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Intuitionistic Fuzzy Soft Hyper BCK Algebras

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Abstract: Maji et al. introduced the concept of fuzzy soft sets as a generalization of the standard soft sets, and presented an application of fuzzy soft sets in a decision-making problem. Maji et al. also introduced the notion of intuitionistic fuzzy soft sets in the paper [P.K. Maji, R. Biswas and A.R. Roy, Intuitionistic fuzzy soft sets, The Journal of Fuzzy Mathematics, **9** (2001), no. 3, 677–692]. The aim of this manuscript is to apply the notion of intuitionistic fuzzy soft set to hyper BCK algebras. The notions of intuitionistic fuzzy soft hyper BCK ideal, intuitionistic fuzzy soft weak hyper BCK ideal, intuitionistic fuzzy soft s-weak hyper BCK-ideal and intuitionistic fuzzy soft strong hyper BCK-ideal are introduced, and related properties and relations are investigated. Characterizations of intuitionistic fuzzy soft (weak) hyper BCK ideal are considered. Conditions for an intuitionistic fuzzy soft weak hyper BCK ideal to be an intuitionistic fuzzy soft s-weak hyper BCK ideal are provided. Conditions for an intuitionistic fuzzy soft set to be an intuitionistic fuzzy soft strong hyper BCK ideal are given.

Keywords: intuitionistic fuzzy soft hyper BCK ideal; intuitionistic fuzzy soft weak hyper BCK ideal; intuitionistic fuzzy soft s-weak hyper BCK-ideal; intuitionistic fuzzy soft strong hyper BCK-ideal

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1. Introduction

Dealing with uncertainties is a major problem in many areas such as economics, engineering, environmental science, medical science, and social science etc. These problems cannot be dealt with by classical methods, because classical methods have inherent difficulties. To overcome these difficulties, Molodtsov [1] proposed a new approach, which was called soft set theory, for modeling uncertainty. In [2], Jun applied the notion of soft sets to the theory of BCK/BCI-algebras, and Jun et al. [3] studied ideal theory of BCK/BCI-algebras based on soft set theory. Maji et al. [4] extended the study of soft sets to fuzzy soft sets. They introduced the concept of fuzzy soft sets as a generalization of the standard soft sets, and presented an application of fuzzy soft sets in a decision-making problem. Maji et al. [5] also introduced the concept of intuitionistic fuzzy soft set which combines the advantage of soft set and Atanassov's intuitionistic fuzzy set. Jun et al. [6] applied fuzzy soft set to BCK/BCI-algebras.

Hyperstructure theory was born in 1934 when Marty defined hypergroups, began to analyze their properties, and applied them to groups and relational algebraic functions (see [7]). Algebraic hyperstructures represent a natural extension of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Many papers and several books have been written on this topic. Presently, hyperstructures have a lot of applications in several branches of mathematics and computer sciences (see [8–19]). In [20], Jun et al. applied the hyperstructures to BCK-algebras, and introduced the concept of a hyper BCK-algebra which is a generalization of a BCK-algebra. Since then, Jun et al. studied more notions and results in [3,21,22]. Also, several fuzzy versions of hyper BCK-algebras have been considered in [23,24]. Recently Davvaz et al. summarize research progress of fuzzy hyperstructures in [25].

In this article, we introduce the notions of intuitionistic fuzzy soft hyper BCK ideal, intuitionistic fuzzy soft weak hyper BCK ideal, intuitionistic fuzzy soft s-weak hyper BCK-ideal and intuitionistic fuzzy soft strong hyper BCK-ideal, and investigate related properties and relations. We discuss characterizations of intuitionistic fuzzy soft (weak) hyper BCK ideal. We find conditions for an intuitionistic fuzzy soft weak hyper BCK ideal to be an intuitionistic fuzzy soft s-weak hyper BCK ideal. We provide conditions for an intuitionistic fuzzy soft set to be an intuitionistic fuzzy soft strong hyper BCK ideal.

2. Preliminaries

Let H be a nonempty set endowed with a hyper operation "$\circ$", that is, $\circ$ is a function from $H \times H$ to $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$. For two subsets A and B of H, denote by $A \circ B$ the set $\cup\{a \circ b \mid a \in A,\ b \in B\}$. We shall use $x \circ y$ instead of $x \circ \{y\}$, $\{x\} \circ y$, or $\{x\} \circ \{y\}$.

By a *hyper BCK algebra* (see [20]) we mean a nonempty set H endowed with a hyper operation "$\circ$" and a constant 0 satisfying the following axioms:

(H1) $(x \circ z) \circ (y \circ z) \ll x \circ y$,
(H2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(H3) $x \circ H \ll \{x\}$,
(H4) $x \ll y$ and $y \ll x$ imply $x = y$,

for all $x, y, z \in H$, where $x \ll y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A \ll B$ is defined by $\forall a \in A, \exists b \in B$ such that $a \ll b$.

In a hyper BCK algebra H, the condition (H3) is equivalent to the condition:

$$x \circ y \ll \{x\} \text{ for all } x, y \in H.$$

In any hyper BCK algebra H, the following hold (see [20]):

$$x \circ 0 \ll \{x\},\ 0 \circ x \ll \{0\},\ 0 \circ 0 \ll \{0\}, \tag{1}$$
$$(A \circ B) \circ C = (A \circ C) \circ B, A \circ B \ll A, 0 \circ A \ll \{0\}, \tag{2}$$
$$0 \ll x,\ x \ll x,\ A \ll A, \tag{3}$$
$$A \subseteq B \ \Rightarrow\ A \ll B, \tag{4}$$
$$0 \circ x = \{0\},\ 0 \circ A = \{0\}, \tag{5}$$
$$A \ll \{0\} \ \Rightarrow\ A = \{0\}, \tag{6}$$
$$x \in x \circ 0, \tag{7}$$

for all $x, y, z \in H$ and for all non-empty subsets A, B and C of H.

