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# Prioritized Aggregation Operators and Correlated Aggregation Operators for Hesitant 2-Tuple Linguistic Variables

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**Abstract:** The aggregation operator is a potential tool to fuse the information derived from multisources, which has been applied in group decision, combination classification and scheduling clusters successfully. To better characterize complex decision situations and capture complex opinions of decision-makers (DMs), aggregation operators are required to be explored from different viewpoints. In view of information fusion of hesitant 2-tuple linguistic variables, this paper establishes four new aggregation operators, which are called the hesitant 2-tuple linguistic prioritized weighted averaging (H2TLPWA) aggregation operator, hesitant 2-tuple linguistic prioritized weighted geometric (H2TLPWG) aggregation operator, hesitant 2-tuple linguistic correlated averaging (H2TLCA) aggregation operator, and hesitant 2-tuple linguistic correlated geometric (H2TLCG) aggregation operator, respectively. The H2TLPWA aggregation operator and H2TLPWG aggregation operator can characterize the prioritization relationship of the aggregated arguments. The H2TLCA aggregation operator and H2TLCG aggregation operator can describe dependencies between criteria in decision-making problem solving. Moreover all aggregation operation operators have the properties of idempotency, boundedness and monotonicity, and the H2TLCA aggregation operator and H2TLCG aggregation operator are also verified to be symmetric functions. In addition, the H2TLPWA aggregation operator and H2TLCA aggregation operator are employed to settle multicriteria decision-making problems with hesitant 2-tuple linguistic terms. By virtue of predefining discrete initial linguistic labels with symmetrical distribution, the detailed steps of the decision-making process with an example are given to illustrate their practicality and effectiveness.

**Keywords:** hesitant linguistic 2-tuple term; H2TLPWA aggregation operator; H2TLPWG aggregation operator; H2TLCA aggregation operator; H2TLCG aggregation operator

## 1. Introduction

Decision-making is a cognitive process to identify the desirable choice among several alternatives. Everyone makes decisions in his/her daily life, such as choosing a suitable car, recruiting excellent staff, choosing a tourist site for enjoying a summer holiday, and so on. Some multicriteria decision-making (MCDM) methods have been established to help people to make decisions, such as TOPSIS (technique for order preference by similarity to an ideal solution), ELECTRE (elimination and choice expressing reality), PROMETHEE (preference ranking organization method for enrichment evaluation), Grey relation analysis, and so forth. Everyone can easily apply these techniques in his/her daily life and benefit from them. The above methods have been successfully used in a quantitative decision context. There has been an increasing growth in generating large volumes of uncertain data for decision environments [1]. In the past, numerical measures were usually employed to characterize decision-makers' (DMs') preferences. Because of the complexity of objective problems and the ambiguity of human thinking, it is hard to respond to the preferences of DMs by using only numerical values. Sometimes, the decision table involves a mixture of quantitative and qualitative uncertain terms; thus fuzzy sets and probability theory are widely employed to deal with both the subjective imprecision of human perception-based information described in natural language and the objective uncertainty of randomness universally existing in the real world. It is the most common situation for experts to reflect their information preferences with a linguistic term, which is introduced to simulate the human decision process on the basis of the expression of cognitive information [2–5]. When evaluating many MCDM problems with qualitative indicators, DMs prefer to express in terms of "general", "good", "very good" and so on. There are some methods to represent the linguistic information, which are mainly classified into symbolic computational methods and membership function methods. Semantically, symbolic methods are simpler than the latter methods but may cause more information loss [6]. In order to further accurately express semantic information, some new methods have been proposed to express experts' preferences by combing symbolic methods and other uncertainty theories [7–9]. Specifically, Herrere and Martínez put forward the 2-tuple linguistic model [10–12] , which is more accurate than conventional linguistic terms. Meanwhile, this model can efficiently avoid distortion and the loss of information [13]. In a nutshell, the 2-tuple linguistic model establishes a bridge to connect the preferences with consecutive values and human natural language interpretations by introducing converting functions.

In order to further characterize the uncertainty at a higher fusion level of human thinking, the quantitative information and 2-tuple linguistic variables are integrated. Atanassove gave the notion of an intuitionistic fuzzy set (IFS) [14]. Beg and Rashid combined the IFS and 2-tuple linguistic model to introduce the intuitionistic 2-tuple correlated averaging aggregation operator [15]. Furthermore, Beg and Rashid generalized the 2-tuple linguistic model, and relevant concepts and operators are proposed [16]. Wei presented a method for MCDM on the basis of the ET-WG (extended 2-tuple weighted geometric) and ET-OWG (extended 2-tuple ordered weighted geometric) aggregation operators with 2-tuple linguistic information [17–19]. Martínez summarized the 2-tuple linguistic model [20]. Xu introduced a method based on 2-tuple linguistic power aggregation operators for MCDM problems and some dependent aggregation operators in a 2-tuple linguistic environment [21,22], respectively. Wang presented an agile evaluation method for developing a mass customized system using the 2-tuple linguistic model [23]. Geng put forward a 2-tuple linguistic DEA (data envelopment analysis) model to solve MCDM problems with unknown experts' weights [24]. Wan presented a novel hybrid method integrating TL-ANP (2-tuple linguistic analytic network process) and IT-ELECTRE II (interval 2-tuple Elimination and Choice Translating Reality II) to solve MCDM problems with two-level criteria on the basis of the interval 2-tuple linguistic model [25]. Dursun presented the DEMATEL (decision-making trial and evaluation laboratory) method and TOPSIS method based on the 2-tuple fuzzy linguistic representation model to address wastewater treatment problems [26]. Santos established a model based on 2-tuple fuzzy linguistic variables and AHP (analytic hierarchy process) to solve supplier segmentation problems [27].

