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Social Network Group Decision-Making Method Based on Q-Rung Orthopair Fuzzy Set and Its Application in the Evaluation of Online Teaching Quality

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Abstract: As q-rung orthopair fuzzy set (q-ROFS) theory can effectively express complex fuzzy information, this study explores its application to social network environments and proposes a social network group decision-making (SNGDM) method based on the q-ROFS. Firstly, the q-rung orthopair fuzzy value is used to represent the trust relationships between experts in the social network, and a trust q-rung orthopair fuzzy value is defined. Secondly, considering the decreasing and multipath of trust in the process of trust propagation, this study designs a trust propagation mechanism by using its multiplication operation in the q-ROFS environment and proposes a trust q-ROFS aggregation approach. Moreover, based on the trust scores and confidence levels of experts, a new integration operator called q-rung orthopair fuzzy-induced ordered weighted average operator is proposed to fuse experts' evaluation information. Additionally, considering the impact of consensus interaction on decision-making results, a consensus interaction model based on the q-ROF distance measure and trust relationship is proposed, including consistency measurement, identification of inconsistent expert decision-making opinions and a personalized adjustment mechanism. Finally, the SNGDM method is applied to solve the problem of evaluating online teaching quality.

Keywords: q-ROFS; trust propagation model; confidence level; consensus interaction model; evaluation of online teaching quality

MSC: 03B52; 47S40; 90B50

Citation: Hu, Y.; Zeng, S.; Carlos, L.-A.; Ullah, K.; Yang, Y. Social Network Group Decision-Making Method Based on Q-Rung Orthopair Fuzzy Set and Its Application in the Evaluation of Online Teaching Quality. *Axioms* **2021**, *10*, 168. https://doi.org/10.3390/axioms10030168

Academic Editor: Goran Ćirović

Received: 27 June 2021
Accepted: 21 July 2021
Published: 28 July 2021

1. Introduction

Online teaching is a new Internet-based teaching mode that can achieve the purpose of teaching through online teaching platforms without face-to-face interactions. Although online teaching is not currently the main method of teaching, it can be used as an alternative emergency teaching mode. For example, the COVID-19 pandemic that occurred at the end of 2019 forced many schools to suspend the traditional classroom-based teaching mode to prevent the spread of the virus. At this point, the online teaching mode largely solves the problem of delays in teaching and ensures the progress of instruction. However, online teaching has its own problems. For example, too many students in the class will cause network freezes and instability of the teaching platform. At the same time, teachers and students cannot communicate face-to-face, and teachers cannot address students' questions in a timely manner. The evaluation of classroom teaching quality can help teachers to fully understand the problems existing in the teaching process and can enhance teachers' teaching ability and improve the quality of teaching. Therefore, it is necessary to propose a scientific and effective method to evaluate the quality of online teaching. The problem of evaluating teaching quality is essentially a multi-attribute group decision-making (MAGDM) problem, because the evaluation process involves multiple evaluation indices, multiple decision-making experts, and multiple decision-making experts that provide the corresponding evaluation information that is used to arrive at a final conclusion regarding the quality level. At present, many scholars have studied the problem of evaluating teaching quality and have proposed various evaluation methods, including offline and online assessment methods. Targeting the imperfect evaluation system used to measure the quality of teaching in the social sports specialty, Liu [1] proposed a novel MAGDM method based on the intuitionistic fuzzy (IF)-TOPSIS method. Carlucci et al. [2] proposed a framework for teaching and curriculum quality evaluation combining u-control chart and fuzzy weight ABC analysis to assess students' evaluation of higher education teaching quality. Using a fuzzy comprehensive evaluation method and combining a fuzzy analytic hierarchy process (AHP) to put forward a new teaching performance evaluation framework, Chen et al. [3] proposed five sub-evaluation factors: planning and preparation, communication and interaction, teaching for learning, managing learning environment, student evaluation, and professionalism. Zhang et al. [4] proposed a new evaluation method based on heterogeneous linguistic information for the MAGDM problem faced in the evaluation of classroom teaching quality. Yu [5] considered teaching attitude, teaching ability, teaching content, and teaching feedback as evaluation indices and proposed a group decision-making (GDM) method based on triangular IF to deal with the evaluation of teaching quality in colleges and universities. Yang and Xiang [6] proposed a multi-attribute decision-making (MADM) method based on the power aggregation operator of fuzzy uncertain linguistic information to solve the problem of assessing teaching quality in higher education. Specifically, the quality of music teaching in colleges and universities was evaluated on four factors: education and teaching services, education and teaching management services, logistics management services, and students' further development services. Considering that the nature of evaluating the quality of teaching is very fuzzy and imprecise, Peng and Dai [7] used q-rung orthopair fuzzy value (q-ROFV) to deal with its uncertainty, and used teaching attitude, teaching ability, teaching content, teaching method and teaching effect as evaluation indices; two algorithms based on distance evaluation and multi-parameter similarity measure based on q-rung orthopair fuzzy set (q-ROFS) were proposed to solve the MADM problem of evaluating classroom teaching quality. Yu [8] improved AprioriTid algorithm and constructed an online evaluation model of teaching quality according to teaching needs and evaluated English online teaching quality through data mining. Liu et al. [9] proposed a new MAGDM method based on the Choquet integral operator and multi-granularity probabilistic linguistic term set, and used it to solve the problem of evaluating the quality of online teaching. Lin et al. [10] proposed an extended linguistic MAGDM framework to solve the problem of evaluating the quality of online teaching. Thus, we can see that existing research mainly used MAGDM/MADM method to solve the problem. As a scientific and effective decision-making method in a complex environment, the MAGDM considers the backgrounds and experiences of multiple decision-making experts, which avoids the subjectivity and one-sidedness presented by a single decision-making expert [11–13].

The traditional evaluation of offline teaching quality has gained increasing attention; however, few studies have focused on the evaluation of online teaching quality. Moreover, in the process of evaluating teaching quality, the influence of the social network relationships between decision-making experts on the evaluation results should be considered. The emergence of the Web2.0 mode has brought decision-making experts closer, and the corresponding social network relationships among experts have also become increasingly prominent. The influence of the Web2.0 on GDM cannot be ignored. Thus, this study combines the social network analysis (SNA) method with GDM method and proposes a new q-ROFS social network group decision-making (SNGDM) method to solve the problem of evaluating online teaching quality.

At present, GDM methods based on social networks mainly focus on the representation of trust, the propagation/aggregation operator of trust, the method of obtaining expert weights, and consensus interaction models (CIMs). The first study focused on the representation of trust. There are discrete values [14–16], continuous values [17,18], fuzzy logic values (including interval values [19–21], intuitionistic fuzzy values (IFVs) [22–25], Pythagorean fuzzy values (PFVs) [26], interval-valued Pythagorean fuzzy values (IVPFVs) [27]), and other trust representations. The second is the trust propagation method. At present, T-norm and T-conorm are used to design trust propagation operators to ensure a decrease in trust and an increase in distrust during the propagation process [24,28,29]. Some authors have designed trust propagation operators based on the Uninorm (U) operator [30,31]. This research on trust aggregation operators is mainly focused on studying the trust relationship integration of multiple propagation paths and the selection of the shortest path. Most researchers select the shortest path trust relationship as the final trust aggregation result, or assign different weights to paths of different lengths, or assign the same weights to paths of the same length, and then integrate the trust relationship on the corresponding paths to obtain the final trust evaluation value [24,30,32,33]. The third research field is on the method of obtaining the weights of experts, which is based mainly on the linguistic quantifier Q [30,32,34] and the SNA method [24,28,35,36] (centrality theory, etc.) to compute the weights of experts. The fourth research topic is the CIM; at present, the CIM focuses mainly on the identification of inconsistent experts and the adjustment of inconsistent expert decision-making opinions [28,32,34,37,38]. In considering of the differences in backgrounds and experience among experts, it is difficult for them to reach an agreement on the initial opinions of the GDM process. In other words, experts do not reach a consensus on decision-making opinions, which will affect the final evaluation and decision-making results, so experts must reach a consensus before making a final decision.