A non-empty subset A of a hyper BCK algebra H is called a

- *hyper BCK ideal* of H (see [20]) if it satisfies

$$0 \in A, \tag{8}$$
$$(\forall x, y \in H)\,(x \circ y \ll A,\ y \in A \ \Rightarrow\ x \in A)\,. \tag{9}$$

- *strong hyper BCK ideal* of H (see [22]) if it satisfies (8) and

$$(\forall x, y \in H)\,((x \circ y) \cap A \neq \emptyset,\ y \in A \ \Rightarrow\ x \in A)\,. \tag{10}$$

- *weak hyper BCK ideal* of H (see [20]) if it satisfies (8) and

$$(\forall x, y \in H)\,(x \circ y \subseteq A,\ y \in A \ \Rightarrow\ x \in A)\,. \tag{11}$$

Recall that every strong hyper BCK ideal is a hyper BCK ideal (see [22]).

Molodtsov [1] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $\mathscr{P}(U)$ denote the power set of U and $A \subseteq E$.

A pair (λ, A) is called a *soft set* over U, where λ is a mapping given by

$$\lambda : A \to \mathscr{P}(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $\lambda(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (λ, A) (see [1]).

Let U be an initial universe set and E be a set of parameters. Let $\mathcal{F}(U)$ denote the set of all fuzzy sets in U. Then (f, A) is called a *fuzzy soft set* over U (see [4]) where $A \subseteq E$ and f is a mapping given by $f : A \to \mathcal{F}(U)$.

In general, for every parameter u in A, $f[u]$ is a fuzzy set in U and it is called *fuzzy value set* of parameter u. If for every $u \in A$, $f[u]$ is a crisp subset of U, then (f, A) is degenerated to be the standard soft set. Thus, from the above definition, it is clear that fuzzy soft set is a generalization of standard soft set.

3. Intuitionistic Fuzzy Soft Hyper BCK Ideals

In what follows let H and E be a hyper BCK algebra and a set of parameters, respectively, unless otherwise specified.

Definition 1. *Let $\mathcal{F}_I(H)$ denote the set of all intuitionistic fuzzy sets in H and $A \subseteq E$. Then a pair $(\tilde{\lambda}, A)$ is called an intuitionistic fuzzy soft set over H, where $\tilde{\lambda}$ is a mapping given by*

$$\tilde{\lambda} : A \to \mathcal{F}_I(H). \tag{12}$$

For any parameter $e \in A$, $\tilde{\lambda}(e)$ is an intuitionistic fuzzy set in H and it is called the *intuitionistic fuzzy value set* of parameter e, which is of the form

$$\tilde{\lambda}(e) = \Big\{ \langle x, \mu_{\tilde{\lambda}(e)}(x), \gamma_{\tilde{\lambda}(e)}(x) \rangle \mid x \in H \Big\}. \tag{13}$$

Definition 2. *An intuitionistic fuzzy soft set $(\tilde{\lambda}, A)$ over H is called an intuitionistic fuzzy soft hyper BCK ideal based on a parameter $e \in A$ over H (briefly, e-intuitionistic fuzzy soft hyper BCK ideal of H) if the intuitionistic fuzzy value set $\tilde{\lambda}(e)$ of e satisfies the following conditions:*

$$(\forall x, y \in H)\left(x \ll y \ \Rightarrow\ \mu_{\tilde{\lambda}(e)}(x) \geq \mu_{\tilde{\lambda}(e)}(y),\ \gamma_{\tilde{\lambda}(e)}(x) \leq \gamma_{\tilde{\lambda}(e)}(y) \right), \tag{14}$$

$$(\forall x, y \in H)\left(\begin{array}{l} \mu_{\tilde{\lambda}(e)}(x) \geq \min\left\{ \inf\limits_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a),\ \mu_{\tilde{\lambda}(e)}(y) \right\} \\ \gamma_{\tilde{\lambda}(e)}(x) \leq \max\left\{ \sup\limits_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a),\ \gamma_{\tilde{\lambda}(e)}(y) \right\} \end{array} \right). \tag{15}$$

If $(\tilde{\lambda}, A)$ is an e-intuitionistic fuzzy soft hyper BCK ideal based on H for all $e \in A$, we say that $(\tilde{\lambda}, A)$ is an *intuitionistic fuzzy soft hyper BCK ideal* of H.

Example 1. *Consider a hyper BCK algebra $H = \{0, a, b\}$ with the hyper operation "$\circ$" which is given in Table 1.*

Table 1. Tabular representation of the binary operation $\circ$.

$\circ$	**0**	***a***	***b***
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0,a\}$	$\{0,a\}$
b	$\{b\}$	$\{a,b\}$	$\{0,a,b\}$

Given a set $A = \{x, y\}$ of parameters, we define an intuitionistic fuzzy soft set $(\tilde{\lambda}, A)$ by Table 2.

Table 2. Tabular representation of $(\tilde{\lambda}, A)$.

$\tilde{\lambda}$	**0**	***a***	***b***
x	$(0.9, 0.05)$	$(0.5, 0.35)$	$(0.3, 0.55)$
y	$(0.8, 0.15)$	$(0.4, 0.45)$	$(0.6, 0.25)$

Then $\tilde{\lambda}(x)$ satisfy conditions (14) and (15). Hence $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft hyper BCK ideal based on x over H. But $\tilde{\lambda}(y)$ does not satisfy the condition (14) since $a \ll b$ and $\mu_{\tilde{\lambda}(y)}(a) \leq \mu_{\tilde{\lambda}(y)}(b)$ and/or $\gamma_{\tilde{\lambda}(y)}(a) \geq \gamma_{\tilde{\lambda}(y)}(b)$, and so it is not an intuitionistic fuzzy soft hyper BCK ideal based on y over H.

