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Article

The Cranks for 5-Core Partitions

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Abstract: It is well known that the number of 5-core partitions of $5^k n + 5^k - 1$ is a multiple of 5^k. In [1] a statistic called a crank was developed to sort the 5-core partitions of $5n + 4$ and $25n + 24$ into 5 and 25 classes of equal size, respectively. In this paper we will develop the cranks that can be used to sort the 5-core partitions of $5^k n + 5^k - 1$ into 5^k classes of equal size.

Keywords: partitions; 5-cores; cranks

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1. Introduction

A t-core partition of n is a partition of n that contains no hook numbers that are multiples of t [2, 2.7.40]. The generating function for t-core partitions is given by $\sum_{\substack{\vec{n}\times\vec{1}=0 \\ \vec{n}\in Z^t}} q^{\frac{t}{2}\|\vec{n}\|+\vec{b}\times\vec{n}}$ where the vector $\vec{1}=(1,1,\ldots,1)$ in Z^t and $\vec{b}=(0,1,\ldots,t-1)$ [3]. In [3] Garvan, Stanton, and Kim showed that the statistic $4n_0 + n_1 + n_3 + 4n_4$ (mod 5), where the n_i's are the components of the vector in the generating function for 5-cores, can be used to sort the 5-cores of $5n + 4$ into 5 classes of equal size. In a sequel to this paper [1] Garvan explicitly describes a crank for the 5-cores of $25n + 24$. In this paper a crank for the 5-cores of $5^k n + 5^k - 1$ will be given using techniques similar to those used by Garvan, Stanton, and Kim.

2. Description of the Crank

For ease of working with the vector $\vec{n}$ we will write it as (*a*, *b*, *c*, *d*, *e*). Using the fact that *a* + *b* + *c* + *d* + *e* = 0, the exponent on *q* in the generating function for the 5-core partitions can be expressed as

$$G(a,b,c,d) = 5a^2 + 5b^2 + 5c^2 + 5d^2 + 5ab + 5ac + 5ad + 5bc + 5bd + 5cd - 4a - 3b - 2c - d \quad (1)$$

Thus the 5-cores of integers of the form 5*n* + 4 are associated with the values of *a*, *b*, *c*, and *d* satisfying $-4a - 3b - 2c - d \equiv 4 (\text{mod } 5)$. Evaluating *G*(*a*, *b*, *c*, *d*) with *a* = *A* − *C* − 2*D*, *b* = −2*A* + *B* − *C* + *D*, *c* = −*B* + 4*C* − *D*, *d* = 2*A* − *B* − *C* + 2*D* + 1, we get an expression in *A*, *B*, *C*, and *D* which we will label as *H*(*A*, *B*, *C*, *D*).

$$H(A,B,C,D) = 25A^2 + 10B^2 + 50C^2 + 25D^2 - 25AB - 25BC - 25CD + 15A - 10B + 15D + 4 \quad (2)$$

Note that *A*, *B*, *C*, and *D* are integers since

$$(A,B,C,D) = ((a,b,c,d) - \gamma)T^{-1} = \left(3a + b + c + 6M, -4a - 5b - 3c - 3d + 3, 2a + b + c + 4M, M\right) \quad (3)$$

where $T = \begin{pmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -2 & 1 & -1 & 2 \end{pmatrix}$, $\gamma = (0,0,0,1)$, and $M = \dfrac{-4a - 3b - 2c - d + 1}{5}$.

Theorem 1.1

The 5-core partitions of 5*n* + 4 corresponding to the vectors (*A*, *B*, *C*, *D*) can be sorted into 5 classes of equal size by looking at the values of *B* modulo 5.

To see this, let (*A*, *B*, *C*, *D*) = $\left(\left(A, \dfrac{B-m}{5}, C, D\right) - \lambda_m\right) U_m$ where $B \equiv m \ (\text{mod } 5)$ and

$$\lambda_0 = (0,0,0,0) \qquad U_0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix} \qquad U_0^{-1} = \begin{pmatrix} 2 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ -3 & -1 & -2 & -1 \\ 2 & 1 & 1 & 1 \end{pmatrix} \quad (4)$$

$$\lambda_1 = (0,0,0,0) \qquad U_1 = \begin{pmatrix} 1 & 0 & -1 & 0 \\ -4 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 \\ 0 & -1 & 0 & 0 \end{pmatrix} \qquad U_1^{-1} = \begin{pmatrix} -2 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 \\ -3 & -1 & -1 & -1 \\ -5 & -2 & -3 & -2 \end{pmatrix} \quad (5)$$

$$\lambda_2 = (-2,-1,-1,-1) \qquad U_2 = \begin{pmatrix} 0 & 1 & 0 & -1 \\ 1 & -4 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{pmatrix} \qquad U_2^{-1} = \begin{pmatrix} 5 & 2 & 3 & 2 \\ 3 & 1 & 2 & 1 \\ 5 & 2 & 3 & 1 \\ 2 & 1 & 2 & 1 \end{pmatrix} \quad (6)$$

$$\lambda_3 = (1,0,1,0) \qquad U_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & -4 & 1 \\ -1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \qquad U_3^{-1} = \begin{pmatrix} -2 & -1 & -2 & -1 \\ 3 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 \end{pmatrix} \tag{7}$$

$$\lambda_4 = (2,0,1,0) \qquad U_4 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & -1 \\ 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & 0 \end{pmatrix} \qquad U_4^{-1} = \begin{pmatrix} -3 & -1 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ -3 & -1 & -1 & -1 \\ 2 & 1 & 2 & 1 \end{pmatrix} \tag{8}$$