However, there are some defects in the above-mentioned social network decision-making methods: (1) using discrete values, continuous values, and interval values to describe trust information does not consider the fuzziness, uncertainty, and subjectivity of trust. Although IFV and PFV can describe the fuzziness and uncertainty of trust information, they express that the scope of fuzzy information is limited (i.e., the membership and non-membership grades are satisfied: $0 \leq \mu + v \leq 1$ or $0 \leq \mu^2 + v^2 \leq 1$); (2) using the U operator to design trust propagation operators will increase the trust value after propagation, which violates the principle of trust decreasing during trust propagation. At the same time, when integrating trust information, the selection of the shortest propagation path will cause a loss of trust information. Assigning the same path weight to the same length of path will reduce the accuracy of the trust value after propagation; (3) solving for the weights of experts ignores the importance of the confidence levels of experts. According to the results of Guha and Chakraborty [39], the evaluation of alternatives by experts is related to the confidence levels of experts. Therefore, in the real decision-making process, we cannot ignore the confidence levels of experts; (4) in the current research on the CIM, the adjustment of expert opinions tends to be based on group preferences, without considering the impact of trust relationships among experts on the adjustment of expert opinions in the social networks. In fact, the experts who need to adjust their opinions are more willing to believe the experts who have direct trust relationships with them than the others. In the study of social network relationships, consider the social network of trust relationships between individuals as a trust relationship network [24,29,32]. The emergence of this network of trust relationships in decision-making has a significant impact on the consensus among experts.

Therefore, according to the analysis of the above four defects, this study proposes a new SNGDM method within the q-ROFS situation to solve the problem of evaluating online teaching quality. Its innovation is reflected in the following four aspects:

(1) In view of the superiority of q-ROFV in expressing fuzzy information, this study uses the q-ROFV to describe the trust relationship among experts in the process of

evaluating online teaching quality, which compensates for the limitations of other data to express trust information, to improve the reliability of decision-making results.

(2) In view of the diminishing principle of trust in the process of propagation, this study uses the multiplication operation of q-ROFS to design the trust propagation operator to ensure a decline in trust.

(3) Considering the importance of the confidence levels of experts in evaluating information, this study introduces the concept of confidence level in the q-ROF environment and uses it to obtain the weights of experts.

(4) A CIM based on trust relationships is proposed to better reflect experts' acceptance of opinion adjustment.

The remainder of this study is organized as follows: Section 2 presents the proposed methods. It mainly introduces the theoretical knowledge of q-ROFS, including the definition of q-ROFS, operation rules, distance measure, score function, and q-rung orthopair fuzzy weighted averaging operator (q-ROFWA). The representation of trust relations, trust networks, and the operators of trust propagation and aggregation are also provided in this section. Section 3 presents the results of this study; it develops a q-ROF aggregation operator based on trust scores and confidence levels of experts, CIM, and decision-making analysis, and a comparison with other methods. Section 4 discusses the results of this study and presents the conclusions.

2. Theoretical Fundamentals

2.1. Theoretical Knowledge of Q-ROFSs

This section briefly reviews the relevant theoretical knowledge of the q-ROFSs.

Definition 1 ([40]). *Let $\aleph = \{Y_1, Y_2, \cdots, Y_n\}$ be a discourse, a q-ROFS defined on $\aleph$ can then be expressed as:*

$$G = \{\langle Y, \mu_G(Y), v_G(Y)\rangle | Y \in \aleph\}, \tag{1}$$

where $\mu_G(Y)$ and $v_G(Y)$ are the membership and non-membership grades of the element $Y \in \aleph$ respectively, and $\mu_G(Y)$ and $v_G(Y)$ satisfy the constraint $\mu_G^q(Y) + v_G^q(Y) \leq 1$ $(\mu_G(Y) \in [0,1], v_G(Y) \in [0,1])$ for all $q \geq 1$. The hesitation grade is expressed as: $\pi_G(Y) = \sqrt[q]{(1 - (\mu_G(Y))^q - (v_G(Y))^q)}$.

In addition, for the convenience of application, Liu and Wang [41] called $\langle \mu_G(Y), v_G(Y)\rangle$ a q-ROFV and denoted it as $G = \langle \mu_G, v_G\rangle$.

Definition 2 ([41]). *Let $G_1 = \langle \mu_1, v_1\rangle$, $G_2 = \langle \mu_2, v_2\rangle$ and $G = \langle \mu, v\rangle$ be three q-ROFVs. Then their operation rules are defined as:*

(i) $G_1 \oplus G_2 = \left(\sqrt[q]{\left(\mu_1^q + \mu_2^q - \mu_1^q\mu_2^q\right)}, v_1 v_2\right)$,

(ii) $G_1 \otimes G_2 = \left(\mu_1\mu_2, \sqrt[q]{\left(v_1^q + v_2^q - v_1^q v_2^q\right)}\right)$,

(iii) $\lambda G = \left(\sqrt[q]{1 - (1 - \mu^q)^\lambda}, v^\lambda\right)$, $\lambda > 0$.

(iv) $G^\lambda = \left(\mu^\lambda, \sqrt[q]{1 - (1 - v^q)^\lambda}\right)$, $\lambda > 0$.

Definition 3 ([42]). *Given any two q-ROFVs $G_1 = \langle \mu_1, v_1\rangle$ and $G_2 = \langle \mu_2, v_2\rangle$, their distances are computed as follows:*

$$d(G_1, G_2) = \frac{1}{2}\left(\left|\mu_1^q - \mu_2^q\right| + \left|v_1^q - v_2^q\right| + \left|\pi_1^q - \pi_2^q\right|\right). \tag{2}$$

Definition 4 ([43]). *Given a q-ROFV $G = \langle \mu, v \rangle$, its score and accuracy functions are defined as $SV(G) = \frac{1+\mu^q - v^q}{2}$ and $AV(G) = \mu^q + v^q$ respectively, and*

(i) If $SV(G_1) > SV(G_2)$, then $G_1 \succ G_2$;
(ii) If $SV(G_1) = SV(G_2)$, then their accuracy function should be further compared as follows:
   (a) If $AV(G_1) > AV(G_2)$, then $G_1 \succ G_2$;
   (b) If $AV(G_1) = AV(G_2)$, then $G_1 \sim G_2$.

Definition 5 ([41]). *Let a series of q-ROFVs be $G_t = \langle \mu_t, v_t \rangle (t = 1, 2, \cdots, n)$, where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weight vector, such that $\omega_t \in [0,1]$, $\sum_{t=1}^{n} \omega_t = 1$. The q-rung orthopair fuzzy weighted average (q-ROFWA) operator is then defined as:*

$$\mathrm{q-ROFWA}(G_1, G_2, \cdots, G_n) = \mathop{\oplus}_{t=1}^{n} \omega_t G_t = \omega_1 G_1 \oplus \omega_2 G_2 \oplus \cdots \oplus \omega_n G_n. \tag{3}$$

2.2. Representation of Trust Relationships

In light of the limitations of crisp value, IFV, and PFV in expressing fuzzy information, this study proposes a trust q-rung orthopair fuzzy value (Tq-ROFV) to represent the complex trust relationships, defined as follows:

Definition 6. *The Tq-ROFV refers to a q-ROFV $G = \langle \mu_G, v_G \rangle$ to represent the trust relationships between experts in a social network, where μ_G represents the membership grade, i.e., the trust degree. v_G represents the non-membership grade, i.e., the distrust degree. $\pi_G = \sqrt[q]{(1 - (\mu_G)^q - (v_G)^q)}$ then represents the uncertainty of trust.*

It follows that when $q = 1$, the Tq-ROFV degenerates into the trust IF value (TIFV) [44].

