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Enhancing Interval-Valued Pythagorean Fuzzy Decision-Making through Dombi-Based Aggregation Operators

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Abstract: The success of any endeavor or process is heavily contingent on the ability to reconcile and satisfy balance requirements, which are often characterized by symmetry considerations. In practical applications, the primary goal of decision-making processes is to efficiently manage the symmetry or asymmetry that exists within different sources of information. In order to address this challenge, the primary aim of this study is to introduce novel Dombi operation concepts that are formulated within the framework of interval-valued Pythagorean fuzzy aggregation operators. In this study, an updated score function is presented to resolve the deficiency of the current score function in an interval-valued Pythagorean fuzzy environment. The concept of Dombi operations is used to introduce some interval-valued Pythagorean fuzzy aggregation operators, including the interval-valued Pythagorean fuzzy Dombi weighted arithmetic (IVPFDWA) operator, the interval-valued Pythagorean fuzzy Dombi ordered weighted arithmetic (IVPFDOWA) operator, the interval-valued Pythagorean fuzzy Dombi weighted geometric (IVPFDWG) operator, and the interval-valued Pythagorean fuzzy Dombi ordered weighted geometric (IVPFDOWG) operator. Moreover, the study investigates many important properties of these operators that provide new semantic meaning to the evaluation. In addition, the suggested score function and newly derived interval-valued Pythagorean fuzzy Dombi aggregation (IVPFDA) operators are successfully employed to select a subject expert in a certain institution. The proposed approach is demonstrated to be successful through empirical validation. Lastly, a comparative study is conducted to demonstrate the validity and applicability of the suggested approaches in comparison with current techniques. This research contributes to the ongoing efforts to advance the field of evaluation and decision-making by providing novel and effective tools and techniques.

Keywords: IVPFDWA operator; IVPFDOWA operator; IVPFDWG operator; IVPFDOWG operator; multi-criteria decision-making problem

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1. Introduction

Selecting the best possible option from a set of candidates based on a number of criteria or qualities is the goal of multi-criteria decision-making. Generally, it is assumed that all data used to evaluate an option in terms of its qualities and the relative weights of those features are presented in the form of crisp numbers. However, in many real-life situations, there are problems taken into account where the objectives and constraints are conventionally vague and imprecise in nature. To cope with this problem, Zadeh [1] introduced the concept of fuzzy sets (FSs) as a generalization of crisp sets.

In fuzzy sets, information about the degree to which certain criteria are met is obtained in the form of membership functions, whose complements are taken as insufficient degrees. In 1970, Bellman and Zadeh [2] discussed the multiple attribute decision-making process in the FS theory. In addition, the theory of solving decision-making problems under fuzzy environments was developed in [3,4]. In 1982, Dombi [5] discussed a general class of fuzzy operators. The theory of intuitionistic fuzzy sets has been widely adopted and has a significant impact on various disciplines, including decision-making, engineering, information technology, pattern recognition, and medical diagnostics. The concept of intuitionistic fuzzy sets, which represents a generalization of fuzzy sets, was first proposed by Atanassov in 1986 [6]. It meets the condition that the sum of the membership and non-membership degrees of a given element must not exceed unity.

In 1986, Turksen [7] put forward the idea of the interval-valued fuzzy set (IVFS) as a modification to fuzzy sets. In real-life situations, where information is often inadequate and imprecise, decision-makers may find it challenging to express their opinions with precise numerical values. To address this challenge, Atanassov [8] introduced the IV IFS concept in 1989. In this approach, the membership and non-membership degrees are represented by an interval within [0, 1] rather than a precise numerical value. In 2000, De et al. [9] introduced three basic operations for an IFS, namely concentration, dilation, and normalization. The intuitionistic fuzzy set theory has attracted more attention from many researchers since its inception. However, later on, many deficiencies were found in this technique, which led the mathematician, Atanassov, to explore some higher order fuzzy sets.

The significance of the concepts of IFSs and IVIFSs is quite evident as many mathematicians have introduced different types of aggregation operators and information measures based on these sets, which have been effectively utilized in addressing MCDM problems in various circumstances [10–15]. Despite the numerous advantages of these theories, there still exist real-world situations that cannot be addressed by the above-mentioned strategies. For instance, if a decision-maker assigns a membership degree of 0.7 and a non-membership degree of 0.5 to an element, their sum would exceed 1. To broaden the scope of membership and non-membership degrees, Yager [16] introduced the concept of Pythagorean Fuzzy Sets (PFSs), where the sum of the squares of the membership and non-membership is less than or equal to 1.

In 2014, Zhang and Xu [17] discussed decision-making using PFSs. In addition, more develop methods in the framework of the interval-valued intuitionistic fuzzy (IVIF) environment can be found in [18,19]. In 2017, Chen and Huang [20] discussed decision-making based on particle swarm optimization techniques. In 2016, Zhang [21] presented the concept of the IV Pythagorean fuzzy set. Moreover, one can view the recent developments of IVPFSs in [22–33]. In 2016, Peng and Yang [34] discussed the fundamental properties of aggregation operators of IVPFSs. Furthermore, different Dombi Operators in various sets were discussed in [35–40]. In addition, many useful strategies were invented to address the issue of energy crises in [41–43]

Dombi operators can take multi-purpose aggregation, decision-making abilities, and operational characteristics and turn them into a highly adaptable tool for gathering imprecise information. The information is converted into a single value with the help of the averaging operators. Dombi operators are incredibly adaptable when it comes to operational conditions, and they are also highly effective at solving decision-making issues. The main benefit of our approach is that it can handle the interrelationship between the arguments, which the other approaches cannot do. Our approach is therefore more versatile. Furthermore, it is important to note that the existing techniques lack the ability to dynamically adjust the parameter in accordance with the decision-makers' risk aversion, which makes the MADM solution difficult to implement in real life. However, the techniques presented in this article are very capable of addressing this weakness in this situation.

Our research work aims to achieve several primary objectives, which are as follows:

1. Develop an updated score function that overcomes the deficiencies of existing score functions in IVPF environment. This will involve the integration of advanced mathematical and statistical techniques to create a more robust and accurate scoring system.
2. Formulate fundamental Dombi operations for IVPFSs. This will involve the development of mathematical models that describe the relationships between different elements of IVPFSs, allowing for more accurate analysis and prediction of outcomes.
3. Initiate the study of IVPFD aggregation operators. This will involve exploring the ways in which different IVPFD operators can be combined to create more effective aggregation methods for IVPFSs.
4. Prove many key properties of the newly defined operators. This will involve rigorous mathematical analysis and formal proofs aimed at demonstrating the validity and effectiveness of the proposed operators.
5. Present an algorithm to solve Multiple Attribute Decision-Making (MADM) problems using IVPFD aggregation operators. This will involve developing a step-by-step process for using the new operators to analyze and evaluate complex decision-making problems.
6. Select the best subject expert in a certain institution using the newly suggested technique. This will involve applying the proposed algorithm to real-world scenarios, such as selecting the most qualified candidate for a job or identifying the most suitable expert for a specific project.
7. Present a comparative analysis to show the validity of the proposed technique compared with existing strategies. This will involve testing the effectiveness of the proposed algorithm against existing methods, using real-world data sets and scenarios.

After a brief discussion of IVPFSs, the rest of the work is organized as: in Section 2, some basic definitions are defined to understand the novelty of the work presented in this article. In Section 3, the deficiency of existing score functions to solve MADM problems is discussed and an improved score function is determined to counter this problem under the IVPF environment. In Section 4, some Dombi aggregation operators for these sets are developed. In Section 5, a numerical example is given to show the validity of this new approach. In addition, a comparative analysis is presented to depict the validity and feasibility of this new strategy with existing methods. Finally, some concrete conclusions about the paper are summarized in Section 6.

2. Preliminaries

In this section, we briefly review some basic concepts, properties, operations, and magnitude comparison methods of IVPFSs over the non-empty universal set X that are important to the achievement of this study.

Definition 1 [7]**.** *An IVFS A defined on X is given by* $A = \{\langle x, [\mu_A{}^L(x), \mu_A{}^U(x)]\rangle : x \in X\}$, *where* $0 \le \mu_A{}^L(x) \le \mu_A{}^U(x) \le 1$.

Definition 2 [14]**.** *Let X be a classical set. A PFS A on X is represented as:* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, *where* μ_A: $X \to [0,1]$ *and* ν_A: $X \to [0,1]$ *denote the membership and non-membership functions, respectively, that satisfy the condition* $0 \le \mu_A^2(x) + \nu_A^2(x) \le 1$. *The hesitancy margin of the PFS A is defined as:* $\pi_A(x) = \sqrt{1 - \mu_A^2(x) - \nu_A^2(x)}$.

Definition 3 [18]**.** *An IVPFS A on X is defined as:* $A = \{\langle x, [\mu_A{}^L(x), \mu_A{}^U(x)], [\nu_A{}^L(x), \nu_A{}^U(x)]\rangle : x \in X\}$, *where* $[\mu_A{}^L(x), \mu_A{}^U(x)]$ *and* $[\nu_A{}^L(x), \nu_A{}^U(x)]$, *respectively, represent the membership and non-membership degree of A that are able to satisfy the condition* $1 \ge \mu_A{}^U(x) \ge \mu_A{}^L(x) \ge 0$ *and* $0 \le$

${\nu_A}^L(x) \le {\nu_A}^U(x) \le 1$. Moreover, $0 \le ({\mu_A}^L(x))^2 + ({\nu_A}^L(x))^2 \le 1$ and $0 \le ({\mu_A}^U(x))^2 + ({\nu_A}^U(x))^2 \le 1$. The hesitancy degree of the IVPFS A is defined as:

$$\pi_A(x) = \left[{\pi_A}^L(x), {\pi_A}^U(x)\right]$$
$$= \left[\sqrt{1 - \left(\mu_A^L(x)\right)^2 - \left(\nu_A^L(x)\right)^2}, \sqrt{1 - \left(\mu_A^U(x)\right)^2 - \left(\nu_A^U(x)\right)^2}\right]$$