Note that *A*, *B*, *C*, and *D* are integers and for each of these changes of variable *H*(*A*, *B*, *C*, *D*) becomes 5*G*(*A*, *B*, *C*, *D*) + 4. Hence *G*(*A*, *B*, *C*, *D*) = *n* and for each solution of this equation we have 5 solutions (*A*, *B*, *C*, *D*) of *H*(*A*, *B*, *C*, *D*) = 5*n* + 4, one with $B \equiv m \pmod 5$ for each choice of *m* = 0, 1, 2, 3, 4, which can be transformed to a solution (*a*, *b*, *c*, *d*) of *G*(*a*, *b*, *c*, *d*) = 5*n* + 4. This completes the proof of the theorem.

Theorem 1.2

The 5-core partitions of $5^k n + 5^k - 1$ can be sorted into 5^k classes of equal size.

From the proof of Theorem 1.1 we can transform a solution of *G*(*a*, *b*, *c*, *d*) = *n* into 5 solutions of *G*(*a*, *b*, *c*, *d*) = 5*n* + 4. Each solution of *G*(*a*, *b*, *c*, *d*) = 5n + 4 can be transformed into 5 solutions of *G*(*a*, *b*, *c*, *d*) = 25*n* + 24. Iterating this process *k* times we easily see that a solution of *G*(*a*, *b*, *c*, *d*) = *n* can be transformed into 5^k solutions of *G*(*a*, *b*, *c*, *d*) = $5^k n + 5^k - 1$. At each stage in the transformation process we can keep track of the congruence class modulo 5 of *B* to get a *k*-tuple of values *m* (mod 5) associated with each solution of *G*(*a*, *b*, *c*, *d*) = $5^k n + 5^k - 1$. These *k*-tuples can be used to sort the solutions of *G*(*a*, *b*, *c*, *d*) = $5^k n + 5^k - 1$ into 5^k classes of equal size.

3. An Illustration of the Crank

The following series of Tables 1–3 show the 2 solutions of *G*(*a*, *b*, *c*, *d*) = 2 transformed into 250 solutions of *G*(*a*, *b*, *c*, *d*) = 374. The intermediate solutions of *H*(*A*, *B*, *C*, *D*) are shown in order to easily see the classes of *B* (mod 5) which can be used to sort these 250 solutions into 125 classes of equal size.

Table 1. Solutions corresponding to 5-cores of 14.

Solutions of ***G*(*a, b, c, d*) = 2**	**Solutions of** ***H*(*A, B, C, D*) = 14**	**Solutions of** ***G*(*a, b, c, d*) = 14**	**Congruence class of** ***B* (mod 5)**
(0, 1, 0, 0)	(−1, 0, 0, 0)	(−1, 2, 0, −1)	0
	(0, 1, 0, −1)	(2, 0, 0, −2)	1
	(1, 2, 1, 0)	(0, −1, 2, 0)	2
	(4, 8, 2, 1)	(0, −1, −1, 1)	3
	(1, 4, 1, 0)	(0, 1, 0, −2)	4
(1, 0, 0, −1)	(0, 0, 0, −1)	(2, −1, 1, −1)	0
	(3, 6, 2, 1)	(−1, −1, 1, 1)	1
	(1, 2, 0, 0)	(1, 0, −2, 1)	2
	(−4, −7, −2, −1)	(0, 2, 0, 0)	3
	(−3, −6, −2, −1)	(1, 1, −1, 1)	4

Table 2. Solutions corresponding to 5-cores of 74.