Definition 7. *Given a Tq-ROFV $G = \langle \mu, v \rangle$, its trust score is defined as: $TS = \frac{\mu^q - v^q + 1}{2}$, where $0 \le TS \le 1$.*

2.3. Q-ROF Trust Network and Trust Propagation Operator

A trust network is composed of nodes and directed edges, where in the GDM process, nodes represent experts, and the directed edges represent the trust relationships between experts. Therefore, the trust network based on GDM can be regarded as being composed of expert sets and trust relationships. In this study, the q-ROFV is introduced into the trust network, and a q-ROF trust network is constructed, a representation of which is shown in Figure 1.

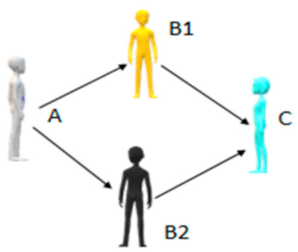

Figure 1. Trust network graph.

In Figure 1, A, B1, B2, and C denote expert nodes, and the directed arrows denote directed edges. For example, A $\rightarrow$ B1 indicates that there is a trust relationship between expert A and expert B1, which is represented by a q-ROFV. It can be seen that there is no directed arrow directly connecting expert A with expert C, but there are directed arrows directly connecting expert A with experts B1 and B2, and experts B1 and B2 with expert C. We call the trust relationships between experts who are directly connected as direct trust, and those who are not directly connected (but indirectly connected through other experts) as indirect trust. In other words, trust relationships between experts can be divided into direct and indirect trust [45].

As shown in Figure 1, there is no direct trust relationship between expert A and expert C, but there is an indirect trust relationship; that is, it is propagated through experts B1 and B2, so we need to calculate the trust relationship between expert A and expert C. Considering the principle that trust does not increase and distrust does not decrease in the process of trust propagation [46], and based on the multiplication operation of q-ROFS, we propose a new trust propagation operator to calculate the indirect trust among experts.

Definition 8. *Let $e_0, e_1, e_2, \cdots, e_{n-1}, e_n$ be the $n+1$ experts, and $G_1, G_2, \cdots, G_{n-1}, G_n$ be the n Tq-ROFVs. The trust evaluation value of expert e_0 to expert e_1 is represented by a Tq-ROFV G_1, the trust evaluation value of expert e_1 to expert e_2 is represented by a Tq-ROFV G_2, and so on. It follows that the trust evaluation value of expert e_{n-1} to expert e_n is expressed by a Tq-ROFV G_n, and the trust evaluation value of expert e_0 to expert e_n is expressed as:*

$$\dot{q}(e_0, e_1, \cdots e_{n-1}, e_n) = G_1 \otimes G_2 \otimes \cdots \otimes G_n. \tag{4}$$

In particular, when $n = 2$, we have $G_1 \otimes G_2 = \left(\mu_1\mu_2, \sqrt[q]{v_1^q + v_2^q - v_1^q v_2^q}\right)$. When $n = 3$, $G_1 \otimes G_2 \otimes G_3 = \left(\mu_1\mu_2\mu_3, \sqrt[q]{1 + \left(v_1^q - 1\right)\left(v_2^q - 1\right)\left(v_3^q - 1\right)}\right)$. Thus, according to the recursive method, we can obtain:

$$\dot{q}(e_0, e_1, \cdots e_{n-1}, e_n) = G_1 \otimes G_2 \otimes \cdots \otimes G_n = \left(\prod_{t=1}^{n} \mu_t, \sqrt[q]{1 + (-1)^{n-1} \prod_{t=1}^{n} \left(v_t^q - 1\right)}\right). \tag{5}$$

It can be proved that $\prod_{t=1}^{n} \mu_t \le \min(\mu_t)$, and $\sqrt[q]{1 + (-1)^{n-1} \prod_{t=1}^{n} \left(v_t^q - 1\right)} \ge \max(v_t)$, i.e., the designed trust propagation operator $\dot{q}$ satisfies the principle that trust does not increase and distrust does not decrease in the process of trust propagation.

Some special cases of the trust propagation operator can be achieved as follows:

(1) When all $G_t = (1, 0)$, then $G_1 \otimes G_2 \otimes \cdots \otimes G_n = (1, 0)$.

(2) If $\exists G_t = (0, 1)$, then $G_1 \otimes G_2 \otimes \cdots \otimes G_n = (0, 1)$. This shows that as long as there is a complete distrust relationship on the propagation path, the final result after propagation is complete distrust, regardless of the other trust relationships.

(3) If $G_1 = G_2 = G_3 = (0.8, 0.2)$, without loss of generality, suppose that $q = 1$, then $G_1 \otimes G_2 \otimes G_3 = (0.51, 0.49)$. The trust value after trust propagation is $0.51 < 0.8$, and the distrust value is $0.49 > 0.2$, which means that the principle of decreasing trust and increasing distrust is satisfied in the process of trust propagation.

(4) If $G_1 = (0.2, 0.8)$, $G_2 = (0.8, 0.2)$, $G_3 = (0.8, 0.1)$, and suppose that $q = 1$, then $G_1 \otimes G_2 \otimes G_3 = (0.13, 0.86)$. Although the trust values of G_2 and G_3 are very high, the trust value of G_1 is low (only 0.2), and so the final trust value after propagation is also low.

The above special cases also show that the trust propagation operator based on the q-ROF multiplication operation is reasonable.

2.4. Trust Aggregation Operator Based on q-ROFS

In the trust network shown in Figure 1, there are two propagation paths for the indirect trust of expert A to expert C. Therefore, it is necessary to integrate the trust relationships of these two propagation paths in order to obtain the final trust evaluation value of expert A with expert C. This study uses the following path weighting method:

$$\mathrm{T} = \sum_{i=1}^{n} w_i G_i = w_1 G_1 \oplus w_2 G_2 \oplus \cdots \oplus w_n G_n, \tag{6}$$

where G_i is a Tq-ROFV, and T is a Tq-ROF weighted average operator. $w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)$ is the path weight, and i is the i-th path. In order from shortest to longest, the shorter the path is, the closer to the front it is, and the longer the path, the further back it is. The path with the larger trust score is at the front when the path lengths are the same. The linguistic quantifiers $Q(r) = r^a (a \geq 0)$, and $Q : [0,1] \to [0,1]$, $Q(0) = 0$, $Q(1) = 1$. Among them, $a = 0.5$ is the fuzzy quantifier for "most" [34].

Example 1. *By the end of 2019, with the emergence of COVID-19, most universities had to terminate their teaching tasks offline. According to the principle of "stopping classes without stopping teaching, and stopping classes without stopping school", universities changed from a traditional offline teaching mode to an online teaching mode. The evaluation of the classroom teaching quality of large-scale online teaching is beneficial for teachers to realize a deficiency in the teaching process, and to improve the quality of teaching. This study evaluates the online teaching quality of teachers in four colleges and universities* $\{Z_1, Z_2, Z_3, Z_4\}$ *during the pandemic. Based on the analysis of the teaching quality evaluation index system in the literature* [7,10,47]*, and by considering the three factors of before class, during class, and after class, this study introduces five evaluation indices to comprehensively evaluate the online teaching quality of teachers in colleges and universities: the stability of teaching platforms* C_1*, the pertinence of teaching resources* C_2*, the timeliness of answering questions and feedback* C_3*, the strictness of teaching attitude* C_4*, the rationality of teaching content* C_5*, and their corresponding index weights* $W = (0.20, 0.25, 0.10, 0.15, 0.30)^T$*. The five attributes are described as follows:*

Stability of teaching platforms C_1: Whether the online teaching platforms used by schools, such as Tencent Classroom and Zoom, are stable, e.g., whether they are stuck, etc.

Pertinence of teaching resources C_2: Whether the PPT courseware and teaching materials provided by teachers before the class are aimed at teaching content in the class, i.e., whether there are differences between the two.

