Sensitivity of High-Scale SUSY in Low Energy Hadronic FCNC

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Abstract: We discuss the sensitivity of the high-scale supersymmetry (SUSY) at 10–1000 TeV in \(B^0, B_s, K^0\) and \(D\) meson systems together with the neutron electric dipole moment (EDM) and the mercury EDM. In order to estimate the contribution of the squark flavor mixing to these flavor changing neutral currents (FCNCs), we calculate the squark mass spectrum, which is consistent with the recent Higgs discovery. The SUSY contribution in \(\epsilon_K\) could be large, around 40\% in the region of the SUSY scale 10–100 TeV. The neutron EDM and the mercury EDM are also sensitive to the SUSY contribution induced by the gluino-squark interaction. The predicted EDMs are roughly proportional to \(|\epsilon_{SU}^K|\). If the SUSY contribution is the level of \(\mathcal{O}(10\%)\) for \(\epsilon_K\), the neutron EDM is expected to be discovered in the region of \(10^{-28}–10^{-26}\) ecm. The mercury EDM also gives a strong constraint for the gluino-squark interaction. The SUSY contribution of \(\Delta M_D\) is also discussed.

Keywords: supersymmetry; high-scale supersymmetry (SUSY); flavor changing neutral current (FCNC); neutron electric dipole moment (EDM)

1. Introduction

Supersymmetry (SUSY) is one of the most attractive theories beyond the standard model (SM). Therefore, SUSY has been expected to be observed at the LHC experiments. However, no signals of
SUSY have been discovered yet. The present searches for SUSY particles give us important constraints for SUSY. Since the lower bounds of the superparticle masses increase gradually, the squark and the gluino masses are supposed to be at a higher scale than 1 TeV [1–3]. On the other hand, the SUSY model has been seriously constrained by the Higgs discovery, in which the Higgs mass is 125 GeV [4–6]. Based on this theoretical and experimental situation, we consider the high-scale SUSY models, which have been widely discussed with a great deal of attention [7–22].

If the squark and slepton masses are at the high-scale $\mathcal{O}(10–1000)$ TeV, the lightest Higgs mass can be pushed up to 125 GeV, whereas SUSY particles are out of the reach of the LHC experiment. Therefore, the indirect search of the SUSY particles becomes important in the low-energy flavor physics [23–25].

The flavor physics is also on the a stage in light of LHCb data. The LHCb collaboration has reported new data of the CP violation of the $B_s$ meson and the branching ratios of rare $B_s$ decays [26–38]. For many years, the CP violation in the $K$ and $B^0$ mesons has been successfully understood within the framework of the standard model (SM), the so-called Kobayashi–Maskawa (KM) model [39], where the source of the CP violation is the KM phase in the quark sector with three families. However, a new physics has been expected to be indirectly discovered in the precise data of $B^0$ and $B_s$ meson decays at the LHCb experiment and the further coming experiment, Belle-II.

There are new sources of the CP violation if the SM is extended to the SUSY models. The soft squark mass matrices contain the CP violating phases, which contribute to the flavor changing neutral current (FCNC) with the CP violation [40]. Therefore, we can expect the SUSY effect in the CP violating phenomena. However, the clear deviation from the SM prediction has not been observed yet in the LHCb experiment [26–38]. Actually, we have found that the CP violation of $B^0$ and $B_s$ meson systems are suppressed if the SUSY scale is above 10 TeV [41]. On the other hand, the CKM fitter group presented the current limits on new physics contributions of $\mathcal{O}(10\%)$ in $B^0$, $B_s$ and $K^0$ systems [42]. They have also estimated the sensitivity to new physics in $B^0$ and $B_s$ mixing achievable with $50 \text{ab}^{-1}$ of Belle-II and $50 \text{fb}^{-1}$ of LHCb data. Therefore, we should carefully study the sensitivity of the high-scale SUSY to the hadronic FCNC.

In this work, we discuss the high-scale SUSY contribution to the $B^0$, $B_s$ and $K^0$ meson systems. Furthermore, we also discuss the sensitivity to the $D$ meson and the electric dipole moment (EDM) of the neutron and mercury. For these modes, the most important process of the SUSY contribution is the gluino-squark-mediated flavor changing process [43–58]. The CP violation of the $K$ meson, $\epsilon_K$, provides a severe constraint to the gluino-squark-mediated FCNC [59,60]. In addition, recent work has found that the chromo-electric dipole moment (cEDM) is sensitive to the high-scale SUSY [61]. It is noted that the upper-bound of the neutron EDM (nEDM) [62] gives a severe constraint for the gluino-squark interaction through the cEDM [63–68]. It is also remarked that the upper bound of the mercury EDM (HgEDM) [69] can give an important constraint [70].

In order to estimate the gluino-squark-mediated FCNC of the $K$, $B^0$, $B_s$ and $D$ mesons, we work in the basis of the squark mass eigenstate with the non-minimal squark (slepton) flavor mixing. There are three reasons why the SUSY contribution to the FCNC considerably depends on the squark mass spectrum. The first one is that the GIM mechanism works in the squark flavor mixing, and the second one is that the loop functions depend on the mass ratio of the squark and gluino. The last one is that we need the mixing angle between the left-handed sbottom and right-handed sbottom, which dominates the
\( \Delta B = 1 \) decay processes. Therefore, we discuss the squark mass spectrum, which is consistent with the recent Higgs discovery. Taking the universal soft parameters at the SUSY breaking scale, we obtain the squark mass spectrum at the matching scale where the SM emerges, by using the renormalization group equations (REGs) of the soft masses. On the other hand, the \( 6 \times 6 \) mixing matrix between squarks and quarks is taken to be free at the low energy.

In Section 2, we discuss the squark and gluino mass spectrum and the squark mixing. In Section 3, we present the formulation of the FCNC with \( \Delta F = 2 \) in the \( K, B^0, B_s \) and \( D \) meson systems together with nEDM and HgEDM. We present numerical results and discussions in Section 4. Section 5 is devoted to the summary. The relevant formulations are presented in Appendices A–D.

2. SUSY Spectrum and Squark Mixing

The low-energy FCNCs depend significantly on the spectrum of the SUSY particles, which depend on the model. As is well known, the lightest Higgs mass can be pushed up to 125 GeV if the squark masses are expected to be \( \mathcal{O}(10) \) TeV. Therefore, let us consider the heavy SUSY particle mass spectrum in the framework of the minimal supersymmetric standard model (MSSM), which is consistent with the observed Higgs mass. The discussion of how to obtain the SUSY spectrum has been given in [71,72].

We outline how to obtain the SUSY spectrum in our work. The details are presented in Appendix A. At the SUSY breaking scale \( \Lambda \), we write the quadratic terms in the MSSM potential as:

\[
V_2 = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 \cdot H_2 + h.c.)
\]  

(1)

Then, the Higgs mass parameter \( m^2 \) is expressed in terms of \( m_1^2, m_2^2 \) and \( \tan \beta \) as:

\[
m^2 = \frac{m_1^2 - m_2^2 \tan^2 \beta}{\tan^2 \beta - 1}
\]  

(2)

After running down to the \( Q_0 \) scale, in which the SM emerges, by the one-loop SUSY renormalization group equations (RGEs) [73], the scalar potential is the SM one as follows:

\[
V_{SM} = -m^2 |H|^2 + \frac{\lambda}{2} |H|^4
\]  

(3)

Here, the Higgs coupling \( \lambda \) is given in terms of the SUSY parameters at the leading order as:

\[
\lambda(Q_0) = \frac{1}{4} (g^2 + g'^2) \cos^2 2\beta + \frac{3h_t^2}{8\pi^2} X_t^2 \left( 1 - \frac{X_t^2}{12} \right), \quad X_t = \frac{A_t(Q_0) - \mu(Q_0) \cot \beta}{Q_0}
\]  

(4)

and \( h_t \) is the top Yukawa coupling of the SM. The parameters \( m_2 \) and \( \lambda \) run with the two-loop SM RGEs with the \( \overline{\text{MS}} \) scheme [74–76] down to the electroweak scale \( Q_{EW} = m_H \) and then give:

\[
m_H^2 = 2m^2(m_H) = \lambda(m_H) v^2
\]  

(5)

When \( m_H = 125 \) GeV is placed, \( \lambda(Q_0) \) and \( m^2(Q_0) \) are obtained. This input constrains the SUSY mass spectrum of the MSSM. In our work, we take the universal soft breaking parameters at the SUSY breaking scale \( \Lambda \) as follows:

\[
m_{\tilde{Q}_i}(\Lambda) = m_{\tilde{U}_i}(\Lambda) = m_{\tilde{D}_i}(\Lambda) = m_{\tilde{L}_i}(\Lambda) = m_{\tilde{E}_i}(\Lambda) = m_0^2 (i = 1, 2, 3)
\]

\[
M_1(\Lambda) = M_2(\Lambda) = M_3(\Lambda) = m_{1/2}, \quad m_{11}^2(\Lambda) = m_{22}^2(\Lambda) = m_0^2
\]

\[
A_U(\Lambda) = A_0 y_U(\Lambda), \quad A_D(\Lambda) = A_0 y_D(\Lambda), \quad A_E(\Lambda) = A_0 y_E(\Lambda)
\]  

(6)
By inputting $m_H = 125$ GeV and taking the heavy scalar mass $m_H \simeq Q_0$ (see Appendix A), we can obtain the SUSY spectrum for the fixed $Q_0$ and $\tan \beta$. The details and numerical results are presented in Appendix A.

