

Supplementary Materials

Statistical Analysis of Alpha Power Exponential Parameters Using Progressive First-Failure Censoring With Applications

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Table S1: The average estimates (1^{st} row), RMSEs (2^{nd} row) and MRABs (3^{rd} row) of α when $k = 2$.

k	(n, m)	Scheme	MLE	SEL		LL			
Prior \rightarrow				1	2	1		2	
$\eta \rightarrow$						-2	+2	-2	+2
2	(30,12)	S1	1.169	0.978	0.601	0.993	0.961	0.604	0.599
			1.153	0.496	0.215	0.494	0.463	0.206	0.193
			1.044	0.957	0.398	0.987	0.923	0.411	0.385
S2		1.005	0.980	0.605	0.995	0.963	0.607	0.602	
		1.447	0.497	0.222	0.495	0.465	0.213	0.198	
		1.182	0.960	0.410	0.989	0.926	0.425	0.396	
S3		1.322	0.972	0.607	0.986	0.957	0.610	0.604	
		1.911	0.488	0.220	0.487	0.458	0.211	0.197	
		1.130	0.944	0.407	0.972	0.913	0.421	0.394	
(30,24)		S1	1.068	0.832	0.589	0.851	0.814	0.598	0.580
			0.600	0.360	0.131	0.352	0.315	0.105	0.099
			0.811	0.664	0.210	0.701	0.628	0.208	0.198
	S2	1.004	0.828	0.589	0.846	0.810	0.598	0.581	
		0.583	0.355	0.130	0.347	0.311	0.108	0.102	
		0.815	0.655	0.209	0.692	0.620	0.215	0.204	
	S3	1.139	0.822	0.594	0.839	0.805	0.603	0.586	
		0.945	0.349	0.134	0.340	0.306	0.110	0.105	
		0.900	0.643	0.214	0.678	0.609	0.219	0.209	
	(80,32)	S1	0.669	0.757	0.640	0.774	0.740	0.647	0.634
			0.429	0.289	0.163	0.275	0.241	0.148	0.134
			0.842	0.534	0.283	0.547	0.480	0.294	0.267
S2		0.678	0.772	0.649	0.790	0.754	0.656	0.642	
		0.429	0.304	0.171	0.291	0.264	0.156	0.142	
		0.841	0.544	0.300	0.580	0.528	0.312	0.284	
S3		0.935	0.796	0.670	0.817	0.776	0.678	0.662	
		0.839	0.330	0.192	0.318	0.277	0.178	0.163	
		1.016	0.593	0.342	0.633	0.552	0.356	0.325	
(80,64)		S1	0.410	0.767	0.699	0.771	0.763	0.706	0.693
			0.417	0.275	0.114	0.271	0.263	0.099	0.081
			0.790	0.514	0.203	0.542	0.526	0.196	0.160
	S2	0.415	0.768	0.705	0.772	0.764	0.712	0.698	
		0.415	0.276	0.118	0.272	0.255	0.099	0.082	
		0.790	0.536	0.200	0.544	0.508	0.197	0.162	
	S3	0.541	0.770	0.704	0.774	0.766	0.711	0.697	
		0.421	0.278	0.119	0.274	0.266	0.104	0.086	
		0.806	0.540	0.206	0.549	0.532	0.206	0.171	

Table S2: The average estimates (1^{st} row), RMSEs (2^{nd} row) and MRABs (3^{rd} row) of α when $k = 5$.