Solutions of ***G*(*a, b, c, d*) = 14**	**Solutions of** ***H*(*A, B, C, D*) = 74**	**Solutions of** ***G*(*a, b, c, d*) = 74**	**2-Tuples showing congruence classes of *B*'s mod 5**
(−1, 2, 0, −1)	(−6, −10, −2, −1)	(−2, 3, 3, −1)	(0, 0)
	(7, 16, 4, 1)	(1, −1, −1, −3)	(0, 1)
	(−3, −8, −2, −2)	(3, −2, 2, 1)	(0, 2)
	(6, 13, 4, 3)	(−4, 0, 0, 2)	(0, 3)
	(1, 4, 0, −1)	(3, 1, −3, −3)	(0, 4)
(2, 0, 0, −2)	(0, 0, 0, −2)	(4, −2, 2, −3)	(1, 0)
	(6, 11, 4, 2)	(−2, −3, 3, 2)	(1, 1)
	(4, 7, 1, 1)	(1, −1, −4, 3)	(1, 2)
	(−9, −17, −5, −2)	(0, 4, −1, 1)	(1, 3)
	(−8, −16, −5, −2)	(1, 3, −2, 2)	(1, 4)
(0, −1, 2, 0)	(−5, −10, −4, −2)	(3, 2, −4, 1)	(2, 0)
	(−6, −9, −2, −1)	(−2, 4, 2, −2)	(2, 1)
	(5, 12, 3, 0)	(2, −1, 0, −4)	(2, 2)
	(0, −2, 0, −1)	(2, −3, 3, 1)	(2, 3)
	(−3, −6, −1, −2)	(2, −1, 4, −2)	(2, 4)
(0, −1, −1, 1)	(6, 10, 3, 2)	(−1, −3, 0, 4)	(3, 0)
	(−2, −4, −2, 0)	(0, 2, −4, 3)	(3, 1)
	(−8, −13, −4, −2)	(0, 5, −1, −2)	(3, 2)
	(0, 3, 1, −1)	(1, 1, 2, −5)	(3, 3)
	(8, 14, 4, 2)	(0, −4, 0, 3)	(3, 4)
(0, 1, 0, −2)	(−5, −10, −2, −2)	(1, 0, 4, −1)	(4, 0)
	(10, 21, 6, 3)	(−2, −2, 0, 0)	(4, 1)
	(−3, −8, −3, −2)	(4, −1, −2, 2)	(4, 2)
	(−2, −2, 0, 1)	(−4, 3, 1, 1)	(4, 3)
	(−3, −6, −3, −2)	(4, 1, −4, 0)	(4, 4)

Table 2. *Cont.*

(2, −1, 1, −1)	(0, 0, −1, −2)	(5, −1, −2, −2)	(0, 0)
	(−2, −4, 0, 0)	(−2, 0, 4, 1)	(0, 1)
	(8, 17, 4, 2)	(0, −1, −3, 0)	(0, 2)
	(−8, −17, −5, −3)	(3, 1, 0, 1)	(0, 3)
	(−8, −16, −4, −2)	(0, 2, 2, 1)	(0, 4)
(−1, −1, 1, 1)	(−2, −5, −2, 0)	(0, 1, −3, 4)	(1, 0)
	(−6, −9, −3, −1)	(−1, 5, −2, −1)	(1, 1)
	(−3, −3, −1, −2)	(2, 2, 1, −5)	(1, 2)
	(4, 8, 3, 0)	(1, −3, 4, −2)	(1, 3)
	(5, 9, 3, 0)	(2, −4, 3, −1)	(1, 4)
(1, 0, −2, 1)	(10, 20, 6, 3)	(−2, −3, 1, 1)	(2, 0)
	(−1, −4, −2, −1)	(3, −1, −3, 3)	(2, 1)
	(−5, −8, −2, 0)	(−3, 4, 0, 1)	(2, 2)
	(0, 3, 0, -1)	(2, 2, −2, −4)	(2, 3)
	(7, 14, 4, 3)	(−3, −1, −1, 3)	(2, 4)
(0, 2, 0, 0)	(−2, 0, 0, 0)	(−2, 4, 0, −3)	(3, 0)
	(0, 1, 0, −2)	(4, −1, 1, −4)	(3, 1)
	(4, 7, 3, 1)	(−1, −3, 4, 1)	(3, 2)
	(7, 13, 3, 2)	(0, −2, −3, 3)	(3, 3)
	(0, 4, 1, 0)	(−1, 3, 0, −4)	(3, 4)
(1, 1, −1, 1)	(6, 15, 4, 2)	(−2, 1, −1, −2)	(4, 0)
	(−4, −9, −3, −3)	(5, −1, 0, −1)	(4, 1)
	(3, 7, 3, 2)	(−4, 0, 3, 1)	(4, 2)
	(4, 8, 1, 0)	(3, −1, −4, 0)	(4, 3)
	(3, 9, 3, 2)	(−4, 2, 1, −1)	(4, 4)

Table 3. Solutions corresponding to 5-cores of 74.

Solutions of $G(a, b, c, d) = 74$	**Solutions of $H(A, B, C, D) = 374$**	**Solutions of $G(a, b, c, d) = 374$**	**3-Tuples showing congruence classes of B's mod 5**
(−2, 3, 3, −1)	(−18, −30, −9, −4)	(−1, 11, −2, −4)	(0, 0, 0)
	(0, 6, 2, −2)	(2, 2, 4, −11)	(0, 0, 1)
	(10, 17, 6, 0)	(4, −9, 7, −2)	(0, 0, 2)
	(14, 23, 7, 5)	(−3, −7, 0, 9)	(0, 0, 3)
	(−6, −6, −2, −4)	(4, 4, 2, −11)	(0, 0, 4)
(1, −1, −1, −3)	(0, −5, 0, −2)	(4, −7, 7, 2)	(0, 1, 0)
	(16, 31, 9, 7)	(−7, −3, −2, 7)	(0, 1, 1)
	(−11, −23, −9, −4)	(6, 4, −9, 3)	(0, 1, 2)
	(−14, −22, −5, −2)	(−5, 9, 4, −4)	(0, 1, 3)
	(−3, −11, −5, −2)	(6, −2, −7, 7)	(0, 1, 4)

