# Massive Spin Zero Fields in Cosmology and the Tail-Free Property

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**Abstract:** Fields of spin $s \geq 1/2$ satisfying wave equations in a curved space obey the Huygens principle under certain conditions clarified by a known theorem. Here, this theorem is generalized to spin zero and applied to an inflaton field in de Sitter-like space, showing that tails of scalar radiation are an unavoidable physical feature. Requiring the absence of tails, on the contrary, necessarily implies an unnatural tuning between cosmological constant, scalar field mass, and coupling constant to the curvature.

## 1. Introduction

There is a long history of studies of massive fields of arbitrary spin which satisfy wave equations in curved space. These studies have approached the subject from both the mathematical and the physical (classical and quantum) sides. An interesting aspect of the physics and the mathematics of the propagation of waves is the validity, or lack thereof, of the Huygens principle. The Huygens principle maintains that a delta-like impulse of radiation travels on a sharp front propagating at the speed of light and reaches an observer all at once. Breaking the Huygens principle corresponds, on the contrary, to this radiation spreading over a finite spacetime region and arriving to the same observer a bit at a time, over an extended period of time. In other words, when the Huygens principle is violated, the wave propagation is not sharp and it exhibits the phenomenon of "tails", i.e., components of the radiation arriving late in time in comparison with what one expects from a delta-like source in flat 3-space and propagation along the characteristic surfaces of the corresponding hyperbolic partial differential equation. There are various possible causes of tails, and one of them is the scattering of radiation by the spacetime curvature [1–4]. Naturally, the high frequency modes (i.e., those with wavelength much smaller than the curvature radius of spacetime) do not "feel" the spacetime curvature and are essentially unaffected by this backscattering phenomenon, which is instead important for wavelengths comparable to, or larger than, the radius of curvature of spacetime. (We use the terminology "mode" and "wavelength" but, of course, in general in a curved space these are not the usual exponential Fourier modes of flat space.) Unless very special conditions are satisfied, a field satisfying some wave-like equation and propagating in a curved spacetime $(\mathcal{M}, g_{ab})$ will have tails. (Here $\mathcal{M}$ denotes a spacetime manifold and $g_{ab}$ is a Lorentzian metric on it.) In general, it is not known which spacetime geometries and which wave equations admit tail-free propagation. Studies of the problem have encountered substantial mathematical difficulties, and only partial results are available for special geometries and for various wave fields (see, e.g., Refs. [1–5]).

Revisiting older literature, one finds a gap in our knowledge in the wave propagation of scalar fields, which are particularly important for inflationary cosmology in the early universe and for the late accelerated era of a quintessence-dominated universe. Specifically, a criterion that must be satisfied by fields of spin $s \geq 1/2$ in order not to have tails was derived long ago and is generalized here to spin zero fields. We allow these scalar fields to couple non-minimally to the Ricci scalar $R$ as is required,

for example in Higgs inflation (see the recent review [6]) and as was considered in many earlier inflationary scenarios ([7] and references therein). We stress the fact that, in general, tails are a generic feature of wave fields propagating in curved spacetime which is introduced by backscattering of the waves off the curvature. With the physics of wave propagation in mind, forbidding tails altogether seems too restrictive and, unless dictated by some special physical motivation, conditions imposing sharp propagation on a generic curved spacetime are unphysical and introduce physical pathologies.

We adopt the metric signature $-+++$ and we use units in which the speed of light and Newton's constant assume the value unity. The Ricci tensor is computed in terms of the Christoffel symbols $\Gamma^{\delta}_{\alpha\beta}$ and its derivatives as

$$R_{\mu\rho} = \Gamma^{\nu}_{\mu\rho,\nu} - \Gamma^{\nu}_{\nu\rho,\mu} + \Gamma^{\alpha}_{\mu\rho}\Gamma^{\nu}_{\alpha\nu} - \Gamma^{\alpha}_{\nu\rho}\Gamma^{\nu}_{\alpha\mu} \tag{1}$$

and we follow the notation of Ref. [8].

## 2. Spin $s \geq 1/2$

Let us begin by considering massive fields of spin $s \geq 1/2$ on a curved spacetime. This situation was analyzed long ago in Ref. [9] (see also [10,11]). This reference proves the

**Theorem 1.** *A solution of the homogeneous wave equation for a massive field of spin $s \geq 1/2$ on the spacetime $(\mathcal{M}, g_{ab})$ obeys the Huygens principle if and only if $(\mathcal{M}, g_{ab})$ has constant curvature and the Ricci scalar is given by*

$$R = \frac{6m^2}{s}, \tag{2}$$

*where m is the field mass.*

The appearance of constant curvature spaces in this theorem is particularly relevant for cosmology and for string theories. In fact, constant curvature spaces include de Sitter spacetime, which is an attractor for early universe inflation in which the cosmos is close (in a phase space sense) to a de Sitter space, and anti-de Sitter space which is important in string theories and for the AdS/CFT correspondence (however, in this article we restrict to four spacetime dimensions and, therefore, we lose this context which would require extra spatial dimensions).