Timeliness of answering questions and feedback C_3: Whether the teacher solves the students' questions in time during and after class.

Strictness of teaching attitude C_4: Whether the teacher has rational and scientific teaching, e.g., whether the words and actions are appropriate, whether the preparation is sufficient, etc.

Rationality of teaching content C_5: Whether the content taught in the class is based mainly on basic knowledge and supplemented by difficult knowledge.

Taking a major of four universities as an example to evaluate the online teaching quality of teachers, we consulted some of the students who had the highest academic achievements in this major as experts. Therefore, the evaluation problem can be regarded as an SNGDM problem. There are five experts $\{e_1, e_2, e_3, e_4, e_5\}$ with a social matrix T_L based on the trust relationships between them. The listed trust values indicate that there is direct trust among the experts, and the unlisted trust value indicates that there is no direct trust between the experts.

$$T_L = \begin{pmatrix} - & (0.6, 0.2) & & & (0.7, 0.1) \\ & - & & (0.6, 0.1) & \\ (0.6, 0.1) & (0.8, 0.1) & - & (0.7, 0.2) & \\ & & (0.5, 0.4) & - & (0.6, 0.3) \\ & & (0.5, 0.2) & & - \end{pmatrix}.$$

In this study, we complete the missing values of the trust evaluation T_L based on the proposed trust propagation operator $\overset{\bullet}{q}$. Taking the missing value of trust evaluation between experts e_1 and e_3 as an example, for the sake of generality, we take $q = 1$. There is no direct trust between experts e_1 and e_3, but there are three indirect trust propagation paths, namely $l_1 : e_1 \to e_5 \to e_3$, $l_2 : e_1 \to e_2 \to e_4 \to e_3$ and $l_3 : e_1 \to e_2 \to e_4 \to e_5 \to e_3$. Considering the importance of each path trust information, this study uses the above path weighting method to solve the indirect trust between experts e_1 and e_3, namely:

$$\overset{\bullet}{q}^{l_1}((0.7, 0.1), (0.5, 0.2)) = (0.35, 0.28),$$
$$\overset{\bullet}{q}^{l_2}((0.6, 0.2), (0.6, 0.1), (0.5, 0.4)) = (0.18, 0.57),$$
$$\overset{\bullet}{q}^{l_3}((0.6, 0.2), (0.6, 0.1), (0.6, 0.3), (0.5, 0.2)) = (0.11, 0.60),$$

where the weights of three paths are given by $w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)$, and $w_1 = 0.58$, $w_2 = 0.24, w_3 = 0.18$.

Therefore, according to Equation (6), the indirect trust of expert e_1 to expert e_3 is:

$$\mathrm{T}(e_1 \to e_3) = 0.58 \cdot (0.35, 0.28) \oplus 0.24 \cdot (0.18, 0.57) \oplus 0.18 \cdot (0.11, 0.60) = (0.27, 0.38).$$

Other missing trust evaluation values can be calculated similarly, so the final social matrix T_L is:

$$T_L = \begin{pmatrix} - & (0.60, 0.20) & (0.27, 0.38) & (0.30, 0.33) & (0.70, 0.10) \\ (0.16, 0.52) & - & (0.27, 0.47) & (0.60, 0.10) & (0.30, 0.42) \\ (0.60, 0.10) & (0.80, 0.10) & - & (0.70, 0.20) & (0.40, 0.27) \\ (0.27, 0.47) & (0.30, 0.50) & (0.50, 0.40) & - & (0.60, 0.30) \\ (0.30, 0.28) & (0.34, 0.32) & (0.50, 0.20) & (0.29, 0.38) & - \end{pmatrix}.$$

Simultaneously, the trust score matrix TS_{hk} can be obtained according to the matrix T_L:

$$TS_{hk} = \begin{pmatrix} - & 0.70 & 0.45 & 0.49 & 0.80 \\ 0.32 & - & 0.40 & 0.75 & 0.44 \\ 0.75 & 0.85 & - & 0.75 & 0.57 \\ 0.40 & 0.40 & 0.55 & - & 0.65 \\ 0.51 & 0.51 & 0.65 & 0.46 & - \end{pmatrix}.$$

Therefore, according to the matrix TS_{hk} and equation $TS_k = \frac{1}{n-1}\sum_{h=1}^{n} TS_{hk}$, the trust scores TS_k of each expert can be obtained, thus:

$$TS_1 = 0.495,\ TS_2 = 0.615,\ TS_3 = 0.513,\ TS_4 = 0.613,\ TS_5 = 0.615.$$

3. Results

3.1. Q-ROF Aggregation Operator Based on Trust Scores and Confidence Levels of Experts

To integrate the evaluation information of the experts, we propose a new q-rung orthopair fuzzy induced ordered weighted average (q-ROFIOWA) operator based on the trust scores and confidence levels (CL) of experts.

Definition 9. *Let a series of q-ROFVs be* $G_t = \langle \mu_t, \nu_t \rangle\,(t = 1, 2, \cdots, n)$, *where* $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ *is the weight vector, and* $\omega_t \in [0, 1]$, $\sum_{t=1}^{n} \omega_t = 1$. *The q-ROFIOWA operator is then defined as:*

$$\mathrm{q-ROFIOWA}\left(\left(TL^1, G_1\right), \left(TL^2, G_2\right), \cdots, (TL^n, G_n)\right) = \overset{n}{\underset{t=1}{\oplus}} \omega_t G_{\sigma(t)} = \omega_1 G_{\sigma(1)} \oplus \omega_2 G_{\sigma(2)} \oplus \cdots \oplus \omega_n G_{\sigma(n)}, \quad (7)$$

where $G_{\sigma(t)}$ *is arranged by* $TL^{\sigma(t)}$ *from largest to smallest, and* $TL^{\sigma(t-1)} \geq TL^{\sigma(t)}$.

Suppose that there are n experts $e_t\,(t=1,2,\cdots,n)$, m alternatives $Z_i\,(i=1,2,\cdots,m)$, s attributes $C_j\,(j=1,2,\cdots,s)$ and the experts' evaluation matrix is $L_t\,(t=1,2,\cdots,n)=\left(G_{ij}^t\right)_{m\times s}$, then the integrated q-ROF matrix is $L^{TL}=\left(G_{ij}^{TL}\right)=\left(\mu_{ij}^{TL},v_{ij}^{TL}\right)$, where μ_{ij}^{TL}, v_{ij}^{TL}, TL are calculated by the following equations, respectively:

$$\mu_{ij}^{TL}=\varphi_{\omega}^{TL}\left(\left(TL^1,\mu_{ij}^1\right),\cdots,\left(TL^n,\mu_{ij}^n\right)\right)=\sqrt[q]{1-\prod_{t=1}^{n}\left(1-\mu_{ij\sigma(t)}^q\right)^{\omega_t}},\tag{8}$$

$$v_{ij}^{TL}=\psi_{\omega}^{TL}\left(\left(TL^1,v_{ij}^1\right),\cdots,\left(TL^n,v_{ij}^n\right)\right)=\prod_{t=1}^{n}v_{ij\sigma(t)}^{\omega_t},\tag{9}$$

$$TL=\eta TS+(1-\eta)CL,\tag{10}$$

where $\omega_{\sigma(t-1)}\geq\omega_{\sigma(t)}\,(t=2,\cdots,n)$, and

$$\omega_t=Q\left(\frac{TL(\sigma(t))}{TL(\sigma(n))}\right)-Q\left(\frac{TL(\sigma(t-1))}{TL(\sigma(n))}\right).\tag{11}$$

The above TL solution also needs to calculate the CL of experts, which is characterized by attributes, alternatives, and experts.

Level 1. Attribute level: the confidence level of the expert e_t on the alternative Z_i under attribute C_j is:

$$CL_{ij}^t=1-\pi_{ij}.\tag{12}$$

where $\pi_{ij}=\sqrt[q]{\left(1-\left(\mu_{ij}\right)^q-\left(v_{ij}\right)^q\right)}$.

