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Article

Study of Dynamical Chiral Symmetry Breaking in $(2+1)$ Dimensional Abelian Higgs Model

Jian-Feng Li [1], **Shi-Song Huang** [2], **Hong-Tao Feng** [3], **Wei-Min Sun** [1,4] **and Hong-Shi Zong** [1,4,*]

[1] Department of Physics, Nanjing University, Nanjing 210093, China

[2] Department of Physics, Nanjing Normal University, Nanjing 210097, China

[3] Department of Physics, Southeast University, Nanjing 211189, China

[4] Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing 210093, China

[*] Author to whom correspondence should be addressed; E-Mail: zonghs@chenwang.nju.edu.cn.

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Abstract: In this paper, we study the dynamical mass generation in the Abelian Higgs model in $2+1$ dimensions. Instead of adopting the approximations in [Jiang H *et al.*, J. Phys. A 41 2008 255402.], we numerically solve the coupled Dyson–Schwinger Equations (DSEs) for the fermion and gauge boson propagators using a specific truncation for the fermion-photon vertex ansatz and compare our results with the corresponding ones in the above mentioned paper. It is found that the results quoted in the above paper remain qualitatively unaffected by refining the truncation scheme of the DSEs, although there exist large quantitative differences between the results presented in the above paper and ours. In addition, our numerical results show that the critical number of fermion flavor N_c decreases steeply with the the gauge boson mass m_a (or the ratio of the Higgs mass m_h to the gauge boson mass m_a, $r = \frac{m_h}{m_a}$) increasing. It is thus easier to generate a finite fermion mass by the mechanism of DCSB for a small ratio r for a given m_a.

Keywords: QED_3; Abelian Higgs model; gauge boson mass; Ginzburg–Landau parameter

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1. Introduction

Quantum electrodynamics in 2+1 dimensions (QED_3) has been extensively studied for over twenty years. It has many features similar to quantum chromodynamics (QCD), such as spontaneous chiral symmetry breaking in the massless fermion limit and confinement [1–13]. Moreover, it is super-renormalizable, so it does not suffer from the ultraviolet divergence which are present in QED_4. Due to these reasons it can serve as a toy model of QCD. In parallel with its relevance as a tool to gain insight into the aspects of QCD, QED_3 is also found to be equivalent to the low-energy effective theories of strongly correlated electronic systems. Recently, QED_3 has been studied in high T_c cuprate superconductors [14–23], quantum Heinsenberg antiferromagnets [24], and fractional quantum Hall effect [25]. In particular, some authors have made progress in studying the property of graphene based on QED_3 [26–28].

Dynamical chiral symmetry breaking (DCSB) occurs when the massless fermion acquires a nonzero mass through non-perturbative effects at low energy, but the Lagrangian keeps chiral symmetry when the fermion mass is neglected. In a four-fermion interaction model [29] Nambu and Jona-Lasinio first adopted the mechanism of DCSB to generate a nonzero mass for the fermion without using the Yukawa type coupling $\phi\overline{\psi}\psi$. In 1988, Appelquist *et al.* [4] studied DCSB in massless QED_3 with N fermion flavors by solving the DSE for fermion self-energy in the lowest-order of $1/N$ expansion and found DCSB occurs when N is less than a critical number N_c. Later Nash showed that the critical number of fermion flavor still exists by considering higher order corrections and he obtained $N_c = \frac{128}{3\pi^2}$ [5]. In 1995, Maris solved the coupled DSEs with a set of simplified vertex functions and obtained the critical number of fermion flavor N_c = 3.3 [7,8]. Soon after that, Fisher *et al.* [30] self-consistently solved a set of coupled DSE and obtained $N_c^{crit} \approx 4$ by using more sophisticated vertex ansatz which satisfies the Ward–Takahashi identity. Here it should be noted that all the above results hold under the condition that the gauge boson is massless. Once a finite gauge boson mass m_a is generated by Anderson–Higgs mechanism, it weakens the strength of interaction and affects DCSB. QED_3 with Abelian Higgs model has been widely studied as the effective theory of the high T_c superconductors [17,18]. Recently, Liu *et al*. [31–33] studied the DSEs for the fermion self-energy in Landau gauge in QED_3 with Abelian Higgs model and found that DCSB occurs only when the gauge boson mass m_a is smaller than a critical value. However, we note that in [31–33], the authors used the so-called nonlocal gauge function approach to solve the nonlinear DSE where the wave function renormalization and the vertex correction are simply absent (in connection to the use of the nonlocal gauge in QED_3, one can see, e.g., [34]). Because of its importance, this problem deserves further study. In this paper, instead of adopting the approximations in [31–33], we numerically solve the coupled DSE for the fermion and gauge boson propagators of QED_3 with Abelian Higgs model using a specific truncation for the fermion-photon vertex ansatz for a range of finite gauge boson mass.