Table 3. *Cont.*

(3, −2, 2, 1)	(4, 10, 0, −1)	(6, 1, −9, −3)	(0, 2, 0)
	(−17, −34, −8, −5)	(1, 3, 7, −1)	(0, 2, 1)
	(19, 42, 12, 6)	(−5, −2, 0, −3)	(0, 2, 2)
	(−6, −17, −6, −5)	(10, −4, −2, 2)	(0, 2, 3)
	(−9, −16, −2, −1)	(−5, 3, 9, −1)	(0, 2, 4)
(−4, 0, 0, 2)	(−4, −10, −2, 2)	(−6, 2, 0, 9)	(0, 3, 0)
	(−2, 1, −2, 0)	(0, 7, −9, −2)	(0, 3, 1)
	(−18, −33, −9, −7)	(5, 5, 4, −7)	(0, 3, 2)
	(15, 33, 11, 4)	(−4, −4, 7, −5)	(0, 3, 3)
	(18, 34, 9, 2)	(5, −9, 0, −2)	(0, 3, 4)
(3, 1, −3, −3)	(8, 15, 6, 0)	(2, −7, 9, −4)	(0, 4, 0)
	(18, 31, 9, 5)	(−1, −9, 0, 7)	(0, 4, 1)
	(−5, −13, −5, 0)	(0, 2, −7, 9)	(0, 4, 2)
	(−14, −22, −7, −2)	(−3, 11, −4, −2)	(0, 4, 3)
	(−5, −11, −5, 0)	(0, 4, −9, 7)	(0, 4, 4)
(4, −2, 2, −3)	(−2, −5, −3, −5)	(11, −3, −2, −5)	(1, 0, 0)
	(1, 1, 3, 2)	(−6, −2, 9, 3)	(1, 0, 1)
	(16, 32, 7, 4)	(1, −3, −8, 2)	(1, 0, 2)
	(−20, −42, −12, −6)	(4, 4, 0, 3)	(1, 0, 3)
	(−20, −41, −11, −5)	(1, 5, 2, 3)	(1, 0, 4)
(−2, −3, 3, 2)	(−6, −15, −6, −1)	(2, 2, −8, 8)	(1, 1, 0)
	(−15, −24, −7, −2)	(−4, 11, −2, −2)	(1, 1, 1)
	(−2, 2, 0, −3)	(4, 3, 1, −11)	(1, 1, 2)
	(5, 8, 4, −1)	(3, −7, 9, −3)	(1, 1, 3)
	(6, 9, 4, −1)	(4, −8, 8, −2)	(1, 1, 4)
(1, −1, −4, 3)	(21, 40, 12, 7)	(−5, −7, 1, 5)	(1, 2, 0)
	(−5, −14, −6, −2)	(5, 0, −8, 7)	(1, 2, 1)
	(−14, −23, −6, −1)	(−6, 10, 0, 0)	(1, 2, 2)
	(1, 8, 1, −2)	(4, 3, −2, −10)	(1, 2, 3)
	(18, 34, 10, 7)	(−6, −5, −1, 7)	(1, 2, 4)
(0, 4, −1, 1)	(1, 10, 3, 2)	(−6, 7, 0, −6)	(1, 3, 0)
	(−2, −4, −2, −5)	(10, −3, 1, −7)	(1, 3, 1)
	(7, 12, 6, 3)	(−5, −5, 9, 3)	(1, 3, 2)
	(15, 28, 6, 4)	(1, −4, −8, 5)	(1, 3, 3)
	(3, 14, 4, 2)	(−5, 6, 0, −7)	(1, 3, 4)
(1, 3, −2, 2)	(9, 25, 7, 4)	(−6, 4, −1, −5)	(1, 4, 0)
	(−6, −14, −5, −6)	(11, −3, 0, −4)	(1, 4, 1)
	(6, 12, 6, 4)	(−8, −2, 8, 3)	(1, 4, 2)
	(12, 23, 4, 2)	(4, −3, −9, 2)	(1, 4, 3)
	(6, 19, 6, 4)	(−8, 5, 1, −4)	(1, 4, 4)
(3, 2, −4, 1)	(18, 40, 12, 5)	(−4, −3, 3, −5)	(2, 0, 0)
	(1, −4, −2, −3)	(9, −7, −1, 3)	(2, 0, 1)
	(1, 2, 2, 4)	(−9, 2, 2, 7)	(2, 0, 2)