Let us consider the squark flavor mixing. As discussed above, there is no flavor mixing at $\Lambda$ in the MSSM. However, in order to consider the non-minimal flavor mixing framework, we allow the off-diagonal components of the squark mass matrices at the 10\% level, which leads to the flavor mixing of order 0.1. We take these flavor mixing angles as free parameters at low energies. Now, we consider the $6 \times 6$ squark mass matrix $M_q$ in the super-CKM basis. In order to move the mass eigenstate basis of squark masses, we should diagonalize the mass matrix by rotation matrix $\Gamma_G^{(q)}$ as:

$$m_q^2 = \Gamma_G^{(q)} M_q^2 \Gamma_G^{(q)\dagger}$$

(7)

where $\Gamma_G^{(q)}$ is the $6 \times 6$ unitary matrix, and we decompose it into the $3 \times 6$ matrices as $\Gamma_G^{(q)} = (\Gamma_{GL}^{(q)}, \Gamma_{GR}^{(q)})^T$ in the following expressions:

$$\Gamma_{GL}^{(d)} = \begin{pmatrix}
  c_{13}^L & 0 & s_{13}^L e^{-i \phi_{13} L} c_\theta & 0 & -s_{13}^L e^{-i \phi_{13} L} s_\theta e^{i \phi} \\
-s_{23}^L s_{13}^L e^{i (\phi_{23} - \phi_{13})} c_{23}^L & c_{23}^L & s_{23}^L e^{-i \phi_{23} L} c_\theta & 0 & -s_{23}^L e^{-i \phi_{23} L} s_\theta e^{i \phi} \\
-s_{13}^L c_{23}^L e^{i \phi_{13}} & -s_{23}^L e^{i \phi_{23}} c_\theta & c_{13}^L c_{23}^L c_\theta & 0 & -c_{13}^L c_{23}^L s_\theta e^{i \phi}
\end{pmatrix}$$

$$\Gamma_{GR}^{(d)} = \begin{pmatrix}
  0 & 0 & s_{13}^R s_\theta e^{-i \phi_{13} R} e^{-i \phi} & 0 & -s_{13}^R s_\theta e^{-i \phi_{13} R} s_\theta e^{i \phi} \\
0 & 0 & s_{23}^R c_{13} s_\theta e^{-i \phi_{23} R} e^{-i \phi} & -s_{23}^R s_{13} e^{-i (\phi_{23} - \phi_{13}) R} & c_{23}^R e^{-i \phi_{23} R} \theta
\end{pmatrix}$$

(8)

where we use abbreviations $c_{ij}^{L,R} = \cos \theta_{ij}^{L,R}$, $s_{ij}^{L,R} = \sin \theta_{ij}^{L,R}$, $c_\theta = \cos \theta$ and $s_\theta = \sin \theta$. Here, $\theta$ is the left-right mixing angle between $b_L$ and $b_R$, which is discussed in Appendix A. It is remarked that we take $s_{12}^{L,R} = 0$ due to the degenerate squark masses of the first and second families, as discussed in Appendix A.

The gluino-squark-quark interaction is given as:

$$\mathcal{L}_{\text{int}}(gq\bar{q}) = -i \sqrt{2} g_s \sum_{\langle q \rangle} \bar{q}_i (T^a) \tilde{G}^a \left[ (\Gamma_{GL}^{(q)})_{ij} L + (\Gamma_{GR}^{(q)})_{ij} R \right] q_j + \text{h.c.}$$

(9)

where $L = (1 - \gamma_5)/2$, $R = (1 + \gamma_5)/2$ and $\tilde{G}^a$ denotes the gluino field; $q^i$ are three left-handed ($i = 1, 2, 3$) and three right-handed quarks ($i = 4, 5, 6$). This interaction leads to the gluino-squark-mediated flavor changing processes with $\Delta F = 2$ and $\Delta F = 1$ through the box and penguin diagrams.

The chargino (neutralino)-squark-quark interaction can be also discussed in a similar way.

3. FCNC of $\Delta F = 2$

In our previous work [41], we have probed the high-scale SUSY, which is at the 10–50 TeV scale, in the CP violations of $K$, $B^0$ and $B_s$ mesons. It is found that $\epsilon_K$ is most sensitive to SUSY, even if the SUSY scale is at 50 TeV. The SUSY contributions for the time-dependent CP asymmetries of $B^0$ and $B_s$ with $\Delta B = 1$ are suppressed at the SUSY scale of 10 TeV. Furthermore, the SUSY contribution
for the $b \to s\gamma$ process is also suppressed, since the left-right mixing angle, which induces the chiral enhancement, is very small, as discussed in Appendix A. Therefore, we discuss the neutral meson mixing $P^0 - \bar{P}^0 (P^0 = K, B^0, B_s, D)$, which are FCNCs with $\Delta F = 2$.

In those FCNCs, the dominant SUSY contribution is given through the gluino-squark interaction. Then, the dispersive part of meson mixing $M_{12}^{P^0} (P^0 = K, B^0, B_s)$ is written as:

$$M_{12}^{P^0} = M_{12}^{P^0, SM} + M_{12}^{P^0, SUSY}$$

(10)

where $M_{12}^{P^0, SUSY}$ are given by the squark mixing parameters in Equation (8), and its explicit formulation is given in Appendices B and C.

At first, we discuss the $\Delta B = 2$ process, that is the mass differences $\Delta M_{B^0}$ and $\Delta M_{B_s}$ and the CP-violating phases $\phi_q$ and $\phi_s$. In general, the contribution of the new physics (NP) to the dispersive part $M_{12}^q$ is parameterized as:

$$M_{12}^{B_q} = M_{12}^{q, SM} + M_{12}^{q, NP} = M_{12}^{q, SM} (1 + h_q e^{2i\sigma_q})$$

(11)

where $M_{12}^{q, NP}$ are the NP contributions. The generic fits for $B^0$ and $B_s$ mixing have given the constraints on $(h_q, \sigma_q)$ [42], where it is assumed that the NP does not significantly affect the SM tree-level charged-current interaction, that is the absorptive part $\Gamma_{12}^q$ is dominated by the decay $b \to c\bar{c}s$. At present, the NP contributions $h_q$ are 10%–35% and 15%–25%, depending on $\sigma_q$ for $B^0$ and $B_s$, respectively. Thus, we can expect the sizable NP contribution of $O(20\%)$. We will discuss whether the high-scale SUSY can fill in the magnitude of the present NP contribution of $O(20\%)$.