k Prior \rightarrow $\eta \rightarrow$	(n, m)	Scheme	MLE	SEL		LL			
				1	2	1		2	
						-2	+2	-2	+2
5	(30,12)	S1	1.296	1.082	0.707	1.104	1.056	0.713	0.702
			1.540	0.603	0.312	0.605	0.558	0.307	0.294
			1.346	1.163	0.600	1.208	1.112	0.614	0.587
		S2	1.488	1.102	0.731	1.125	1.076	0.736	0.725
			1.877	0.623	0.329	0.626	0.578	0.325	0.312
			1.528	1.205	0.636	1.250	1.152	0.650	0.623
		S3	1.941	1.112	0.738	1.135	1.084	0.744	0.732
			1.972	0.633	0.333	0.636	0.586	0.330	0.316
			1.853	1.223	0.645	1.270	1.169	0.659	0.631
	(30,24)	S1	0.584	0.985	0.710	1.007	0.962	0.720	0.699
			0.633	0.508	0.233	0.507	0.464	0.220	0.203
			0.865	0.969	0.419	1.013	0.925	0.440	0.404
		S2	0.613	0.983	0.712	1.004	0.961	0.722	0.702
			0.649	0.438	0.243	0.505	0.462	0.237	0.226
			0.881	0.864	0.461	1.008	0.922	0.473	0.450
		S3	1.371	0.975	0.720	0.996	0.955	0.730	0.710
			1.618	0.497	0.250	0.496	0.456	0.244	0.233
			1.725	0.951	0.475	0.991	0.910	0.487	0.464
	(80,32)	S1	0.716	0.853	0.738	0.872	0.834	0.746	0.730
			0.432	0.422	0.254	0.421	0.409	0.246	0.231
			0.831	0.830	0.477	0.841	0.819	0.491	0.461
		S2	0.952	0.863	0.756	0.881	0.845	0.764	0.749
			0.442	0.506	0.271	0.438	0.427	0.264	0.249
			0.829	0.965	0.513	0.875	0.853	0.528	0.497
		S3	1.020	0.894	0.781	0.912	0.875	0.789	0.772
			1.183	0.436	0.295	0.435	0.423	0.289	0.273
			1.137	0.859	0.561	0.870	0.847	0.577	0.545
	(80,64)	S1	0.477	0.915	0.800	0.920	0.909	0.807	0.793
			0.430	0.380	0.221	0.373	0.335	0.214	0.200
			0.789	0.706	0.414	0.744	0.668	0.425	0.399
		S2	0.488	0.932	0.818	0.938	0.926	0.825	0.811
			0.430	0.388	0.235	0.381	0.346	0.223	0.203
			0.791	0.726	0.424	0.761	0.690	0.444	0.404
		S3	0.757	0.929	0.822	0.935	0.923	0.829	0.816
			0.432	0.418	0.243	0.413	0.376	0.231	0.210
			0.799	0.788	0.440	0.825	0.750	0.460	0.419

Table S3: The average estimates (1^{st} row), RMSEs (2^{nd} row) and MRABs (3^{rd} row) of δ when $k = 2$.

k Prior \rightarrow $\eta \rightarrow$	(n, m)	Scheme	MLE	SEL		LL			
				1	2	1		2	
						-2	+2	-2	+2
2	(30,12)	S1	0.804	0.703	0.870	0.724	0.681	0.883	0.856
			0.541	0.388	0.376	0.365	0.385	0.353	0.372
			0.498	0.375	0.363	0.365	0.385	0.352	0.372
		S2	0.924	0.716	0.867	0.737	0.696	0.880	0.853
			0.687	0.376	0.366	0.351	0.373	0.344	0.362
			0.534	0.361	0.353	0.351	0.373	0.343	0.361
		S3	1.540	0.726	0.870	0.746	0.705	0.883	0.856
			1.368	0.363	0.353	0.342	0.359	0.323	0.347
			1.021	0.351	0.335	0.342	0.359	0.323	0.347
	(30,24)	S1	0.651	0.694	0.697	0.712	0.676	0.717	0.681
			0.535	0.334	0.332	0.288	0.319	0.284	0.320
			0.456	0.306	0.303	0.288	0.319	0.283	0.319
		S2	0.674	0.696	0.697	0.714	0.679	0.716	0.680
			0.534	0.332	0.331	0.286	0.322	0.285	0.320
			0.493	0.304	0.303	0.286	0.321	0.284	0.320
		S3	0.746	0.699	0.700	0.716	0.682	0.719	0.684
			0.540	0.329	0.325	0.284	0.318	0.282	0.317
			0.485	0.301	0.300	0.284	0.318	0.281	0.316
	(80,32)	S1	0.584	0.625	0.637	0.635	0.615	0.648	0.628
			0.504	0.332	0.176	0.276	0.324	0.157	0.162
			0.451	0.299	0.159	0.276	0.324	0.157	0.161
		S2	0.627	0.639	0.647	0.649	0.627	0.657	0.639
			0.510	0.318	0.178	0.263	0.304	0.155	0.159
			0.452	0.285	0.156	0.263	0.304	0.154	0.158
		S3	0.995	0.665	0.649	0.677	0.653	0.658	0.641
			0.736	0.310	0.175	0.254	0.295	0.153	0.157
			0.616	0.276	0.155	0.254	0.295	0.153	0.157
	(80,64)	S1	0.506	0.816	0.841	0.831	0.799	0.843	0.839
			0.488	0.225	0.166	0.170	0.202	0.118	0.145
			0.426	0.187	0.135	0.169	0.201	0.117	0.144
		S2	0.513	0.821	0.844	0.836	0.806	0.846	0.842
			0.495	0.218	0.163	0.165	0.195	0.121	0.148
			0.431	0.182	0.138	0.164	0.194	0.120	0.147
		S3	0.542	0.825	0.845	0.840	0.810	0.847	0.843
			0.526	0.215	0.162	0.161	0.191	0.118	0.145
			0.448	0.178	0.135	0.160	0.190	0.117	0.144