Level 2. Alternative level: the confidence level of the expert e_t on the alternative Z_i is:

$$CL_i^t=\frac{1}{s}\sum_{j=1}^{s}CL_{ij}^t.\tag{13}$$

Level 3. Expert level: the confidence level of the expert e_t is:

$$CL^t=\frac{1}{m}\sum_{i=1}^{m}CL_i^t.\tag{14}$$

Thus, the confidence level of the expert e_t is:

$$CL^t=\frac{1}{ms}\sum_{i=1}^{m}\sum_{j=1}^{s}CL_{ij}^t.\tag{15}$$

From Equation (12), we can see that the confidence level of the expert e_t is based on the grade of hesitation when the expert e_t evaluates information. When $\pi_{ij}=0$ and $CL_{ij}^t=1$, there is no hesitation. When the q value increases, the hesitation grade increases, and the corresponding confidence level of the expert e_t decreases.

Example 2 (Continuation of Example 1). *Suppose that for teachers of a particular major in four universities, there are five experts' evaluation matrices:*

$$A^1 = \begin{pmatrix} [0.7, 0.1] & [0.3, 0.5] & [0.4, 0.5] & [0.5, 0.3] & [0.5, 0.1] \\ [0.5, 0.4] & [0.4, 0.2] & [0.6, 0.1] & [0.6, 0.2] & [0.5, 0.2] \\ [0.4, 0.3] & [0.5, 0.3] & [0.4, 0.2] & [0.5, 0.3] & [0.4, 0.4] \\ [0.6, 0.4] & [0.3, 0.4] & [0.3, 0.6] & [0.6, 0.2] & [0.3, 0.6] \end{pmatrix}$$

$$A^2 = \begin{pmatrix} [0.3, 0.4] & [0.5, 0.2] & [0.4, 0.6] & [0.7, 0.1] & [0.8, 0.1] \\ [0.7, 0.2] & [0.6, 0.4] & [0.3, 0.5] & [0.4, 0.3] & [0.5, 0.2] \\ [0.5, 0.1] & [0.3, 0.6] & [0.8, 0.2] & [0.6, 0.3] & [0.5, 0.3] \\ [0.4, 0.3] & [0.7, 0.2] & [0.6, 0.3] & [0.3, 0.4] & [0.9, 0.1] \end{pmatrix}$$

$$A^3 = \begin{pmatrix} [0.5, 0.2] & [0.6, 0.1] & [0.3, 0.5] & [0.4, 0.1] & [0.4, 0.5] \\ [0.8, 0.1] & [0.3, 0.2] & [0.6, 0.2] & [0.5, 0.3] & [0.5, 0.2] \\ [0.4, 0.2] & [0.2, 0.5] & [0.5, 0.4] & [0.3, 0.4] & [0.6, 0.1] \\ [0.7, 0.1] & [0.5, 0.3] & [0.4, 0.2] & [0.8, 0.1] & [0.3, 0.3] \end{pmatrix}$$

$$A^4 = \begin{pmatrix} [0.5, 0.4] & [0.4, 0.3] & [0.3, 0.6] & [0.2, 0.4] & [0.5, 0.4] \\ [0.3, 0.5] & [0.5, 0.2] & [0.2, 0.5] & [0.7, 0.1] & [0.5, 0.3] \\ [0.6, 0.2] & [0.7, 0.1] & [0.4, 0.5] & [0.3, 0.6] & [0.7, 0.2] \\ [0.4, 0.2] & [0.6, 0.1] & [0.5, 0.3] & [0.7, 0.2] & [0.5, 0.2] \end{pmatrix}$$

$$A^5 = \begin{pmatrix} [0.5, 0.3] & [0.6, 0.2] & [0.3, 0.5] & [0.7, 0.1] & [0.5, 0.4] \\ [0.6, 0.3] & [0.5, 0.2] & [0.4, 0.6] & [0.8, 0.2] & [0.7, 0.2] \\ [0.4, 0.5] & [0.8, 0.1] & [0.5, 0.5] & [0.6, 0.2] & [0.4, 0.5] \\ [0.3, 0.4] & [0.7, 0.1] & [0.6, 0.3] & [0.4, 0.4] & [0.5, 0.1] \end{pmatrix}$$

Then the corresponding confidence levels of experts are:

$$CL^1 = 0.78,\, CL^2 = 0.83,\, CL^3 = 0.73,\, CL^4 = 0.79,\, CL^5 = 0.85.$$

By combining the above TS and CL, and setting $\eta = 0.5$, TL can be obtained:

$$TL_1 = 0.64,\, TL_2 = 0.72,\, TL_3 = 0.62,\, TL_4 = 0.70,\, TL_5 = 0.73.$$

Therefore, the weights of the experts can be obtained according to Equation (11):

$$\omega_1 = 0.111,\, \omega_2 = 0.189,\, \omega_3 = 0.095,\, \omega_4 = 0.142,\, \omega_5 = 0.463.$$

Combined with Equation (7), the comprehensive evaluation matrix $\overline{A}$ can be given as:

$$\overline{A} = \begin{pmatrix} (0.497, 0.281) & (0.530, 0.220) & (0.332, 0.531) & (0.610, 0.138) & (0.572, 0.270) \\ (0.606, 0.278) & (0.495, 0.228) & (0.408, 0.417) & (0.693, 0.203) & (0.605, 0.212) \\ (0.453, 0.281) & (0.661, 0.185) & (0.560, 0.372) & (0.532, 0.282) & (0.495, 0.334) \\ (0.423, 0.301) & (0.641, 0.148) & (0.543, 0.312) & (0.518, 0.294) & (0.605, 0.149) \end{pmatrix}.$$

3.2. CIM and Decision-Making Analysis Based on Q-ROF with the Paragraphs

Considering the influence of the CIM on the final results, this study proposes a CIM based on q-ROF distance measure and trust relationships between experts, including consistency measurement, identification of inconsistent expert decision-making opinions, and a personalized adjustment mechanism (based on direct trust relationships between experts).

3.2.1. Calculation of Consistency Degree

The consistency index is divided into three levels:

Level 1. Attribute level: the consistency degree of the expert e_t on the alternative Z_i under attribute C_j is:

$$CE_{ij}^t = 1 - d(G_{ij}^t, \overline{G}_{ij}) = 1 - \frac{1}{2}\left(\left|\left(\mu_{ij}^t\right)^q - \left(\overline{\mu}_{ij}\right)^q\right| + \left|\left(v_{ij}^t\right)^q - (\overline{v}_{ij})^q\right| + \left|\left(\pi_{ij}^t\right)^q - (\overline{\pi}_{ij})^q\right|\right). \tag{16}$$