Definition 4 [35]**.** *Consider the three IVPF $g = \langle[\mu_g^L, \mu_g^U],[\nu_g^L, \nu_g^U]\rangle$, $g_1 = \langle[\mu_{g1}^L, \mu_{g1}^U],[\nu_{g1}^L, \nu_{g1}^U]\rangle$ and $g_2 = \langle[\mu_{g2}^L, \mu_{g2}^U],[\nu_{g2}^L, \nu_{g2}^U]\rangle$. The basic operations on these numbers are defined in the subsequent way:*

i. $g_1 \cup g_2 = \langle[max\{\mu_{g1}^L, \mu_{g2}^L,\}, max\{\mu_{g1}^U, \mu_{g2}^U,\}], [min\{\nu_{g1}^L, \nu_{g2}^L\}, min\{\nu_{g1}^U, \nu_{g2}^U,\}]\rangle$

ii. $g_1 \cap g_2 = \langle[min\{\mu_{g1}^L, \mu_{g2}^L,\}, min\{\mu_{g1}^U, \mu_{g2}^U,\}], [max\{\nu_{g1}^L, \nu_{g2}^L\}, max\{\nu_{g1}^U, \nu_{g2}^U,\}]\rangle$

iii. $g_1 \oplus g_2 = \left\langle \begin{array}{c} \left[\sqrt{\left(\mu_{g_1}^L\right)^2 + \left(\mu_{g_2}^L\right)^2 - \left(\mu_{g_1}^L\right)^2\left(\mu_{g_2}^L\right)^2}, \sqrt{\left(\mu_{g_1}^U\right)^2 + \left(\mu_{g_2}^U\right)^2 - \left(\mu_{g_1}^U\right)^2\left(\mu_{g_2}^U\right)^2}\right], \\ \left[\nu_{g_1}^L \nu_{g_2}^L, \nu_{g_1}^U \nu_{g_2}^U\right] \end{array} \right\rangle$

iv. $g_1 \otimes g_2 = \left\langle \begin{array}{c} \left[\mu_{g_1}^L \mu_{g_2}^L, \mu_{g_1}^U \mu_{g_2}^U\right], \\ \left[\sqrt{\left(\nu_{g_1}^L\right)^2 + \left(\nu_{g_2}^L\right)^2 - \left(\nu_{g_1}^L\right)^2\left(\nu_{g_2}^L\right)^2}, \sqrt{\left(\nu_{g_1}^U\right)^2 + \left(\nu_{g_2}^U\right)^2 - \left(\nu_{g_1}^U\right)^2\left(\nu_{g_2}^U\right)^2}\right] \end{array} \right\rangle$

v. $\chi g = \left\langle \left[\sqrt{1 - \left(1 - \left(\mu_g^L\right)^2\right)^{\mathcal{X}}}, \sqrt{1 - \left(1 - \left(\mu_g^U\right)^2\right)^{\mathcal{X}}}\right], \left[\left(\nu_g^L\right)^{\mathcal{X}}, \left(\nu_g^U\right)^{\mathcal{X}}\right] \right\rangle$*, for all $\chi > 0$.*

vi. $g^{\chi} = \left\langle \left[\left(\mu_g^L\right)^{\mathcal{X}}, \left(\mu_g^U\right)^{\mathcal{X}}\right], \left[\sqrt{1 - \left(1 - \left(\nu_g^L\right)^2\right)^{\mathcal{X}}}, \sqrt{1 - \left(1 - \left(\nu_g^U\right)^2\right)^{\mathcal{X}}}\right] \right\rangle$*, for all $\chi > 0$.*

vii. $g^C = \left\langle \left[\nu_g^L, \nu_g^U\right], \left[\mu_g^L, \mu_g^U\right] \right\rangle$.

Some certain types of triangular norms and conorms are discussed in the following definition.

Definition 5 [5]**.** *Let ã and $\widetilde{b}$ be any two real numbers. The Dombi t-norms and Dombi t-conorms are described in the following way:*

i. $Dom(\tilde{a}, \widetilde{b}) = \frac{1}{1+\left\{\left\{\left(\frac{1-}{\,}\right)^{Ḱ}+\left(\frac{1-\widetilde{b}}{\widetilde{b}}\right)^{Ḱ}\right\}\right\}^{\frac{1}{Ḱ}}}$

ii. $Dom^C(\tilde{a}, \widetilde{b}) = \frac{1}{1+\left\{\left\{\left(\frac{\,}{1-}\right)^{Ḱ}+\left(\frac{\widetilde{b}}{1-\widetilde{b}}\right)^{Ḱ}\right\}\right\}^{\frac{1}{Ḱ}}}$

where $Ḱ \ge 1$ and $\left(\tilde{a}, \widetilde{b}\right) \in [0,1] \times [0,1]$. In the above discussion, (i) represents the Dombi product and (ii) represents the Dombi sum.

Definition 6 [35]**.** *Let $g_{\dot{c}} = (\mu_{\dot{c}}, \nu_{\dot{c}})$ ($\dot{c} = 1, 2, 3, \ldots, j$) be j number of PFEs. A Pythagorean fuzzy Dombi weighted averaging (PFDWA) operator is characterized by a function PFDWA: $PFE^j \to PFE$ such that*

$$PFDWA_{\wp}\left(g_1, g_2, \ldots, g_j\right) = \oplus_{\dot{c}=1}^{j}\left(\wp_{\dot{c}} g_{\dot{c}}\right)$$
$$= \left(\sqrt{1 - \frac{1}{1+\left\{\sum_{\dot{c}=1}^{j} \wp_{\dot{c}}\left(\frac{\mu_{\dot{c}}^2}{1-\mu_{\dot{c}}^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{j} \wp_{\dot{c}}\left(\frac{1-\nu_{\dot{c}}^2}{\nu_{\dot{c}}^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}\right)$$

where $\mathcal{P} = \left(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots, \mathcal{P}_j\right)^T$ is the weight vector of $g_{\dot{c}}(\dot{c} = 1, 2, 3, \ldots, j)$ with $\mathcal{P}_{\dot{c}} > 0$, $\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} = 1$ and $\vartheta \geq 1$.

Definition 7 [35]**.** *Let $g_{\dot{c}} = (\mu_{\dot{c}}, \nu_{\dot{c}})$ ($\dot{c}$ = 1, 2, 3, ..., j) be j number of PFEs. A PFD ordered weighted averaging operator of dimension j is characterized by a function PFDOWA: PFE^j → PFE with the associated weighted vector $\mathcal{P} = \left(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots, \mathcal{P}_j\right)^T$ such that $\mathcal{P}_{\dot{c}} > 0$, $\sum\limits_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} = 1$ and $\vartheta \geq 1$. Then,*

$$PFDOWA_{\mathcal{P}}(g_1, g_2, \ldots, g_j) = \oplus_{\dot{c}=1}^{j}\left(\mathcal{P}_{\dot{c}} g_{\rho(\dot{c})}\right)$$
$$= \left(\sqrt{1 - \frac{1}{1 + \left\{\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}}\left(\frac{\mu^2_{\rho(\dot{c})}}{1-\mu^2_{\rho(\dot{c})}}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1 + \left\{\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}}\left(\frac{1-\nu^2_{\rho(\dot{c})}}{\nu^2_{\rho(\dot{c})}}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}\right)$$

where $(\rho(1), \rho(2), \ldots, \rho(j))$ represents the permutations of $\dot{c} = 1, 2, 3, \ldots, j$, and $g_{\rho(\dot{c}-1)} \geq g_{\rho(\dot{c})} \forall\ \dot{c} = 1, 2, \ldots, j$.

Definition 8 [35]**.** *Let $g_{\dot{c}} = (\mu_{\dot{c}}, \nu_{\dot{c}})$ ($\dot{c}$ = 1, 2, 3, ..., j) be j number of PFEs. A PFD weighted geometric operator is characterized by a function PFDWG: PFE^j→PFE such that*

$$PFDWG_{\mathcal{P}}(g_1, g_2, \ldots, g_j) = \otimes_{\dot{c}=1}^{j}(g_{\dot{c}})^{\mathcal{P}_{\dot{c}}}$$
$$= \left(\sqrt{\frac{1}{1 + \left\{\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}}\left(\frac{1-\mu^2_{\dot{c}}}{\mu^2_{\dot{c}}}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1 + \left\{\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}}\left(\frac{\nu^2_{\dot{c}}}{1-\nu^2_{\dot{c}}}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}\right)$$

where $\mathcal{P} = \left(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots, \mathcal{P}_j\right)^T$ is the weight vector of $g_{\dot{c}}(\dot{c} = 1, 2, 3, \ldots, j)$ with $\mathcal{P}_{\dot{c}} > 0$, $\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} = 1$ and $\vartheta \geq 1$.

Definition 9 [34]**.** *Let $g_{\dot{c}} = (\mu_{\dot{c}}, \nu_{\dot{c}})$ ($\dot{c}$ = 1, 2, 3, ..., j) be j number of PFEs. A PFD ordered weighted geometric operator of dimension j is characterized by a function PFDOWG: PFE^j →PFE with the associated weighted vector $\mathcal{P} = \left(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots, \mathcal{P}_j\right)^T$ such that $\mathcal{P}_{\dot{c}} > 0$, $\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} = 1$ and $\vartheta \geq 1$. Then,*

$$PFDOWG_{\mathcal{P}}(g_1, g_2, \ldots, g_j) = \otimes_{\dot{c}=1}^{j}\left(g_{\rho(\dot{c})}{}^{\mathcal{P}_{\dot{c}}}\right)$$
$$= \left(\sqrt{\frac{1}{1 + \left\{\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}}\left(\frac{1-\mu^2_{\rho(\dot{c})}}{\mu^2_{\rho(\dot{c})}}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1 + \left\{\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}}\left(\frac{\nu^2_{\rho(\dot{c})}}{1-\nu^2_{\rho(\dot{c})}}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}\right)$$

where $(\rho(1), \rho(2), \ldots, \rho(j))$ represents the permutations of $\dot{c} = 1, 2, 3, \ldots, j$, and $g_{\rho(\dot{c}-1)} \geq g_{\rho(\dot{c})}\ \forall\ \dot{c} = 1, 2, \ldots, j$.

Definition 10 [27]**.** *The score function for IVPF number $\alpha = \langle [a, b],\ [c, d] \rangle$ is defined in the following way:*

$$S(\alpha) = \frac{(a^2 - c^2)\left(1 + \sqrt{1 - a^2 - c^2}\right) + (b^2 - d^2)\left(1 + \sqrt{1 - b^2 - d^2}\right)}{4}$$

where $S(\alpha) \in [-1, 1]$. Particulary, if $S(\alpha) = 1$, then α is the largest IVPFN, i.e., $\langle[1, 1], [0, 0]\rangle$; if $S(\alpha) = -1$, then α is the least IVPFN, i.e., $\langle[0, 0], [1, 1]\rangle$. Moreover, the score function satisfies the following properties for any two IVPFNs α and β

i. If $S(\alpha) < S(\beta)$, then $\alpha \prec \beta$
ii. If $S(\alpha) > S(\beta)$, then $\alpha \succ \beta$
iii. If $S(\alpha) = S(\beta)$, then $\alpha \sim \beta$

3. Deficiency of the Existing Score Function of IVPFS and Its Improvement

In this section, we provide an example that indicates the deficiencies in the score function of the IVPFS developed in [28] and improve it in the subsequent discussion.

Example 1. *Let $\varsigma_1 = \langle[0.4, 0.7], [0.4, 0.7]\rangle$ and $\varsigma_2 = \langle[0.3, 0.4], [0.3, 0.4]\rangle$ be any two IVPFNs. Then, by applying Definition 10 on ς_1 and ς_2, we obtain*

$$S(\varsigma_1) = S(\varsigma_2) = 0$$

Then, in view of the property *(iii)* of Definition 10, we have $\varsigma_1 \sim \varsigma_2$, but $\varsigma_1 \neq \varsigma_2$.

This indicates the deficiency in the score function under consideration. The above discussion leads us to improve this score function in the following definition.

Definition 11. *Consider the IVPFN $\alpha = \langle[a, b], [c, d]\rangle$. The improved score function $S(\alpha)$ is defined for α as:*

$$\begin{aligned} S(\alpha) = \tfrac{1}{4}\,&((a^2 - c^2 + \tfrac{1}{2})(1 + \sqrt{1 - a^2 - c^2}) \\ &+ (b^2 - d^2 + \tfrac{1}{3})(1 + \sqrt{1 - b^2 - d^2})) \end{aligned} \tag{1}$$

where $h(\alpha)$ is the range of the score function and $-0.29 \leq h(\alpha) \leq 0.58$.