Table S4: The average estimates (1^{st} row), RMSEs (2^{nd} row) and MRABs (3^{rd} row) of δ when $k = 5$.

k Prior \rightarrow $\eta \rightarrow$	(n, m)	Scheme	MLE	SEL		LL			
				1	2	1		2	
						-2	+2	-2	+2
5	(30,12)	S1	0.408	0.416	0.593	0.449	0.388	0.639	0.549
			0.807	0.732	0.694	0.719	0.733	0.653	0.696
			0.798	0.727	0.678	0.719	0.733	0.651	0.696
		S2	0.484	0.428	0.608	0.461	0.401	0.653	0.564
			0.802	0.721	0.685	0.709	0.722	0.645	0.687
			0.793	0.716	0.669	0.708	0.722	0.644	0.686
		S3	0.995	0.439	0.618	0.471	0.411	0.661	0.575
			0.923	0.703	0.671	0.691	0.704	0.633	0.672
			0.760	0.699	0.656	0.690	0.704	0.632	0.672
	(30,24)	S1	0.301	0.331	0.389	0.348	0.318	0.434	0.357
			0.730	0.681	0.642	0.653	0.682	0.568	0.644
			0.701	0.669	0.611	0.652	0.682	0.566	0.643
		S2	0.317	0.335	0.392	0.352	0.323	0.435	0.359
			0.722	0.676	0.640	0.649	0.678	0.567	0.641
			0.689	0.665	0.608	0.648	0.677	0.565	0.641
		S3	0.384	0.339	0.395	0.357	0.326	0.437	0.363
			0.687	0.672	0.636	0.644	0.674	0.565	0.638
			0.636	0.621	0.605	0.623	0.614	0.563	0.607
	(80,32)	S1	0.257	0.273	0.322	0.281	0.267	0.349	0.304
			0.761	0.651	0.621	0.632	0.652	0.598	0.622
			0.743	0.643	0.601	0.631	0.652	0.597	0.621
		S2	0.284	0.284	0.331	0.292	0.278	0.356	0.314
			0.746	0.645	0.612	0.625	0.645	0.590	0.613
			0.719	0.636	0.602	0.624	0.645	0.589	0.612
		S3	0.579	0.301	0.345	0.309	0.296	0.368	0.328
			0.779	0.641	0.608	0.621	0.641	0.586	0.608
			0.752	0.632	0.598	0.620	0.641	0.585	0.608
	(80,64)	S1	0.202	0.357	0.389	0.369	0.348	0.403	0.379
			0.676	0.610	0.461	0.552	0.612	0.363	0.452
			0.625	0.584	0.407	0.551	0.612	0.361	0.451
		S2	0.207	0.364	0.398	0.376	0.355	0.411	0.388
			0.683	0.598	0.448	0.540	0.599	0.348	0.437
			0.618	0.572	0.392	0.539	0.599	0.347	0.436
		S3	0.241	0.368	0.402	0.380	0.359	0.415	0.392
			0.670	0.588	0.438	0.530	0.589	0.340	0.426
			0.590	0.562	0.382	0.529	0.589	0.339	0.425