Next, we discuss the $\Delta S = 2$ process, $\Delta M_{K^{*0}}$ and the CP-violating parameter in the $K$ meson, $\epsilon_K$. By the similar parametrization in Equation (11), the allowed region of $(h_K, \sigma_K)$ has been estimated in [42]. The NP contribution is at least 50%, although there is the strong $\sigma_K$ dependence. Therefore, it is important to examine carefully the CP violating parameter $\epsilon_K$, which is given as follows:

$$\epsilon_K = e^{i\phi_K} \sin \phi_e \left( \frac{\text{Im}(M_{12}^K)}{\Delta M_K} + \xi \right), \quad \xi = \frac{\text{Im}A_{12}^K}{\text{Re}A_{12}^K}, \quad \phi_e = \tan^{-1} \left( \frac{2\Delta M_K}{\Delta \Gamma_K} \right)$$

(12)

with $A_{12}^K$ being the isospin zero amplitude in $K \to \pi\pi$ decays. Here, $M_{12}^K$ is the dispersive part of the $K^{0}\bar{K}^{0}$ mixing, and $\Delta M_K$ is the mass difference in the neutral $K$ meson. The effects of $\xi \neq 0$ and $\phi_e < \pi/4$ give a suppression effect in $\epsilon_K$, and it is parameterized as $\kappa_e$ and estimated by Buras and Guadagnoli [77] as:

$$\kappa_e = 0.92 \pm 0.02$$

(13)

In the SM, the dispersive part $M_{12}^K$ is given as follows,

$$M_{12}^K = \langle K|H_{\Delta F=2}|\bar{K} \rangle$$

$$= -\frac{4}{3} \left( \frac{G_F}{4\pi} \right)^2 M_{\tilde{W}_B}^2 F_K^2 M_K \left( \eta_{cc} \lambda^2_e E(x_c) + \eta_{tt} \lambda^2_t E(x_t) + 2\eta_{cd} \lambda_c \lambda_t E(x_c, x_t) \right)$$

(14)

where $\lambda_c = V_{cs}V_{cd}^*$, $\lambda_t = V_{ts}V_{td}^*$. The $E(x)$’s are the one-loop functions [78], and $\eta_{cc,tt,cd}$ are the QCD corrections [77]. Then, $|\epsilon_{K}^{SM}|$ is given in terms of the Wolfenstein parameters $\lambda, \overline{\rho}$ and $\overline{\eta}$ as follows:

$$|\epsilon_{K}^{SM}| = \kappa_e C_e \tilde{B}_K |V_{cb}|^2 \lambda^2 \overline{\eta} \left( |V_{cb}|^2 (1 - \overline{\rho}) \eta_{tt} E(x_t) - \eta_{cc} E(x_c) + \eta_{cd} E(x_c, x_t) \right)$$

(15)
with
\[ C_\epsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6 \sqrt{2} \pi^2 \Delta M_K} \] (16)

Note that \(|\epsilon_K^{\text{SM}}|\) depends on the non-perturbative parameter \(\hat{B}_K\) in Equation (15). Recently, the error of this parameter shrank dramatically in the lattice calculations [79]. In our calculation, we use the updated value by the flavor Lattice averaging group [80]:
\[ \hat{B}_K = 0.766 \pm 0.010 \] (17)

Let us write down \(\epsilon_K\) as:
\[ \epsilon_K = \epsilon_K^{\text{SM}} + \epsilon_K^{\text{SUSY}} \] (18)

where \(\epsilon_K^{\text{SUSY}}\) is induced by the imaginary part of the gluino-squark box diagram, which is presented in Appendices B and C. Since \(s_{12}^{L(R)}\) vanishes in our scheme, \(\epsilon_K^{\text{SUSY}}\) is given in the second order of the squark mixing \(s_{13}^{L(R)} \times s_{23}^{L(R)}\).

In addition to the above FCNC processes, the neutron EDM, \(d_n\), arises through the cEDM of the quarks, \(d_q^C\), due to the gluino-squark mixing [63–68]. By using the QCD sum rules, \(d_n\) is given as:
\[ d_n = (0.79d_d - 0.20d_u) + e(0.3d_u^C + 0.59d_d^C) \] (19)

where \(d_q\) and \(d_q^C\) denote the EDM and cEDM of quarks \(d_q^C\) defined in Appendix D. On the other hand, by using the chiral perturbation theory:
\[ d_n = e(3.0d_u^C + 2.5d_d^C + 0.5d_s^C) \] (20)

Therefore, the experimental upper bound [62]:
\[ |d_n| < 0.29 \times 10^{-25}\text{ecm} \] (21)

provides us a strong constraint to the gluino-squark mixing.

The HgEDM can also probe the gluino-squark mixing [70]. The QCD sum rule approach gives [81]:
\[ d_{Hg} = e(d_u^C - d_d^C + 0.012d_s^C) \times 3.2 \times 10^{-2} \] (22)

and the chiral Lagrangian method gives [82]:
\[ d_{Hg} = e(d_u^C - d_d^C + 0.0051d_s^C) \times 8.7 \times 10^{-3} \] (23)

The experimental upper bound [69]:
\[ |d_{Hg}| < 3.1 \times 10^{-29}\text{ecm} \] (24)

constrains the gluino-squark mixing.

At the last step, we discuss the charm sector, which is a promising field to probe for the new physics beyond the SM. The \(D^0 - \bar{D}^0\) mixing is now well established [83] as follows:
\[ x_D = \frac{\Delta M_D}{\Gamma_D} = (3.6 \pm 1.6) \times 10^{-3} , \quad y_D = \frac{\Delta \Gamma_D}{2\Gamma_D} = (6.1 \pm 0.7) \times 10^{-3} \] (25)
where $\Delta M_D$ and $\Delta \Gamma_D$ are the differences of the masses and the decay widths between the mass eigenstates of the $D$ meson, respectively, and $\Gamma_D$ is the averaged decay width of the $D$ meson. Since the SM prediction of $\Delta M_D$ at the short distance is much suppressed compared with the experimental value due to the bottom quark loop, the SUSY contribution may be enhanced.

4. Results and Discussions

Let us estimate the SUSY contribution of the low-energy FCNC. We calculate the SUSY mass spectrum at $Q_0 = 10, 50, 100, 1000$ TeV and interpolate the each mass of the SUSY particle in the region of $Q_0 = 10–1000$ TeV. This approximation is satisfied within $\mathcal{O}(10\%)$. Therefore, our numerical results should be taken with the ambiguity of $\mathcal{O}(10\%)$. The mass spectrum at $Q_0 = 10$ TeV is presented in Appendix A. See [41,60] for the mass spectrum at $Q_0 = 50$ TeV.

Then, we have four mixing angles $\theta_{13}^{L(R)}$ and $\theta_{23}^{L(R)}$, five phase $\phi_{13}^{L(R)}$, $\phi_{23}^{L(R)}$, $\phi$. We reduce the number of parameters by taking $\sin \theta_{ij} = \sin \theta_{ij} \equiv s_{ij}$ for simplicity. In the numerical calculations, we scan the phases of Equation (8) in the region of $0 \sim 2\pi$ for fixed $s_{ij}$, where the Cabibbo angle 0.22 and the large angle 0.5 are taken as the typical mixing. Other relevant input parameters, such as quark masses $m_c$, $m_b$, the CKM parameters $V_{us}$, $V_{cb}$, $\bar{\rho}$, $\bar{\eta}$ and $f_B$, $f_K$, etc., have been presented in our previous paper [57], which are referred from the UTfit Collaboration [84] and PDG [62].

4.1. $B^0$ and $B_s$ Meson Systems

At first, we examine the SUSY contribution in the $\Delta B = 2$ process. We show the SUSY scale $m_{\tilde{Q}} \equiv Q_0$ dependence of the SUSY contributions of $\Delta M_{B^0}$ and $\Delta M_{B_s}$ in Figure 1a,b, where the experimental central value is shown by the red line. The experimental error bars are the 1% and 0.1% levels for $\Delta M_{B^0}$ and $\Delta M_{B_s}$, respectively. We take $s_{13} = s_{23} = 0.22, 0.5$. There is no phase dependence in our predictions. It is found that the SUSY contributions in $\Delta M_{B^0}$ and $\Delta M_{B_s}$ are at most 1.5% and 0.1% at $m_{\tilde{Q}} = 10$ TeV, respectively. Namely, the high-scale SUSY cannot explain the NP contributions of $h_d = 0.1–0.35$ and $h_s = 0.15–0.25$, which have been discussed in Equation (11). As $m_{\tilde{Q}}$ increases, the SUSY contributions of both $\Delta M_{B^0}$ and $\Delta M_{B_s}$ decrease approximately with the power of $1/m_{\tilde{Q}}^2$. Thus, there is no hope to observe the SUSY contribution in the $\Delta B = 2$ process for the high-scale SUSY. It should be noted that the SM predictions are comparable to these experimental data.

The related phenomena are the CP violations of the non-leptonic decays $B^0 \to J/\psi K_S$ and $B_s \to J/\psi \phi$. The recent experimental data of these phases are [29,36–38]:

\[
\sin \phi_d = 0.679 \pm 0.020, \quad \phi_s = 0.07 \pm 0.09 \pm 0.01
\]

in which the contribution of the gluino-squark-quark interaction may be included. The NP contributions in $\phi_d$ and $\phi_s$ are expressed in terms of the parameters of Equation (11) as [57]:

\[
\phi_d = 2\beta_d + \arg(1 + h_d e^{2i\sigma_d}), \quad \phi_s = -2\beta_s + \arg(1 + h_s e^{2i\sigma_s})
\]

where $\beta_d(\beta_s)$ is the one angle of the unitarity triangle giving by the CKM matrix elements of the SM. However, $h_d$ and $h_s$ in the high-scale SUSY are much suppressed compared with $h_d = 0.1–0.35$ and
$h_s = 0.15-0.25$ of Equation (11), and one cannot find signals of the high-scale SUSY in the CP violating decays $B^0 \to J/\psi K_S$ and $B_s \to J/\psi \phi$.