Moreover, the above score function satisfies the following comparison law for any two IVPFNs α and β

i. If $S(\alpha) < S(\beta)$, then $\alpha \prec \beta$
ii. If $S(\alpha) > S(\beta)$, then $\alpha \succ \beta$
iii. If $S(\alpha) = S(\beta)$, then $\alpha \sim \beta$

Consider the following example to demonstrate the effectiveness of the score function proposed for IVPFN.

Example 2. *The application of the proposed score function $S(\alpha)$ in Equation (1) of Example 1 gives that $S(\varsigma_1) = 0.392$ and $S(\varsigma_2) = 0.386$. Thus, in view of property ii of Definition 11, we have $\varsigma_1 \succ \varsigma_2$. This fact suggests that the alternative ς_1 is better than the alternative ς_2.*

So, we conclude that the suggested score function offers a more appropriate and efficient approach for decision analysis compared with the current score function, which has proven to be inadequate.

4. Dombi Operations on Interval-Valued Pythagorean Fuzzy Numbers

In this section, we present the Dombi operations in the framework of the IVPF environment.

Definition 12. *Let $1 \leq \vartheta$, $0 \leq \mathcal{X}$ and $g_1 = \left\langle \left[\mu^L_{g_1}, \mu^U_{g_1}\right], \left[\nu^L_{g_1}, \nu^U_{g_1}\right] \right\rangle$ and $g_2 = \left\langle \left[\mu^L_{g_2}, \mu^U_{g_2}\right], \left[\nu^L_{g_2}, \nu^U_{g_2}\right] \right\rangle$ be any two IVPFEs. The Dombi operations of t-norms and t-conorms of IVPFEs are explained in the following way:*

i. $$g_1 \oplus g_2 = \left(\begin{array}{c} \left[\sqrt{1 - \frac{1}{1+\left\{ \left(\frac{\left(\mu_{g_1}^L\right)^2}{1-\left(\mu_{g_1}^L\right)^2} \right)^{\vartheta} + \left(\frac{\left(\mu_{g_2}^L\right)^2}{1-\left(\mu_{g_2}^L\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1+\left\{ \left(\frac{\left(\mu_{g_1}^U\right)^2}{1-\left(\mu_{g_1}^U\right)^2} \right)^{\vartheta} + \left(\frac{\left(\mu_{g_2}^U\right)^2}{1-\left(\mu_{g_2}^U\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{\frac{1}{1+\left\{ \left(\frac{1-\left(\nu_{g_1}^L\right)^2}{\left(\nu_{g_1}^L\right)^2} \right)^{\vartheta} + \left(\frac{1-\left(\nu_{g_2}^L\right)^2}{\left(\nu_{g_2}^L\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{ \left(\frac{1-\left(\nu_{g_1}^U\right)^2}{\left(\nu_{g_1}^U\right)^2} \right)^{\vartheta} + \left(\frac{1-\left(\nu_{g_2}^U\right)^2}{\left(\nu_{g_2}^U\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right)$$

ii. $$g_1 \otimes g_2 = \left(\begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{ \left(\frac{1-\left(\mu_{g_1}^L\right)^2}{\left(\mu_{g_1}^L\right)^2} \right)^{\vartheta} + \left(\frac{1-\left(\mu_{g_2}^L\right)^2}{\left(\mu_{g_2}^L\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{ \left(\frac{1-\left(\mu_{g_1}^U\right)^2}{\left(\mu_{g_1}^U\right)^2} \right)^{\vartheta} + \left(\frac{1-\left(\mu_{g_2}^U\right)^2}{\left(\mu_{g_2}^U\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{1 - \frac{1}{1+\left\{ \left(\frac{\left(\nu_{g_1}^L\right)^2}{1-\left(\nu_{g_1}^L\right)^2} \right)^{\vartheta} + \left(\frac{1-\left(\nu_{g_2}^L\right)^2}{\left(\nu_{g_2}^L\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1+\left\{ \left(\frac{\left(\nu_{g_1}^U\right)^2}{1-\left(\nu_{g_1}^U\right)^2} \right)^{\vartheta} + \left(\frac{\left(\nu_{g_2}^U\right)^2}{1-\left(\nu_{g_2}^U\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right)$$

iii. $$\mathcal{X}.\, g_1 = \left(\begin{array}{c} \left[\sqrt{1 - \frac{1}{1+\left\{ \mathcal{X} \left(\frac{\left(\mu_{g_1}^L\right)^2}{1-\left(\mu_{g_1}^L\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1+\left\{ \mathcal{X} \left(\frac{\left(\mu_{g_1}^U\right)^2}{1-\left(\mu_{g_1}^U\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{\frac{1}{1+\left\{ \mathcal{X} \left(\frac{1-\left(\nu_{g_1}^L\right)^2}{\left(\nu_{g_1}^L\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{ \mathcal{X} \left(\frac{1-\left(\nu_{g_1}^U\right)^2}{\left(\nu_{g_1}^U\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right)$$

iv. $${g_1}^{\mathcal{X}} = \left(\begin{array}{c} \left[\sqrt{1 - \frac{1}{1+\left\{ \mathcal{X} \left(\frac{1-\left(\mu_{g_1}^L\right)^2}{\left(\mu_{g_1}^L\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1+\left\{ \mathcal{X} \left(\frac{1-\left(\mu_{g_1}^U\right)^2}{\left(\mu_{g_1}^U\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{\frac{1}{1+\left\{ \mathcal{X} \left(\frac{\left(\nu_{g_1}^L\right)^2}{1-\left(\nu_{g_1}^L\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{ \mathcal{X} \left(\frac{\left(\nu_{g_1}^U\right)^2}{1-\left(\nu_{g_1}^U\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right)$$

In the following definition, we propose a Dombi arithmetic aggregation operator with IVPFEs, namely, the IVPFDWA operator.

Definition 13. *Let* $\dot{c} = 1, 2, 3, \ldots, j$ *and* $g_{\dot{c}} = \left\langle \left[\mu_{g_{\dot{c}}}^L, \mu_{g_{\dot{c}}}^U \right], \left[\nu_{g_{\dot{c}}}^L, \nu_{g_{\dot{c}}}^U \right] \right\rangle$*) be j number of IVPFEs. The IVPFDWA operator is characterized by a function \$IVPFDWA:* $IVPFE^j \rightarrow IVPFE$ *such that*

$$IVPFDWA_{\wp}(g_1, g_2, g_3, \ldots, g_j) = \oplus_{\dot{c}=1}^{j} (\wp_{\dot{c}} g_{\dot{c}})$$

where $\wp = (\wp_1, \wp_2, \wp_3, \ldots, \wp_j)^T$ is the weight vector of $g_{\dot{c}}(\dot{c} = 1, 2, 3, \ldots, j)$ with $\wp_{\dot{c}} > 0$ and $\sum_{\dot{c}=1}^{j} \wp_{\dot{c}} = 1$.

Theorem 1. *Let* $\dot{c} = 1, 2, 3, \ldots, j$ *and* $g_{\dot{c}} = \left\langle \left[\mu^L_{g_{\dot{c}}}, \mu^U_{g_{\dot{c}}}\right], \left[\nu^L_{g_{\dot{c}}}, \nu^U_{g_{\dot{c}}}\right] \right\rangle$ *be j number of IVPFEs. The aggregated value of these IVPFEs in the framework of the IVPFDWA operator is an IVPFE and is determined in the following way:*

$$IVPFDWA_{\wp}(g_1, g_2, g_3, \ldots, g_j) = \oplus_{\dot{c}=1}^{j}(\wp_{\dot{c}} g_{\dot{c}})$$
$$= \left(\begin{array}{c} \left[\sqrt{1 - \frac{1}{1+\left\{\sum_{\dot{c}=1}^{j} \wp_{\dot{c}} \left(\frac{\left(\mu^L_{g_{\dot{c}}}\right)^2}{1-\left(\mu^L_{g_{\dot{c}}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1+\left\{\sum_{\dot{c}=1}^{j} \wp_{\dot{c}} \left(\frac{\left(\mu^U_{g_{\dot{c}}}\right)^2}{1-\left(\mu^U_{g_{\dot{c}}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{j} \wp_{\dot{c}} \left(\frac{1-\left(\nu^L_{g_{\dot{c}}}\right)^2}{\left(\nu^L_{g_{\dot{c}}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{j} \wp_{\dot{c}} \left(\frac{1-\left(\nu^U_{g_{\dot{c}}}\right)^2}{\left(\nu^U_{g_{\dot{c}}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right) \quad (2)$$

where $\wp = (\wp_1, \wp_2, \wp_3, \ldots, \wp_j)^T$ is the weight vector of $g_{\dot{c}}$ and $0 \leq \wp_{\dot{c}}$, $\sum_{\dot{c}=1}^{j} \wp_{\dot{c}} = 1$.

Proof. The proof of this result is established through the use of mathematical induction. □

The application of Definition 12 for ċ = 2 gives the following outcome:

$$IVPFDWA_{\wp}(g_1, g_2) = \wp_1 g_1 \oplus \wp_2 g_2$$
$$= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt{1 - \frac{1}{1+\left\{\wp_1 \left(\frac{\left(\mu^L_{g_1}\right)^2}{1-\left(\mu^L_{g_1}\right)^2} \right)^{\vartheta} + \wp_2 \left(\frac{\left(\mu^L_{g_2}\right)^2}{1-\left(\mu^L_{g_2}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \\ \sqrt{1 - \frac{1}{1+\left\{\wp_1 \left(\frac{\left(\mu^U_{g_1}\right)^2}{1-\left(\mu^U_{g_1}\right)^2} \right)^{\vartheta} + \wp_2 \left(\frac{\left(\mu^U_{g_2}\right)^2}{1-\left(\mu^U_{g_2}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \end{array} \right], \\ \left[\begin{array}{c} \sqrt{\frac{1}{1+\left\{\wp_1 \left(\frac{1-\left(\nu^L_{g_1}\right)^2}{\left(\nu^L_{g_1}\right)^2} \right)^{\vartheta} + \wp_2 \left(\frac{1-\left(\nu^L_{g_2}\right)^2}{\left(\nu^L_{g_2}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \\ \sqrt{\frac{1}{1+\left\{\wp_1 \left(\frac{1-\left(\nu^U_{g_1}\right)^2}{\left(\nu^U_{g_1}\right)^2} \right)^{\vartheta} + \wp_2 \left(\frac{1-\left(\nu^U_{g_2}\right)^2}{\left(\nu^U_{g_2}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \end{array} \right] \end{array} \right)$$

This means that

$$IVPFDWA_{\wp}(g_1, g_2) = \left(\begin{array}{c} \left[\sqrt{1 - \frac{1}{1+\left\{ \sum_{\dot{c}=1}^{2} \wp_{\dot{c}} \left(\frac{\left(\mu^L_{g_{\dot{c}}}\right)^2}{1-\left(\mu^L_{g_{\dot{c}}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1+\left\{ \sum_{\dot{c}=1}^{2} \wp_{\dot{c}} \left(\frac{\left(\mu^U_{g_{\dot{c}}}\right)^2}{1-\left(\mu^U_{g_{\dot{c}}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{\frac{1}{1+\left\{ \sum_{\dot{c}=1}^{2} \wp_{\dot{c}} \left(\frac{1-\left(\nu^L_{g_1}\right)^2}{\left(\nu^L_{g_{\dot{c}}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{ \sum_{\dot{c}=1}^{2} \wp_{\dot{c}} \left(\frac{1-\left(\nu^U_{g_1}\right)^2}{\left(\nu^U_{g_{\dot{c}}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right)$$