Table S5: The average estimates (1^{st} row), RMSEs (2^{nd} row) and MRABs (3^{rd} row) of $R(t)$ when $k = 2$.

k Prior \rightarrow $\eta \rightarrow$	(n, m)	Scheme	MLE	SEL		LL			
				1	2	1		2	
						-2	+2	-2	+2
2	(30,12)	S1	0.940	0.931	0.896	0.931	0.930	0.896	0.895
			0.072	0.061	0.054	0.059	0.059	0.052	0.051
			0.078	0.067	0.059	0.067	0.067	0.059	0.059
		S2	0.937	0.929	0.896	0.930	0.929	0.896	0.896
			0.069	0.060	0.053	0.058	0.058	0.051	0.051
			0.075	0.066	0.058	0.066	0.066	0.058	0.058
		S3	0.941	0.928	0.896	0.929	0.928	0.896	0.896
			0.073	0.059	0.054	0.056	0.056	0.052	0.051
			0.079	0.064	0.059	0.064	0.064	0.059	0.059
	(30,24)	S1	0.939	0.927	0.914	0.927	0.926	0.914	0.913
			0.069	0.056	0.046	0.054	0.054	0.042	0.041
			0.077	0.062	0.047	0.062	0.062	0.048	0.047
		S2	0.939	0.926	0.914	0.926	0.926	0.914	0.913
			0.068	0.056	0.046	0.054	0.054	0.042	0.041
			0.072	0.062	0.048	0.062	0.061	0.048	0.047
		S3	0.940	0.926	0.914	0.926	0.925	0.914	0.913
			0.070	0.057	0.046	0.054	0.053	0.042	0.041
			0.078	0.061	0.047	0.061	0.061	0.048	0.047
	(80,32)	S1	0.938	0.931	0.924	0.931	0.931	0.924	0.924
			0.067	0.060	0.034	0.058	0.058	0.033	0.033
			0.075	0.064	0.037	0.066	0.066	0.038	0.037
		S2	0.936	0.930	0.923	0.930	0.930	0.923	0.923
			0.065	0.059	0.034	0.057	0.056	0.033	0.032
			0.073	0.064	0.038	0.065	0.064	0.038	0.037
		S3	0.938	0.928	0.924	0.929	0.928	0.924	0.924
			0.067	0.055	0.033	0.052	0.055	0.033	0.032
			0.076	0.061	0.037	0.063	0.061	0.037	0.037
	(80,64)	S1	0.937	0.912	0.905	0.912	0.911	0.905	0.905
			0.066	0.042	0.028	0.040	0.039	0.024	0.023
			0.074	0.045	0.027	0.045	0.045	0.027	0.026
		S2	0.937	0.911	0.905	0.911	0.911	0.905	0.905
			0.062	0.041	0.028	0.039	0.038	0.024	0.023
			0.071	0.044	0.027	0.045	0.044	0.028	0.027
		S3	0.938	0.911	0.905	0.911	0.911	0.905	0.905
			0.066	0.041	0.028	0.039	0.038	0.024	0.024
			0.075	0.044	0.027	0.044	0.044	0.027	0.027

Table S6: The average estimates (1^{st} row), RMSEs (2^{nd} row) and MRABs (3^{rd} row) of $R(t)$ when $k = 5$.