4.2. Neutral $K$ Meson System

At the second step, we examine the neutral $K$ meson. We show the SUSY contributions of $\Delta M_{K^0}$ and $\epsilon_K$ versus $m_{\tilde{Q}} \equiv Q_0$ in Figure 2a,b, where the experimental central value is shown by the horizontal red line. The experimental error bars are the 0.2% and 0.5% levels for $\Delta M_{K^0}$ and $\epsilon_K$, respectively. Since $\theta^{L,R}_{12} = 0$, the SUSY flavor mixing arises from the second order of $s_{13} \times s_{23}$, where $s_{13} = s_{23} = 0.22, 0.5$ are placed.

It is found in Figure 2a that the SUSY contribution in $\Delta M_{K^0}$ can be comparable to the experimental value in the case of $s_{13} = s_{23} = 0.5$, whereas it is suppressed in the case of $s_{13} = s_{23} = 0.22$ at $m_{\tilde{Q}} = 10$ TeV. Thus, $\Delta M_{K^0}$ constrains the squark mixing of $s_{13}$ and $s_{23}$ around $m_{\tilde{Q}} = 10$ TeV. When the SUSY scale increases to more than 20 TeV, no SUSY contribution is expected.

On the other hand, $\epsilon_K$ is very sensitive to the SUSY contribution up to 100 TeV, as seen in Figure 2b. The plot is scattered due to the random phases of the squark mixing. The experimental data of $\epsilon_K$ constrain the squark mixing and phases considerably. Actually, we have already pointed out that the SUSY contribution in $\epsilon_K$ could be 40% and 35% at $m_{\tilde{Q}} = 10, 50$ TeV, respectively [41]. It is found that this sizable SUSY contribution still exist up to 100 TeV in this work.

In the SM, there is only one CP violating phase. Therefore, the observed value of $\phi_d$ in Equation (27), should be correlated with $\epsilon_K$ in the SM. According to the recent experimental results, it is found that the consistency between the SM prediction and the experimental data of $\sin \phi_d$ and $\epsilon_K$ is marginal. This fact was pointed out by Buras and Guadagnoli [77] and called the tension between $\epsilon_K$ and $\sin \phi_d$. Considering the effect of the SUSY contribution $O(10\%)$ in $\epsilon_K$, this tension can be relaxed even if $m_{\tilde{Q}} = 100$ TeV. The precise determination of the unitarity triangle of $B^0$ is required in order to find the SUSY contribution of this level.
It is noted that the SUSY contribution of both $\Delta M_{K^0}$ and $\epsilon_K$ also decrease approximately with the power of $1/m_{\tilde{Q}}^2$ as $m_{\tilde{Q}}$ increases up to 1000 TeV.

![Figure 2](image1.png)

**Figure 2.** The SUSY components of (a) $\Delta M_{K^0}$ and (b) $|\epsilon_K|$ versus $m_{\tilde{Q}}$ for $s_{13} = s_{23} = 0.22$ (cyan) and 0.5 (blue). The horizontal red line denotes the experimental central value.

### 4.3. The nEDM and HgEDM with $\epsilon_K$

The nEDM and HgEDM are also sensitive to the SUSY contribution [61,70]. The gluino-squark interaction leads to the cEDM of quarks, which give the nEDM as shown in Equations (19) and (20). We show the predicted nEDM versus $m_{\tilde{Q}}$ for the case of the QCD sum rules of Equation (19) in Figure 3a, where the upper bound of $|d_n|$ is shown by the red line. The plot is scattered due to the random phases of the squark mixing, as well as in the case of $\epsilon_K$. We find that the contributions of EDM, $d_d$ and $d_u$ occupy around 25% of the neutron EDM. The SUSY contribution is close to the experimental upper bound up to 50 TeV. Since the predicted nEDM depends on the phases of the squark mixing matrix significantly, we plot the nEDM versus $|\epsilon_{K}^{SUSY}|$ in Figure 3b. It is found that the predicted nEDM is roughly proportional to $|\epsilon_{K}^{SUSY}|$. If the SUSY contribution is the level of $O(10\%)$ for $\epsilon_K$, the nEDM is expected to be discovered in the region of $10^{-27}$–$10^{-26}$ cm. On the other hand, if the nEDM is not observed above $10^{-28}$ cm, the SUSY contribution of $\epsilon_K$ is below a few %. Thus, there is the correlation between $d_n$ and $\epsilon_{K}^{SUSY}$.

We also show the predicted HgEDM versus $m_{\tilde{Q}}$ for the case of the QCD sum rules of Equation (22) in Figure 4a, where the upper bound of $|d_{Hg}|$ is shown by the red line. The SUSY contribution is close to the experimental upper bound up to 200 TeV, which is much higher than the one of the nEDM. In Figure 4b, we plot the HgEDM versus $|\epsilon_{K}^{SUSY}|$. It is found that the experimental upper bound of the HgEDM excludes completely $|\epsilon_{K}^{SUSY}|$, which is inconsistent with the experimental data. If the SUSY contribution is the level of $O(10\%)$ for $\epsilon_K$, the nEDM is expected to be discovered in the region of $10^{-27}$–$10^{-26}$ cm. If the HgEDM is not observed above $10^{-29}$ cm, the SUSY contribution of $\epsilon_K$ is below a few %. Thus, the mercury EDM gives more significant information for the gluino-squark interaction compared with the neutron EDM.

However, these correlations strongly depend on the assumptions of $\theta_{L_{23}} = \theta_{L_{13}}$ and $\theta_{L_{ij}} = \theta_{R_{ij}}$. The deviation from these relations destroys these correlations. For instance, for the case of $\theta_{L_{23}} \gg \theta_{L_{13}}$ with $\theta_{L_{ij}} = \theta_{R_{ij}}$, $\epsilon_{K}^{SUSY}$ is much suppressed, whereas the nEDM and HgEDM are still sizable. On the other
hand, if $\theta_{ij}^L \gg \theta_{ij}^R$ or $\theta_{ij}^L \gg \theta_{ij}^R$ is realized, the cEDMs are suppressed, because they require the chirality flipping. In conclusion, careful studies of the mixing angle relations are required to test the correlations between EDMs and $\epsilon_{K}^{\text{SUSY}}$.

Figure 3. The the neutron EDM versus (a) $m_{\tilde{Q}}$ and (b) versus $|\epsilon_{K}^{\text{SUSY}}|$ for $s_{13} = s_{23} = 0.22$ (cyan) and 0.5 (blue) for the case of the QCD sum rule. The horizontal red line denotes the experimental upper bound of $|d_n|$, and the vertical one is the experimental central value of $|\epsilon_K|$.

Figure 4. The mercury EDM versus (a) $m_{\tilde{Q}}$ and (b) versus $|\epsilon_{K}^{\text{SUSY}}|$ for $s_{13} = s_{23} = 0.22$ (cyan) and 0.5 (blue) for the case of the QCD sum rule. The horizontal red line denotes the experimental upper bound of $|d_{Hg}|$, and the vertical one is the experimental central value of $|\epsilon_K|$.

We should comment on the hadronic model dependence of our numerical result. For both nEDM and HgEDM, we show the numerical result by using the hadronic model of the QCD sum rules in Equations (19) and (22). We have also calculated the EDMs by using the hadronic model of the chiral perturbation theory in Equations (20) and (23). For the neutron EDM, the prediction of the chiral perturbation theory is larger than the one of the QCD sum rule at most of a factor of two. However, for the mercury EDM, the prediction of the QCD sum rule is more than three-times larger compared...
with the one of the chiral perturbation theory. Thus, predicted EDMs have ambiguity with a factor of 2–3 from the hadronic model.

4.4. $D$-$\bar{D}$ Mixing

Since the SM prediction of $\Delta M_D$ at the short distance is $\mathcal{O}(10^{-18})$ GeV, which is very small compared with the experimental value due to the bottom quark loop, it is important to estimate the SUSY contribution of $\Delta M_D$. The mixing angle $\theta_{ij}^{L(R)}$ also appears in the up-type squark mixing matrix, whereas the down-type squark mixing matrix contributes to the $K^0$, $B^0$ and $B_s$ meson systems induced by the gluino-squark-quark interaction.