Thus, Equation (2) holds for $\dot{c} = 2$.
Suppose that Equation (2) holds for $\dot{c} = p$. Therefore, we have

$$IVPFDWA_{\wp}(g_1, g_2, g_3, \ldots, g_p) = \oplus_{\dot{c}=1}^{p} (\wp_{\dot{c}} g_{\dot{c}})$$

$$= \left(\begin{array}{c} \left[\sqrt{1 - \frac{1}{1+\left\{ \sum_{\dot{c}=1}^{p} \wp_{\dot{c}} \left(\frac{\left(\mu^L_{g_{\dot{c}}}\right)^2}{1-\left(\mu^L_{g_{\dot{c}}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1+\left\{ \sum_{\dot{c}=1}^{p} \wp_{\dot{c}} \left(\frac{\left(\mu^U_{g_{\dot{c}}}\right)^2}{1-\left(\mu^U_{g_{\dot{c}}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{\frac{1}{1+\left\{ \sum_{\dot{c}=1}^{p} \wp_{\dot{c}} \left(\frac{1-\left(\nu^L_{g_1}\right)^2}{\left(\nu^L_{g_{\dot{c}}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{ \sum_{\dot{c}=1}^{p} \wp_{\dot{c}} \left(\frac{1-\left(\nu^U_{g_1}\right)^2}{\left(\nu^U_{g_{\dot{c}}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right)$$

Moreover, for $\dot{c} = p + 1$, we have

$$IVPFDWA_{\mathcal{P}}(g_1, g_2, g_3, \ldots, g_p,\ g_{p+1}) = \oplus_{\dot{c}=1}^{p}(\mathcal{P}_{\dot{c}} g_{\dot{c}}) \oplus (\mathcal{P}_{p+1} g_{p+1})$$

$$= \left(\begin{array}{c} \left[\sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p}\mathcal{P}_{\dot{c}}\left(\frac{\left(\mu^L_{g_{\dot{c}}}\right)^2}{1-\left(\mu^L_{g_{\dot{c}}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}},\ \sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p}\mathcal{P}_{\dot{c}}\left(\frac{\left(\mu^U_{g_{\dot{c}}}\right)^2}{1-\left(\mu^U_{g_{\dot{c}}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p}\mathcal{P}_{\dot{c}}\left(\frac{1-\left(\nu^L_{g_1}\right)^2}{\left(\nu^L_{g_{\dot{c}}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}},\ \sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p}\mathcal{P}_{\dot{c}}\left(\frac{1-\left(\nu^U_{g_1}\right)^2}{\left(\nu^U_{g_{\dot{c}}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right)$$

$$\oplus \left(\begin{array}{c} \left[\sqrt{1-\frac{1}{1+\left\{\mathcal{P}_{p+1}\left(\frac{\left(\mu^L_{g_{p+1}}\right)^2}{1-\left(\mu^L_{g_{p+1}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}},\ \sqrt{1-\frac{1}{1+\left\{\mathcal{P}_{p+1}\left(\frac{\left(\mu^U_{g_{p+1}}\right)^2}{1-\left(\mu^U_{g_{p+1}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{\frac{1}{1+\left\{\mathcal{P}_{p+1}\left(\frac{1-\left(\nu^L_{p+1}\right)^2}{\left(\nu^L_{g_{p+1}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}},\ \sqrt{\frac{1}{1+\left\{\mathcal{P}_{p+1}\left(\frac{1-\left(\nu^U_{g_{p+1}}\right)^2}{\left(\nu^U_{g_{p+1}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right)$$

This means that

$$IVPFDWA_{\mathcal{P}}(g_1, g_2, g_3, \ldots, g_p,\ g_{p+1})$$

$$= \left(\begin{array}{c} \left[\sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p+1}\mathcal{P}_{\dot{c}}\left(\frac{\left(\mu^L_{g_{\dot{c}}}\right)^2}{1-\left(\mu^L_{g_{\dot{c}}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}},\ \sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p+1}\mathcal{P}_{\dot{c}}\left(\frac{\left(\mu^U_{g_{\dot{c}}}\right)^2}{1-\left(\mu^U_{g_{\dot{c}}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p+1}\mathcal{P}_{\dot{c}}\left(\frac{1-\left(\nu^L_{g_1}\right)^2}{\left(\nu^L_{g_{\dot{c}}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}},\ \sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p+1}\mathcal{P}_{\dot{c}}\left(\frac{1-\left(\nu^U_{g_1}\right)^2}{\left(\nu^U_{g_{\dot{c}}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right)$$

Thus, Equation (2) is true for $\dot{c} = p + 1$. Hence, we conclude that Equation (2) is true for any $\dot{c} \in N$.

The following example describes the above-stated fact.

Example 3. *Consider the IVPFEs* $\varsigma_1 = \langle [0.1, 0.5], [0.4, 0.6] \rangle$, $\varsigma_2 = \langle [0.3, 0.4], [0.5, 0.7] \rangle$, *and* $\varsigma_3 = \langle [0.0, 0.3], [0.1, 0.6] \rangle$. *Let* $\mathcal{P}$ *=(0.2,0.5,0.3)*^{T} *be the weighted vector of* $\varsigma_{\dot{c}}$ $(\dot{c} = 1, 2, 3)$ *and* $\vartheta = 4$. *Then,*

$$IVPFDWA_{\mathcal{P}}(\varsigma_1, \varsigma_2, \varsigma_3) = \oplus_{\dot{c}=1}^{3}(\mathcal{P}_{\dot{c}}\varsigma_{\dot{c}})$$

$$= \left(\begin{array}{c} \left[\sqrt{1 - \frac{1}{1 + \left\{ \sum_{\dot{c}=1}^{3} \mathcal{P}_{\dot{c}} \left(\frac{\left(\mu_{g_{\dot{c}}}^{L}\right)^2}{1-\left(\mu_{g_{\dot{c}}}^{L}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1 + \left\{ \sum_{\dot{c}=1}^{3} \mathcal{P}_{\dot{c}} \left(\frac{\left(\mu_{g_{\dot{c}}}^{U}\right)^2}{1-\left(\mu_{g_{\dot{c}}}^{U}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{\frac{1}{1 + \left\{ \sum_{\dot{c}=1}^{3} \mathcal{P}_{\dot{c}} \left(\frac{1-\left(\nu_{g_1}^{L}\right)^2}{\left(\nu_{g_{\dot{c}}}^{L}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{\dot{c}=1}^{3} \mathcal{P}_{\dot{c}} \left(\frac{1-\left(\nu_{g_1}^{U}\right)^2}{\left(\nu_{g_{\dot{c}}}^{U}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right)$$

Consequently,

$$IVPFDWA_{\mathcal{P}}(\varsigma_1, \varsigma_2, \varsigma_3) = \langle [0.28, 0.39], [0.12, 0.62] \rangle$$

In the following definition, we propose a Dombi arithmetic aggregation operator with IVPFEs, namely, the IVPFDOWA operator.

Definition 14. *Let* $\dot{c} = 1, 2, 3, \ldots, j$ *and* $g_{\dot{c}} = \left\langle \left[\mu_{g_{\dot{c}}}^{L}, \mu_{g_{\dot{c}}}^{U}\right], \left[\nu_{g_{\dot{c}}}^{L}, \nu_{g_{\dot{c}}}^{U}\right] \right\rangle$ *be j number of IVPFEs. A IVPFDOWA operator of dimension j is characterized by a function.*

IVPFDOWA: $IVPFE^j \rightarrow IVPFE$ with the associated weighted vector $\mathcal{P} = \left(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots, \mathcal{P}_j\right)^T$ such that with $\mathcal{P}_{\dot{c}} > 0$ and $\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} = 1$. Therefore, $IVPFDOWA_{\mathcal{P}}(g_1, g_2, g_3, \ldots, g_j) = \oplus_{\dot{c}=1}^{j}\left(\mathcal{P}_{\dot{c}} g_{\rho(\dot{c})}\right)$, where $(\rho(1), \rho(2), \rho(3), \ldots, \rho(j))$ represents the permutations of $1, 2, 3, \ldots, j$, respectively, and $g_{\rho(\dot{c}-1)} \geq g_{\rho(\dot{c})}$ $\forall$ $\dot{c} = 1, 2, \ldots, j$.

Theorem 2. *Let* $\dot{c} = 1, 2, 3, \ldots, j$ *and* $g_{\dot{c}} = \left\langle \left[\mu_{g_{\dot{c}}}^{L}, \mu_{g_{\dot{c}}}^{U}\right], \left[\nu_{g_{\dot{c}}}^{L}, \nu_{g_{\dot{c}}}^{U}\right] \right\rangle$ *be the j number of IVPFEs. Then, the aggregated value of these IVPFEs in the framework of IVPFDOWA operator is an IVPFE and is determined in the following way:*

$$IVPFDOWA_{\mathcal{P}}(g_1, g_2, g_3, \ldots, g_j) = \oplus_{\dot{c}=1}^{j}\left(\mathcal{P}_{\dot{c}} g_{\rho(\dot{c})}\right)$$

$$= \left(\begin{array}{c} \left[\sqrt{1 - \frac{1}{1 + \left\{ \sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} \left(\frac{\left(\mu_{g_{\rho(\dot{c})}}^{L}\right)^2}{1-\left(\mu_{g_{\rho(\dot{c})}}^{L}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1 + \left\{ \sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} \left(\frac{\left(\mu_{g_{\rho(\dot{c})}}^{U}\right)^2}{1-\left(\mu_{g_{\rho(\dot{c})}}^{U}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{\frac{1}{1 + \left\{ \sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} \left(\frac{1-\left(\nu_{g_{\rho(\dot{c})}}^{L}\right)^2}{\left(\nu_{g_{\rho(\dot{c})}}^{L}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} \left(\frac{1-\left(\nu_{g_{\rho(\dot{c})}}^{U}\right)^2}{\left(\nu_{g_{\rho(\dot{c})}}^{U}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right),$$

where $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots, \mathcal{P}_j)^T$ is the weighted vector of $g_{\dot{c}}$ with $0 \leq \mathcal{P}_{\dot{c}}$, $\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} = 1$ and $\vartheta \geq 1$. Moreover, $(\rho(1), \rho(2), \rho(3), \ldots, \rho(j))$ represent the permutations of 1, 2, 3, … , j, respectively, and $g_{\rho(\dot{c}-1)} \geq g_{\rho(\dot{c})}$ $\forall$ $\dot{c} = 1, 2, \ldots, j$.

Proof. The proof of this theorem is analogous to Theorem 1. □

The following example describes the above-stated fact.