k Prior \rightarrow $\eta \rightarrow$	(n, m)	Scheme	MLE	SEL		LL			
				1	2	1		2	
						-2	+2	-2	+2
5	(30,12)	S1	0.976	0.960	0.932	0.960	0.959	0.932	0.931
			0.105	0.099	0.093	0.098	0.098	0.091	0.090
			0.119	0.113	0.104	0.113	0.113	0.104	0.103
		S2	0.975	0.959	0.931	0.959	0.958	0.932	0.931
			0.103	0.098	0.092	0.097	0.097	0.090	0.090
			0.118	0.112	0.103	0.112	0.111	0.103	0.103
		S3	0.976	0.958	0.930	0.958	0.958	0.931	0.930
			0.104	0.097	0.091	0.096	0.096	0.089	0.089
			0.119	0.110	0.102	0.110	0.110	0.102	0.102
	(30,24)	S1	0.976	0.967	0.954	0.967	0.967	0.955	0.953
			0.104	0.096	0.086	0.095	0.094	0.082	0.081
			0.118	0.108	0.094	0.108	0.108	0.094	0.093
		S2	0.976	0.966	0.954	0.966	0.966	0.955	0.953
			0.102	0.095	0.086	0.094	0.094	0.082	0.081
			0.116	0.108	0.094	0.108	0.107	0.094	0.093
		S3	0.976	0.966	0.954	0.966	0.966	0.954	0.953
			0.103	0.095	0.085	0.094	0.093	0.082	0.081
			0.118	0.107	0.093	0.107	0.107	0.094	0.093
	(80,32)	S1	0.976	0.971	0.963	0.971	0.970	0.963	0.962
			0.103	0.092	0.086	0.091	0.091	0.085	0.085
			0.117	0.104	0.097	0.104	0.104	0.097	0.097
		S2	0.974	0.970	0.962	0.970	0.970	0.963	0.962
			0.102	0.091	0.085	0.091	0.090	0.085	0.084
			0.115	0.104	0.097	0.104	0.104	0.097	0.097
		S3	0.975	0.968	0.961	0.968	0.968	0.962	0.961
			0.103	0.091	0.085	0.090	0.090	0.084	0.084
			0.118	0.103	0.096	0.103	0.103	0.097	0.096
	(80,64)	S1	0.975	0.963	0.957	0.963	0.963	0.957	0.957
			0.101	0.090	0.065	0.088	0.087	0.060	0.059
			0.114	0.100	0.068	0.100	0.100	0.069	0.067
		S2	0.975	0.963	0.957	0.963	0.963	0.957	0.957
			0.101	0.089	0.064	0.087	0.086	0.060	0.058
			0.112	0.099	0.067	0.100	0.099	0.068	0.067
		S3	0.975	0.962	0.956	0.963	0.962	0.957	0.956
			0.102	0.088	0.063	0.086	0.085	0.059	0.058
			0.117	0.098	0.067	0.099	0.098	0.067	0.066

Table S7: The average estimates (1^{st} row), RMSEs (2^{nd} row) and MRABs (3^{rd} row) of $h(t)$ when $k = 2$.

k Prior \rightarrow $\eta \rightarrow$	(n, m)	Scheme	MLE	SEL		LL			
				1	2	1		2	
						-2	+2	-2	+2
2	(30,12)	S1	0.631	0.720	1.092	0.759	0.687	1.119	1.065
			0.760	0.655	0.581	0.621	0.659	0.531	0.582
			0.536	0.470	0.415	0.461	0.490	0.393	0.432
		S2	0.673	0.733	1.087	0.770	0.702	1.113	1.059
			0.729	0.634	0.573	0.613	0.636	0.524	0.573
			0.526	0.464	0.409	0.455	0.472	0.388	0.426
		S3	0.675	0.745	1.088	0.781	0.713	1.114	1.061
			0.743	0.629	0.580	0.595	0.633	0.534	0.580
			0.537	0.450	0.415	0.442	0.470	0.396	0.431
	(30,24)	S1	0.631	0.762	0.898	0.792	0.737	0.953	0.853
			0.738	0.608	0.502	0.556	0.644	0.396	0.494
			0.532	0.434	0.334	0.412	0.479	0.292	0.366
		S2	0.640	0.766	0.897	0.795	0.741	0.950	0.853
			0.717	0.604	0.502	0.552	0.605	0.398	0.494
			0.506	0.431	0.334	0.409	0.449	0.294	0.366
		S3	0.624	0.771	0.898	0.800	0.747	0.950	0.854
			0.726	0.598	0.501	0.548	0.599	0.398	0.493
			0.510	0.427	0.334	0.406	0.445	0.294	0.365
	(80,32)	S1	0.641	0.713	0.788	0.725	0.702	0.817	0.765
			0.721	0.642	0.365	0.588	0.609	0.348	0.362
			0.523	0.466	0.263	0.436	0.452	0.258	0.269
		S2	0.667	0.722	0.796	0.734	0.711	0.824	0.773
			0.704	0.621	0.365	0.577	0.615	0.349	0.362
			0.517	0.456	0.264	0.428	0.459	0.259	0.269
		S3	0.663	0.740	0.787	0.752	0.729	0.813	0.766
			0.713	0.615	0.362	0.567	0.617	0.347	0.360
			0.524	0.445	0.262	0.420	0.458	0.257	0.267
	(80,64)	S1	0.645	0.921	0.991	0.943	0.899	0.998	0.984
			0.710	0.451	0.304	0.404	0.447	0.229	0.284
			0.521	0.316	0.189	0.299	0.332	0.169	0.209
		S2	0.650	0.927	0.991	0.947	0.906	0.998	0.984
			0.693	0.444	0.309	0.400	0.441	0.234	0.289
			0.504	0.312	0.193	0.296	0.327	0.173	0.213
		S3	0.641	0.930	0.993	0.951	0.909	1.000	0.987
			0.697	0.441	0.307	0.396	0.437	0.234	0.287
			0.507	0.309	0.192	0.294	0.325	0.173	0.212

