in \mathcal{H}(d, n)$ and*

$$r(z) + \frac{zr'(z)}{\nu} \prec \ell(z),$$

then

$$r(z) \prec q(z) \prec \ell(z),$$

where

$$q(z) = \frac{\nu}{n\, z^{\nu/n}} \int_0^z t^{(\nu/n)-1} \ell(t) dt.$$

The function q is convex and is the best (d, n)-dominant.

Theorem 2. *Let $\varphi, \chi \in \Lambda$ with $\varphi(z)$, $\varphi'(z)$ and $\chi(z) \neq 0$ for all $z \in \Pi \setminus \{0\}$. Further, let $\Gamma(Q, R; 1; \sigma; \Psi)$ be convex univalent in Π with $[\Gamma(Q, R; 1; \sigma; \Psi)]_{z=0} = 1$ and $\mathrm{Re}\ \Gamma(Q, R; 1; \sigma; \Psi) > 0$. Further, suppose that $\chi \in \mathcal{S}^*(0)$ and*

$$\left(\frac{z\left[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z)\right]'}{[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z)]^{1-\delta}\,[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\chi(z)]^{\delta}} \right)^2 \left[3 + 2\left\{ \frac{z\left[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z)\right]''}{\left[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z)\right]'} - \right.\right.$$
$$\left.\left. (1-\delta)\frac{z\left[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z)\right]'}{\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z)} - \delta\frac{z\left[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\chi(z)\right]'}{\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\chi(z)} \right\} \right] \prec \Gamma(Q, R; 1; \sigma; \Psi). \tag{20}$$

Then

$$\frac{z\left[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z)\right]'}{[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z)]^{1-\delta}\,[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\chi(z)]^{\delta}} \prec \sqrt{\Omega(z)} \tag{21}$$

where

$$\Omega(z) = \frac{1}{z}\int_0^z \Gamma(Q, R; 1; \sigma; \Psi)\, dt$$

and K is convex and is the best dominant.

Proof. Let

$$r(z) = \frac{z\left[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z)\right]'}{[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z)]^{1-\delta}\,[\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\chi(z)]^{\delta}} \quad (z \in \Pi;\ \delta \geq 0),$$

then $r(z) \in \mathcal{H}(1, 1)$ with $r(z) \neq 0$.

By assumption, $\Gamma(Q, R; 1; \sigma; \Psi)$ is convex univalent in Π, which, in turn, implies Ω is convex and univalent in Π. Suppose $T(z) = r^2(z)$, then $T(z) \in \mathcal{H}$ with $T(z) \neq 0$ in Π.

Using logarithmic differentiation, we have

$$\frac{zT'(z)}{T(z)} = 2\left[1 + \frac{z(\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z))''}{(\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z))'} - (1-\delta)\frac{z\left(\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z)\right)'}{\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\varphi(z)} \right.$$
$$\left. - \delta\frac{z\left(\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\chi(z)\right)'}{\mathfrak{I}^m_{k,\lambda}(\alpha, \beta, \gamma)\chi(z)} \right].$$

Thus by (2), we have

$$T(z) + zT'(z) \prec \ell(z) \quad (z \in \Pi). \tag{22}$$

Now, by Lemma 3, we deduce that

$$T(z) \prec \Omega(z) \prec \ell(z).$$

Since $Re\,\ell(z) > 0$ and $\Omega(z) \prec \ell(z)$, we also have $Re\,\Omega(z) > 0$. $\sqrt{\Omega(z)}$ is univalent by virtue of Ω being univalent and $r^2(z) \prec \Omega(z)$ implies that $r(z) \prec \sqrt{\Omega(z)}$, which establishes the assertion. $\square$