Example 4. *Three researchers $\varsigma_1, \varsigma_2, \varsigma_3$ of mathematics want to estimate the performance of a student. The estimated values from researchers with respect to research work for a student P specified by IVPF information such as:*

$\varsigma_1 = \langle [0.1, 0.3], [0.2, 0.5] \rangle, \varsigma_2 = \langle [0.3, 0.4], [0.4, 0.7] \rangle$, $\varsigma_3 = \langle [0.4, 0.4], [0.3, 0.7] \rangle$ where the corresponding weighted vector is $\mathcal{P} = (0.3, 0.3, 0.4)^{\mathrm{T}}$ and the operational parameter $\vartheta = 3$. To aggregate these values by the IVPDOWA operator, we firstly permute these numbers by using Equation (1) and obtain the following information

$$\varsigma_1 = 0.31,\ \varsigma_2 = 0.2,\ \text{and}\ \varsigma_3 = 0.27.$$

By applying Definition 14, we then permuted values of IVPEs as follows:
$\varsigma_{\rho(1)} = \langle [0.1, 0.3], [0.2, 0.5] \rangle, \varsigma_{\rho(2)} = \langle [0.4, 0.4], [0.3, 0.7] \rangle$ and $\varsigma_{\rho(3)} = \langle [0.3, 0.4], [0.4, 0.7] \rangle$.
Now,

$$IVPFDOWA_{\mathcal{P}}(\varsigma_1, \varsigma_2, \varsigma_3) = \oplus_{\dot{c}=1}^{j} \left(\mathcal{P}_{\dot{c}} \varsigma_{\rho(\dot{c})} \right)$$

$$= \left(\begin{array}{c} \left[\sqrt{1 - \frac{1}{1 + \left\{ \sum_{\dot{c}=1}^{3} \mathcal{P}_{\dot{c}} \left(\frac{\left(\mu^L_{g_{\rho(\dot{c})}} \right)^2}{1 - \left(\mu^L_{g_{\rho(\dot{c})}} \right)^2} \right)^3 \right\}^{\frac{1}{3}}}}, \sqrt{1 - \frac{1}{1 + \left\{ \sum_{\dot{c}=1}^{3} \mathcal{P}_{\dot{c}} \left(\frac{\left(\mu^U_{g_{\rho(\dot{c})}} \right)^2}{1 - \left(\mu^U_{g_{\rho(\dot{c})}} \right)^2} \right)^3 \right\}^{\frac{1}{3}}}} \right], \\ \left[\sqrt{\frac{1}{1 + \left\{ \sum_{\dot{c}=1}^{3} \mathcal{P}_{\dot{c}} \left(\frac{1 - \left(\nu^L_{g_{\rho(\dot{c})}} \right)^2}{\left(\nu^L_{g_{\rho(\dot{c})}} \right)^2} \right)^3 \right\}^{\frac{1}{3}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{\dot{c}=1}^{3} \mathcal{P}_{\dot{c}} \left(\frac{1 - \left(\nu^U_{g_{\rho(\dot{c})}} \right)^2}{\left(\nu^U_{g_{\rho(\dot{c})}} \right)^2} \right)^3 \right\}^{\frac{1}{3}}}} \right] \end{array} \right)$$

Consequently, $IVPFDOWA_{\mathcal{P}}(\varsigma_1, \varsigma_2, \varsigma_3) = ([0.34, 0.39], [0.24, 0.57])$.
In the following definition, we propose a Dombi geometric aggregation operator with IVPFEs, namely, the IVPFDWG operator.

Definition 15. *Let $\dot{c} = 1, 2, 3, \ldots, j$ and $g_{\dot{c}} = \left\langle \left[\mu^L_{g_{\dot{c}}}, \mu^U_{g_{\dot{c}}} \right], \left[\nu^L_{g_{\dot{c}}}, \nu^U_{g_{\dot{c}}} \right] \right\rangle$ be the j number of IVPFEs. The IVPFDWG operator is characterized by a function IVPFDWG: IVPFE^j $\rightarrow$ IVPFE such that*

$$IVPFDWG_{\mathcal{P}}(g_1, g_2, g_3, \ldots, g_j) = \otimes_{\dot{c}=1}^{j} (g_{\dot{c}})^{\mathcal{P}_{\dot{c}}}$$

where $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots, \mathcal{P}_j)^T$ is the weight vector of $g_{\dot{c}}$ such that $\mathcal{P}_{\dot{c}} > 0$ and $\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} = 1$..

Theorem 3. *Let* $\dot{c} = 1, 2, 3, \ldots, j$ *and* $g_{\dot{c}} = \left\langle \left[\mu^L_{g_{\dot{c}}}, \mu^U_{g_{\dot{c}}}\right], \left[\nu^L_{g_{\dot{c}}}, \nu^U_{g_{\dot{c}}}\right] \right\rangle$ *be the j number of IVPFEs. The aggregated value of these IVPFEs in the framework of the IVPFDWG operator is an IVPFE and is determined in the following way:*

$$IVPFDWG_{\wp}(g_1, g_2, g_3, \ldots, g_j) = \otimes^j_{\dot{c}=1} (g_{\dot{c}})^{\wp_{\dot{c}}}$$
$$= \left(\begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{\sum^j_{\dot{c}=1} \wp_{\dot{c}} \left(\frac{1-\left(\mu^L_{g_{\dot{c}}}\right)^2}{\left(\mu^L_{g_{\dot{c}}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{\sum^j_{\dot{c}=1} \wp_{\dot{c}} \left(\frac{1-\left(\mu^U_{g_{\dot{c}}}\right)^2}{\left(\mu^U_{g_{\dot{c}}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{1-\frac{1}{1+\left\{\sum^j_{\dot{c}=1} \wp_{\dot{c}} \left(\frac{\left(\nu^L_{g_{\dot{c}}}\right)^2}{\left(1-\nu^L_{g_{\dot{c}}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{1-\frac{1}{1+\left\{\sum^j_{\dot{c}=1} \wp_{\dot{c}} \left(\frac{\left(\nu^U_{g_{\dot{c}}}\right)^2}{1-\left(\nu^U_{g_{\dot{c}}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right),$$

where $\wp = (\wp_1, \wp_2, \wp_3, \ldots, \wp_j)^T$ is the weight vector of $g_{\dot{c}}$, $0 \leq \wp_{\dot{c}}$, $\sum^j_{\dot{c}=1} \wp_{\dot{c}} = 1$ and $\vartheta \geq 1$.

Proof. Proof of this result is analogous to Theorem 1. □

The following example describes the above-stated fact.

Example 5. *The three analyzers* $\varsigma_1, \varsigma_2, \varsigma_3$ *want to check the working ability of a certain machine. The estimated values of the working ability of a certain machine are specified by IVPF information such as:*

$\varsigma_1 = ([0.4, 0.6], [0.5, 0.6]), \varsigma_2 = ([0.5, 0.7], [0.2, 0.5]), \varsigma_3 = ([0.3, 0.4], [0.6, 0.77])$ with the corresponding weighted vector $\wp = (0.3, 0.5, 0.2)^{\mathrm{T}}$ and the operational parameter $\vartheta = 5$. Then,

$$IVPFDWG_{\wp}(\varsigma_1, \varsigma_2, \varsigma_3) = \otimes^3_{\dot{c}=1} (\varsigma_{\dot{c}})^{\wp_{\dot{c}}}$$
$$= \left(\begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{\sum^3_{\dot{c}=1} \wp_{\dot{c}} \left(\frac{1-\left(\mu^L_{g_{\dot{c}}}\right)^2}{\left(\mu^L_{g_{\dot{c}}}\right)^2}\right)^{5}\right\}^{\frac{1}{5}}}}, \sqrt{\frac{1}{1+\left\{\sum^3_{\dot{c}=1} \wp_{\dot{c}} \left(\frac{1-\left(\mu^U_{g_{\dot{c}}}\right)^2}{\left(\mu^U_{g_{\dot{c}}}\right)^2}\right)^{5}\right\}^{\frac{1}{5}}}} \right], \\ \left[\sqrt{1-\frac{1}{1+\left\{\sum^3_{\dot{c}=1} \wp_{\dot{c}} \left(\frac{\left(\nu^L_{g_{\dot{c}}}\right)^2}{\left(1-\nu^L_{g_{\dot{c}}}\right)^2}\right)^{5}\right\}^{\frac{1}{5}}}}, \sqrt{1-\frac{1}{1+\left\{\sum^3_{\dot{c}=1} \wp_{\dot{c}} \left(\frac{\left(\nu^U_{g_{\dot{c}}}\right)^2}{1-\left(\nu^U_{g_{\dot{c}}}\right)^2}\right)^{5}\right\}^{\frac{1}{5}}}} \right] \end{array} \right)$$

Consequently, $IVPFDWG_{\wp}(\varsigma_1, \varsigma_2, \varsigma_3) = ([0.34, 0.46], [0.54, 0.72])$.

In the following definition, we propose a Dombi geometric aggregation operator with IVPFEs, namely, the IVPFDOWG operator.

Definition 16. *Let* $\dot{c} = 1, 2, 3, \ldots, j$ *and* $g_{\dot{c}} = \left\langle \left[\mu^L_{g_{\dot{c}}}, \mu^U_{g_{\dot{c}}}\right], \left[\nu^L_{g_{\dot{c}}}, \nu^U_{g_{\dot{c}}}\right] \right\rangle$ *be the j number of IVPFEs. An IVPFDOWG operator of dimension j is characterized by a function \$IVPFDOWG: IVPFE*^{j} $\rightarrow$ *IVPFE with the associated weighted vector* $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots, \mathcal{P}_j)^T$ *such that* $\mathcal{P}_{\dot{c}} > 0$ *and* $\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} = 1$. *Therefore,*
where $\rho(1), \rho(2), \rho(3), \ldots, \rho(j)$ represent the permutations of $1, 2, 3, \ldots, j$, respectively, and $g_{\rho(\dot{c}-1)} \geq g_{\rho(\dot{c})}$ $\forall$ $\dot{c} = 1, 2, \ldots, j$.

Theorem 4. *Let* $g_{\dot{c}} = \left\langle \left[\mu^L_{g_{\rho(\dot{c})}}, \mu^U_{g_{\rho(\dot{c})}}\right], \left[\nu^L_{g_{\rho(\dot{c})}}, \nu^U_{g_{\rho(\dot{c})}}\right] \right\rangle$ *(1, 2, 3, ... , j) be j number of IVPFEs. Then. The amassed value of these IVPFEs in the framework of the IVPFDOWA operator is an IVPFE and is determined in the following way:*

$$IVPFDOWG_{\mathcal{P}}(g_1, g_2, g_3, \ldots, g_j) = \otimes_{\dot{c}=1}^{j} \left(g_{\rho(\dot{c})}\right)^{\mathcal{P}_{\dot{c}}}$$
$$= \left(\begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} \left(\frac{1-\left(\mu^L_{g_{\rho(\dot{c})}}\right)^2}{\left(\mu^L_{g_{\rho(\dot{c})}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} \left(\frac{1-\left(\mu^U_{g_{\rho(\dot{c})}}\right)^2}{\left(\mu^U_{g_{\rho(\dot{c})}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} \left(\frac{\left(\nu^L_{g_{\rho(\dot{c})}}\right)^2}{1-\left(\nu^L_{g_{\rho(\dot{c})}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} \left(\frac{\left(\nu^U_{g_{\rho(\dot{c})}}\right)^2}{1-\left(\nu^U_{g_{\rho(\dot{c})}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right) \quad (3)$$

where $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots, \mathcal{P}_j)^T$ is the weighted vector of $g_{\dot{c}}$ such that $0 \leq \mathcal{P}_{\dot{c}}$, $\sum_{\dot{c}=1}^{j} \mathcal{P}_{\dot{c}} = 1$ and $\vartheta \geq 1$. Moreover, $\rho(1), \rho(2), \rho(3), \ldots, \rho(j)$ represent the permutations of $1, 2, 3, \ldots, j$, respectively, where $g_{\rho(\dot{c}-1)} \geq g_{\rho(\dot{c})}$ $\forall$ $\dot{c} = 1, 2, \ldots, j$.