Proposition 1. *For every intuitionistic fuzzy soft hyper BCK ideal $(\tilde{\lambda}, A)$ of H and any parameter $e \in A$, the following assertions are valid.*

(1) *$(\tilde{\lambda}, A)$ satisfies the condition*

$$(\forall x \in H)\left(\mu_{\tilde{\lambda}(e)}(0) \geq \mu_{\tilde{\lambda}(e)}(x),\ \gamma_{\tilde{\lambda}(e)}(0) \leq \gamma_{\tilde{\lambda}(e)}(x)\right) \tag{16}$$

(2) *If $(\tilde{\lambda}, A)$ satisfies the condition*

$$(\forall T, S \in 2^H)(\exists (x_0, y_0) \in T \times S)\left(\begin{array}{l} \mu_{\tilde{\lambda}(e)}(x_0) = \inf\limits_{a \in T} \mu_{\tilde{\lambda}(e)}(a) \\ \gamma_{\tilde{\lambda}(e)}(y_0) = \sup\limits_{b \in S} \gamma_{\tilde{\lambda}(e)}(b) \end{array}\right), \tag{17}$$

then the following assertion is valid.

$$(\forall x, y \in H)(\exists a, b \in x \circ y)\left(\begin{array}{l} \mu_{\tilde{\lambda}(e)}(x) \geq \min\{\mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\} \\ \gamma_{\tilde{\lambda}(e)}(x) \leq \max\{\gamma_{\tilde{\lambda}(e)}(b), \gamma_{\tilde{\lambda}(e)}(y)\} \end{array}\right). \tag{18}$$

Proof. Since $0 \ll x$ for all $x \in H$, we have $\mu_{\tilde{\lambda}(e)}(0) \geq \mu_{\tilde{\lambda}(e)}(x)$ and $\gamma_{\tilde{\lambda}(e)}(0) \leq \gamma_{\tilde{\lambda}(e)}(x)$ by (14). For any $x, y \in H$, there exists $x_0, y_0 \in x \circ y$ such that $\mu_{\tilde{\lambda}(e)}(x_0) = \inf\limits_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a)$ and $\gamma_{\tilde{\lambda}(e)}(y_0) = \sup\limits_{b \in x \circ y} \gamma_{\tilde{\lambda}(e)}(b)$ by (17). It follows from (15) that

$$\mu_{\tilde{\lambda}(e)}(x) \geq \min\left\{\inf_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\right\} = \min\left\{\mu_{\tilde{\lambda}(e)}(x_0), \mu_{\tilde{\lambda}(e)}(y)\right\}$$

and

$$\gamma_{\tilde{\lambda}(e)}(x) \leq \max\left\{\sup_{b \in x \circ y} \gamma_{\tilde{\lambda}(e)}(b), \gamma_{\tilde{\lambda}(e)}(y)\right\} = \max\left\{\gamma_{\tilde{\lambda}(e)}(y_0), \gamma_{\tilde{\lambda}(e)}(y)\right\}$$

which is the desired result. □

Lemma 1 ([21]). *Let A be a subset of a hyper BCK algebra H. If I is a hyper BCK ideal of H such that $A \ll I$, then A is contained in I.*

Given an intuitionistic fuzzy soft set $(\tilde{\lambda}, A)$ over H and $(\varepsilon, \delta) \in [0,1] \times [0,1]$ with $\varepsilon + \delta \leq 1$, we consider the following sets.

$$U_\varepsilon := \left\{x \in H \mid \mu_{\tilde{\lambda}(e)}(x) \geq \varepsilon\right\}$$
$$L_\delta := \left\{x \in H \mid \gamma_{\tilde{\lambda}(e)}(x) \leq \delta\right\} \tag{19}$$

where e is a parameter in A.

Theorem 1. *An intuitionistic fuzzy soft set $(\tilde{\lambda}, A)$ over H is an intuitionistic fuzzy soft hyper BCK ideal of H if and only if the nonempty sets U_ε and L_δ are hyper BCK ideals of H for all $(\varepsilon, \delta) \in [0,1] \times [0,1]$ with $\varepsilon + \delta \leq 1$.*

Proof. Let e be a parameter in A. Assume that $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft hyper BCK ideal of H and U_ε and L_δ are nonempty for all $(\varepsilon, \delta) \in [0,1] \times [0,1]$ with $\varepsilon + \delta \leq 1$. Then there exist $a \in U_\varepsilon$ and $b \in L_\delta$, and so $\mu_{\tilde{\lambda}(e)}(a) \geq \varepsilon$ and $\gamma_{\tilde{\lambda}(e)}(b) \leq \delta$. It follows from (16) that

$$\mu_{\tilde{\lambda}(e)}(0) \geq \mu_{\tilde{\lambda}(e)}(a) \geq \varepsilon \text{ and } \gamma_{\tilde{\lambda}(e)}(0) \leq \gamma_{\tilde{\lambda}(e)}(b) \leq \delta.$$

Hence $0 \in U_\varepsilon \cap L_\delta$. Let $x, y \in H$ be such that $x \circ y \ll U_\varepsilon$ and $y \in U_\varepsilon$. Then for any $a \in x \circ y$ there exists $a_0 \in U_\varepsilon$ such that $a \ll a_0$. Thus $\mu_{\tilde{\lambda}(e)}(a) \geq \mu_{\tilde{\lambda}(e)}(a_0) \geq \varepsilon$ by (14), which implies from (15) that

$$\mu_{\tilde{\lambda}(e)}(x) \geq \min\left\{\inf_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\right\} \geq \min\left\{\varepsilon, \mu_{\tilde{\lambda}(e)}(y)\right\} \geq \varepsilon.$$