Corollary 3. *Let $\varphi, \chi \in \Lambda$ with $\varphi(z)$, $\varphi'(z)$ and $\chi(z) \neq 0$ for all $z \in \Pi \setminus \{0\}$. If $\chi \in \mathcal{S}^*(0)$ and*

$$\mathrm{Re}\left\{\left(\frac{z\left[\mathfrak{I}^m_{k,\lambda}(\alpha,\beta,\gamma)\varphi(z)\right]'}{[\mathfrak{I}^m_{k,\lambda}(\alpha,\beta,\gamma)\varphi(z)]^{1-\delta}\,[\mathfrak{I}^m_{k,\lambda}(\alpha,\beta,\gamma)\chi(z)]^{\delta}}\right)^2\left[2\left\{\frac{z\left[\mathfrak{I}^m_{k,\lambda}(\alpha,\beta,\gamma)\varphi(z)\right]''}{\left[D^{m,q}_{\lambda,s}(\alpha_1,\beta_1)\varphi(z)\right]'} - (1-\delta)\frac{z\left[\mathfrak{I}^m_{k,\lambda}(\alpha,\beta,\gamma)\varphi(z)\right]'}{\mathfrak{I}^m_{k,\lambda}(\alpha,\beta,\gamma)\varphi(z)} - \delta\frac{z\left[\mathfrak{I}^m_{k,\lambda}(\alpha,\beta,\gamma)\chi(z)\right]'}{\mathfrak{I}^m_{k,\lambda}(\alpha,\beta,\gamma)\chi(z)}\right\} + 3\right]\right\} > \xi$$

then

$$Re\left[\frac{z\left[\mathfrak{I}^m_{k,\lambda}(\alpha,\beta,\gamma)\varphi(z)\right]'}{[\mathfrak{I}^m_{k,\lambda}(\alpha,\beta,\gamma)\varphi(z)]^{1-\delta}\,[\mathfrak{I}^m_{k,\lambda}(\alpha,\beta,\gamma)\chi(z)]^{\delta}}\right] > \omega(\xi),$$

where $\omega(\xi) = \sqrt{[2(1-\xi)\cdot\log 2 + (2\xi-1)]}$, $(0 \le \xi < 1)$. The inequality is sharp.

Proof. Let $\sigma = 0$, $Q = 1$, $R = -1$ and $\Psi(z) = \frac{1+(2\xi-1)z}{1+z}$, $0 \le \xi < 1$ in Theorem 2, we can easily get the desired result. $\square$

If we let $m = \delta = \alpha = 0$ and $k = 1$ in Corollary 3, then we have the following

Corollary 4. *Let $\varphi \in \Lambda$ with $\varphi'(z)$ and $\varphi(z) \neq 0$ for all $z \in \Pi \setminus \{0\}$. If*

$$Re\left\{\left(\frac{z\varphi'(z)}{\varphi(z)}\right)^2\left[3 + \frac{2z\varphi''(z)}{\varphi'(z)} - \frac{2z\varphi'(z)}{\varphi(z)}\right]\right\} > \xi,$$

then

$$Re\,\frac{z\varphi'(z)}{\varphi(z)} > \omega(\xi),$$

where $\omega(\xi) = \sqrt{[2(1-\xi)\cdot\log 2 + (2\xi-1)]}$, $(0 \le \xi < 1)$. This inequality is sharp.

If we let $\delta = 1$, $m = \alpha = 0$ and $k = 1$ in Corollary 3, then we have the following

Corollary 5. *If $\varphi, \chi \in \Lambda$ satisfies*

$$Re\left\{\left(\frac{z\varphi'(z)}{\chi(z)}\right)^2\left[3 + \frac{2z\varphi''(z)}{\varphi'(z)} - \frac{2z\chi'(z)}{\chi(z)}\right]\right\} > \xi,$$

with $\chi \in \mathcal{S}^*(0)$ *and* $\chi(z) \neq 0$ *for all* $z \in \Pi \setminus \{0\}$, *then*

$$Re \frac{z\varphi'(z)}{\chi(z)} > \omega(\xi),$$

where $\omega(\xi) = \sqrt{[2(1-\xi) \cdot \log 2 + (2\xi - 1)]}$, $(0 \leq \xi < 1)$. *The inequality is sharp.*

5. Conclusions

The main purpose of this present study is to obtain the coefficient inequality for the class of Bazilevič functions, which is computationally cumbersome. To add more versatility to our study, we have studied a class of Bazilevič functions involving the Mittag-Leffler functions. Coefficient inequality, solutions to the Fekete–Szegö problem and sufficient conditions for starlikeness are the primary results of this paper. We have pointed out appropriate connections that we investigated here, together with those in several interconnected earlier works.

We note that this study can be extended by taking a trigonometric function, exponential function, Legendre polynomial, Chebyshev polynomial, Fibonacci sequence or q-Hermite polynomial instead of considering $\psi(z)$ as in (7).

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