We show the SUSY component of $\Delta M_D$ and $x_D$ versus $m_{\tilde{Q}}$ for $s_{13} = s_{23} = 0.22, 0.5$ in Figure 5. At the SUSY scale of 10 TeV, the SUSY component may be comparable to the observed value. Although the accurate estimate of the long-distance effect is difficult, Cheng and Chiang estimated $x_D$ of order $10^{-3}$ from the two body hadronic modes [85]. This obtained value is consistent with the experimental one. Therefore, we should take into account the long-distance effect properly in order to constrain the SUSY contribution from $\Delta M_D$.

![Figure 5](image.png)

**Figure 5.** The SUSY component of (a) $\Delta M_D$ and (b) $x_D$ versus $m_{\tilde{Q}}$ for $s_{13} = s_{23} = 0.22$ (cyan) and 0.5 (blue). The horizontal red line denotes the experimental central value.

Before closing the presentation of the numerical results, we add a comment on the other gaugino contribution. There are additional contributions to the FCNC induced by chargino exchanging diagrams. The chargino contribution to the gluino one is approximately 10% in the above numerical study of $\Delta F = 2$. Thus, the chargino contributions are the sub-leading ones.

5. Summary

We discussed the sensitivity of the high-scale SUSY at 10–1000 TeV in the $B^0$, $B_s$ and $K^0$ meson systems. Furthermore, we have also discussed the sensitivity to the $D$-$\bar{D}$ mixing, the neutron EDM and the mercury EDM. In order to estimate the contribution of the squark flavor mixing to these FCNC, we calculate the squark mass spectrum, which is consistent with the recent Higgs discovery.
The SUSY contributions in $\Delta M_{B^0}$ and $\Delta M_{B_s}$ are at most 1.5% and 0.1% at $m_{\tilde{Q}} = 10$ TeV, respectively. As $m_{\tilde{Q}}$ increases, the SUSY contributions of both $\Delta M_{B^0}$ and $\Delta M_{B_s}$ decrease approximately with the power of $1/m_{\tilde{Q}}^2$. Therefore, the SUSY scale increases to more than 10 TeV, and no signal of SUSY is expected. On the other hand, the SUSY contribution in $\Delta M_{K^0}$ can be comparable to the experimental value in the case of $s_{13} = s_{23} = 0.5$, whereas it is suppressed in the case of $s_{13} = s_{23} = 0.22$ at $m_{\tilde{Q}} = 10$ TeV. Furthermore, the SUSY contribution in $\epsilon_K$ could be large, around 40% in the region of the SUSY scale 10–100 TeV. By considering the effect of the SUSY contribution $O(10\%)$ in $\epsilon_K$, the tension between $\epsilon_K$ and $\sin \phi_d$ can be relaxed even if the SUSY scale is 100 TeV.

The neutron EDM and the mercury EDM are also sensitive to the SUSY contribution induced by the gluino-squark interaction. The $|d_n|$ is expected to be close to the experimental upper bound, even if the SUSY scale is 50 TeV. The predicted nEDM is roughly proportional to $|\epsilon_{K,\text{SUSY}}|$. If the SUSY contribution is the level of $O(10\%)$ for $\epsilon_K$, the $|d_n|$ is expected to be discovered in the region of $10^{-27}–10^{-26}$ cm. For the $|d_{Hg}|$, the SUSY contribution is close to the experimental upper bound up to 200 TeV, which is much higher than the one of the nEDM. If the HgEDM is not observed above $10^{-29}$ cm, the SUSY contribution of $\epsilon_K$ is below a few %. Thus, the mercury EDM gives more significant information for the gluino-squark interaction compared with the neutron EDM. It may be important to give a comment that these predictions depend strongly on the assumptions of $\theta^{L}_{23} = \theta^{L}_{13}$ and $\theta^{R}_{ij} = \theta^{R}_{ij}$. The deviation from these relations destroys these correlations. In conclusion, careful studies of the mixing angle relations are required to test the correlations between EDMs and $\epsilon_{K,\text{SUSY}}$. The predicted EDMs have also ambiguity with the factor of 2–3 from the hadronic model.

Since the SM prediction of $\Delta M_D$ at the short distance is $O(10^{-18})$ GeV, which is very small compared with the experimental value, it is important to estimate the SUSY contribution of $\Delta M_D$.

In conclusion, more detailed studies of the $K^0$ meson system, the EDMs of the neutron and mercury are required in order to probe the high-scale SUSY at 10–1000 TeV.

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Author Contributions

Morimitsu Tanimoto and Kei Yamamoto conceived of and discussed ideas of this work, did the calculations and wrote the paper.

Appendix

A. Running of SUSY Particle Masses

In the framework of the MSSM, one obtains the SUSY particle spectrum, which is consistent with the observed Higgs mass. The numerical analyses have been given in [71,72]. At the SUSY breaking scale $\Lambda$, the quadratic terms in the MSSM potential are given as:

$$V_2 = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 \cdot H_2 + h.c.)$$

(A1)
The mass eigenvalues at the $H_1$ and $\tilde{H}_2 \equiv \epsilon H_2^*$ system are given:

$$m_+^2 = \frac{m_1^2 + m_2^2}{2} \pm \sqrt{\left(\frac{m_1^2 - m_2^2}{2}\right)^2 + m_3^4}$$

Suppose that the MSSM matches the SM at the SUSY mass scale $Q_0 \equiv m_0$. Then, the smaller one $m_-^2$ is identified to be the mass squared of the SM Higgs $H$ with the tachyonic mass. The larger one $m_+^2$ is the mass squared of the orthogonal combination $\mathcal{H}$, which is decoupled from the SM at $Q_0$, that is $m_H \simeq Q_0$. Therefore, we have:

$$m_-^2 = -m^2(Q_0), \quad m_+^2 = m_H^2(Q_0) = m_1^2 + m_2^2 + m^2$$

with

$$m_3^4 = (m_1^2 + m_2^2)(m_2^2 + m^2)$$

which leads to the mixing angle between $H_1$ and $\tilde{H}_2$, $\beta$ as follows:

$$\tan^2 \beta = \frac{m_1^2 + m_2^2}{m_2^2 + m^2}, \quad H = \cos \beta H_1 + \sin \beta \tilde{H}_2, \quad \mathcal{H} = -\sin \beta H_1 + \cos \beta \tilde{H}_2$$

Thus, the Higgs mass parameter $m^2$ is expressed in terms of $m_1^2$, $m_2^2$ and $\tan \beta$:

$$m^2 = \frac{m_1^2 - m_2^2 \tan^2 \beta}{\tan^2 \beta - 1}$$

Below the $Q_0$ scale, in which the SM emerges, the scalar potential is the SM one as follows:

$$V_{SM} = -m^2|H|^2 + \frac{\lambda}{2}|H|^4$$

Here, the Higgs coupling $\lambda$ is given in terms of the SUSY parameters at the leading order as:

$$\lambda(Q_0) = \frac{1}{4}(g^2 + g'^2)\cos^2\beta + \frac{3h_t^2}{8\pi^2}X_i^2 \left(1 - \frac{X_i^2}{12}\right), \quad X_i = \frac{A_i(Q_0) - \mu(Q_0) \cot \beta}{Q_0}$$

and $h_t$ is the top Yukawa coupling of the SM. The parameters $m_2$ and $\lambda$ run with the SM renormalization group equation down to the electroweak scale $Q_{EW} = m_H$ and then give:

$$m_H^2 = 2m^2(m_H) = \lambda(m_H)v^2$$

It is easily seen that the VEV of Higgs, $\langle H \rangle$, is $v$, and $\langle \mathcal{H} \rangle = 0$, taking account of $\langle H_1 \rangle = v \cos \beta$ and $\langle H_2 \rangle = v \sin \beta$, where $v = 246$ GeV.

Let us fix $m_H = 125$ GeV, which gives $\lambda(Q_0)$ and $m^2(Q_0)$. This experimental input constrains the SUSY mass spectrum of the MSSM. We consider the some universal soft breaking parameters at the SUSY breaking scale $\Lambda$ as follows:

$$m_{\tilde{Q}_i}(\Lambda) = m_{\tilde{U}_i}(\Lambda) = m_{\tilde{D}_i}(\Lambda) = m_{\tilde{E}_i}(\Lambda) = m_0^2 \quad (i = 1, 2, 3)$$

$$M_i(\Lambda) = M_2(\Lambda) = M_3(\Lambda) = m_{3/2} \quad m_1^2(\Lambda) = m_2^2(\Lambda) = m_0^2$$

$$A_U(\Lambda) = A_0y_U(\Lambda), \quad A_D(\Lambda) = A_0y_D(\Lambda), \quad A_E(\Lambda) = A_0y_E(\Lambda)$$
Therefore, there is no flavor mixing at $\Lambda$ in the MSSM. However, in order to consider the non-minimal flavor mixing framework, we allow the off-diagonal components of the squark mass matrices at the 10% level, which leads to the flavor mixing of order 0.1. We take these flavor mixing angles as free parameters at low energies.