Table S8: The average estimates (1^{st} row), RMSEs (2^{nd} row) and MRABs (3^{rd} row) of $h(t)$ when $k = 5$.

k Prior \rightarrow $\eta \rightarrow$	(n, m)	Scheme	MLE	SEL		LL			
				1	2	1		2	
						-2	+2	-2	+2
5	(30,12)	S1	0.243	0.415	0.706	0.469	0.376	0.782	0.634
			1.107	1.053	0.988	1.032	1.057	0.917	0.997
			0.819	0.778	0.718	0.766	0.785	0.678	0.740
		S2	0.261	0.422	0.713	0.475	0.385	0.786	0.643
			1.100	1.044	0.981	1.024	1.047	0.914	0.990
			0.816	0.771	0.714	0.760	0.778	0.676	0.735
		S3	0.268	0.430	0.721	0.482	0.393	0.793	0.653
			1.104	1.030	0.970	1.012	1.033	0.908	0.978
			0.818	0.761	0.707	0.751	0.767	0.672	0.727
	(30,24)	S1	0.244	0.339	0.472	0.368	0.321	0.558	0.414
			1.104	1.019	0.916	0.979	1.025	0.791	0.932
			0.818	0.748	0.650	0.726	0.761	0.585	0.692
		S2	0.248	0.344	0.473	0.373	0.326	0.557	0.417
			1.095	1.013	0.914	0.975	1.020	0.792	0.930
			0.813	0.744	0.649	0.723	0.757	0.586	0.690
		S3	0.245	0.350	0.474	0.378	0.331	0.557	0.419
			1.084	1.009	0.912	0.970	1.015	0.792	0.928
			0.804	0.740	0.648	0.719	0.754	0.586	0.689
	(80,32)	S1	0.248	0.299	0.380	0.315	0.290	0.433	0.350
			1.100	0.979	0.921	0.955	0.983	0.886	0.926
			0.816	0.722	0.675	0.709	0.730	0.657	0.688
		S2	0.260	0.308	0.385	0.323	0.299	0.436	0.357
			1.089	0.975	0.915	0.951	0.979	0.881	0.921
			0.807	0.719	0.671	0.706	0.727	0.653	0.684
		S3	0.264	0.321	0.395	0.335	0.313	0.441	0.368
			1.096	0.970	0.911	0.947	0.974	0.878	0.916
			0.814	0.715	0.668	0.703	0.724	0.651	0.680
	(80,64)	S1	0.249	0.375	0.437	0.392	0.363	0.462	0.421
			1.098	0.957	0.699	0.880	0.970	0.566	0.714
			0.815	0.692	0.475	0.652	0.720	0.419	0.529
		S2	0.252	0.379	0.442	0.396	0.367	0.467	0.426
			1.088	0.948	0.691	0.874	0.961	0.562	0.704
			0.806	0.686	0.470	0.647	0.714	0.416	0.522
		S3	0.250	0.383	0.446	0.400	0.372	0.470	0.430
			1.081	0.940	0.682	0.866	0.953	0.556	0.695
			0.801	0.680	0.464	0.642	0.708	0.411	0.515