Hence $x \in U_\varepsilon$, and therefore U_ε is a hyper BCK ideal of H. Now suppose that $a \circ b \ll L_\delta$ and $b \in L_\delta$ for all $a, b \in H$. Then for any $x \in a \circ b$ there exists $x_0 \in L_\delta$ such that $x \ll x_0$. Thus $\gamma_{\tilde{\lambda}(e)}(x) \leq \gamma_{\tilde{\lambda}(e)}(x_0) \leq \delta$ by (14), which implies from (15) that

$$\gamma_{\tilde{\lambda}(e)}(a) \leq \max\left\{\sup_{x \in a \circ b} \gamma_{\tilde{\lambda}(e)}(x), \gamma_{\tilde{\lambda}(e)}(b)\right\} \leq \max\left\{\delta, \gamma_{\tilde{\lambda}(e)}(b)\right\} \leq \delta.$$

Hence $a \in L_\delta$, and therefore L_δ is a hyper BCK ideal of H.

Conversely, suppose that the nonempty sets U_ε and L_δ are hyper BCK ideals of H for all $(\varepsilon, \delta) \in [0,1] \times [0,1]$ with $\varepsilon + \delta \leq 1$. Let $x, y, u, v \in H$ be such that $x \ll y$, $\mu_{\tilde{\lambda}(e)}(y) = \varepsilon$, $u \ll v$ and $\gamma_{\tilde{\lambda}(e)}(v) = \delta$. Then $y \in U_\varepsilon$ and $v \in L_\delta$, which imply that $x \ll U_\varepsilon$ and $u \ll L_\delta$. It follows from Lemma 1 that $x \in U_\varepsilon$ and $u \in L_\delta$. Thus $\mu_{\tilde{\lambda}(e)}(x) \geq \varepsilon = \mu_{\tilde{\lambda}(e)}(y)$ and $\gamma_{\tilde{\lambda}(e)}(u) \leq \delta = \gamma_{\tilde{\lambda}(e)}(v)$. Now, for any $x, y, u, v \in H$, let $\varepsilon := \min\left\{\inf_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\right\}$ and $\delta := \max\left\{\sup_{b \in u \circ v} \gamma_{\tilde{\lambda}(e)}(b), \gamma_{\tilde{\lambda}(e)}(v)\right\}$. Then $y \in U_\varepsilon$ and $v \in L_\delta$, and for each $a \in x \circ y$ and $b \in u \circ v$ we have

$$\mu_{\tilde{\lambda}(e)}(a) \geq \inf_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a) \geq \min\left\{\inf_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\right\} = \varepsilon$$

and

$$\gamma_{\tilde{\lambda}(e)}(b) \leq \sup_{b \in u \circ v} \gamma_{\tilde{\lambda}(e)}(b) \leq \max\left\{\sup_{b \in u \circ v} \gamma_{\tilde{\lambda}(e)}(b), \gamma_{\tilde{\lambda}(e)}(v)\right\} = \delta.$$

Thus, $a \in U_\varepsilon$ and $b \in L_\delta$, and so $x \circ y \subseteq U_\varepsilon$ and $u \circ v \subseteq L_\delta$. Hence $x \circ y \ll U_\varepsilon$ and $u \circ v \ll L_\delta$ by (4). Since $y \in U_\varepsilon$, $v \in L_\delta$ and U_ε and L_δ are hyper BCK ideal of H, it follows that $x \in U_\varepsilon$ and $u \in L_\delta$. Therefore

$$\mu_{\tilde{\lambda}(e)}(x) \geq \varepsilon = \min\left\{\inf_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\right\}$$

and

$$\gamma_{\tilde{\lambda}(e)}(u) \leq \delta = \max\left\{\sup_{b \in u \circ v} \gamma_{\tilde{\lambda}(e)}(b), \gamma_{\tilde{\lambda}(e)}(v)\right\}.$$

Consequently, $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft hyper BCK ideal of H. □

Definition 3. *An intuitionistic fuzzy soft set $(\tilde{\lambda}, A)$ over H is called an*

- *intuitionistic fuzzy soft weak hyper BCK ideal based on a parameter $e \in A$ over H (briefly, e-intuitionistic fuzzy soft weak hyper BCK ideal of H) if the intuitionistic fuzzy value set $\tilde{\lambda}(e)$ of e satisfies conditions (15) and (16).*
- *intuitionistic fuzzy soft s-weak hyper BCK ideal based on a parameter $e \in A$ over H (briefly, e-intuitionistic fuzzy soft s-weak hyper BCK ideal of H) if the intuitionistic fuzzy value set $\tilde{\lambda}(e)$ of e satisfies conditions (16) and (18).*

If $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft weak (resp., s-weak) hyper BCK ideal based on e over H for all $e \in A$, we say that $(\tilde{\lambda}, A)$ is an *intuitionistic fuzzy soft weak (resp., s-weak) hyper BCK ideal* of H.

Example 2. *The intuitionistic fuzzy soft set $(\tilde{\lambda}, A)$ in Example 1 is an intuitionistic fuzzy soft weak hyper BCK ideal of H.*

Obviously, every intuitionistic fuzzy soft hyper BCK ideal is an intuitionistic fuzzy soft weak hyper BCK ideal. However, the converse is not true in general. In fact, the intuitionistic fuzzy soft weak hyper BCK ideal of H in Example 2 is not an intuitionistic fuzzy soft hyper BCK ideal of H since it is not an intuitionistic fuzzy soft hyper BCK ideal based on parameter y over H.