Now, we have the SUSY five parameters, $\Lambda$, $\tan \beta$, $m_0$, $m_{1/2}$, $A_0$, where $Q_0 = m_0$. In addition to these parameters, we take $\mu = Q_0$. Inputting $m_H = 125$ GeV and taking $m_{1/2} \simeq Q_0$, we can obtain the SUSY spectrum for the fixed $Q_0$ and $\tan \beta$.

We present the SUSY mass spectrum at $Q_0 = 10$ TeV. The input parameter set and the obtained SUSY mass spectra at $Q_0$ are summarized in Table A1, where we use $m_t(m_t) = 163.5 \pm 2$ GeV \cite{62,84}. These parameter sets are easily found from the work in \cite{71}.

**Table A1.** Input parameters at $\Lambda$ and the obtained SUSY spectra at $Q_0 = 10$ TeV.

<table>
<thead>
<tr>
<th>Input at $\Lambda$ and $Q_0$</th>
<th>Output at $Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>at $\Lambda = 10^{17}$ GeV</td>
<td>$m_\tilde{b} = 12.8$ TeV, $m_{\tilde{t}<em>L} = 5.2$ TeV, $m</em>{\tilde{B}} = 2.9$ TeV</td>
</tr>
<tr>
<td>$m_0 = 10$ TeV</td>
<td>$m_{\tilde{b}<em>L} = m</em>{\tilde{t}_L} = 12.2$ TeV</td>
</tr>
<tr>
<td>$m_{1/2} = 6.2$ TeV</td>
<td>$m_{\tilde{b}<em>R} = 14.1$ TeV, $m</em>{\tilde{t}_R} = 8.4$ TeV</td>
</tr>
<tr>
<td>$A_0 = 25.803$ TeV</td>
<td>$m_{\tilde{\chi}_L, \tilde{\chi}_R} = 15.1$ TeV</td>
</tr>
<tr>
<td>at $Q_0 = 10$ TeV</td>
<td>$m_{\tilde{b}_L, \tilde{b}<em>R} \simeq m</em>{\tilde{t}_L, \tilde{t}_R} = 14.6$ TeV, $m_H = 13.7$ TeV</td>
</tr>
<tr>
<td>$\mu = 10$ TeV</td>
<td>$m_{\tilde{\chi}<em>L} = m</em>{\tilde{\tau}<em>L} = 10.4$ TeV, $m</em>{\tilde{\tau}_R} = 9.3$ TeV</td>
</tr>
<tr>
<td>$\tan \beta = 10$</td>
<td>$m_{\tilde{\mu}<em>L, \tilde{\mu}<em>R} = m</em>{\tilde{\nu}</em>{L,R}} = 10.8$ TeV, $m_{\tilde{\mu}_R, \tilde{\tau}_R} = 10.3$ TeV</td>
</tr>
<tr>
<td></td>
<td>$X_t = -0.22$, $\lambda_H = 0.126$</td>
</tr>
</tbody>
</table>

As seen in Table A1, the first and second family squarks are degenerate in their masses; on the other hand, the third ones split due to the large RGE effect. Therefore, the mixing angle between the first and second family squarks vanishes, but the mixing angles between the first-third and the second-third family squarks are produced at the $Q_0$ scale. The left-right mixing angle between $\tilde{b}_L$ and $\tilde{b}_R$ is given as:

$$\theta \simeq \frac{m_b(A_b(Q_0) - \mu \tan \beta)}{m_{\tilde{b}_L}^2 - m_{\tilde{b}_R}^2} \quad (A11)$$

which is very small, $\mathcal{O}(0.01)$ at 10 TeV. The lightest squark is the right-handed stop, and the lightest gaugino is the Bino.

**B. Squark Contribution in the $\Delta F = 2$ Process**

The $\Delta F = 2$ effective Lagrangian from the gluino-bottom-quark interaction is given as \cite{86}:

$$\mathcal{L}^{\Delta F=2} = -\frac{1}{2} [C_{VLL}O_{VLL} + C_{VRR}O_{VRR}] - \frac{1}{2} \sum_{i=1}^{2} \left[ C_{SLL}^{(i)}O_{SLL}^{(i)} + C_{SRR}^{(i)}O_{SRR}^{(i)} + C_{SLR}^{(i)}O_{SLR}^{(i)} \right] \quad (B1)$$

where
\[ O_{VLL} = (\bar{q}_a \gamma^\mu L Q^a)(\bar{q}_b \gamma^\mu L Q^b), \quad O_{VRR} = (\bar{q}_a \gamma^\mu R Q^a)(\bar{q}_b \gamma^\mu R Q^b) \]
\[ O_{SLL}^{(1)} = (\bar{q}_a L Q^a)(\bar{q}_b L Q^b), \quad O_{SLL}^{(2)} = (\bar{q}_a L Q^a)(\bar{q}_b L Q^b) \]
\[ O_{SRR}^{(1)} = (\bar{q}_a R Q^a)(\bar{q}_b R Q^b), \quad O_{SRR}^{(2)} = (\bar{q}_a R Q^a)(\bar{q}_b R Q^b) \]
\[ O_{SLR}^{(1)} = (\bar{q}_a L Q^a)(\bar{q}_b R Q^b), \quad O_{SLR}^{(2)} = (\bar{q}_a L Q^a)(\bar{q}_b R Q^b) \]  
(B2)

with \((P, Q, q) = (B^3, b, d), (B_s, b, s), (K^0, s, d)\). The \(L, R\) denote \((1 \pm \gamma_5)/2\), and \(a, b\) are color indices. Then, the \(P^0 - \bar{P}^0\) mixing, \(M_{12}\), is written as:
\[ M_{12} = -\frac{1}{2 m_P} \langle P^0 | \mathcal{L}_{\text{eff}}^{F=2} | \bar{P}^0 \rangle \]  
(B3)

The hadronic matrix elements are given in terms of the non-perturbative parameters \(B_i\) as:
\[ \langle P^0 | \mathcal{O}_{VLL} | \bar{P}^0 \rangle = \frac{2}{3} m_P^2 f_P^2 B_1, \quad \langle P^0 | \mathcal{O}_{VRR} | \bar{P}^0 \rangle = \langle P^0 | \mathcal{O}_{VLL} | \bar{P}^0 \rangle \]
\[ \langle P^0 | \mathcal{O}_{SLL}^{(1)} | \bar{P}^0 \rangle = -\frac{5}{12} m_P^2 f_P^2 R_P B_2, \quad \langle P^0 | \mathcal{O}_{SLL}^{(2)} | \bar{P}^0 \rangle = \langle P^0 | \mathcal{O}_{SRR}^{(1)} | \bar{P}^0 \rangle \]
\[ \langle P^0 | \mathcal{O}_{SRR}^{(2)} | \bar{P}^0 \rangle = \frac{1}{12} m_P^2 f_P^2 R_P B_3, \quad \langle P^0 | \mathcal{O}_{SLR}^{(1)} | \bar{P}^0 \rangle = \langle P^0 | \mathcal{O}_{SLR}^{(2)} | \bar{P}^0 \rangle = \frac{1}{6} m_P^2 f_P^2 R_P B_5 \]  
(B4)

where
\[ R_P = \left( \frac{m_P}{m_Q + m_q} \right)^2 \]  
(B5)