The exact formulation of the Huygens principle adopted by the author of Ref. [9] is crucial for the validity of the theorem. The Huygens principle holding for solutions of wave equations was already formulated in a clear and physically meaningful way by Hadamard [1] as the absence of "tails" of radiation generated by an impulsive source. However, several inequivalent definitions have mushroomed over the years, including the characteristic propagation property, the progressing-wave property, and the tail-free property. These inequivalent definitions are all loosely referred to as "Huygens' principle", creating considerable confusion in the literature. Some order was brought to this area of research by the detailed discussion of the relations between the several characterizations of wave tails given in Ref. [12].

## 3. Spin Zero Fields

In order to fix the ideas and to generalize the result of Ref. [9] to the zero spin case relevant for cosmology, consider the Klein-Gordon equation for a zero spin field $\phi$,

$$\Box\phi - m^2\phi - \xi R\,\phi = 0, \tag{3}$$

where $\Box\phi \equiv g^{ab}\nabla_a\nabla_b\phi$ is the curved spacetime d'Alembertian and the dimensionless constant $\xi$ describes the direct coupling between the field $\phi$ and the Ricci curvature $R$ of spacetime. (The reader should keep in mind that we are only considering the $s = 0$ case here, while Ref. [9] studies the more complicated wave equations for fields of spin $s \geq 1/2$. Direct couplings between the field and the curvature tensor or its contractions (such as, e.g., those of Refs. [8,13,14]) do not appear there).

The solutions of Equation (3) can be expressed by a formal Green function representation in a normal domain $\mathcal{N}$ of spacetime that does not contain sources:

$$\phi(x) = \int_{\partial\mathcal{N}} dS^{a'}(x')\, G\left(x',x\right) \overleftrightarrow{\nabla}_{a'}\, \phi(x')\,, \tag{4}$$

where $\partial\mathcal{N}$ is the boundary of $\mathcal{N}$, $dS^{a'}(x')$ is the oriented volume element on the hypersurface $\partial\mathcal{N}$ at $x'$, whereas the operator $\overleftrightarrow{\nabla}$ is defined as

$$f_1 \overleftrightarrow{\nabla} f_2 \equiv f_1 \nabla f_2 - f_2 \nabla f_1 \tag{5}$$

for any pair of differentiable functions $(f_1, f_2)$. As is natural, we take into account only the retarded Green function $G_R\left(x',x\right)$, which is a solution of the wave Equation (3) with an impulsive source at the point $x$, so that

$$\left[g^{a'b'}(x')\nabla_{a'}\nabla_{b'} - m^2 - \xi R(x')\right] G\left(x',x\right) = -\delta\left(x',x\right)\,, \tag{6}$$

where $\delta\left(x',x\right)$ is the spacetime Dirac delta [15,16]. This is defined such that, for each test function $f$,

$$\int d^4x'\,\sqrt{-g(x')}\, f(x')\,\delta\left(x',x\right) = f(x)\,. \tag{7}$$

The retarded Green function $G_R\left(x',x\right)$ can be decomposed as [1–4]

$$G_R\left(x',x\right) = \Sigma\left(x',x\right)\,\delta_R(\Gamma(x',x)) + V\left(x',x\right)\,\Theta_R(-\Gamma(x',x))\,, \tag{8}$$

where $\Gamma\left(x',x\right)$ is the square of the proper distance between $x'$ and $x$ evaluated along the unique geodesic that connects $x'$ and $x$ in the normal domain $\mathcal{N}$ [4]. In our notation $\delta_R$ and $\Theta_R$ are the Dirac delta and the Heaviside step function which has support in the past of the spacetime point $x'$. In a fixed spacetime geometry, the coefficient functions $\Sigma$ and $V$ are determined uniquely [3,4]. The non-vanishing of $V(x',x)$ corresponds to the presence of wave tails propagating *inside* the light cone, while the part of the Green function proportional to $\Sigma(x',x)$ describes sharp propagation along the light cone [1–4]. The fact that Green functions and propagators of massless fields do not have support strictly along the light cone is familiar from quantum field theory in curved spacetimes [15,16].