Table 3. *Cont.*

	(0, 3, −2, −1)	(4, 4, −10, −2)	(2, 0, 3)
	(5, 14, 4, 5)	(−9, 5, −3, 3)	(2, 0, 4)
(−2, 4, 2, −2)	(−18, −30, −8, −4)	(−2, 10, 2, −5)	(2, 1, 0)
	(8, 21, 6, 0)	(2, −1, 3, −10)	(2, 1, 1)
	(6, 7, 3, −1)	(5, −9, 6, 1)	(2, 1, 2)
	(13, 23, 7, 6)	(−6, −4, −1, 9)	(2, 1, 3)
	(−6, −6, −3, −4)	(5, 5, −2, −10)	(2, 1, 4)
(2, −1, 0, −4)	(−3, −10, −2, −4)	(7, −6, 6, −1)	(2, 2, 0)
	(16, 31, 10, 7)	(−8, −4, 2, 6)	(2, 2, 1)
	(−3, −8, −5, −2)	(6, 1, −10, 4)	(2, 2, 2)
	(−18, −32, −8, −3)	(−4, 9, 3, −1)	(2, 2, 3)
	(−11, −26, −9, −4)	(6, 1, −6, 6)	(2, 2, 4)
(2, −3, 3, 1)	(0, 0, −3, −2)	(7, 1, −10, 0)	(2, 3, 0)
	(−18, −34, −8, −4)	(−2, 6, 6, −1)	(2, 3, 1)
	(16, 37, 10, 4)	(−2, −1, −1, −6)	(2, 3, 2)
	(−6, −17, −5, −5)	(9, −5, 2, 1)	(2, 3, 3)
	(−8, −16, −2, −2)	(−2, 0, 10, −1)	(2, 3, 4)
(2, −1, 4, −2)	(−11, −20, −8, −6)	(9, 4, −6, −5)	(2, 4, 0)
	(−6, −9, 0, −1)	(−4, 2, 10, −4)	(2, 4, 1)
	(21, 42, 11, 4)	(2, −7, −2, −2)	(2, 4, 2)
	(−8, −22, −6, −3)	(4, −3, 1, 7)	(2, 4, 3)
	(−19, −36, −9, −6)	(2, 5, 6, −4)	(2, 4, 4)
(−1, −3, 0, 4)	(9, 15, 3, 4)	(−2, −2, −7, 9)	(3, 0, 0)
	(−18, −34, −11, −4)	(1, 9, −6, 2)	(3, 0, 1)
	(−8, −8, −2, −2)	(−2, 8, 2, −9)	(3, 0, 2)
	(6, 13, 4, −2)	(6, −5, 5, −8)	(3, 0, 3)
	(16, 29, 10, 4)	(−2, −9, 7, 2)	(3, 0, 4)
(0, 2, −4, 3)	(16, 35, 11, 7)	(−9, −1, 2, 1)	(3, 1, 0)
	(−3, −9, −5, −4)	(10, −2, −7, 1)	(3, 1, 1)
	(−10, −18, −3, 0)	(−7, 5, 6, 2)	(3, 1, 2)
	(12, 28, 6, 2)	(2, 0, −6, −5)	(3, 1, 3)
	(18, 39, 11, 7)	(−7, −1, −2, 1)	(3, 1, 4)
(0, 5, −1, −2)	(−6, −5, 0, −1)	(−4, 6, 6, −8)	(3, 2, 0)
	(13, 26, 7, 0)	(6, −7, 2, −6)	(3, 2, 1)
	(4, 2, 2, 1)	(0, −7, 5, 7)	(3, 2, 2)
	(9, 18, 4, 5)	(−5, 1, −7, 7)	(3, 2, 3)
	(−4, −1, −2, −1)	(0, 8, −6, −6)	(3, 2, 4)
(1, 1, 2, −5)	(−15, −30, −8, −7)	(7, 1, 5, −5)	(3, 3, 0)
	(17, 36, 12, 6)	(−7, −4, 6, −1)	(3, 3, 1)