2. Results and Discussions

In Euclidean space, the total Lagrangian of QED$_3$ with N massless fermion flavors and N scalar boson flavors is $\mathcal{L}$=$\mathcal{L}_F$+$\mathcal{L}_B$ [7,33], where

$$\mathcal{L}_F = \sum_{i=1}^{N} \bar{\psi}_i(\partial\!\!\!/ + \mathrm{ie}\, A\!\!\!/)\psi_i + \frac{1}{4}F_{\rho\nu}^2 + \frac{1}{2\xi}(\partial_\rho\, A_\rho)^2 \tag{1}$$

$$\mathcal{L}_B = \sum_{i=1}^{N} \left[|(\partial_\mu - ieA_\mu)\phi_i|^2 + \mu^2|\phi_i|^2 + \lambda|\phi_i|^4\right] \tag{2}$$

Here $\mathcal{L}_F$ contains the coupling between massless Dirac fermions and the $U(1)$ gauge field. In Abelian Higggs model $\mathcal{L}_B$ is added to describe the additional interaction between the complex scalar fields and the gauge field, which will change the gauge boson propagator. The 4×1 spinor ψ_i represents the fermion field, the 4×4 γ_μ matrices obey the Clifford algebra, $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ and $i = 1, \cdots, N$ are the flavor indices. For physical reasons, the number of flavors of fermion or scalar boson N equals to 2, and ξ is the gauge parameter. The bare propagator $S_0 = \frac{1}{i\gamma\cdot p}$ and the full fermion propagator is $S = \frac{1}{i\gamma\cdot pA(p^2)+B(p^2)}$, where $A(p^2)$ is the wave-function renormalization and $B(p^2)$ is the fermion self-energy function. The full fermion propagator satisfies the DSE

$$S^{-1}(p) = i\gamma\cdot p + e^2\int\frac{d^3k}{(2\pi)^3}\gamma_\rho S(k)\Gamma_\nu(p,k)D_{\rho\nu}(q) \tag{3}$$

where $q = p - k$. In QED$_3$ with Abelian Higgs model the gauge field couples to both the fermion field and the scalar boson field. $\Pi_{\mu\nu}(q) = \Pi^F_{\mu\nu}(q) + \Pi^B_{\mu\nu}(q)$ is the total vacuum polarization tensor and the full inverse gauge boson propagator is

$$D^{-1}_{\mu\nu}(q) = D^{(0)-1}_{\mu\nu}(q) + \Pi_{\mu\nu}(q) \tag{4}$$

where $D^{(0)-1}_{\mu\nu}(q)$ is the free inverse gauge boson propagator. The gauge boson propagator in Landau gauge is

$$D_{\mu\nu}(q) = \frac{1}{q^2[1+\Pi(q)] + m_a^2}(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) \tag{5}$$

$$\Pi(q) = \Pi_F(q) + \Pi_B(q) \tag{6}$$

where $\Pi_F(q)$ and $\Pi_B(q)$ are the polarization function from the fermion part and the boson part, respectively.