In Ref. [17], an interesting property of the scalar field with regard to the Huygens principle was reported. By expanding the retarded Green function $G_R(x',x)$ around a point $x$ to approximate the four-dimensional curved manifold with its tangent space, it was shown that the absence of wave tails, i.e., the condition $V \equiv 0$, is equivalent to

$$m^2 + \left(\xi - \frac{1}{6}\right) R(x) = 0\,. \tag{9}$$

The characterization of the Huygens principle adopted in Ref. [9] actually embodies the requirement that tails be absent. Let us discuss what this requirement would amount to for a scalar field $\phi$ relevant in cosmology. By imposing this condition on a spin zero field, Equation (6) provides us with the condition to be satisfied at the spacetime point $x$ in order to have no tails at $x$. If this requirement is extended to *every* point of spacetime one obtains, as a straightforward consequence of Equation (9),

**Theorem 2.** *The solution of Equation (3) in the spacetime $(\mathcal{M}, g_{ab})$ propagates without tails (i.e., $V(x',x) = 0$ in Equation (8) for any pair of spacetime points $x'$ and $x$) if and only if $(\mathcal{M}, g_{ab})$ is a constant curvature space and*

$$R = \frac{6m^2}{1 - 6\xi}\,. \tag{10}$$

A special case is given by the parameter value $\xi = 1/6$, for which Equation (9) implies that there are no tails of the scalar field if and only if $m = 0$, regardless of the curvature. As is well known [8], if $\xi = 1/6$ and $m = 0$, then Equation (3) is conformally invariant [8] and the absence of tails for a free scalar field propagating in empty spacetime carries over to (anti-)de Sitter space, which has constant curvature and is conformally flat, like all Friedmann-Lemaître-Robertson-Walker spaces [8].

For $s = 0$, the case $\xi = 0$ in which the scalar field couples minimally to the curvature mimics the equations studied in Ref. [9], and there is some analogy with the theorem proven in Ref. [9]. Most likely, the inclusion of non-minimal coupling terms between the wave field and the Riemann tensor or its contractions in the wave equations of [9] would modify the condition (2).

In general relativity and for $\xi \neq 1/6$, the spacetime metric $g_{ab}$ which determines sharp propagation of the scalar field $\phi$ at all spacetime points is obtained by imposing that a cosmological constant $\Lambda$ be present (which ensures that the Ricci curvature $R$ does not depend on the spacetime point) and that its value is related with the scalar field mass and with the coupling constant by

$$\Lambda = \frac{R}{4} = \frac{3m^2}{2(1 - 6\xi)} \, . \tag{11}$$

Therefore, by imposing tail-free propagation (in the sense described above), one necessarily obtains the de Sitter space if $\xi > 1/6$ or the anti-de Sitter space if $\xi < 1/6$. These two constant curvature spaces are extremely important for cosmology and for string theories and the AdS/CFT correspondence, respectively. Requiring the absence of tails, which would be interesting from the mathematical point of view, would necessarily imply the unnatural tuning relation (11) between cosmological constant $\Lambda$, scalar field mass $m$, and coupling constant $\xi$, which cannot be justified on a physical basis.

It is interesting that a relation similar to Equation (11) appears in theories in which gravity is described by a non-linear $\sigma$-model, and in massive gravity in particular. Ref. [18] studied the propagation of the massive graviton in this type of theory in anti-de Sitter space, and derived a relation similar to Equation (11) between the graviton mass $m$, the cosmological constant $\Lambda$, and another parameter $\alpha$ of the theory. The graviton mass $m$ is certainly a necessary parameter in the study of the validity or violation of the Huygens principle in this class of theories. To be more precise, the theory is described by the action [18–20]

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R + m^2 U \left( g, \phi \right) \right] , \tag{12}$$

where the effective potential $U$ contains two free parameters $\alpha_{3,4}$ [18–20],

$$U \left( g, \phi \right) = U_2 + \alpha_3 U_3 + \alpha_4 U_4 \, . \tag{13}$$

and a relation similar to Equation (11) involving $m$, $\Lambda$, and $\alpha \equiv 1 + 3\alpha_3$ emerges in anti-de Sitter space (see Equation (19) of Ref. [18]). The two free parameters play a role similar to that of the coupling parameter $\xi$ in our discussion. In the theory (12) the vacuum can be single or degenerate, depending on the relation between these two free parameters. In this sense, these free parameters (resp., $\xi$) control the background vacuum experienced by the massive graviton (resp. scalar) as it propagates in the constant curvature anti-de Sitter (resp., de Sitter) background. This interesting coincidence will be explored in the future.