Theorem 2. *An intuitionistic fuzzy soft set $(\tilde{\lambda}, A)$ over H is an intuitionistic fuzzy soft weak hyper BCK ideal of H if and only if the nonempty sets U_ε and L_δ are weak hyper BCK ideals of H for all $(\varepsilon, \delta) \in [0,1] \times [0,1]$ with $\varepsilon + \delta \leq 1$ where e is any parameter in A.*

Proof. It is similar to the proof of Theorem 1. □

Theorem 3. *Every intuitionistic fuzzy soft s-weak hyper BCK ideal is an intuitionistic fuzzy soft weak hyper BCK ideal.*

Proof. Let $(\tilde{\lambda}, A)$ be an intuitionistic fuzzy soft s-weak hyper BCK ideal of H. Let $x, y \in H$ and $e \in A$. Then there exists $a, b \in x \circ y$ such that $\mu_{\tilde{\lambda}(e)}(x) \geq \min\{\mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\}$ and $\gamma_{\tilde{\lambda}(e)}(x) \leq \max\{\gamma_{\tilde{\lambda}(e)}(b), \gamma_{\tilde{\lambda}(e)}(y)\}$ by (18). Since $\mu_{\tilde{\lambda}(e)}(a) \geq \inf_{c \in x \circ y} \mu_{\tilde{\lambda}(e)}(c)$ and $\gamma_{\tilde{\lambda}(e)}(b) \leq \sup_{d \in x \circ y} \mu_{\tilde{\lambda}(e)}(d)$, it follows that

$$\mu_{\tilde{\lambda}(e)}(x) \geq \min\left\{\inf_{c \in x \circ y} \mu_{\tilde{\lambda}(e)}(c), \mu_{\tilde{\lambda}(e)}(y)\right\}$$

and

$$\gamma_{\tilde{\lambda}(e)}(x) \leq \max\left\{\sup_{d \in x \circ y} \gamma_{\tilde{\lambda}(e)}(d), \gamma_{\tilde{\lambda}(e)}(y)\right\}.$$

Therefore $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft weak hyper BCK ideal of H. □

Question 1. *Is the converse of Theorem 3 true?*

It is not easy to find an example of an intuitionistic fuzzy soft weak hyper BCK ideal which is not an intuitionistic fuzzy soft s-weak hyper BCK ideal. However, we have the following theorem.

Theorem 4. *If an intuitionistic fuzzy soft weak hyper BCK ideal $(\tilde{\lambda}, A)$ of H satisfies the condition (17) then $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft s-weak hyper BCK ideal of H.*

Proof. Let e be a parameter in A. For any $x, y \in H$, there exists $x_0, y_0 \in x \circ y$ such that $\mu_{\tilde{\lambda}(e)}(x_0) = \inf_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a)$ and $\gamma_{\tilde{\lambda}(e)}(y_0) = \sup_{b \in x \circ y} \gamma_{\tilde{\lambda}(e)}(b)$ by (17). It follows from (15) that

$$\mu_{\tilde{\lambda}(e)}(x) \geq \min\left\{ \inf_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y) \right\} = \min\left\{ \mu_{\tilde{\lambda}(e)}(x_0), \mu_{\tilde{\lambda}(e)}(y) \right\}$$

and

$$\gamma_{\tilde{\lambda}(e)}(x) \leq \max\left\{ \sup_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a), \gamma_{\tilde{\lambda}(e)}(y) \right\} = \max\left\{ \gamma_{\tilde{\lambda}(e)}(y_0), \gamma_{\tilde{\lambda}(e)}(y) \right\}$$

Therefore $(\tilde{\lambda}, A)$ is an e-intuitionistic fuzzy soft s-weak hyper BCK ideal of H, and hence $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft s-weak hyper BCK ideal of H since e is arbitrary. □

The condition (17) is always true in a finite hyper BCK algebra. Hence the notion of intuitionistic fuzzy soft s-weak hyper BCK ideal is in accord with the notion of intuitionistic fuzzy soft weak hyper BCK ideal in a finite hyper BCK algebra.

Definition 4. *An intuitionistic fuzzy soft set $(\tilde{\lambda}, A)$ over H is called an intuitionistic fuzzy soft strong hyper BCK ideal over H based on a parameter e in A (briefly, e-intuitionistic fuzzy soft strong hyper BCK ideal of H) if the intuitionistic fuzzy value set $\tilde{\lambda}(e) : H \to [0, 1]$ of e satisfies the condition*

$$(\forall x, y \in H) \left(\begin{array}{l} \mu_{\tilde{\lambda}(e)}(x) \geq \min\left\{ \sup_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a),\ \mu_{\tilde{\lambda}(e)}(y) \right\} \\ \gamma_{\tilde{\lambda}(e)}(x) \leq \max\left\{ \inf_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a),\ \gamma_{\tilde{\lambda}(e)}(y) \right\} \end{array} \right). \tag{20}$$

and

$$(\forall x \in H) \left(\inf_{a \in x \circ x} \mu_{\tilde{\lambda}(e)}(a) \geq \mu_{\tilde{\lambda}(e)}(x),\ \sup_{a \in x \circ x} \gamma_{\tilde{\lambda}(e)}(a) \leq \gamma_{\tilde{\lambda}(e)}(x) \right). \tag{21}$$

If $(\tilde{\lambda}, A)$ is an e-intuitionistic fuzzy soft strong hyper BCK ideal of H for all $e \in A$, we say that $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft strong hyper BCK ideal of H.

Proposition 2. *Every intuitionistic fuzzy soft strong hyper BCK ideal $(\tilde{\lambda}, A)$ of H satisfies the following assertions.*

(1) *$(\tilde{\lambda}, A)$ satisfies the condition (16) for all $e \in A$.*

(2) $(\forall x, y \in H)(\forall e \in A) \left(x \ll y \Rightarrow \begin{cases} \mu_{\tilde{\lambda}(e)}(x) \geq \mu_{\tilde{\lambda}(e)}(y) \\ \gamma_{\tilde{\lambda}(e)}(x) \leq \gamma_{\tilde{\lambda}(e)}(y) \end{cases} \right)$.