The Wilson coefficients for the gluino contribution in Equation (B1) are written as [86]:
\[ C_{VLL}(m_\tilde{g}) = \frac{\alpha_s^2}{m_\tilde{g}^2} \sum_{I,J=1}^{6} (\lambda_{GLL}^{(d)})_{ij} \sum_{I,J=1}^{6} (\lambda_{GLL}^{(d)})_{ij} \left[ \frac{11}{18} g_{2|I|}(\bar{x}_I, \bar{x}_J) + \frac{2}{9} g_{1|I|}(\bar{x}_I, \bar{x}_J) \right] \]
\[ C_{VRR}(m_\tilde{g}) = C_{VLL}(m_\tilde{g})(L \leftrightarrow R), \]
\[ C_{SRR}^{(1)}(m_\tilde{g}) = \frac{\alpha_s^2}{m_\tilde{g}^2} \sum_{I,J=1}^{6} (\lambda_{GLR}^{(d)})_{ij} \sum_{I,J=1}^{6} (\lambda_{GLR}^{(d)})_{ij} \frac{17}{9} g_{1|I|}(\bar{x}_I, \bar{x}_J) \]
\[ C_{SRR}^{(1)}(m_\tilde{g}) = C_{SRR}^{(1)}(m_\tilde{g})(L \leftrightarrow R) \]
\[ C_{SRR}^{(2)}(m_\tilde{g}) = \frac{\alpha_s^2}{m_\tilde{g}^2} \sum_{I,J=1}^{6} (\lambda_{GLR}^{(d)})_{ij} \sum_{I,J=1}^{6} (\lambda_{GLR}^{(d)})_{ij} \left( -\frac{1}{3} \right) g_{1|I|}(\bar{x}_I, \bar{x}_J) \]
\[ C_{SRR}^{(2)}(m_\tilde{g}) = C_{SRR}^{(2)}(m_\tilde{g})(L \leftrightarrow R) \]
Equation (B1) at the leading order of QCD as follows: the effective Wilson coefficients are given by using the RGEs for higher-dimensional operators in Symmetry where

\begin{align}
C_{SLR}(m_{\tilde{g}}) &= \frac{\alpha_s^2}{m_{\tilde{g}}^2} \sum_{i,j=1}^{6} \left\{ (\lambda_{GLR}^{(d)})_{ij} (\lambda_{GRL}^{(d)})_{ij} \left( -\frac{11}{9} \right) g_{2[1]}(x_{i}^{\tilde{g}}, x_{j}^{\tilde{g}}) 
+ (\lambda_{GLL}^{(d)})_{ij} (\lambda_{GRR}^{(d)})_{ij} \left[ \frac{14}{3} g_{1[1]}(x_{i}^{\tilde{g}}, x_{j}^{\tilde{g}}) - \frac{2}{3} g_{2[1]}(x_{i}^{\tilde{g}}, x_{j}^{\tilde{g}}) \right] \right\} \\
C_{SLL}(m_{\tilde{g}}) &= \frac{\alpha_s^2}{m_{\tilde{g}}^2} \sum_{i,j=1}^{6} \left\{ (\lambda_{GLR}^{(d)})_{ij} (\lambda_{GRL}^{(d)})_{ij} \left( -\frac{5}{3} \right) g_{2[1]}(x_{i}^{\tilde{g}}, x_{j}^{\tilde{g}}) 
+ (\lambda_{GLL}^{(d)})_{ij} (\lambda_{GRR}^{(d)})_{ij} \left[ \frac{2}{9} g_{1[1]}(x_{i}^{\tilde{g}}, x_{j}^{\tilde{g}}) + \frac{10}{9} g_{2[1]}(x_{i}^{\tilde{g}}, x_{j}^{\tilde{g}}) \right] \right\}
\end{align}

(B6)

where

\begin{align}
(\lambda_{GLL}^{(d)})_{ij} &= (\Gamma_{GL})_{ij}^{K} (\lambda_{GL}^{(d)})_{ij}^{K}, \\
(\lambda_{GRR}^{(d)})_{ij} &= (\Gamma_{GR})_{ij}^{K} (\lambda_{GR}^{(d)})_{ij}^{K}, \\
(\lambda_{GLR}^{(d)})_{ij} &= (\Gamma_{GL})_{ij}^{K} (\lambda_{GR}^{(d)})_{ij}^{K}, \\
(\lambda_{GRL}^{(d)})_{ij} &= (\Gamma_{GR})_{ij}^{K} (\lambda_{GL}^{(d)})_{ij}^{K}.
\end{align}

(B7)

Here, we take \((i, j) = (1, 3), (2, 3), (1, 2)\) which correspond to \(B^0, B_s\), and \(K^0\) mesons, respectively. The loop functions are given as follows:

- If \(x_{i}^{\tilde{g}} \neq x_{j}^{\tilde{g}} \ (x_{i,j}^{\tilde{g}} = m_{\tilde{g}}^2 / m_{\tilde{g}}^2),\)

\begin{align}
g_{1[1]}(x_{i}^{\tilde{g}}, x_{j}^{\tilde{g}}) &= \frac{1}{x_{i}^{\tilde{g}} - x_{j}^{\tilde{g}}} \left( \frac{x_{i}^{\tilde{g}} \log x_{i}^{\tilde{g}}}{(x_{i}^{\tilde{g}} - 1)^2} - \frac{1}{x_{i}^{\tilde{g}} - 1} - \frac{x_{j}^{\tilde{g}} \log x_{j}^{\tilde{g}}}{(x_{j}^{\tilde{g}} - 1)^2} + \frac{1}{x_{j}^{\tilde{g}} - 1} \right) \\
g_{2[1]}(x_{i}^{\tilde{g}}, x_{j}^{\tilde{g}}) &= \frac{1}{x_{i}^{\tilde{g}} - x_{j}^{\tilde{g}}} \left( \frac{(x_{i}^{\tilde{g}})^2 \log x_{i}^{\tilde{g}}}{(x_{i}^{\tilde{g}} - 1)^2} - \frac{1}{x_{i}^{\tilde{g}} - 1} - \frac{(x_{j}^{\tilde{g}})^2 \log x_{j}^{\tilde{g}}}{(x_{j}^{\tilde{g}} - 1)^2} + \frac{1}{x_{j}^{\tilde{g}} - 1} \right)
\end{align}

(B8)

- If \(x_{i}^{\tilde{g}} = x_{j}^{\tilde{g}}\)

\begin{align}
g_{1[1]}(x_{i}^{\tilde{g}}, x_{j}^{\tilde{g}}) &= -\frac{\log x_{i}^{\tilde{g}}}{x_{i}^{\tilde{g}} - 1} + \frac{2}{(x_{i}^{\tilde{g}} - 1)^2} \\
g_{2[1]}(x_{i}^{\tilde{g}}, x_{j}^{\tilde{g}}) &= -\frac{2x_{i}^{\tilde{g}} \log x_{i}^{\tilde{g}}}{(x_{i}^{\tilde{g}} - 1)^3} + \frac{x_{i}^{\tilde{g}} + 1}{(x_{i}^{\tilde{g}} - 1)^2}
\end{align}

(B9)

Taking account of the case that the gluino mass is much smaller than the squark mass scale \(Q_0\), the effective Wilson coefficients are given by using the RGEs for higher-dimensional operators in Equation (B1) at the leading order of QCD as follows:

\begin{align}
C_{VLL}(m_{\tilde{b}}(\Lambda = 2 \text{ GeV})) &= \eta_{VLL}^{B(K)} C_{VLL}(Q_0) \quad C_{VRR}(m_{\tilde{b}}(\Lambda = 2 \text{ GeV})) = \eta_{VRR}^{B(K)} C_{VLL}(Q_0) \\
C_{SLR}^{(1)}(m_{\tilde{b}}(\Lambda = 2 \text{ GeV})) &= C_{SLR}^{(1)}(Q_0) X_{LL}^{-1} \eta_{LL}^{B(K)} X_{LL} \\
C_{SLR}^{(2)}(m_{\tilde{b}}(\Lambda = 2 \text{ GeV})) &= C_{SLR}^{(2)}(Q_0) X_{RR}^{-1} \eta_{RR}^{B(K)} X_{RR} \\
C_{SLL}^{(1)}(m_{\tilde{b}}(\Lambda = 2 \text{ GeV})) &= C_{SLL}^{(1)}(Q_0) X_{LR}^{-1} \eta_{LR}^{B(K)} X_{LR} \\
C_{SLL}^{(2)}(m_{\tilde{b}}(\Lambda = 2 \text{ GeV})) &= C_{SLL}^{(2)}(Q_0) X_{LR}^{-1} \eta_{LR}^{B(K)} X_{LR}
\end{align}