Table 3. *Cont.*

	(6, 7, 0, −1)	(8, −6, −6, 4)	(3, 3, 2)
	(−11, −22, −5, 0)	(−6, 5, 2, 6)	(3, 3, 3)
	(−18, −36, −12, −7)	(8, 5, −5, −1)	(3, 3, 4)
(0, −4, 0, 3)	(10, 15, 3, 3)	(1, −5, −6, 9)	(3, 4, 0)
	(−15, −29, −9, −2)	(−2, 8, −5, 5)	(3, 4, 1)
	(−8, −8, −3, −2)	(−1, 9, −2, −8)	(3, 4, 2)
	(−2, −2, 0, −4)	(6, −2, 6, −9)	(3, 4, 3)
	(12, 19, 7, 3)	(−1, −9, 6, 5)	(3, 4, 4)
(1, 0, 4, −1)	(−12, −20, −8, −5)	(6, 7, −7, −5)	(4, 0, 0)
	(−9, −14, −2, −3)	(−1, 3, 9, −7)	(4, 0, 1)
	(21, 42, 12, 4)	(1, −8, 2, −3)	(4, 0, 2)
	(0, −7, −2, −1)	(4, −6, 0, 8)	(4, 0, 3)
	(−15, −26, −6, −5)	(1, 5, 7, −7)	(4, 0, 4)
(−2, −2, 0, 0)	(−2, −10, −2, 0)	(0, −4, 2, 9)	(4, 1, 0)
	(4, 11, 2, 4)	(−6, 5, −7, 4)	(4, 1, 1)
	(−18, −33, −11, −7)	(7, 7, −4, −5)	(4, 1, 2)
	(−1, 3, 3, 0)	(−4, 2, 9, −7)	(4, 1, 3)
	(10, 14, 3, 0)	(7, −9, −2, 4)	(4, 1, 4)
(4, −1, −2, 2)	(19, 40, 10, 4)	(1, −4, −4, −3)	(4, 2, 0)
	(−12, −29, −8, −5)	(6, −2, 2, 4)	(4, 2, 1)
	(9, 22, 7, 6)	(−10, 3, 0, 2)	(4, 2, 2)
	(−6, −12, −6, −5)	(10, 1, −7, −3)	(4, 2, 3)
	(1, 4, 3, 4)	(−10, 3, 4, 4)	(4, 2, 4)
(−4, 3, 1, 1)	(−12, −20, −5, 0)	(−7, 9, 0, 2)	(4, 3, 0)
	(0, 6, 0, −2)	(4, 4, −4, −9)	(4, 3, 1)
	(−6, −13, −2, −4)	(4, −3, 9, −4)	(4, 3, 2)
	(22, 43, 13, 7)	(−5, −7, 2, 3)	(4, 3, 3)
	(10, 24, 6, 0)	(4, −2, 0, −9)	(4, 3, 4)
(4, 1, −4, 0)	(19, 40, 12, 4)	(−1, −6, 4, −5)	(4, 4, 0)
	(4, 1, 0, −1)	(6, −8, 0, 6)	(4, 4, 1)
	(1, 2, 1, 4)	(−8, 3, −2, 8)	(4, 4, 2)
	(−8, −12, −6, −3)	(4, 7, −9, −3)	(4, 4, 3)
	(1, 4, 1, 4)	(−8, 5, −4, 6)	(4, 4, 4)
(5, −1, −2, −2)	(13, 25, 7, 0)	(6, −8, 3, −5)	(0, 0, 0)
	(6, 6, 3, 2)	(−1, −7, 4, 8)	(0, 0, 1)
	(6, 12, 2, 4)	(−4, 2, −8, 7)	(0, 0, 2)
	(−20, −37, −12, −6)	(4, 9, −5, −2)	(0, 0, 3)
	(−10, −21, −6, 0)	(−4, 5, −3, 8)	(0, 0, 4)

Table 3. *Cont.*

(−2, 0, 4, 1)	(−14, −25, −9, −3)	(1, 9, −8, 1)	(0, 1, 0)
	(−13, 19, −5, −4)	(0, 8, 3, −9)	(0, 1, 1)
	(10, 22, 7, 0)	(3, −5, 6, −8)	(0, 1, 2)
	(12, 18, 6, 2)	(2, −10, 4, 5)	(0, 1, 3)
	(–2, –1, 1, –3)	(3, –1, 8, –9)	(0, 1, 4)
(0, −1, −3, 0)	(10, 15, 6, 3)	(−2, −8, 6, 6)	(0, 2, 0)
	(9, 16, 3, 4)	(−2, −1, −8, 8)	(0, 2, 1)
	(−20, −38, −12, −5)	(2, 9, −5, 1)	(0, 2, 2)
	(−5, −2, 0, −1)	(−3, 7, 3, −9)	(0, 2, 3)
	(12, 19, 4, 3)	(2, −6, −6, 8)	(0, 2, 4)
(3, 1, 0, 1)	(7, 20, 4, 1)	(1, 3, −5, −7)	(0, 3, 0)
	(−11, −24, −6, −6)	(7, −2, 6, −3)	(0, 3, 1)
	(18, 37, 12, 7)	(−8, −4, 4, 2)	(0, 3, 2)
	(1, −2, −3, −2)	(8, −3, −8, 4)	(0, 3, 3)
	(−6, −6, 0, 1)	(−8, 7, 5, −3)	(0, 3, 4)
(0, 2, 2, 1)	(−6, −5, −3, −1)	(−1, 9, −6, −5)	(0, 4, 0)
	(−11, −19, −5, −6)	(6, 2, 5, −9)	(0, 4, 1)
	(16, 32, 11, 4)	(−3, −7, 8, −2)	(0, 4, 2)
	(12, 18, 4, 2)	(4, −8, −4, 7)	(0, 4, 3)
	(−4, −1, 1, −1)	(−3, 5, 6, −9)	(0, 4, 4)
(0, 1, −3, 4)	(16, 35, 10, 7)	(−8, 0, −2, 2)	(1, 0, 0)
	(−11, −24, −9, −6)	(10, 1, −6, 0)	(1, 0, 1)
	(−6, −8, 0, 1)	(−8, 5, 7, −1)	(1, 0, 2)
	(13, 28, 6, 1)	(5, −3, −5, −5)	(1, 0, 3)
	(18, 39, 12, 7)	(−8, −2, 2, 0)	(1, 0, 4)
(−1, 5, −2, −1)	(−3, 0, 2, 1)	(−7, 5, 7, −5)	(1, 1, 0)
	(13, 26, 6, 0)	(7, −6, −2, −5)	(1, 1, 1)
	(−4, −13, −2, −1)	(0, −4, 6, 6)	(1, 1, 2)
	(13, 28, 7, 6)	(−6, 1, −6, 4)	(1, 1, 3)
	(4, 14, 2, 1)	(0, 5, −7, −5)	(1, 1, 4)
(2, 2, 1, −5)	(−11, −20, −5, −6)	(6, 1, 6, −8)	(1, 2, 0)
	(18, 36, 12, 5)	(−4, −7, 7, −1)	(1, 2, 1)
	(9, 12, 2, 1)	(5, −7, −5, 7)	(1, 2, 2)
	(−11, −22, −6, 0)	(−5, 6, −2, 7)	(1, 2, 3)
	(−19, −36, −12, −6)	(5, 8, −6, −1)	(1, 2, 4)
(1, −3, 4, −2)	(−11, −25, −9, −6)	(10, 0, −5, 1)	(1, 3, 0)
	(−4, −4, 1, 2)	(−9, 5, 6, 0)	(1, 3, 1)
	(10, 22, 4, 0)	(6, −2, −6, −5)	(1, 3, 2)
	(−12, −27, −6, −4)	(2, −1, 7, 2)	(1, 3, 3)
	(−14, −31, −8, −6)	(6, −1, 5, 0)	(1, 3, 4)