Table S9: The ACLs (1^{st} column) and CPs (2^{nd} column) of ACI/HPD credible intervals of α .

k Prior \rightarrow	(n, m)	Scheme	ACI		HPD			
					1		2	
2	(30,12)	S1	3.278	0.922	0.464	0.938	0.293	0.951
		S2	3.926	0.915	0.468	0.940	0.293	0.951
		S3	3.984	0.912	0.472	0.942	0.297	0.950
	(30,24)	S1	1.426	0.933	0.451	0.953	0.272	0.967
		S2	1.733	0.928	0.445	0.945	0.280	0.962
		S3	1.889	0.925	0.453	0.947	0.275	0.965
	(80,32)	S1	1.378	0.942	0.426	0.962	0.250	0.977
		S2	1.151	0.946	0.439	0.956	0.255	0.976
		S3	1.522	0.937	0.435	0.957	0.268	0.969
	(80,64)	S1	1.035	0.956	0.242	0.970	0.198	0.984
		S2	1.126	0.954	0.241	0.970	0.202	0.984
		S3	1.321	0.948	0.243	0.969	0.200	0.985
5	(30,12)	S1	3.869	0.894	0.576	0.923	0.326	0.943
		S2	4.045	0.888	0.583	0.919	0.324	0.942
		S3	4.738	0.885	0.595	0.916	0.325	0.942
	(30,24)	S1	1.896	0.907	0.499	0.939	0.288	0.954
		S2	2.752	0.898	0.494	0.937	0.290	0.951
		S3	2.270	0.901	0.477	0.933	0.292	0.951
	(80,32)	S1	1.629	0.922	0.447	0.954	0.278	0.967
		S2	2.062	0.903	0.433	0.946	0.281	0.966
		S3	1.942	0.914	0.444	0.954	0.287	0.964
	(80,64)	S1	1.349	0.934	0.277	0.962	0.274	0.977
		S2	1.488	0.926	0.279	0.962	0.273	0.977
		S3	1.528	0.923	0.284	0.964	0.281	0.975

Table S10: The ACLs (1^{st} column) and CPs (2^{nd} column) of ACI/HPD credible intervals of δ .

k Prior \rightarrow	(n, m)	Scheme	ACI		HPD			
					1		2	
2	(30,12)	S1	1.418	0.932	0.444	0.964	0.422	0.970
		S2	1.644	0.928	0.442	0.960	0.423	0.967
		S3	2.942	0.917	0.456	0.959	0.426	0.964
	(30,24)	S1	1.012	0.945	0.434	0.969	0.368	0.979
		S2	1.066	0.942	0.440	0.968	0.367	0.978
		S3	2.331	0.928	0.439	0.965	0.365	0.976
	(80,32)	S1	0.889	0.952	0.422	0.975	0.340	0.989
		S2	1.001	0.950	0.435	0.973	0.332	0.985
		S3	1.593	0.935	0.421	0.970	0.326	0.982
	(80,64)	S1	0.673	0.968	0.333	0.983	0.184	0.991
		S2	0.707	0.963	0.341	0.981	0.187	0.992
		S3	1.101	0.957	0.351	0.980	0.195	0.995
5	(30,12)	S1	2.019	0.917	0.622	0.951	0.576	0.962
		S2	2.395	0.914	0.634	0.950	0.581	0.962
		S3	4.102	0.902	0.634	0.945	0.586	0.958
	(30,24)	S1	1.445	0.930	0.612	0.955	0.438	0.966
		S2	1.532	0.924	0.614	0.952	0.439	0.964
		S3	3.028	0.913	0.620	0.951	0.441	0.962
	(80,32)	S1	1.254	0.935	0.488	0.968	0.271	0.976
		S2	1.462	0.929	0.500	0.962	0.375	0.974
		S3	2.204	0.920	0.508	0.967	0.371	0.972
	(80,64)	S1	0.946	0.948	0.376	0.980	0.266	0.989
		S2	0.993	0.944	0.381	0.975	0.267	0.987
		S3	1.590	0.936	0.386	0.973	0.352	0.993