(B10)
where

\[
\eta_{VLL}^B = \eta_{VRR}^B = \left( \frac{\alpha_s(Q_0)}{\alpha_s(m_g)} \right)^\frac{1}{\eta_{s_{LL}}} \left( \frac{\alpha_s(m_{\bar{g}})}{\alpha_s(m_{\ell})} \right)^\frac{1}{\eta_{s_{LR}}} \left( \frac{\alpha_s(m_{\ell})}{\alpha_s(m_b)} \right)^\frac{1}{\eta_{s_{LR}}}
\]

\[
\eta_{\bar{g}g}^B = \eta_{\bar{g}g}^B = S_{LL} \left( \frac{d_{LL}^\bar{g}}{d_{LL}^g} \right) S_{LL}^{-1}, \quad \eta_{LR}^B = S_{LR} \left( \frac{d_{LR}^\bar{g}}{d_{LR}^g} \right) S_{LR}^{-1}
\]

\[
\eta_{\Lambda_{\bar{g}}}^B = \left( \frac{\alpha_s(Q_0)}{\alpha_s(m_g)} \right)^\frac{1}{\eta_{\Lambda_{LL}}} \left( \frac{\alpha_s(m_{\bar{g}})}{\alpha_s(m_{\ell})} \right)^\frac{1}{\eta_{\Lambda_{LR}}} \left( \frac{\alpha_s(m_{\ell})}{\alpha_s(m_b)} \right)^\frac{1}{\eta_{\Lambda_{LR}}}
\]

\[
d_{LL}^\bar{g} = \frac{2}{3}(1 - \sqrt{241}), \quad d_{LL}^g = \frac{2}{3}(1 + \sqrt{241}), \quad d_{LR}^\bar{g} = -16, \quad d_{LR}^g = 2
\]

\[
S_{LL} = \left( \begin{array}{cc} \frac{16+\sqrt{241}}{60} & \frac{16-\sqrt{241}}{60} \\ 1 & 1 \end{array} \right), \quad S_{LR} = \left( \begin{array}{cc} -2 & 1 \\ 3 & 0 \end{array} \right)
\]

\[
X_{LL} = X_{RR} = \left( \begin{array}{cc} 1 & 0 \\ 4 & 8 \end{array} \right), \quad X_{LR} = \left( \begin{array}{cc} 0 & -2 \\ 1 & 0 \end{array} \right)
\]

(B11)

For the parameters \( B_1^{(d)}(i = 2 - 5) \) of \( B \) mesons, we use values in [87] as follows:

\[
B_2^{(B_d)}(m_b) = 0.79(2)(4), \quad B_3^{(B_d)}(m_b) = 0.92(2)(4)
\]

\[
B_4^{(B_d)}(m_b) = 1.15(3)(+5^{-7}), \quad B_5^{(B_d)}(m_b) = 1.72(4)(+20^{-6})
\]

\[
B_2^{(B_s)}(m_b) = 0.80(1)(4), \quad B_3^{(B_s)}(m_b) = 0.93(3)(8)
\]

\[
B_4^{(B_s)}(m_b) = 1.16(2)(+5^{-7}), \quad B_5^{(B_s)}(m_b) = 1.75(3)(+21^{-6})
\]

(B12)

On the other hand, we use the most updated values for \( \hat{B}_1^{(d)} \) and \( \hat{B}_1^{(s)} \) as [84]:

\[
\hat{B}_1^{(B_d)} = 1.33 \pm 0.06, \quad \hat{B}_1^{(B_s)}/\hat{B}_1^{(B_d)} = 1.05 \pm 0.07 \]

(B13)

For the parameters \( B_i^{K}(i = 2 - 5) \), we use the following values [88],

\[
B_2^{(K)}(2\text{GeV}) = 0.66 \pm 0.04, \quad B_3^{(K)}(2\text{GeV}) = 1.05 \pm 0.12
\]

\[
B_4^{(K)}(2\text{GeV}) = 1.03 \pm 0.06, \quad B_5^{(K)}(2\text{GeV}) = 0.73 \pm 0.10
\]

(B14)

and we take the recent value of Equation (17) for deriving \( B_1^{(K)}(2\text{GeV}) \).

For the parameters \( B_i^{D}(i = 1 - 5) \), we use the following values [89,90],

\[
B_1^{(D)}(3\text{GeV}) = 0.75 \pm 0.02, \quad B_2^{(D)}(3\text{GeV}) = 0.66 \pm 0.02, \quad B_3^{(D)}(3\text{GeV}) = 0.96 \pm 0.05
\]

\[
B_4^{(D)}(3\text{GeV}) = 0.91 \pm 0.04, \quad B_5^{(D)}(3\text{GeV}) = 1.10 \pm 0.05
\]

(B15)
C. The Loop Functions $F_i$

The loop functions $F_i(x^I_g)$ are given in terms of $x^I_g = m^2_{3I}/m^2_{d_I}$ ($I = 3, 6$) as follows:

\begin{align*}
F_1(x^I_g) &= \frac{x^I_g \log x^I_g}{2(x^I_g - 1)^4} + \frac{(x^I_g)^2 - 5x^I_g - 2}{12(x^I_g - 1)^3}, \quad F_2(x^I_g) = -\frac{(x^I_g)^2 \log x^I_g}{2(x^I_g - 1)^4} + \frac{2(x^I_g)^2 + 5x^I_g - 1}{12(x^I_g - 1)^3}
F_3(x^I_g) &= \frac{\log x^I_g}{(x^I_g - 1)^3} + \frac{x^I_g - 3}{2(x^I_g - 1)^2}, \quad F_4(x^I_g) = -\frac{x^I_g \log x^I_g}{2(x^I_g - 1)^2} + \frac{x^I_g + 1}{2(x^I_g - 1)^2} = \frac{1}{2}g_2[1](x^I_g, x^I_g) \quad \text{(C1)}
\end{align*}

D. EDM and Chromo-EDM of Quarks

We present the EDM of the strange quark from the gluino contribution as the typical example [86]:

\[ d_s(Q_0) = -2\sqrt{4\pi\alpha(m_\tilde{g})} \text{Im}[A_s^{22}(Q_0)] \quad \text{(D1)} \]

where

\[ A_s^{22}(Q_0) = \frac{Q_s\alpha_s(m_\tilde{g})}{4\pi} \sum_{I=1}^{6} \frac{1}{2m^2_{d_I}} \left\{ \left(m_s(\lambda^{(d)}_{GFF})_{3I} + m_s(\lambda^{(d)}_{GFF})_{2I} \right) F_2(x^I_g) + m_\tilde{g}(\lambda^{(d)}_{GFF})^{22}_{I} F_4(x^I_g) \right\} \quad \text{(D2)} \]

On the other hand, the chromo-EDM (cEDM) of the strange quark from gluino contribution is given as:

\[ d_s^{C}(Q_0) = -2\sqrt{4\pi\alpha_s(m_\tilde{g})} \text{Im}[A_s^{22}(Q_0)] \quad \text{(D3)} \]

where

\begin{align*}
A_s^{22}(Q_0) &= -\alpha_s(m_\tilde{g}) \frac{1}{3} \sum_{I=1}^{6} \frac{1}{2m^2_{d_I}} \left\{ \left(m_s(\lambda^{(d)}_{GFF})_{2I} + m_s(\lambda^{(d)}_{GFF})_{2I} \right) (9F_1(x^I_g) + F_2(x^I_g)) \\
&\quad + m_\tilde{g}(\lambda^{(d)}_{GFF})^{22}_{I} (9F_3(x^I_g) + F_4(x^I_g)) \right\} \quad \text{(D4)}
\end{align*}

Including the RGE effect of QCD [91], the cEDM of the strange quark is given as:

\[ d_s^{C}(2\text{GeV}) = d_s^{C}(Q_0) \left( \frac{\alpha_s(Q_0)}{\alpha_s(m_t)} \right)^{\frac{14}{7}} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{\frac{14}{7}} \left( \frac{\alpha_s(m_b)}{\alpha_s(2\text{GeV})} \right)^{\frac{14}{7}} \quad \text{(D5)} \]

On the other hand, the EDM operator is mixed with the cEDM operator during RGE evolution. Then, one obtains:

\[ d_s(2\text{GeV}) = d_s(Q_0) \left( \frac{\alpha_s(Q_0)}{\alpha_s(m_t)} \right)^{\frac{14}{7}} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{\frac{14}{7}} \left( \frac{\alpha_s(m_b)}{\alpha_s(2\text{GeV})} \right)^{\frac{14}{7}} + \frac{8}{g_s} d_s^{c}(Q_0) \times \quad \text{(D6)} \]

\[ \left[ \left( \frac{\alpha_s(Q_0)}{\alpha_s(m_t)} \right)^{\frac{14}{7}} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{\frac{14}{7}} \left( \frac{\alpha_s(m_b)}{\alpha_s(2\text{GeV})} \right)^{\frac{14}{7}} - \left( \frac{\alpha_s(Q_0)}{\alpha_s(m_t)} \right)^{\frac{14}{7}} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{\frac{14}{7}} \left( \frac{\alpha_s(m_b)}{\alpha_s(2\text{GeV})} \right)^{\frac{14}{7}} \right] \]

The EDMs and cEDMs of the down- and up-quarks induced by the gluino interaction are also given by the similar formulas.
Conflicts of Interest

The authors declare no conflict of interest.

References


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