Table 3. *Cont.*

(2, −4, 3, −1)	(−3, −10, −5, −4)	(10, −3, −6, 2)	(1, 4, 0)
	(−8, −14, −2, 1)	(−8, 5, 5, 3)	(1, 4, 1)
	(9, 22, 4, 1)	(3, 1, −7, −5)	(1, 4, 2)
	(−15, −32, −8, −6)	(5, 0, 6, −1)	(1, 4, 3)
	(−11, −26, −6, −4)	(3, −2, 6, 3)	(1, 4, 4)
(−2, −3, 1, 1)	(−2, −10, −3, 0)	(1, −3, −2, 10)	(2, 0, 0)
	(−4, −4, −2, 2)	(−6, 8, −6, 3)	(2, 0, 1)
	(−14, −23, −8, −6)	(6, 7, −3, −8)	(2, 0, 2)
	(0, 3, 3, −1)	(−1, −1, 10, −7)	(2, 0, 3)
	(10, 14, 4, 0)	(6, −10, 2, 3)	(2, 0, 4)
(3, −1, −3, 3)	(22, 45, 12, 6)	(−2, −5, −3, 0)	(2, 1, 0)
	(−12, −29, −9, −5)	(7, −1, −2, 5)	(2, 1, 1)
	(1, 7, 3, 4)	(−10, 6, 1, 1)	(2, 1, 2)
	(−2, −2, −3, −4)	(9, 1, −6, −6)	(2, 1, 3)
	(9, 19, 7, 6)	(−10, 0, 3, 5)	(2, 1, 4)
(−3, 4, 0, 1)	(−8, −10, −2, 1)	(−8, 9, 1, −1)	(2, 2, 0)
	(1, 6, 0, −3)	(7, 1, −3, −9)	(2, 2, 1)
	(−3, −8, 0, −2)	(1, −4, 10, −1)	(2, 2, 2)
	(22, 43, 12, 7)	(−4, −6, −2, 4)	(2, 2, 3)
	(9, 24, 6, 1)	(1, 1, −1, −9)	(2, 2, 4)
(2, 2, −2, −4)	(0, 0, 2, −2)	(2, −4, 10, −5)	(2, 3, 0)
	(22, 41, 12, 6)	(−2, −9, 1, 4)	(2, 3, 1)
	(−4, −13, −5, −1)	(3, −1, −6, 9)	(2, 3, 2)
	(−11, −17, −5, 0)	(−6, 10, −3, 1)	(2, 3, 3)
	(−8, −16, −7, −2)	(3, 5, −10, 4)	(2, 3, 4)
(−3, −1, −1, 3)	(4, 5, 2, 4)	(−6, −1, −1, 10)	(2, 4, 0)
	(−6, −9, −5, −1)	(1, 7, −10, 1)	(2, 4, 1)
	(−19, −33, −9, −6)	(2, 8, 3, −7)	(2, 4, 2)
	(12, 28, 9, 2)	(−1, −3, 6, −8)	(2, 4, 3)
	(21, 39, 11, 4)	(2, −10, 1, 1)	(2, 4, 4)
(−2, 4, 0, −3)	(−14, −25, −5, −3)	(−3, 5, 8, −3)	(3, 0, 0)
	(19, 41, 11, 4)	(0, −4, −1, −5)	(3, 0, 1)
	(−6, −18, −5, −4)	(7, −5, 2, 4)	(3, 0, 2)
	(8, 18, 6, 6)	(−10, 2, 0, 5)	(3, 0, 3)
	(−2, −1, −3, −3)	(7, 3, −8, −5)	(3, 0, 4)
(4, −1, 1, −4)	(−2, −5, −2, −5)	(10, −4, 2, −6)	(3, 1, 0)
	(9, 16, 7, 4)	(−6, −5, 8, 4)	(3, 1, 1)
	(12, 22, 4, 3)	(2, −3, −9, 5)	(3, 1, 2)
	(−21, −42, −12, −5)	(1, 7, −1, 3)	(3, 1, 3)
	(−20, −41, −12, −5)	(2, 6, −2, 4)	(3, 1, 4)