Next we decompose the scalar field as follows:

$$\phi(x) = \frac{v + h(x) + i\varphi(x)}{\sqrt{2}} \tag{7}$$

For $\mu^2 < 0$, in Anderson–Higgs model the nonzero vacuum expectation value $\langle\phi\rangle_0 = \sqrt{-\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$. In fact, a nonzero $\langle\phi\rangle_0$ induces the spontaneous breaking of gauge symmetry and the gauge boson acquires a nonzero mass m_a via the Anderson–Higgs mechanism. However, the finite gauge boson mass will suppress the occurrence of DCSB. Now the boson Lagrangian can be written in the form:

$$\mathcal{L}_B = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu\varphi)^2 + \frac{1}{2}e^2(v+h)^2A_\mu^2 + \frac{1}{2}e^2\varphi^2A_\mu^2 + e\varphi A_\mu\partial_\mu h - e(v+h)A_\mu\partial_\mu\varphi$$
$$-\frac{\lambda}{4}(h^4 + \varphi^4 + 4v^2h^2 + 4vh^3 + 4vh\varphi^2 + 2h^2\varphi^2) \tag{8}$$

The mass of gauge boson is $m_a = ev$ and $m_h = \sqrt{2\lambda}v$. ¿From these two mass scales we obtain the Ginzburg–Landau parameter $r = \frac{m_h}{m_a}$. In high temperature superconductors r is generally about 100 [35]. The one-loop vacuum polarization $\Pi_B(q)$ has been calculated by evaluating four Feynman diagrams [33,35] and the result is

$$\Pi_B = \frac{e^2}{4\pi q^2}[m_a - m_h + \frac{m_a}{q^2}(m_h^2 - m_a^2) + \frac{m_h}{q^2}(m_a^2 - m_h^2)] + \frac{e^2}{4\pi}\frac{(q^2+m_h^2-m_a^2)^2 - 4m_a^2q^2}{2q^5}\zeta \tag{9}$$

where $\zeta = \arctan\frac{q^2+m_a^2-m_h^2}{2m_hq} + \arctan\frac{q^2+m_h^2-m_a^2}{2m_aq}$.

In order to obtain a closed system of coupled DSEs which can be solved by iteration method, one should specify the fermion-photon vertex function. In the literature, there are several attempts to determine the form of the fermion-photon vertex [36–46], for instance, the Ball–Chiu (BC) [45] and Curtis–Pennington (CP) vertex [39]. In this paper we shall follow [8] to choose the vertex ansatz $\Gamma_\nu(p,k) = \frac{1}{2}[A(p^2) + A(k^2)]\gamma_\nu$ (the BC_1 vertex). This choice has the advantage that the equations are simplified significantly and it already contains all qualitative features of the solution employing the CP/BC vertex in the infrared region, as was demonstrated by the numerical calculations given in [30]. Based on the above discussion, we obtain the coupled DSEs with gauge boson mass m_a and Higgs mass m_h [47,48]:

$$A(p^2) = 1 + \frac{1}{p^2}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{A(p^2)+A(k^2)}{A^2(k^2)k^2+B(p^2)}\frac{A(k^2)(p\cdot q)(k\cdot q)/q^2}{[q^2(1+\Pi(q^2))+m_a^2]} \tag{10}$$

$$B(p^2) = \int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{B(k^2)[A(p^2)+A(k^2)]}{[A^2(k^2)k^2+B^2(k^2)][q^2(1+\Pi(q^2))+m_a^2]} \tag{11}$$

$$\Pi(q^2) = N\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{A(k^2)A(p^2)[A(p^2)+A(k^2)]}{q^2[A^2(k^2)k^2+B^2(k^2)]}\frac{[2k^2-4k\cdot q-6(k\cdot q)^2/q^2]}{[A^2(p^2)p^2+B^2(p^2)]} \tag{12}$$

It is well known that one can obtain two types of solution by iterating the above coupled DSEs, the Nambu solution and the Wigner solution. If Equations (10-12) has a nontrivial solution, *i.e.*, the Nambu solution, then the fermions can acquire a nonzero mass by DCSB. In DCSB phase ($N < N_c$, the fermion mass function $M(p^2) = \frac{B(p^2)}{A(p^2)} > 0$) the attractive force between a pair of fermion and anti-fermion becomes weak with m_a increasing.