Level 2. Alternative level: the consistency degree of the expert e_t on the alternative Z_i is:

$$CA_i^t = \frac{1}{s}\sum_{j=1}^{s} CE_{ij}^t. \tag{17}$$

Level 3. Expert level: the consistency degree of the expert e_t is:

$$CI^t = \frac{1}{m}\sum_{i=1}^{m} CA_i^t. \tag{18}$$

Example 3 (Continuation of Example 2). *The evaluation matrix for each expert is known. According to Equations (16)–(18), the degree of consistency at the attribute level is calculated as:*

$$CE^1 = \begin{pmatrix} 0.797 & 0.720 & 0.932 & 0.838 & 0.758 \\ 0.878 & 0.877 & 0.683 & 0.904 & 0.883 \\ 0.947 & 0.839 & 0.669 & 0.968 & 0.906 \\ 0.724 & 0.660 & 0.712 & 0.906 & 0.549 \end{pmatrix}, CE^2 = \begin{pmatrix} 0.804 & 0.951 & 0.863 & 0.910 & 0.772 \\ 0.906 & 0.723 & 0.892 & 0.707 & 0.883 \\ 0.820 & 0.585 & 0.760 & 0.914 & 0.966 \\ 0.976 & 0.887 & 0.943 & 0.782 & 0.705 \end{pmatrix},$$

$$CE^3 = \begin{pmatrix} 0.919 & 0.880 & 0.937 & 0.752 & 0.770 \\ 0.806 & 0.777 & 0.783 & 0.807 & 0.883 \\ 0.867 & 0.539 & 0.940 & 0.768 & 0.766 \\ 0.723 & 0.848 & 0.745 & 0.718 & 0.695 \end{pmatrix}, CE^4 = \begin{pmatrix} 0.878 & 0.870 & 0.931 & 0.590 & 0.870 \\ 0.694 & 0.972 & 0.792 & 0.897 & 0.895 \\ 0.853 & 0.915 & 0.840 & 0.682 & 0.795 \\ 0.876 & 0.913 & 0.945 & 0.818 & 0.895 \end{pmatrix},$$

$$CE^5 = \begin{pmatrix} 0.978 & 0.930 & 0.937 & 0.910 & 0.870 \\ 0.978 & 0.972 & 0.817 & 0.893 & 0.905 \\ 0.781 & 0.861 & 0.872 & 0.918 & 0.834 \\ 0.877 & 0.940 & 0.943 & 0.882 & 0.846 \end{pmatrix}.$$

The degree of consistency at the alternative level:

$$CA^1 = (0.809, 0.845, 0.866, 0.710),\ CA^2 = (0.860, 0.822, 0.809, 0.859),$$
$$CA^3 = (0.852, 0.811, 0.776, 0.746),\ CA^4 = (0.828, 0.850, 0.817, 0.889),$$
$$CA^5 = (0.925, 0.913, 0.853, 0.898).$$

And the consistency at the expert level:

$$CI^1 = 0.807, CI^2 = 0.837, CI^3 = 0.796, CI^4 = 0.846, CI^5 = 0.897.$$

3.2.2. Identification of Inconsistent Expert Decision-Making Opinions and a Personalized Adjustment Mechanism

After obtaining the consistency degree of the above three levels, we need to identify the decision-making information of inconsistent experts and then modify their corresponding decision-making opinions. According to the literature [34], a three-level recognition method is proposed:

Level 1. Determine all experts whose consistency is lower than the consensus threshold δ, which is defined as:

$$EXPCH = \left\{t \middle| CI^t < \delta\right\}.$$

Level 2. On the basis of obtaining all inconsistent experts, determine all alternatives whose consistency is lower than the consensus threshold δ, which is defined as:

$$ALT = \left\{(t,i) \middle| t \in EXPCH \wedge CA_i^t < \delta\right\}.$$

Level 3. Determine all attribute evaluation information with a degree of consistency lower than the consensus threshold δ:

$$APS = \left\{(t,i,j)\middle|(t,i) \in ALT \wedge CE_{ij}^t < \delta\right\}.$$

As mentioned above, the experts who need to adjust their opinions are more willing to trust the decision-making opinions of the experts who have a direct trust relationship with them. Therefore, this study modifies the expert's decision-making opinions according to the following adjustment advice:

$$GG_{ij}^t = (1-\vartheta)\cdot G_{ij}^t + \vartheta \cdot \widetilde{G}, \tag{19}$$

where $\widetilde{G} = \left(\widetilde{\mu}_{ij}, \widetilde{v}_{ij}\right)$, $\widetilde{\mu}_{ij} = \frac{1}{l}\sum_{r=1}^{l} \widetilde{\mu}_{ij}^r$, $\widetilde{v}_{ij} = \frac{1}{l}\sum_{r=1}^{l} \widetilde{v}_{ij}^r$, and $r = 1,2,\cdots,l$ means the expert e_t has l directly trusted experts.

Example 4 (Continuation of Example 3). *Assuming $\delta = 0.8$, the expert e_3 needs to be adjusted. The specific evaluation opinions that need to be adjusted at the attribute level are:*

$$G_{32}^3, G_{34}^3, G_{35}^3, G_{41}^3, G_{43}^3, G_{44}^3, G_{45}^3.$$

Let $\vartheta = 0.5$, then the opinions of the expert e_3 are modified to become:
$G_{32}^3 = (0.39, 0.36)$, $G_{34}^3 = (0.40, 0.39)$, $G_{35}^3 = (0.58, 0.17)$, $G_{41}^3 = (0.60, 0.17)$,
$G_{43}^3 = (0.44, 0.27)$, $G_{44}^3 = (0.70, 0.16)$, $G_{45}^3 = (0.52, 0.26)$.

3.2.3. Decision-Making Analysis after Reaching Consensus (Alternatives Ranking)

After adjusting the evaluation opinions, the new consistency degrees of the experts are calculated as follows:

$$CI^1 = 0.806,\ CI^2 = 0.838,\ CI^3 = 0.836,\ CI^4 = 0.844,\ CI^5 = 0.900.$$

The corresponding weights of experts are:

$$\omega_1 = 0.110,\ \omega_2 = 0.189,\ \omega_3 = 0.097,\ \omega_4 = 0.142,\ \omega_5 = 0.462.$$

Therefore, the adjusted aggregation evaluation matrix is:

$$\overline{A} = \begin{pmatrix} (0.496, 0.281) & (0.530, 0.219) & (0.332, 0.531) & (0.610, 0.137) & (0.572, 0.270) \\ (0.607, 0.277) & (0.495, 0.228) & (0.408, 0.417) & (0.692, 0.204) & (0.605, 0.212) \\ (0.453, 0.280) & (0.669, 0.179) & (0.560, 0.372) & (0.538, 0.282) & (0.493, 0.350) \\ (0.408, 0.316) & (0.640, 0.148) & (0.546, 0.321) & (0.499, 0.307) & (0.619, 0.147) \end{pmatrix}.$$

According to Equation (3) and attribute weights, we can get

$$G_{Z_1} = [0.533, 0.250],\ G_{Z_2} = [0.579, 0.242],\ G_{Z_3} = [0.550, 0.276],\ G_{Z_4} = [0.565, 0.207].$$

The trust scores are thus calculated as:

$$TS_1 = 0.642,\ TS_2 = 0.669,\ TS_3 = 0.637,\ TS_4 = 0.679.$$

Therefore, the ranking of alternatives is: $Z_4 \succ Z_2 \succ Z_1 \succ Z_3$.

In summary, the specific steps of the proposed SNGDM method based on q-ROFS are as follows.

Step 1: Based on the proposed trust propagation and aggregation operators in Equations (4) and (6), the missing q-ROF trust social matrix is completed, and the corresponding trust score matrix and the trust score of each expert have been calculated.

Step 2: Calculate the confidence levels of experts based on Equations (12)–(15) and calculate the weights of experts combined with the trust scores of experts. Based on Equation (7), a comprehensive decision-making matrix is obtained by integrating the expert evaluation matrices.

Step 3: Calculate the consistency degree of the three levels based on Equations (16)–(18). If the consistency of all experts is greater than the threshold, we skip to Step 4. Otherwise, identify the inconsistent experts and their decision-making opinions, adjust the inconsistent decision-making opinions according to Equation (19), and then return to Step 2.

Step 4: Calculate the evaluation value of each alternative based on Equation (3) and attribute weight and sort the alternatives according to Definition 4.