Proof. The proof of this result is established through the use of mathematical induction. □

The application of definition 12 for ċ = 2 gives the following outcome:

$$IVPFDOWG_{\mathcal{P}}(g_1, g_2) = g_{\rho(1)}^{\mathcal{P}_1} \otimes g_{\rho(2)}^{\mathcal{P}_2}$$
$$= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt{\frac{1}{1+\left\{\mathcal{P}_1 \left(\frac{1-\left(\mu^L_{g_{\rho(1)}}\right)^2}{\left(\mu^L_{g_{\rho(1)}}\right)^2}\right)^{\vartheta} + \mathcal{P}_2 \left(\frac{1-\left(\mu^L_{g_{\rho(2)}}\right)^2}{\left(\mu^L_{g_{\rho(2)}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \\ \sqrt{\frac{1}{1+\left\{\mathcal{P}_1 \left(\frac{1-\left(\mu^U_{g_{\rho(1)}}\right)^2}{\left(\mu^U_{g_{\rho(1)}}\right)^2}\right)^{\vartheta} + \mathcal{P}_2 \left(\frac{1-\left(\mu^U_{g_{\rho(2)}}\right)^2}{\left(\mu^U_{g_{\rho(2)}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}} \end{array} \right], \\ \left[\begin{array}{c} \sqrt{1-\frac{1}{1+\left\{\mathcal{P}_1 \left(\frac{\left(\nu^L_{g_{\rho(1)}}\right)^2}{1-\left(\nu^L_{g_{\rho(1)}}\right)^2}\right)^{\vartheta} + \mathcal{P}_2 \left(\frac{\left(\nu^L_{g_{\rho(2)}}\right)^2}{1-\left(\nu^L_{g_{\rho(2)}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \\ \sqrt{1-\frac{1}{1+\left\{\mathcal{P}_1 \left(\frac{\left(\nu^U_{g_{\rho(1)}}\right)^2}{1-\left(\nu^U_{g_{\rho(1)}}\right)^2}\right)^{\vartheta} + \mathcal{P}_2 \left(\frac{\left(\nu^U_{g_{\rho(2)}}\right)^2}{1-\left(\nu^U_{g_{\rho(2)}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}} \end{array} \right] \end{array} \right)$$

This means that

$$IVPFDOWG_{\mathcal{P}}(g_1, g_2)$$
$$= \left(\begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{2} \mathcal{P}_{\dot{c}} \left(\frac{1-\left(\mu^L_{g_{\rho(\dot{c})}}\right)^2}{\left(\mu^L_{g_{\rho(\dot{c})}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{2} \mathcal{P}_{\dot{c}} \left(\frac{1-\left(\mu^U_{g_{\rho(\dot{c})}}\right)^2}{\left(\mu^U_{g_{\rho(\dot{c})}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right], \\ \left[\sqrt{1 - \frac{1}{1+\left\{\sum_{\dot{c}=1}^{2} \mathcal{P}_{\dot{c}} \left(\frac{\left(\nu^L_{g_{\rho(\dot{c})}}\right)^2}{1-\left(\nu^L_{g_{\rho(\dot{c})}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1+\left\{\sum_{\dot{c}=1}^{2} \mathcal{P}_{\dot{c}} \left(\frac{\left(\nu^U_{g_{\rho(\dot{c})}}\right)^2}{1-\left(\nu^U_{g_{\rho(\dot{c})}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right)$$

Thus, Equation (3) holds for $\dot{c} = 2$.
Let us assume that Equation (3) holds for $\dot{c} = p$. Therefore, we have

$$IVPFDOWG_{\mathcal{P}}(g_1, g_2, g_3, \ldots, g_p) = \oplus_{\dot{c}=1}^{p} \left(g_{\rho(\dot{c})}\right)^{\mathcal{P}_{\dot{c}}}$$
$$= \left(\begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p} \mathcal{P}_{\dot{c}} \left(\frac{1-\left(\mu^L_{g_{\rho(\dot{c})}}\right)^2}{\left(\mu^L_{g_{\rho(\dot{c})}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p} \mathcal{P}_{\dot{c}} \left(\frac{1-\left(\mu^U_{g_{\rho(\dot{c})}}\right)^2}{\left(\mu^U_{g_{\rho(\dot{c})}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right] \\ \left[\sqrt{1 - \frac{1}{1+\left\{\sum_{\dot{c}=1}^{p} \mathcal{P}_{\dot{c}} \left(\frac{\left(\nu^L_{g_{\rho(\dot{c})}}\right)^2}{1-\left(\nu^L_{g_{\rho(\dot{c})}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}}, \sqrt{1 - \frac{1}{1+\left\{\sum_{\dot{c}=1}^{p} \mathcal{P}_{\dot{c}} \left(\frac{\left(\nu^U_{g_{\rho(\dot{c})}}\right)^2}{1-\left(\nu^U_{g_{\rho(\dot{c})}}\right)^2} \right)^{\vartheta} \right\}^{\frac{1}{\vartheta}}}} \right] \end{array} \right)$$

Moreover, for $\dot{c} = p + 1$, we have

$$IVPFDOWG_{\mathcal{P}}(g_1, g_2, g_3, \ldots, g_p, g_{p+1}) = \oplus_{\dot{c}=1}^{p}\left(g_{\rho(\dot{c})}\right)^{\mathcal{P}_{\dot{c}}} \oplus \left(g_{\rho(p+1)}\right)^{\mathcal{P}_{p+1}}$$

$$= \left(\begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p}\mathcal{P}_{\dot{c}}\left(\frac{1-\left(\mu^L_{g_{\rho(\dot{c})}}\right)^2}{\left(\mu^L_{g_{\rho(\dot{c})}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p}\mathcal{P}_{\dot{c}}\left(\frac{1-\left(\mu^U_{g_{\rho(\dot{c})}}\right)^2}{\left(\mu^U_{g_{\rho(\dot{c})}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}\right], \\ \left[\sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p}\mathcal{P}_{\dot{c}}\left(\frac{\left(\nu^L_{g_{\rho(\dot{c})}}\right)^2}{1-\left(\nu^L_{g_{\rho(\dot{c})}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p}\mathcal{P}_{\dot{c}}\left(\frac{\left(\nu^U_{g_{\rho(\dot{c})}}\right)^2}{1-\left(\nu^U_{g_{\rho(\dot{c})}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}\right] \end{array}\right)$$

$$\otimes \left(\begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{\mathcal{P}_{p+1}\left(\frac{1-\left(\mu^L_{g_{\rho(p+1)}}\right)^2}{\left(\mu^L_{g_{\rho(p+1)}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{\frac{1}{1+\left\{\mathcal{P}_{p+1}\left(\frac{1-\left(\mu^U_{g_{\rho(p+1)}}\right)^2}{\left(\mu^U_{g_{\rho(p+1)}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}\right], \\ \left[\sqrt{1-\frac{1}{1+\left\{\mathcal{P}_{p+1}\left(\frac{\left(\nu^L_{\rho(p+1)}\right)^2}{1-\left(\nu^L_{g_{\rho(p+1)}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{1-\frac{1}{1+\left\{\mathcal{P}_{p+1}\left(\frac{\left(\nu^U_{g_{\rho(p+1)}}\right)^2}{1-\left(\nu^U_{g_{\rho(p+1)}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}\right] \end{array}\right)$$

This means that

$$IVPFDOWG_{\mathcal{P}}(g_1, g_2, g_3, \ldots, g_p,\ g_{p+1})$$

$$= \left(\begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p+1}\mathcal{P}_{\dot{c}}\left(\frac{1-\left(\mu^L_{g_{\rho(\dot{c})}}\right)^2}{\left(\mu^L_{g_{\rho(\dot{c})}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p+1}\mathcal{P}_{\dot{c}}\left(\frac{1-\left(\mu^U_{g_{\rho(\dot{c})}}\right)^2}{\left(\mu^U_{g_{\rho(\dot{c})}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}\right], \\ \left[\sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p+1}\mathcal{P}_{\dot{c}}\left(\frac{\left(\nu^L_{g_{\rho(\dot{c})}}\right)^2}{1-\left(\nu^L_{g_{\rho(\dot{c})}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}, \sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{p+1}\mathcal{P}_{\dot{c}}\left(\frac{1-\left(\nu^U_{g_{\rho(\dot{c})}}\right)^2}{1-\left(\nu^U_{g_{\rho(\dot{c})}}\right)^2}\right)^{\vartheta}\right\}^{\frac{1}{\vartheta}}}}\right] \end{array}\right)$$

Thus, Equation (3) is true for $\dot{c} = p + 1$. Hence, we conclude that Equation (3) is true for any $\dot{c} \in N$.

The following example describes the above-stated fact.

Example 6. *Experts want to check the style and appearance of the fabric of an industry. The estimated values from the two experts are specified by IVPF information as* $\varsigma_1 = ([0.4, 0.6], [0.5, 0.7])$ *and* $\varsigma_2 = ([0.1,0.4],[0.4,0.5])$*. The corresponding weighted vector is* $\Psi = (0.6,0.4)^T$ *with the operational parameter* $\vartheta = 1$*. To aggregate these values by the IVPDOWG operator, we firstly permute these numbers by using Equation (1) and obtain the following information:*

$$S(\varsigma_1) = 0.25 \text{ and } S(\varsigma_2) = 0.27.$$

By applying Definition 16, the permuted values of IVPFEs are calculated as follows:

$$\varsigma_{\rho(1)} = ([0.1, 0.4], [0.4, 0.5]) and \ \varsigma_{\rho(2)} = ([0.4, 0.6], [0.5, 0.7]).$$

Also,

$$IVPFDOWG_{\Psi}(\varsigma_1, \varsigma_2) = \otimes_{\dot{c}=1}^{2} \left(\varsigma_{\rho(\dot{c})}\right)^{\Psi_{\dot{c}}}$$

$$= \left(\begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{2} \Psi_{\dot{c}} \left(\frac{1-\left(\mu^L_{g_{\rho(\dot{c})}}\right)^2}{\left(\mu^L_{g_{\rho(\dot{c})}}\right)^2}\right)^1\right\}^{\frac{1}{1}}}}, \sqrt{\frac{1}{1+\left\{\sum_{\dot{c}=1}^{2} \Psi_{\dot{c}} \left(\frac{1-\left(\mu^U_{g_{\rho(\dot{c})}}\right)^2}{\left(\mu^U_{g_{\rho(\dot{c})}}\right)^2}\right)^1\right\}^{\frac{1}{1}}}} \right], \\ \left[\sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{2} \Psi_{\dot{c}} \left(\frac{\left(\nu^L_{g_{\rho(\dot{c})}}\right)^2}{1-\left(\nu^L_{g_{\rho(\dot{c})}}\right)^2}\right)^1\right\}^{\frac{1}{1}}}}, \sqrt{1-\frac{1}{1+\left\{\sum_{\dot{c}=1}^{2} \Psi_{\dot{c}} \left(\frac{\left(\nu^U_{g_{\rho(\dot{c})}}\right)^2}{1-\left(\nu^U_{g_{\rho(\dot{c})}}\right)^2}\right)^1\right\}^{\frac{1}{1}}}} \right] \end{array} \right)$$

Consequently, $IVPFDOWG_{\Psi}(\varsigma_1, \varsigma_2)$ = ([0.13,0.45],[0.45,0.6]).