Here it is interesting to look at the quantitative impact of refining the truncation scheme used in this paper. In order to show a comparison of our result with that of the bare vertex approximation, in Figure 1, taking the $N = 1$ case as an example, we draw the curves of the mass function versus p^2 for $m_h = 0.001$ and $r = 20$ for both the bare vertex and the BC_1 vertex ansatz. It can be seen that for both the bare vertex and the BC_1 vertex ansatz, the mass function almost remains constant for small p^2, and decreases monotonously with p^2 increasing after p^2 reaches a certain value. In the whole range of p^2, the mass function obtained using the BC_1 vertex is much larger than that obtained using the bare vertex. This shows that the dressing effect of the fermion-photon vertex is very important in the study of dynamical mass generation in the Abelian Higgs Model in 2 + 1 dimensions. In addition, due to the fact that increasing gauge boson mass weakens the attractive force between a pair of fermion and antifermion, the larger infrared value of the mass function for the case of BC_1 ansatz implies that the critical gauge boson mass for the case of BC_1 ansatz should be larger that the one for the case of bare vertex.

Figure 1. The mass function $M(p^2)$ versus p^2 at $r = \frac{m_h}{m_a} = 20$ and $m_h = 0.001$ for $N = 1$ case calculated using both the bare vertex and the BC_1 vertex ansatz.

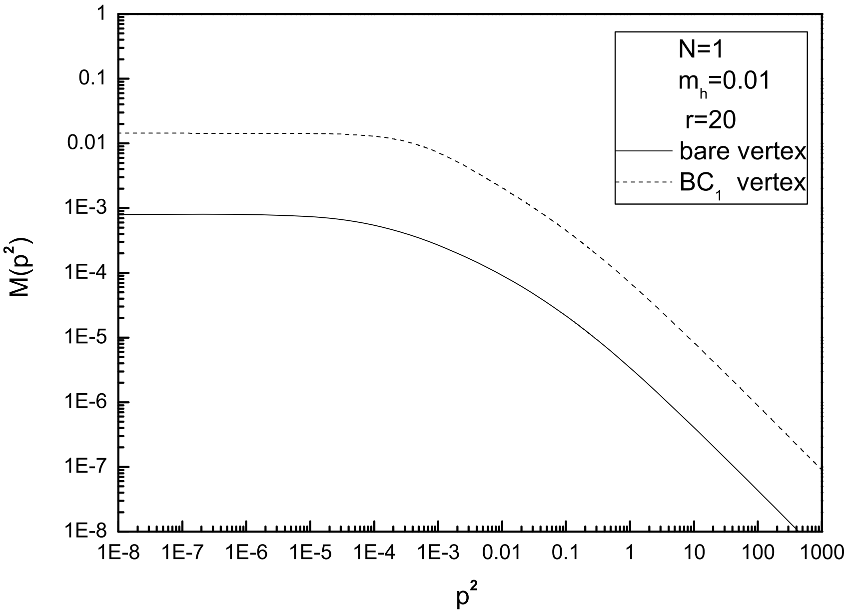
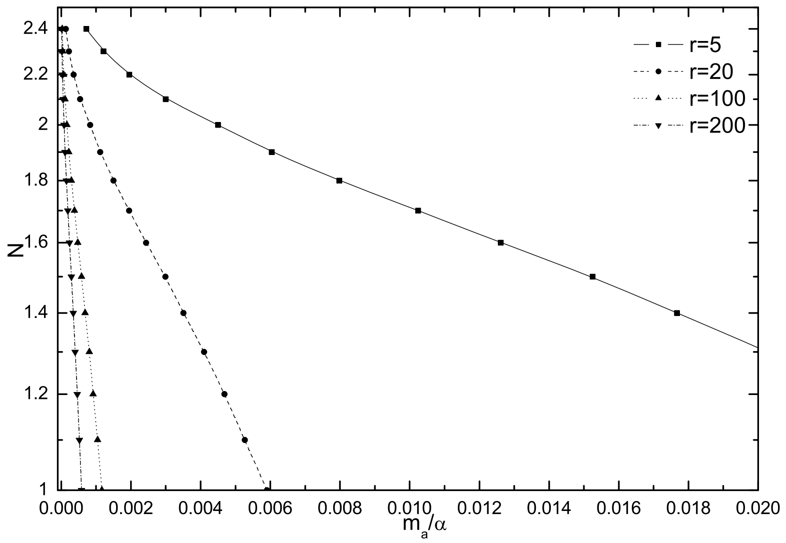