Accordingly, the SNGDM method can be represented by the following flow chart (Figure 2):

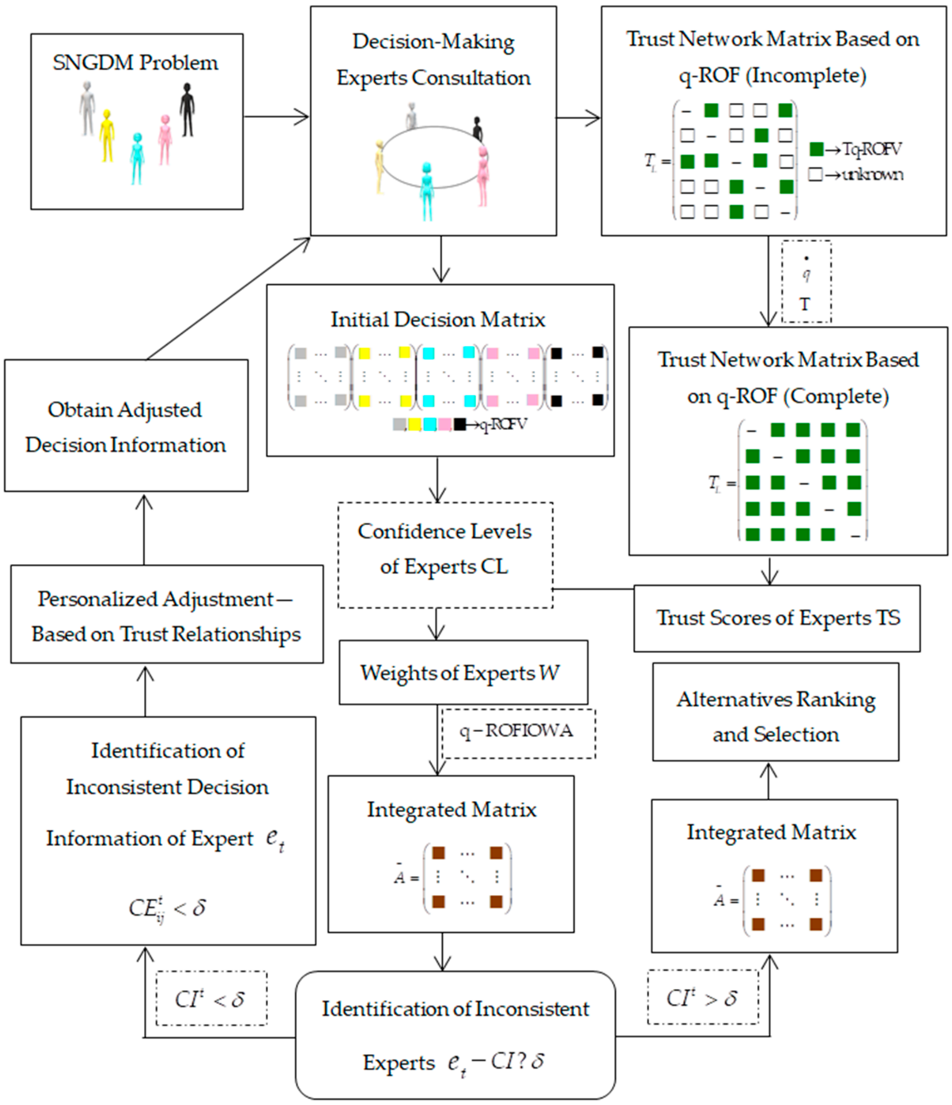

Figure 2. Framework of SNGDM method based on q-ROFS.

3.2.4. Analysis of the Influence of the Parameter q on Alternative Ranking

In this study, we consider the influence of the change in parameter q on the ranking of alternatives and take $q = 1, 2, 3, 5, 10$ and 20, respectively. The trust scores and ranking results of the alternatives are presented in Table 1; it can be seen that with the increase in the parameter q, the trust score TS of each alternative Z_i presents a decreasing trend. Moreover, when q takes different values, the ranking results of the alternatives are not the same. Specifically, when $q = 1$, the ranking of the alternatives is $Z_4 \succ Z_2 \succ Z_1 \succ Z_3$; when $q = 2, 3$ and 5, the ranking of the alternatives is $Z_4 \succ Z_1 \succ Z_2 \succ Z_3$; when $q = 10$ and 20, the ranking of the alternatives is $Z_4 \succ Z_1 \succ Z_3 \succ Z_2$. Therefore, the value of q has a definite influence on the trust scores and ranking of the alternatives. However, regardless of how q changes, the optimal alternative does not change, that is, the alternative Z_4, which indicates that the operator q-ROFIOWA is relatively stable. At the same time, the decision-making experts can choose an appropriate value of q to express their preferences according to the actual situation, which highlights the flexibility of the proposed method.

Table 1. Alternative ranking under parameter q.

q	TS	Ranking
1	$TS_1 = 0.642, TS_2 = 0.669$ $TS_3 = 0.637, TS_4 = 0.679$	$Z_4 \succ Z_2 \succ Z_1 \succ Z_3$
2	$TS_1 = 0.638, TS_2 = 0.628$ $TS_3 = 0.613, TS_4 = 0.686$	$Z_4 \succ Z_1 \succ Z_2 \succ Z_3$
3	$TS_1 = 0.596, TS_2 = 0.588$ $TS_3 = 0.579, TS_4 = 0.643$	$Z_4 \succ Z_1 \succ Z_2 \succ Z_3$
5	$TS_1 = 0.545, TS_2 = 0.538$ $TS_3 = 0.536, TS_4 = 0.584$	$Z_4 \succ Z_1 \succ Z_2 \succ Z_3$
10	$TS_1 = 0.510, TS_2 = 0.506$ $TS_3 = 0.507, TS_4 = 0.532$	$Z_4 \succ Z_1 \succ Z_3 \succ Z_2$
20	$TS_1 = 0.501, TS_2 = 0.500$ $TS_3 = 0.501, TS_4 = 0.509$	$Z_4 \succ Z_1 \succ Z_3 \succ Z_2$

Note: when $q = 20$, $TS_1 = TS_3$, and $H_1 > H_3$, so $Z_1 \succ Z_3$.

3.3. Comparative Analysis

3.3.1. Feasibility Analysis

It can be seen in Table 2 that the ranking results of the existing methods are not the same as the method proposed in this study. For example, based on IFVs and Frank operators, Zheng and Xu [24] proposed a new trust propagation operator that contains an IF trust weighted average (IFTWA) operator and length (L-IFTWA) operator. The final result was the same as that of the proposed method ($q = 1$), but different from that obtained by $q = 2$ and 10. Wu et al. [30] used representable uninorm U to propagate trust, and the final result was the same as that obtained by the proposed method ($q = 2$), but different from that obtained by $q = 1$ and 10. Although the ranking of alternatives is different, the optimal alternative is Z_4, which shows that the proposed method is feasible.

Table 2. Different methods of alternative ranking.

Methods	Ranking
IFTWA/L-IFTWA [24]	$Z_4 \succ Z_2 \succ Z_1 \succ Z_3$
Uninorm [30]	$Z_4 \succ Z_1 \succ Z_2 \succ Z_3$
q-ROFIOWA [the proposed method, q = 1]	$Z_4 \succ Z_2 \succ Z_1 \succ Z_3$
q-ROFIOWA [the proposed method, q = 2]	$Z_4 \succ Z_1 \succ Z_2 \succ Z_3$
q-ROFIOWA [the proposed method, q = 10]	$Z_4 \succ Z_1 \succ Z_3 \succ Z_2$

3.3.2. Superiority Analysis

To demonstrate the advantages of the proposed method, we carry out a detailed comparative analysis using the methods in literatures [24,30]; the results are given in Table 3.

Table 3. Comparative analysis.

Methods	Method of Literature [24]	Method of Literature [30]	The Proposed Method
Problems Solved	SNGDM	SNGDM	SNGDM
The Representation of Trust	IFVs	Trust decision space	q-ROFVs
Trust Paopagation (Diminishing Trust)	Frank operator (Decrease)	Uninorm (Increase)	$\dot{q}$ (Decrease)
Trust Aggregation	L-IFTWA	Shortest path	T
Experts' Weights	Centrality theory	Q(TS)	Q(TS + CL)
CIM	No considered	No considered	Considered

(1) Comparison with the method proposed in [24]. First, the condition $0 \leq \mu + v \leq 1$ must always be maintained in the range of IFVs expressing the trust relationship, but in reality, the situation of $\mu + v > 1$ often appears. This study proposes a Tq-ROFV, which uses q-ROFV to represent the trust relationships among experts and express trust information over a wider range. Naturally, the TIFV is a special case of the Tq-ROFV; i.e., when $q = 1$, the Tq-ROFV degenerates into the TIFV.