Table 3. *Cont.*

(−1, −3, 4, 1)	(−9, −20, −8, −3)	(5, 3, −9, 5)	(3, 2, 0)
	(−15, −24, −6, −2)	(−5, 10, 2, −3)	(3, 2, 1)
	(6, 17, 4, −1)	(4, 0, 0, −10)	(3, 2, 2)
	(1, −2, 1, −2)	(4, −7, 8, 0)	(3, 2, 3)
	(−2, −6, 0, −3)	(4, −5, 9, −3)	(3, 2, 4)
(0, −2, −3, 3)	(17, 30, 9, 6)	(−4, −7, 0, 8)	(3, 3, 0)
	(−6, −14, −6, −1)	(2, 3, −9, 7)	(3, 3, 1)
	(−17, −28, −8, −3)	(−3, 11, −1, −3)	(3, 3, 2)
	(1, 8, 2, −2)	(3, 2, 2, −11)	(3, 3, 3)
	(19, 34, 10, 6)	(−3, −8, 0, 7)	(3, 3, 4)
(−1, 3, 0, −4)	(−13, −25, −5, −4)	(0, 2, 9, −3)	(3, 4, 0)
	(22, 46, 13, 6)	(−3, −5, 0, −2)	(3, 4, 1)
	(−6, −18, −6, −4)	(8, −4, −2, 5)	(3, 4, 2)
	(0, 3, 2, 4)	(−10, 5, 1, 4)	(3, 4, 3)
	(−6, −11, −6, −4)	(8, 3, −9, −2)	(3, 4, 4)
(−2, 1, −1, −2)	(−6, −15, −2, −1)	(−2, −2, 8, 4)	(4, 0, 0)
	(17, 36, 9, 6)	(−4, −1, −6, 2)	(4, 0, 1)
	(−18, −38, −12, −7)	(8, 3, −3, 1)	(4, 0, 2)
	(1, 8, 4, 3)	(−9, 5, 5, −3)	(4, 0, 3)
	(6, 9, 0, −1)	(8, −4, −8, 2)	(4, 0, 4)
(5, −1, 0, −1)	(9, 20, 4, −1)	(7, −3, −3, −7)	(4, 1, 0)
	(−5, −14, −2, −2)	(1, −4, 8, 3)	(4, 1, 1)
	(18, 37, 10, 7)	(−6, −2, −4, 4)	(4, 1, 2)
	(−15, −32, −11, −6)	(8, 3, −6, 2)	(4, 1, 3)
	(−14, −26, −6, −1)	(−6, 7, 3, 3)	(4, 1, 4)
(−4, 0, 3, 1)	(−15, −30, −9, −2)	(−2, 7, −4, 6)	(4, 2, 0)
	(−6, −4, −2, −1)	(−2, 9, −3, −7)	(4, 2, 1)
	(−5, −8, −2, −5)	(7, −1, 5, −9)	(4, 2, 2)
	(15, 28, 10, 4)	(−3, −8, 8, 1)	(4, 2, 3)
	(7, 14, 4, −2)	(7, −6, 4, −7)	(4, 2, 4)
(3, −1, −4, 0)	(19, 35, 11, 4)	(0, −10, 5, 1)	(4, 3, 0)
	(6, 6, 1, 2)	(1, −5, −4, 10)	(4, 3, 1)
	(−10, −18, −6, 0)	(−4, 8, −6, 5)	(4, 3, 2)
	(−12, −17, −6, −4)	(2, 9, −3, −8)	(4, 3, 3)
	(6, 9, 2, 4)	(−4, −1, −5, 10)	(4, 3, 4)
(−4, 2, 1, −1)	(−15, −30, −7, −2)	(−4, 5, 4, 4)	(4, 4, 0)
	(10, 26, 6, 3)	(−2, 3, −5, −5)	(4, 4, 1)
	(−13, −28, −8, −7)	(9, −1, 3, −3)	(4, 4, 2)
	(13, 28, 10, 6)	(−9, −2, 6, 1)	(4, 4, 3)
	(7, 14, 2, −2)	(9, −4, −4, −5)	(4, 4, 4)

4. Conclusion

Though this crank is not explicit like the ones presented by Garvan, Stanton, and Kim, its iterative nature makes it easy to program using a computer algebra system. I used a simple routine in MAPLE to generate the information included in the tables in the previous section.

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