5. Application of Proposed Model in MADM

In this area, we can also endorse a multiple attribute decision-making method manipulating the IVPFD aggregation operator, where criteria weights are real numbers and criteria values are IVPFEs. Let $A = A_1, A_2, \ldots, A_m$ be a set of alternatives, and $C = C_1, C_2, C_3, \ldots, C_n$ be a set of attributes. Moreover, $\Psi = (\Psi_1, \Psi_2, \Psi_3, \ldots, \Psi_n)^T$ denotes the weighted vector of attributes, where $\Psi_i \in [0, 1]$ for all $I = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} \Psi_i, = 1$. Suppose that $M_{ij} = ([\mu^L_{ij}, \mu^U_{ij}], [\nu^L_{ij}, \nu^U_{ij}])$ is the IVPFE, where $[\mu^L_{ij}, \mu^U_{ij}]$ manifest the distinct set of degrees that the alternatives A_i meet the criteria C_j specified by the decision-maker and $\left[\nu^L_{ij}, \nu^U_{ij}\right]$ denotes the distinct set of the degrees to which the alternative A_i does not fulfill the criteria C_j. Then, the information provided from the decision-maker is summarized in the form of the IVPF decision matrices $W = (M_{ij})_{m\times n}$ as:

$$W = \begin{pmatrix} M_{11} & \cdots & M_{1n} \\ \vdots & \ddots & \vdots \\ M_{m1} & \cdots & M_{mn} \end{pmatrix}$$

The proposed approach based on the IVPFDWA operator for designing the MCDM problem mainly comprises the following steps.

Step 1.

Convert the decision matrix W to a normalized matrix $X = (x_{ij})\ m \times n$, where x_{ij} is computed by the following equation:

$$X_{ij} = \begin{cases} \left(\left[\mu_{ij}^{L}, \mu_{ij}^{U}\right], \left[\nu_{ij}^{L}, \nu_{ij}^{U}\right]\right); \text{ for benefit type criteria} \\ \left(\left[\nu_{ij}^{L}, \nu_{ij}^{U}\right], \left[\mu_{ij}^{L}, \mu_{ij}^{U}\right]\right); \text{ for loss type criteria} \end{cases}$$

Step 2.

By utilizing the decision information and the IVPFDWA operator

$$b_i = IVPFDWA_{\mathcal{P}}(A_1,\ A_2,\ A_3, \ldots,\ A_n) = \oplus_{\dot{c}=1}^{n}(\mathcal{P}_{\dot{c}} A_{\dot{c}})$$

or by applying the IVPFDOWA operator, we have

$$b_i = IVPFDOWA_{\mathcal{P}}(A_1,\ A_2,\ A_3, \ldots,\ A_n) = \oplus_{\dot{c}=1}^{n}\left(\mathcal{P}_{\dot{c}} A_{\rho(\dot{c})}\right)$$

or by applying the IVPFDWG operator, we have

$$b_i = IVPFDWG_{\mathcal{P}}(A_1,\ A_2,\ A_3, \ldots,\ A_n) = \otimes_{\dot{c}=1}^{n}(A_{\dot{c}})^{\mathcal{P}_{\dot{c}}}$$

or by applying the IVPFDWG operator, we have

$$b_i = IVPFDOWG_{\mathcal{P}}(A_1,\ A_2,\ A_3, \ldots,\ A_n) = \otimes_{\dot{c}=1}^{n}\left(A\rho_{(\dot{c})}\right)^{\mathcal{P}_{\dot{c}}}$$

to acquire the comprehensive values b_i, where i = 1, 2, . . . , n of the alternatives A_i.

Step 3.

Compute the scores $S(b_i)$, where i = 1, 2, . . . , n of the alternatives A_i.

Step 4.

Rank all the alternatives and *determine the most favorable option* with the help of the score function.

5.1. Numerical Application of Decision-Making; A Case Study: (Selection of an Expert in a Medical University)

The university of XYZ is a leading educational institution renowned for its academic excellence in various fields. The university offers undergraduate and graduate programs in several disciplines such as Engineering, Medicine, Arts, Sciences, and Business. The university has recently received funding to establish a new research center in the field of Medicine. As a part of this initiative, the university is looking to hire a subject expert in the field of medicine to lead the process that requires careful consideration of various factors such as qualifications, expertise, and research output. The university faces the following challenges in selecting a subject expert in the medical field:

i. Identifying the right candidate: The field of medicine is highly specialized and requires a deep understanding of various sub-fields. The university needs to identify a candidate who has a strong background in one or more of these sub-fields and can lead research in medicine.

ii. Competition from other institutions: The university faces competition from other leading educational institutions that are also looking to hire subject experts in medicine. The university needs to offer competitive compensation and benefits to attract top candidates.

iii. Limited pool of candidates: The pool of candidates with a strong background in the medical field is limited. The university needs to expand its search to include international candidates and those from non-traditional academic backgrounds.

To address these challenges, the university adopted the following approach:

i. Expert panel: The university formed an expert panel comprising professors and experts in the field of medicine. The panel reviewed the job description and identified the key qualifications and experience required for the position.

ii. Recruitment campaign: The university launched a comprehensive recruitment campaign to attract top talent from around the word. The campaign included expert selection for university.

iii. Expert selection: The university developed a rigorous selection process to identify the best candidate for the position. The process included initial screening of resumes, followed by multiple rounds of interviews with the expert panel and stakeholders. The final selection was based on the candidate's qualifications, experience, research output, and ability to lead a research team.

Results The university successfully identified and hired a subject expert in medicine to lead the new research center. The expert has a strong background in medicine and has published several research papers in top-tier academic journals. The expert has also worked in leading medical companies and has experience leading research teams. The expert is now leading a team of researchers in developing new medical algorithms and applications that have the potential to transform various industries.

Conclusion The selection of a subject expert is a critical process that requires careful consideration of various factors. By adopting a comprehensive approach that involved an expert panel, a recruitment campaign, and a rigorous selection process, the university of XYZ successfully identified and hired a subject expert in medicine to lead the new research center. The expert's background, experience, and research output have positioned the university to become a leading institution in the field of medicine.

The following discussion presents a numerical example and comparative analysis to showcase the effectiveness of this innovative approach.

The School of Medicine at a Pakistani university is pursuing the addition of exceptional teachers to revitalize academic efficiency and upgrade teaching quality. This initiative has received widespread interest, and a panel consisting of the President of the university, the Dean of the School of Medicine, and other faculty members has been established to assume primary responsibility for its implementation. The panel conducts a rigorous evaluation of six alternatives, identified as A_1, A_2, A_3, A_4, A_5, and A_6, based on four key attributes: morality (C_1), educational background and research capability (C_2), teaching skills and experience (C_3), and relevant qualifications (C_4), which are weighted according to the vector $\mathcal{P}$ = (0.25,0.2,0.25,0.3)^T. The evaluation of these six candidates A_i, where i = 1, 2, 3, 4, 5, 6, is done by using IVPF information.

Step 1.

The information given by the decision-makers for the above five criteria of these six alternatives are evaluated under the IVPF information in Table 1.

Table 1. Decision matrix representing the opinion of the decision-makers about the physical situation in terms of IVPF values.

	C_1	C_2	C_3	C_4
A_1	$([0.4,0.5],[0.1,0.2])$	$([0.5,0.6],[0.2,0.3])$	$([0.3,0.6],[0.4,0.6])$	$([\mathbf{0.5},\mathbf{0.6}],[\mathbf{0.3},\mathbf{0.7}])$
A_2	$([0.6,0.6],[0.4,0.5])$	$([0.3,0.4],[0.5,0.6])$	$([0.5,0.7],[0.7,0.7])$	$([\mathbf{0.3},\mathbf{0.4}],[\mathbf{0.4},\mathbf{0.5}])$
A_3	$([0.6,0.7],[0.1,0.2])$	$([0.2,0.4],[0.4,0.5])$	$([0.1,0.7],[0.1,0.4])$	$([\mathbf{0.5},\mathbf{0.7}],[\mathbf{0.3},\mathbf{0.6}])$
A_4	$([0.3,0.5],[0.2,0.4])$	$([0.3,0.5],[0.5,0.5])$	$([0.2,0.7],[0.1,0.2])$	$([\mathbf{0},\mathbf{0.1}],[\mathbf{0.2},\mathbf{0.3}])$
A_5	$([0.2,0.3],[0.7,0.7])$	$([0.4,0.5],[0.5,0.6])$	$([0.1,0.3],[0.4,0.5])$	$([\mathbf{0.2},\mathbf{0.4}],[\mathbf{0.4},\mathbf{0.6}])$
A_6	$([0,0.1],[0.3,0.4])$	$([0.2,0.5],[0.5,0.5])$	$([0.4,0.5],[0.4,0.5])$	$([\mathbf{0.3},\mathbf{0.4}],[\mathbf{0.4},\mathbf{0.5}])$

The decision matrix does not require being normalized because all the attributes are benefitting attributes.

Step 2.

For $\vartheta = 1$, use the decision-makers' weight vector $\mathcal{P}$ = (0.25,0.2,0.25,0.3)^T and the IVPFDWA operator to obtain the overall preference of alternatives $\mathrm{A_i}$(I = 1, 2, ... , 6). We have A_1 = ([0.44,0.58], [0.17,0.32]), A_2 = ([0.46,0.57], [0.14,0.55]), A_3 = ([0.45,0.67],[0.14,0.33]), A_4 = ([0.23,0.53], [0.16,0.29]), A_5 = ([0.24,0.38], [0.46,0.59]), and A_6 = ([0.28,0.42],[0.38,0.47]). As the parameter ϑ changes, we are able to get various outputs.

Step 3.

Using the score function, we determine the score values $S(A_i)$, where I = 1, 2, 3, 4, 5, 6, as $S(A_1) = 0.56$, $S(A_2) = 0.47$, $S(A_3) = 0.6$, $S(A_4) = 0.5$, $S(A_5) = 0.22$, and $S(A_6) = 0.33$.

Step 4.

Since $S(A_3) \geq S(A_1) \geq S(A_4) \geq S(A_2) \geq S(A_6) \geq S(A_5)$, we have $A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_6 \succ A_5$. According to the ranking order, A_3 is the most suitable option.

5.2. Comparative Analysis

A comparative evaluation is conducted to demonstrate the effectiveness and practicality of the proposed method in the following discussion. The comparison between our techniques and existing strategies, including the Interval-valued Pythagorean Fuzzy Weighted Geometric (IVPFWG) operator and the Interval-valued Pythagorean Fuzzy Ordered Weighted Geometric (IVPFOWG) operator, is presented in the table below. Our method offers a distinct advantage as it is capable of handling the interdependence between arguments, while the existing techniques are limited in this regard. Consequently, our approach possesses a greater degree of generality. As a result, the analysis above highlights the superiority of the proposed methods in resolving MADM problems, as they provide a more adaptable and flexible solution. Table 2 describes the comparative analysis of the newly defined techniques with the existing methods. Based on this analysis, it is quite evident that the optimal solution is consistent with the outcomes of the currently used ways, and that the newly obtained IVPFN is more pessimistic than the aggregated values currently used in the existing employed decision-making methodology. This fact is highlighted in Table 2.

Table 2. Comparative analysis of IVPFD operators with the existing methods.

Techniques	Preference Order	Optimal Alternative
IVPFWG [25]	$A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_6 \succ A_5$	A_3
IVPFOWG [25]	$A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_6 \succ A_5$	A_3
IVPFDWA	$A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_6 \succ A_5$	A_3
IVPFDOWA	$A_3 \succ A_1 \succ A_4 \succ A_6 \succ A_2 \succ A_5$	A_3
IVPFDWG	$A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_6 \succ A_5$	A_3
IVPFDOWG	$A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_6 \succ A_5$	A_3

5.3. Operational Applications of IVPF Dombi Aggregation Operators

It is important to note that the geometric aggregation operators utilized in IVPF lack the capability to adaptively modify their parameters based on the risk tolerance of the decision-maker. This deficiency limits the effectiveness of IVPF in addressing multiple attribute decision-making (MADM) challenges in practical scenarios.