Second, when integrating the trust information of multiple propagation paths, Zheng and Xu [24] make full use of each path, but it falls short when the lengths of the paths are the same and the weights are the same. When the lengths of the paths are the same, the greater the trust score, and the greater the weight. Therefore, the method for determination of the experts' weights based on a Q quantifier in this study is more reasonable. Moreover, the method proposed in [24] does not consider the impact of the CIM on the decision-making results.

(2) Comparison with the method proposed in [30]. First, the distrust values of the previous trust relationships are not considered in [30], and only the distrust value of the last trust relationship is considered. Simultaneously, the values of trust increase during the trust propagation process. For example, if there are two direct trust relationships $G_1 = (0.55, 0.35)$ and $G_2 = (0.75, 0.20)$ on the propagation path, then the U operator is used for trust propagation, and we get:

$$U(G_1, G_2) = \left(\frac{0.55 \times 0.75}{0.55 \times 0.75 + 0.45 \times 0.25}, \frac{0.55 \times 0.20}{0.55 \times 0.20 + 0.45 \times 0.80} \right) = (0.79, 0.23).$$

It is clear that the trust value after propagation is $0.79 > 0.75$, $0.79 > 0.55$; the trust value increases, which does not conform to the principle of trust diminishing in the process of trust propagation. Moreover, the distrust value of G_1 is not used in the process of trust propagation, and only the distrust value of G_2 is considered. This study proposes a trust propagation operator based on q-ROF multiplication operation to compensate for this defect, that is, (assuming $q = 1$):

$$q(G_1, G_2) = (0.55 \times 0.75, 0.35 + 0.20 - 0.35 \times 0.20) = (0.41, 0.48).$$

Given $0.41 < 0.75$, $0.41 < 0.55$ and $0.48 > 0.35$, $0.48 > 0.20$, the trust relationship after propagation satisfies the principles of diminishing trust and increasing distrust. At the same time, the distrust value of each trust relationship was considered.

Second, when integrating the trust information of multiple paths, Wu et al. [30] directly selects the trust relationship of the shortest path as the ultimate indirect trust relationship, ignoring the trust information of other paths. However, in this study, we use the trust information of each path, taking into account and assigning the corresponding weights. The shorter the path, the greater the weight, and the longer the path, the smaller the weight, and when the path length is the same, the greater the trust score, the greater the weight of the path, and so the path weighting method in this study is more reasonable.

Third, Wu et al. [30] use the TS as the guidance to solve the weights of experts, and the importance of experts' confidence level in the process of GDM is ignored. This study not only considers the trust score TS, but also considers the confidence level CL of experts, and further enables solving for the weights of experts through the combination of the two.

Fourth, considering the difference in professional backgrounds among experts, it is difficult to reach an agreement on the initial opinion, and a consensus needs to be reached through mutual coordination and adjustment of opinion. Therefore, considering the importance of consensus building, this study proposes a CIM based on the q-ROF distance measure and trust relationship, whereas the literature [30] does not consider the CIM.

4. Discussion and Conclusions

At present, there are many GDM methods for teaching quality evaluation [1,4,5,9,10], but most of them do not consider the influence of trust networks among experts on the evaluation results. This study extends the SNA method to the GDM method and proposes a new SNGDM method based on q-ROFS for the evaluation of online teaching quality, which makes the evaluation results more reasonable. At the same time, considering the subjectivity, fuzziness, and uncertainty of trust and the vagueness of evaluation indices, this study uses q-ROFV to describe them in a more comprehensive and detailed manner. Thus, decision-makers can choose the appropriate value of the parameter q to evaluate the indices according to the actual situation in order to minimize the loss of information. The example presented in this study shows that the alternative ranking of different methods varies greatly, but the optimal alternative remains unchanged. Additionally, a comparison with the existing SNGDM methods in [24] and [30] verifies that the proposed method is more advantageous and feasible. Specifically, the advantages and contributions of the proposed SNGDM method can be summarized as follows:

First, this study combines q-ROF theory with the SNA method and proposes a Tq-ROFV to express the trust relationships between experts, which can describe the subjectivity, ambiguity, and uncertainty of trust more comprehensively. Second, a trust propagation operator based on the q-ROF multiplication operator is proposed to obtain an indirect trust between experts, which fully considers the principle of diminishing trust values in the process of trust propagation. In view of the flexibility of the trust propagation operator (including varying parameter q), decision-makers can choose the appropriate parameters according to the actual situation, and the greater the value of q, the greater the uncertainty. Moreover, considering the existence of multiple trust paths, a trust aggregation operator based on q-ROFS is proposed to integrate the trust relationships of multiple paths. Additionally, to integrate the evaluation information of experts, a q-ROFIOWA operator is proposed, which considers the confidence levels of experts when evaluating the information. Finally, given the impact of the CIM and the trust relationships among experts on decision-making results, a CIM based on q-ROF distance measure and trust relationships among experts is introduced.

The SNGDM method not only provides a new method for the evaluation of online teaching quality, but also enables teachers to discover issues in the teaching process in time and make improvements. However, the SNGDM method has some limitations. First, the evaluation information of experts in the process of evaluating online teaching quality is known, and there is no missing evaluation information. In fact, some or all experts involved in the evaluation process may not have a thorough understanding of the issues involved, so it is often impossible to give all the evaluation information. Therefore, we will consider the situation in which evaluation information is missing in future work. Second, the SNGDM method is implemented in a multi-attribute environment, and whether it can be extended to other decision-making environments remains to be seen, such as fuzzy preference relationships [22,48–50]. At the same time, it is yet to be determined as to whether the SNGDM method can be extended to solve evaluation decision-making problems in other industries, such as venture capital evaluation, green supply chain management, e-learning course selection, selection of the best substitutes for biopesticides, site assessment, digitalization in logistics and retail, and weapon selection decisions [27,51–57]. Third, when adjusting expert opinions, this study takes the decision-making opinions of trusted experts as a simple weighted average. In fact, among all trusted experts, inconsistent decision-makers have different trust levels for different experts. That is, when the expert's opinions are adjusted, the adjustment of weights should be different for different levels of trust, which is our future avenue of research.

Author Contributions: Conceptualization, Y.H. and S.Z.; Funding acquisition, S.Z.; Investigation, Y.H., S.Z. and Y.Y.; Methodology, Y.H., S.Z., L.-A.C. and K.U.; Project administration, Y.H., S.Z., and K.U.; Software, Y.H., S.Z., L.-A.C., K.U. and Y.Y.; Validation, Y.H., S.Z. and Y.Y.; Writing—original draft, Y.H., and Y.Y.; Writing—review and editing, S.Z., L.-A.C., and K.U. All authors have read and agreed to the published version of the manuscript.

Funding: This work is supported by the Social Sciences Planning Projects of Zhejiang (21QNYC11ZD), Statistical Scientific Key Research Project of China (2021LZ33), Major Humanities and Social Sciences Research Projects in Zhejiang Universities (2018QN058), Fundamental Research Funds for the Provincial Universities of Zhejiang (SJWZ2020002), Longyuan Construction Financial Research Project of Ningbo University (LYYB2002), Statistical Scientifific Key Research Project of China and the First Class Discipline of Zhejiang-A (Zhejiang Gongshang University-Statistics).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors are extremely grateful for the editorial team's valuable comments on improving the quality of this article.

Conflicts of Interest: The authors declare no conflict of interest.

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