(3) $(\forall a, x, y \in H)(\forall e \in A) \left(a \in x \circ y \Rightarrow \begin{cases} \mu_{\tilde{\lambda}(e)}(x) \geq \min\{\mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\} \\ \gamma_{\tilde{\lambda}(e)}(x) \leq \max\{\gamma_{\tilde{\lambda}(e)}(a), \gamma_{\tilde{\lambda}(e)}(y)\} \end{cases} \right)$.

Proof. (1) Let $e \in A$. Since $x \ll x$, i.e., $0 \in x \circ x$ for all $x \in H$, we have

$$\mu_{\tilde{\lambda}(e)}(0) \geq \inf_{a \in x \circ x} \mu_{\tilde{\lambda}(e)}(a) \geq \mu_{\tilde{\lambda}(e)}(x)$$

and

$$\gamma_{\tilde{\lambda}(e)}(0) \leq \sup_{a \in x \circ x} \gamma_{\tilde{\lambda}(e)}(a) \leq \gamma_{\tilde{\lambda}(e)}(x)$$

for all $x \in H$ by (21).

(2) Let $e \in A$ and $x, y \in H$ be such that $x \ll y$. Then $0 \in x \circ y$, and so $\mu_{\tilde{\lambda}(e)}(0) \leq \sup_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a)$ and $\gamma_{\tilde{\lambda}(e)}(0) \geq \inf_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a)$. It follows from (20) and (16) that

$$\mu_{\tilde{\lambda}(e)}(x) \geq \min\left\{ \sup_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y) \right\} \geq \min\left\{ \mu_{\tilde{\lambda}(e)}(0), \mu_{\tilde{\lambda}(e)}(y) \right\} = \mu_{\tilde{\lambda}(e)}(y)$$

and

$$\gamma_{\tilde{\lambda}(e)}(x) \leq \max\left\{ \inf_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a), \gamma_{\tilde{\lambda}(e)}(y) \right\} \leq \max\left\{ \gamma_{\tilde{\lambda}(e)}(0), \gamma_{\tilde{\lambda}(e)}(y) \right\} = \gamma_{\tilde{\lambda}(e)}(y).$$

(3) Let $e \in A$ and $a, x, y \in H$ be such that $a \in x \circ y$. Then $\sup_{b \in x \circ y} \mu_{\tilde{\lambda}(e)}(b) \geq \mu_{\tilde{\lambda}(e)}(a)$ and $\inf_{c \in x \circ y} \gamma_{\tilde{\lambda}(e)}(c) \leq \gamma_{\tilde{\lambda}(e)}(a)$, which imply from (20) that

$$\mu_{\tilde{\lambda}(e)}(x) \geq \min\left\{ \sup_{b \in x \circ y} \mu_{\tilde{\lambda}(e)}(b), \mu_{\tilde{\lambda}(e)}(y) \right\} \geq \min\left\{ \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y) \right\}$$

and

$$\gamma_{\tilde{\lambda}(e)}(x) \leq \max\left\{ \inf_{c \in x \circ y} \gamma_{\tilde{\lambda}(e)}(c), \gamma_{\tilde{\lambda}(e)}(y) \right\} \leq \max\left\{ \gamma_{\tilde{\lambda}(e)}(a), \gamma_{\tilde{\lambda}(e)}(y) \right\}.$$

This proves (3). □

Please note that if $a \in x \circ y$ for all $a, x, y \in H$, then $\mu_{\tilde{\lambda}(e)}(a) \geq \inf_{b \in x \circ y} \mu_{\tilde{\lambda}(e)}(b)$ and $\gamma_{\tilde{\lambda}(e)}(a) \leq \sup_{b \in x \circ y} \gamma_{\tilde{\lambda}(e)}(b)$ for all $e \in A$. Hence, we have the following corollary.

Corollary 1. *Every intuitionistic fuzzy soft strong hyper BCK ideal $(\tilde{\lambda}, A)$ of H satisfies the following condition:*

$$(\forall e \in A)(\forall x, y \in H) \left(\begin{array}{l} \mu_{\tilde{\lambda}(e)}(x) \geq \min\left\{ \inf_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y) \right\} \\ \gamma_{\tilde{\lambda}(e)}(x) \leq \max\left\{ \sup_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a), \gamma_{\tilde{\lambda}(e)}(y) \right\} \end{array} \right). \tag{22}$$

Corollary 2. *Every intuitionistic fuzzy soft strong hyper BCK ideal is both an intuitionistic fuzzy soft s-weak hyper BCK ideal and an intuitionistic fuzzy soft hyper BCK ideal.*

Proof. Straightforward. □

The following example shows that there is an intuitionistic fuzzy soft hyper BCK ideal (and hence an intuitionistic fuzzy soft weak hyper BCK ideal) which is not an intuitionistic fuzzy soft strong hyper BCK ideal.

Example 3. *Consider the hyper BCK algebra $H = \{0, a, b\}$ in Example 1. Given a set $E = \{x, y, z\}$ of parameters, let $(\tilde{\lambda}, A)$ be an intuitionistic fuzzy soft set over H defined by Table 3.*

Table 3. Tabular representation of $(\tilde{\lambda}, A)$.