Table S11: The ACLs (1^{st} column) and CPs (2^{nd} column) of ACI/HPD credible intervals of $R(t)$.

k Prior \rightarrow	(n, m)	Scheme	ACI		HPD			
					1		2	
2	(30,12)	S1	0.099	0.925	0.066	0.950	0.057	0.956
		S2	0.090	0.930	0.065	0.951	0.056	0.955
		S3	0.090	0.930	0.065	0.950	0.053	0.959
	(30,24)	S1	0.071	0.941	0.059	0.962	0.050	0.970
		S2	0.068	0.946	0.057	0.964	0.050	0.971
		S3	0.070	0.942	0.057	0.964	0.049	0.972
	(80,32)	S1	0.060	0.952	0.052	0.970	0.038	0.980
		S2	0.054	0.955	0.051	0.971	0.037	0.981
		S3	0.056	0.954	0.051	0.970	0.036	0.982
	(80,64)	S1	0.049	0.959	0.042	0.976	0.029	0.986
		S2	0.049	0.959	0.041	0.978	0.029	0.987
		S3	0.048	0.960	0.040	0.977	0.029	0.987
5	(30,12)	S1	0.080	0.936	0.067	0.961	0.037	0.969
		S2	0.079	0.937	0.066	0.962	0.035	0.972
		S3	0.078	0.938	0.066	0.962	0.037	0.973
	(30,24)	S1	0.079	0.950	0.049	0.974	0.026	0.980
		S2	0.078	0.952	0.049	0.973	0.025	0.985
		S3	0.077	0.953	0.050	0.973	0.027	0.978
	(80,32)	S1	0.064	0.957	0.040	0.980	0.022	0.985
		S2	0.062	0.959	0.040	0.980	0.021	0.987
		S3	0.060	0.961	0.039	0.979	0.023	0.984
	(80,64)	S1	0.047	0.966	0.032	0.984	0.015	0.990
		S2	0.046	0.968	0.031	0.985	0.015	0.991
		S3	0.046	0.968	0.030	0.986	0.016	0.990

Table S12: The ACLs (1^{st} column) and CPs (2^{nd} column) of ACI/HPD credible intervals of $h(t)$.

k Prior \rightarrow	(n, m)	Scheme	ACI		HPD			
					1	2		
2	(30,12)	S1	1.003	0.898	0.708	0.925	0.629	0.931
		S2	0.920	0.900	0.701	0.927	0.616	0.934
		S3	0.882	0.906	0.698	0.930	0.610	0.938
	(30,24)	S1	0.831	0.912	0.620	0.941	0.543	0.944
		S2	0.828	0.927	0.610	0.938	0.543	0.943
		S3	0.817	0.925	0.605	0.932	0.537	0.946
	(80,32)	S1	0.670	0.923	0.566	0.950	0.528	0.957
		S2	0.653	0.934	0.546	0.953	0.519	0.960
		S3	0.633	0.939	0.518	0.958	0.508	0.963
	(80,64)	S1	0.488	0.934	0.401	0.958	0.317	0.966
		S2	0.486	0.935	0.395	0.956	0.312	0.968
		S3	0.480	0.938	0.383	0.952	0.307	0.971
5	(30,12)	S1	0.839	0.907	0.704	0.930	0.368	0.949
		S2	0.830	0.912	0.693	0.932	0.348	0.957
		S3	0.824	0.915	0.689	0.934	0.350	0.956
	(30,24)	S1	0.716	0.926	0.515	0.944	0.258	0.956
		S2	0.690	0.931	0.515	0.943	0.252	0.964
		S3	0.693	0.929	0.518	0.941	0.261	0.950
	(80,32)	S1	0.614	0.933	0.415	0.956	0.220	0.973
		S2	0.553	0.940	0.416	0.955	0.206	0.974
		S3	0.551	0.942	0.411	0.961	0.206	0.974
	(80,64)	S1	0.432	0.948	0.328	0.967	0.151	0.983
		S2	0.413	0.957	0.322	0.969	0.147	0.985
		S3	0.417	0.954	0.315	0.970	0.153	0.981