## 4. Conclusions

Thus far, we have restricted our considerations to the mathematical aspects of the propagation of a scalar field in a curved space. At this point, it is appropriate to remember the physical reasons for the occurrence of tails. The possible causes are [12,17]:

- The presence of a mass term ($m \neq 0$) or of a potential $V(\phi)$ in the wave equation (here we restrict the potential to a mass term $V = m^2\phi^2/2$ that keeps the Klein-Gordon equation linear). As an example, the wave-like solutions of the Klein-Gordon Equation (2) in four-dimensional flat spacetime $\left(\mathbb{R}^4, \eta_{ab}\right)$ exhibit tails whenever $m \neq 0$.
- The spacetime dimension is another important factor for the presence or absence of tails. For example, in $d$-dimensional Minkowski spacetime, the solutions of Equation (3) have tails for odd $d$, but not for even $d$ [2,21]. Here, however, we limit ourselves to considering four spacetime dimensions.
- Scattering of the waves off the background spacetime curvature—this is the situation in which the most interesting physics appears.

The mathematical literature has focussed on looking for the conditions and geometries for which tails of radiation are absent, presumably because sharp propagation without tails seems a desirable feature in physics and in other areas of research. Indeed, sharp propagation is required for transmitting efficiently information carried by wave signals. For example, the absence of tails in the transmission of electric signals along nerves is necessary for the functioning of complex neural networks and for the existence of sophisticated organisms, and it has been used as an anthropic argument to discriminate the dimensionality of space (in general, the Huygens principle is violated by any linear partial differential equation in which the solution depends on an odd number of variables [1,2,22]. For even number of variables (as in a four-dimensional spacetime); the Huygens principle may or may not hold, which may be difficult to assess. The tail-free property can be seen as being equivalent to the Huygens principle [2,12,21]. It is natural, therefore, that there is renewed interest about tails of radiation in curved space, and in cosmology in particular, by the information theory community [14,23–28]).

Studying the various conditions for the absence of wave tails of massive fields of arbitrary spin (as in Ref. [9]) is certainly legitimate and well motivated from the mathematical point of view. However, it is not easily justifiable from the physical point of view. In fact, a field with mass $m \neq 0$ has tails due the fact that it is massive (or that it has a potential $V(\phi)$, which amounts to a field-dependent mass) and to the backscattering off the background curvature of spacetime. The absence of tails for such a field means that the two effects cancel out, and this cancellation is completely unphysical. From the point of view of field theory, this situation would correspond to the case of a field $\phi$ with non-zero mass which propagates locally with the speed of light. This rather fictional, and clearly fine-tuned, phenomenon goes against our experience in flat space and does not have any experimental support. The case, contemplated above, of a massive scalar field with coupling constant $\xi \neq 1/6$ travelling in de Sitter or anti-de Sitter space is an example in which this pathological behaviour occurs at *every* point of spacetime. In light of these considerations, one concludes that a wave tail is indeed a natural, and even desirable, feature for a massive field of any spin in four spacetime dimensions. Therefore, imposing the absence of tails (as done in Ref. [9]) may have considerable mathematical interest but is not significant, nor realistic, from the physical point of view. Indeed, scalar wave tails may have effects that are important for cosmological applications [29–32] and are relevant for the propagation of super-horizon modes and nearly horizon-sized modes of an inflation field. Moreover, the violation of the Huygens principle has consequences for relativistic quantum communication, as pointed out in recent studies [23–26]. Tails of gravitational (spin two) radiation emitted from compact objects have also been studied [33–37] in relation with the *LIGO* and *VIRGO* laser interferometric detectors which have recently discovered gravitational waves [38–40], although in that case tails are a higher order effect in the wave amplitude and, therefore, not of immediate interest [33–37]. For physical applications, it seems that only spacetimes and wave fields which have non-pathological behaviour with respect to wave tails are admissible (by non-pathological we mean that a true field mass can never be removed by a tail which is caused by scattering off the curvature of spacetime) [17,41].

In Ref. [17] it was argued that, to ensure that tails of a scalar field are present when $m \neq 0$ and to preclude physical pathologies, only the value $\xi = 1/6$ of the coupling constant between $\phi$ and $R$ should be allowed. This result was later confirmed in Ref. [42], and its consequences for inflationary cosmology and for the late, quintessence-dominated universe, were explored in Refs. [7,43–49]. In the

future, it will be interesting to explore the generalization of the wave equations obtained by including explicit couplings between fields of various spins and the Riemann tensor $R_{abcd}$ or its contractions (e.g., [8,13,14]). Perhaps the imposition that there are no causal pathologies such as massive fields propagating exactly on the light cone will fix the values of the coupling constants (perhaps, again, to nonzero values), as it happens in the $s = 0$ case [17]. In the case of spin 1/2, this approach may potentially be of interest for neutrinos emitted in supernova collapse or for the cosmological neutrino background in the early universe.

**Conflicts of Interest:** The author declares no conflict of interest.

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