$\tilde{\lambda}$	**0**	***a***	***b***
x	$(0.8, 0.15)$	$(0.7, 0.25)$	$(0.6, 0.35)$
y	$(0.5, 0.35)$	$(0.3, 0.45)$	$(0.2, 0.45)$
z	$(0.9, 0.05)$	$(0.6, 0.35)$	$(0.1, 0.65)$

Then $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft (weak) hyper BCK ideal of H. However, it is not an intuitionistic fuzzy soft strong hyper BCK ideal of H since

$$\mu_{\tilde{\lambda}(y)}(b) = 0.2 < 0.3 = \min\left\{\sup_{c \in b \circ a} \mu_{\tilde{\lambda}(y)}(c), \mu_{\tilde{\lambda}(y)}(a)\right\}$$

and

$$\gamma_{\tilde{\lambda}(y)}(b) = 0.45 = 0.45 = \max\left\{\inf_{c \in b \circ a} \gamma_{\tilde{\lambda}(y)}(c), \gamma_{\tilde{\lambda}(y)}(a)\right\}.$$

Theorem 5. *If $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft strong hyper BCK ideal of H, then the nonempty sets U_ε and L_δ are strong hyper BCK ideals of H for all $(\varepsilon, \delta) \in [0,1] \times [0,1]$ with $\varepsilon + \delta \leq 1$.*

Proof. Let $(\tilde{\lambda}, A)$ be an intuitionistic fuzzy soft strong hyper BCK ideal of H and $(\varepsilon, \delta) \in [0,1] \times [0,1]$ be such that $\varepsilon + \delta \leq 1$ and $U_\varepsilon \neq \emptyset \neq L_\delta$ where e is any parameter in A. Then there exist $a \in U_\varepsilon$ and $b \in L_\delta$, and thus $\mu_{\tilde{\lambda}(e)}(a) \geq \varepsilon$ and $\gamma_{\tilde{\lambda}(e)}(b) \leq \delta$. By Proposition 2(1), $\mu_{\tilde{\lambda}(e)}(0) \geq \mu_{\tilde{\lambda}(e)}(a) \geq \varepsilon$ and $\gamma_{\tilde{\lambda}(e)}(0) \leq \gamma_{\tilde{\lambda}(e)}(b) \leq \delta$, and thus $0 \in U_\varepsilon \cap L_\delta$. Let $x, y \in H$ be such that $(x \circ y) \cap U_\varepsilon \neq \emptyset$ and $y \in U_\varepsilon$. Then $\mu_{\tilde{\lambda}(e)}(y) \geq \varepsilon$ and there exists $a_0 \in (x \circ y) \cap U_\varepsilon$. It follows from (20) that

$$\mu_{\tilde{\lambda}(e)}(x) \geq \min\left\{\sup_{c \in x \circ y} \mu_{\tilde{\lambda}(e)}(c), \mu_{\tilde{\lambda}(e)}(y)\right\} \geq \min\left\{\mu_{\tilde{\lambda}(e)}(a_0), \mu_{\tilde{\lambda}(e)}(y)\right\} \geq \varepsilon.$$

Hence $x \in U_\varepsilon$, and therefore U_ε is a strong hyper BCK ideal of H. Now assume that $(x \circ y) \cap L_\delta \neq \emptyset$ and $y \in L_\delta$ for all $x, y \in H$. Then there exists $b_0 \in (x \circ y) \cap L_\delta$ and $\gamma_{\tilde{\lambda}(e)}(y) \leq \delta$. Using (20), we get

$$\gamma_{\tilde{\lambda}(e)}(x) \leq \max\left\{\inf_{c \in x \circ y} \gamma_{\tilde{\lambda}(e)}(c), \gamma_{\tilde{\lambda}(e)}(y)\right\} \leq \max\left\{\gamma_{\tilde{\lambda}(e)}(b_0), \gamma_{\tilde{\lambda}(e)}(y)\right\} \leq \delta.$$

Thus, $x \in L_\delta$, and so L_δ is a strong hyper BCK ideal of H. □

We provide conditions for an intuitionistic fuzzy soft set to be an intuitionistic fuzzy soft strong hyper BCK ideal.

Theorem 6. *Let $(\tilde{\lambda}, A)$ be an intuitionistic fuzzy soft set over H such that*

$$(\forall T \subseteq H)(\exists x_0, y_0 \in T)\left(\mu_{\tilde{\lambda}(e)}(x_0) = \sup_{a \in T} \mu_{\tilde{\lambda}(e)}(a),\ \gamma_{\tilde{\lambda}(e)}(y_0) = \inf_{b \in T} \gamma_{\tilde{\lambda}(e)}(b)\right) \tag{23}$$

where e is any parameter in A. If the sets U_ε and L_δ in (19) are nonempty strong hyper BCK ideals of H for all $(\varepsilon, \delta) \in [0,1] \times [0,1]$ with $\varepsilon + \delta \leq 1$, then $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft strong hyper BCK ideal of H.

Proof. For any parameter e in A and $x \in H$, let $\mu_{\tilde{\lambda}(e)}(x) = \varepsilon$ and $\gamma_{\tilde{\lambda}(e)}(x) = \delta$. Then $x \in U_\varepsilon$ and $x \in L_\delta$. Since $x \circ x \ll x$ by (H3), it follows from Lemma 1 that $x \circ x \subseteq U_\varepsilon$. Hence $\mu_{\tilde{\lambda}(e)}(a) \geq \varepsilon$ and $\gamma_{\tilde{\lambda}(e)}(a) \leq \delta$ for all $a \in x \circ x$, and so $\inf\limits_{a \in x \circ x} \mu_{\tilde{\lambda}(e)}(a) \geq \varepsilon = \mu_{\tilde{\lambda}(e)}(x)$ and $\sup\limits_{a \in x \circ x} \gamma_{\tilde{\lambda}(e)}(a) \leq \delta = \gamma_{\tilde{\lambda}(e)}(x)$. For any $x, y \in H$, let $k = \min\left\{\sup\limits_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\right\}$ and $r = \max\left\{\inf\limits_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a), \gamma_{\tilde{\lambda}(e)}(y)\right\}$. Then U_k and L_r are nonempty and are strong hyper BCK ideals of H by hypothesis. Using the condition (23) implies that $\mu_{\tilde{\lambda}(e)}(a_0) = \sup\limits_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a)$ and $\gamma_{\tilde{\lambda}(e)}(b_0) = \inf\limits_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a)$ for some $a_0, b_0 \in x \circ y$. Hence