To overcome this limitation, novel techniques have been introduced that exhibit a high level of proficiency in providing a more flexible and resilient method of information fusion. These new techniques offer dynamic parameter adjustment, which enables decision-makers to tailor their approach based on their risk preferences and the specific demands of the decision-making scenario.

The ability to dynamically adjust parameters allows for a more tailored and personalized approach to decision-making, resulting in more precise and effective outcomes. This can be particularly valuable in complex decision-making scenarios where multiple factors need to be considered, and the stakes are high. The new techniques offer a robust solution that is capable of handling a wide range of MADM challenges, making it a valuable addition to the decision-making toolkit.

6. Conclusions

In the context of Multiple Attribute Decision-Making (MADM), the IVPFSs exhibit greater efficacy than IVIFSs due to their ability to handle inherent uncertainty. Consequently, they have wider applicability in real-world scenarios. In this research, novel approaches to address MADM problems in IVPF environments have been proposed. The study focuses on developing a new score function to overcome limitations of existing score functions in the IVPFS framework. The aim is to capture interconnections between individual arguments.

To integrate opinions of multiple experts on various aspects of a physical phenomenon, four operators, namely IVPFDWA, IVPFDOWA, IVPFDWG, and IVPFDOWG, have been implemented. Additionally, other desirable qualities of these operators have been investigated. Furthermore, the proposed strategies have been employed to address the MADM problem of selecting the best subject expert in an institution.

The proposed techniques can be effectively applied to solve MADM problems in fields such as supplier selection, project portfolio management, and renewable energy selection in future work. Additionally, the credibility of Dombi operators will be explored in interval-valued q-rung orthopair and interval-valued spherical fuzzy sets. These efforts will enable the effective resolution of a wide variety of essential MADM problems at an affordable cost.

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References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Bellman, R.E.; Zadeh, L.A. Decision-making in a fuzzy environment. *Manag. Sci.* **1970**, *17*, 141–164. [CrossRef]
3. Yager, R.R. Fuzzy decision making including unequal objectives. *Fuzzy Sets Syst.* **1978**, *1*, 87–95. [CrossRef]
4. Zimmermann, H.J. Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets Syst.* **1978**, *1*, 45–55. [CrossRef]
5. Dombi, J. A general class of fuzzy operators, the DeMorgan class of fuzzy operators and fuzziness measures induced by fuzzy operators. *Fuzzy Sets Syst.* **1978**, *8*, 149–163. [CrossRef]
6. Atanassov, K. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96. [CrossRef]
7. Turksen, I.B. Interval valued fuzzy sets based on normal forms. *Fuzzy Sets Syst.* **1986**, *20*, 191–210. [CrossRef]
8. Atanassov, K.T. More on intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1989**, *33*, 37–45. [CrossRef]
9. De, S.K.; Biswas, R.; Roy, A.R. Some operations on intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **2000**, *114*, 477–484. [CrossRef]
10. Xu, Z. On consistency of the weighted geometric mean complex judgement matrix in AHP. *Eur. J. Oper. Res.* **2000**, *126*, 683–687. [CrossRef]
11. Wei, G.; Wang, X. Some geometric aggregation operators based on interval-valued intuitionistic fuzzy sets and their application to group decision making. In Proceedings of the 2007 International Conference on Computational Intelligence and Security (CIS 2007), Harbin, China, 15–19 December 2007; pp. 495–499.
12. Wang, Z.; Li, K.W.; Wang, W. An approach to multiattribute decision making with interval-valued intuitionistic fuzzy assessments and incomplete weights. *Inf. Sci.* **2009**, *179*, 3026–3040. [CrossRef]

13. Su, Z.X.; Xia, G.P.; Chen, M.Y. Some induced intuitionistic fuzzy aggregation operators applied to multi-attribute group decision making. *Int. J. Gen. Syst.* **2011**, *40*, 805–835. [CrossRef]
14. Garg, H. Generalized intuitionistic fuzzy interactive geometric interaction operators using Einstein t-norm and t-conorm and their application to decision making. *Comput. Ind. Eng.* **2016**, *101*, 53–69. [CrossRef]
15. Cavallaro, F.; Zavadskas, E.K.; Streimikiene, D.; Mardani, A. Assessment of concentrated solar power (CSP) technologies based on a modified intuitionistic fuzzy topsis and trigonometric entropy weights. *Technol. Forecast. Soc. Chang.* **2019**, *140*, 258–270. [CrossRef]
16. Yager, R.R. Pythagorean membership grades in multicriteria decision making. *IEEE Trans. Fuzzy Syst.* **2013**, *22*, 958–965. [CrossRef]
17. Zhang, X.; Xu, Z. Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets. *Int. J. Intell. Syst.* **2014**, *29*, 1061–1078. [CrossRef]
18. Chen, S.M.; Chiou, C.H. Multiattribute decision making based on interval-valued intuitionistic fuzzy sets, PSO techniques, and evidential reasoning methodology. *IEEE Trans. Fuzzy Syst.* **2014**, *23*, 1905–1916. [CrossRef]
19. Yang, Y.R.; Yuan, S. Induced interval-valued intuitionistic fuzzy Einstein ordered weighted geometric operator and their application to multiple attribute decision making. *J. Intell. Fuzzy Syst.* **2014**, *26*, 2945–2954. [CrossRef]
20. Chen, S.M.; Huang, Z.C. Multiattribute decision making based on interval-valued intuitionistic fuzzy values and particle swarm optimization techniques. *Inf. Sci.* **2017**, *397*, 206–218. [CrossRef]
21. Zhang, X. Multicriteria Pythagorean fuzzy decision analysis: A hierarchical QUALIFLEX approach with the closeness index-based ranking methods. *Inf. Sci.* **2016**, *330*, 104–124. [CrossRef]
22. Garg, H. A novel accuracy function under interval-valued Pythagorean fuzzy environment for solving multicriteria decision making problem. *J. Intell. Fuzzy Syst.* **2016**, *31*, 529–540. [CrossRef]
23. Garg, H. A novel improved accuracy function for interval valued Pythagorean fuzzy sets and its applications in the decision-making process. *Int. J. Intell. Syst.* **2017**, *32*, 1247–1260. [CrossRef]
24. Garg, H. A new improved score function of an interval-valued Pythagorean fuzzy set based TOPSIS method. *Int. J. Uncertain. Quantif.* **2017**, *7*, 463–473. [CrossRef]
25. Rahman, K.; Abdullah, S.; Khan, M.S.A. Some interval-valued Pythagorean fuzzy Einstein weighted averaging aggregation operators and their application to group decision making. *J. Intell. Syst.* **2020**, *29*, 393–408. [CrossRef]
26. Rahman, K.; Abdullah, S.; Shakeel, M.; Ali Khan, M.S.; Ullah, M. Interval-valued Pythagorean fuzzy geometric aggregation operators and their application to group decision making problem. *Cogent Math.* **2017**, *4*, 1338638. [CrossRef]
27. Garg, H. A linear programming method based on an improved score function for interval-valued Pythagorean fuzzy numbers and its application to decision-making. *Int. J. Uncertain. Fuzziness Knowl.-Based Syst.* **2018**, *26*, 67–80. [CrossRef]
28. Garg, H. Generalised Pythagorean fuzzy geometric interactive aggregation operators using Einstein operations and their application to decision making. *J. Exp. Theor. Artif. Intell.* **2018**, *30*, 763–794. [CrossRef]
29. Yüksel, S.; Dinçer, H. Identifying the strategic priorities of nuclear energy investments using hesitant 2-tuple interval-valued Pythagorean fuzzy DEMATEL. *Prog. Nucl. Energy* **2022**, *145*, 104103. [CrossRef]
30. Al-Barakati, A.; Mishra, A.R.; Mardani, A.; Rani, P. An extended interval-valued Pythagorean fuzzy WASPAS method based on new similarity measures to evaluate the renewable energy sources. *App. Soft Comput.* **2022**, *120*, 108689. [CrossRef]
31. Rahman, K.; Abdullah, S.; Ali, A. Some induced aggregation operators based on interval-valued Pythagorean fuzzy numbers. *Granul. Comput.* **2019**, *4*, 53–62. [CrossRef]
32. Peng, X. New operations for interval-valued Pythagorean fuzzy set. *Sci. Iran.* **2019**, *26*, 1049–1076. [CrossRef]
33. Peng, X.; Li, W. Algorithms for interval-valued pythagorean fuzzy sets in emergency decision making based on multiparametric similarity measures and WDBA. *IEEE Access* **2019**, *7*, 7419–7441. [CrossRef]
34. Peng, X.; Yang, Y. Fundamental properties of interval-valued Pythagorean fuzzy aggregation operators. *Int. J. Intell. Syst.* **2016**, *31*, 444–487. [CrossRef]
35. Liu, P.; Liu, J.; Chen, S.M. Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making. *J. Oper. Res. Soc.* **2018**, *69*, 1–24. [CrossRef]
36. Wu, L.; Wei, G.; Wu, J.; Wei, C. Some interval-valued intuitionistic fuzzy dombi heronian mean operators and their application for evaluating the ecological value of forest ecological tourism demonstration areas. *Int. J. Environ. Res. Public Health* **2020**, *17*, 829. [CrossRef]
37. Khan, A.A.; Ashraf, S.; Abdullah, S.; Qiyas, M.; Luo, J.; Khan, S.U. Pythagorean fuzzy Dombi aggregation operators and their application in decision support system. *Symmetry* **2019**, *11*, 383. [CrossRef]
38. Mahmood, T.; Rehman, U.U. A method to multi-attribute decision making technique based on Dombi aggregation operators under bipolar complex fuzzy information. *Comput. Appl. Math.* **2022**, *41*, 47. [CrossRef]
39. Waqar, M.; Ullah, K.; Pamucar, D.; Jovanov, G.; Vranješ, Đ. An Approach for the Analysis of Energy Resource Selection Based on Attributes by Using Dombi T-Norm Based Aggregation Operators. *Energies* **2022**, *15*, 3939. [CrossRef]
40. Zhang, H.; Wei, G.; Chen, X. Spherical fuzzy Dombi power Heronian mean aggregation operators for multiple attribute group decision-making. *Comput. Appl. Math.* **2022**, *41*, 98. [CrossRef]

41. Almutairi, K.; Hosseini Dehshiri, S.J.; Hosseini Dehshiri, S.S.; Mostafaeipour, A.; Hoa, A.X.; Techato, K. Determination of optimal renewable energy growth strategies using SWOT analysis, hybrid MCDM methods, and game theory: A case study. *Int. J. Energy Res.* **2022**, *46*, 6766–6789. [CrossRef]
42. Hosseini Dehshiri, S.J.; Zanjirchi, S.M. Comparative analysis of multicriteria decision-making approaches for evaluation hydrogen projects development from wind energy. *Int. J. Energy Res.* **2022**, *46*, 13356–13376. [CrossRef]
43. Hosseini Dehshiri, S.J.; Amiri, M. An integrated multi-criteria decision-making framework under uncertainty for evaluating sustainable hydrogen production strategies based on renewable energies in Iran. *Environ. Sci. Pollut. Res.* **2023**, 1–16. [CrossRef] [PubMed]