Figure 2. Dependence of the number of fermion flavor N_c on m_a for several values of the ratio $r = \frac{m_h}{m_a}$.



In Figure 2, the variation of N_c on m_a for several values of the ratio r is shown. From the obtained numerical results one finds that DCSB is completely suppressed when m_a exceeds a critical value m_a^{crit} for a fixed number of fermion flavor N (here the mass terms are scaled by $\alpha = Ne^2$). From Figure 2 one finds that N_c decreases monotonically with the gauge boson mass m_a increasing for fixed value of r and it decreases monotonically with the Ginzburg–Landau parameter r increasing for fixed value of m_a. It can be seen that the corresponding critical number of fermion flavor N_c is about 2.4

when the gauge boson mass m_a tends to zero. So the originally massless fermion acquires a dynamical mass (the physical number of fermion flavor $N = 2 < 2.4$). But in Figure 2 the $N_c - m_a$ curve is steeper and for the same value of N_c and r, m_a is much larger than the value of [33]. In fact, for $m_h = 0.001$ and $N = 2$, we find that the critical gauge boson mass m_a^{c0} is too small and DCSB can be hardly observed when we numerically solve the coupled DSEs by means of iteration method using the bare vertex ansatz. Here we note that QED_3 theory (and its generalization to the Abelian Higgs model) with $N = 2$ is employed as an effective continuum theory for the 2D quantum antiferromagnetic (Neel) ordering corresponding to dynamical fermion mass generation [49–52]. So a reliable value of the critical number of fermion flavors is important for the study of dynamical fermion mass generation and chiral symmetry breaking.

3. Conclusions

In this paper, the DSE for the fermion self-energy in QED_3 with Abelian Higgs model is studied. We numerically solve the coupled DSEs by means of iteration method using the BC_1 vertex ansatz and compare our result with that obtained using the bare vertex. It is found that the mass function obtained using the BC_1 vertex ansatz is much larger than the one obtained using the bare vertex. This shows that the dressing effect of the fermion-photon vertex is very important in the study of dynamical mass generation in the Abelian Higgs model in $2+1$ dimensions. It is also found that the gauge boson mass m_a suppresses the critical number of fermion flavor N_c for a fixed ratio $r = \frac{m_h}{m_a}$. When m_a exceeds a critical value m_a^{crit}, DCSB will be completely suppressed. One the other hand, the gauge boson mass is reduced rapidly as r increases. These results imply that for a fixed value of m_a, the smaller is the ratio r, the easier is it to generate a finite fermion mass by the mechanism of DCSB. The above results qualitatively accord with the conclusion of Liu *et al.* [33]. Finally, we note that the BC_1 vertex ansatz employed in our calculation does not satisfy the Ward–Takahashi identity when the dynamical mass function $B(p^2)$ is present in the fermion propagator. In a more reliable calculation the CP [39] vertex should be employed. This work will be done in the future.

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