$$\mu_{\tilde{\lambda}(e)}(a_0) = \sup_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a) \geq \min\left\{\sup_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\right\} = k$$

and

$$\gamma_{\tilde{\lambda}(e)}(b_0) = \inf_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a) \leq \max\left\{\inf_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a), \gamma_{\tilde{\lambda}(e)}(y)\right\} = r,$$

which imply that $a_0 \in U_k$ and $b_0 \in L_r$. It follows that $(x \circ y) \cap U_k \neq \emptyset$ and $(x \circ y) \cap L_r \neq \emptyset$. Since U_k and L_r are strong hyper BCK ideals of H, we have $x \in U_k$ and $x \in L_r$. Thus

$$\mu_{\tilde{\lambda}(e)}(x) \geq k = \min\left\{\sup_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\right\}$$

and

$$\gamma_{\tilde{\lambda}(e)}(x) \leq r = \max\left\{\inf_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a), \gamma_{\tilde{\lambda}(e)}(y)\right\}$$

Therefore $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft strong hyper BCK ideal of H. □

Theorem 7. *Let H satisfy the following condition:*

$$(\forall x, y \in H)\,(|x \circ y| < \infty)\,. \tag{24}$$

Given an intuitionistic fuzzy soft set $(\tilde{\lambda}, A)$ over H, if the nonempty sets U_ε and L_δ in (19) are strong hyper BCK ideals of H for all $(\varepsilon, \delta) \in [0,1] \times [0,1]$ with $\varepsilon + \delta \leq 1$, then $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft strong hyper BCK ideal of H.

Proof. Assume that U_ε and L_δ in (19) are nonempty strong hyper BCK ideals of H for all $(\varepsilon, \delta) \in [0,1] \times [0,1]$ with $\varepsilon + \delta \leq 1$. Then U_ε and L_δ are hyper BCK ideals of H, and so $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft hyper BCK ideal of H by Theorem 1. Please note that $x \circ x \subseteq x \circ H \ll \{x\}$ for all $x \in H$. Hence $a \ll x$ for every $a \in x \circ x$, which implies from (14) that $\mu_{\tilde{\lambda}(e)}(a) \geq \mu_{\tilde{\lambda}(e)}(x)$ and $\gamma_{\tilde{\lambda}(e)}(a) \leq \gamma_{\tilde{\lambda}(e)}(x)$ for all $a \in x \circ x$ and any parameter e in A. Thus $\mu_{\tilde{\lambda}(e)}(x) \leq \inf\limits_{a \in x \circ x} \mu_{\tilde{\lambda}(e)}(a)$ and $\gamma_{\tilde{\lambda}(e)}(x) \geq \sup\limits_{a \in x \circ x} \gamma_{\tilde{\lambda}(e)}(a)$. Let $\min\left\{\sup\limits_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\right\} = \varepsilon$ and $\max\left\{\inf\limits_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a), \gamma_{\tilde{\lambda}(e)}(y)\right\} = \delta$. Then $\sup\limits_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a) \geq \varepsilon$, $\mu_{\tilde{\lambda}(e)}(y) \geq \varepsilon$, $\inf\limits_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a) \leq \delta$ and $\gamma_{\tilde{\lambda}(e)}(y) \leq \delta$. Since $|x \circ y| < \infty$ for all $x, y \in H$, there exists $b \in x \circ y$ such that $\mu_{\tilde{\lambda}(e)}(b) \geq \varepsilon$, $\mu_{\tilde{\lambda}(e)}(y) \geq \varepsilon$, $\gamma_{\tilde{\lambda}(e)}(b) \leq \delta$ and $\gamma_{\tilde{\lambda}(e)}(y) \leq \delta$. It follows that $(x \circ y) \cap U_\varepsilon \neq \emptyset$, $y \in U_\varepsilon$, $(x \circ y) \cap L_\delta \neq \emptyset$ and $y \in L_\delta$. Since U_ε and L_δ are strong hyper BCK ideal of H, we have $x \in U_\varepsilon \cap L_\delta$. Consequently, $\mu_{\tilde{\lambda}(e)}(x) \geq \varepsilon = \min\left\{\sup\limits_{a \in x \circ y} \mu_{\tilde{\lambda}(e)}(a), \mu_{\tilde{\lambda}(e)}(y)\right\}$ and

$\gamma_{\tilde{\lambda}(e)}(x) \leq \delta = \max\left\{\inf_{a \in x \circ y} \gamma_{\tilde{\lambda}(e)}(a), \gamma_{\tilde{\lambda}(e)}(y)\right\}$. Therefore $(\tilde{\lambda}, A)$ is an intuitionistic fuzzy soft strong hyper BCK ideal of H. $\square$

4. Conclusions

We have introduced the notions of intuitionistic fuzzy soft hyper BCK ideal, intuitionistic fuzzy soft weak hyper BCK ideal, intuitionistic fuzzy soft s-weak hyper BCK-ideal and intuitionistic fuzzy soft strong hyper BCK-ideal, and have investigated related properties and relations. We have discussed characterizations of intuitionistic fuzzy soft (weak) hyper BCK ideal, and have found conditions for an intuitionistic fuzzy soft weak hyper BCK ideal to be an intuitionistic fuzzy soft s-weak hyper BCK ideal. We have provided conditions for an intuitionistic fuzzy soft set to be an intuitionistic fuzzy soft strong hyper BCK ideal. In future work, different types of intuitionistic fuzzy soft hyper BCK ideals will be defined and discussed.

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