The above methods can be used in the MCDM problems expressed in a single linguistic term for the evaluation of information. However, in true MCDM problems, a single linguistic term set is not enough to express the DM's cognitive process with hesitation. To settle this problem, Rodríguez defined the hesitant fuzzy linguistic term set (HFLTS) [28] based on hesitant fuzzy set theory [29]. By using this method, the expert can evaluate a criterion by using several possible linguistic terms, which are characterized by a HFLTS. Some MCDM methods based on HFLTSs and aggregation operators of HFLTSs are established from a different point of view [30–37]. Besides the HFLTS, the dual hesitant fuzzy set (DHFS), which is a generation of fuzzy sets, can also describe the perception of the DM with hesitation. Some aggregation operators are also presented to deal with MCDM problems based on the DHFS [38–42].

In the complex MCDM environment, it is obligatory to collect and integrate multiple opinions of DMs or experts from different domains. Experts can identify all relevant factors of MCDM problems; how to integrate the information from different experts or criteria is a crucial step. In the aggregation process, the collective result can be obtained from the different experts by establishing suitable aggregation operators. Many operators are proposed, such as the 2-tuple weighted averaging aggregation operator, the 2-tuple ordered weighted averaging aggregation operator, the extended 2-tuple weighted geometric aggregation operator, the extended 2-tuple ordered weighted geometric aggregation operator and so on [11,12,43]. These operators can only deal with some single 2-tuple linguistic problems. Hesitant 2-tuple linguistic aggregation operators are introduced into the MCDM process [43–45] and can avoid information distortion, making the results more accurate. Furthermore, hesitant 2-tuple linguistic term sets are also broadly applied in practical life. For example, Xue proposed an integrated model and extended the QUALIFLEX (qualitative flexible multiple criteria method) approach to handle robot selection problems on the basis of the hesitant 2-tuple linguistic term sets [46].

In a real decision-making process, to make the MCDM process become closer to reality, we need to consider the prioritization relationship of the experts or the dependencies between criteria. Some existing aggregation operators do not consider these in hesitant 2-tuple linguistic problems solving. Thus it is necessary to develop some aggregation operators to solve these problems and further characterize the human decision process, by which we can shorten the gap between the theoretical results and experimental results. In this paper, we propose the hesitant 2-tuple linguistic prioritized weighted averaging (H2TLPWA) aggregation operator, the hesitant 2-tuple linguistic prioritized weighted geometric (H2TLPWG) aggregation operator, the hesitant 2-tuple linguistic correlated averaging (H2TLCA) aggregation operator and the hesitant 2-tuple linguistic correlated geometric (H2TLCG) aggregation operator.

The rest of the paper is structured as follows. In Section 2, we review some related definitions of hesitant 2-tuple linguistic variables and fuzzy measures. In Section 3, four new aggregation operators are established to aggregate hesitant 2-tuple linguistic term sets, and their properties are explored. In Section 4, we develop a method to solve MCDM problems with hesitant 2-tuple linguistic term sets. In Section 5, an example is employed to show the above decision method and prove that the method we introduced is effective and feasible. Section 6 concludes the paper.

## 2. Preliminaries

In this part, some related definitions, operations and comparison rules of 2-tuple linguistic variables are introduced.

**Definition 1.** *[47,48] Let $S = \{s_p | p = 0, 1, \ldots, l\}$ be a linguistic term set, where $l$ is a positive integer, and $s_p$ represents a linguistic variable with the following characteristics:*

(1) $s_p > s_q \Leftrightarrow p > q$.
(2) For each $s_p$, there always exist $neg(s_p) = s_q, q = l - p$.
(3) If $s_p < s_q$, then $\max(s_p, s_q) = s_q$ and $\min(s_p, s_q) = s_p$.

The 2-tuple linguistic variable is a new development of linguistic variables, which is made up of $(s_p, a_p)$, where $s_p \in S = \{s_p | p = 0, 1, \ldots, l\}$ is a linguistic term and $a_p$ is a numerical value; $a_p$ represents the deviation between the evaluation value and $s_p$ [11,12].

**Definition 2.** *[49] Let $S = \{s_p | p = 0, 1, \ldots, l\}$ be a linguistic term set and $\beta \in [0, 1]$ be a real number that can be converted into an equivalent 2-tuple linguistic variable by the following function:*

$$\triangle : [0, l] \longrightarrow S \times [-\frac{1}{2}, \frac{1}{2})$$
$$\triangle(\beta) = (s_p, a_p) \ with \begin{cases} s_p, & p = round(\beta) \\ a_p = \beta - p, & a_p \in [-\frac{1}{2}, \frac{1}{2}) \end{cases}$$

*where round is the function that returns the positive number rounded to $\beta$. On the contrary, $\triangle^{-1}$ is the function, which is given as follows:*

$$\triangle^{-1} : S \times [-\frac{1}{2}, \frac{1}{2}) \rightarrow [0, l]$$
$$\triangle^{-1} : (s_p, a_p) = (p + a_p) = \beta$$

**Definition 3.** *[50] Let $S = \{s_p | p = 0, 1, \ldots, l\}$ be a linguistic term set; define a transform function from $s_p$ to a 2-tuple linguistic variable:*

$$\begin{aligned} &G : \ S \rightarrow S \times [-\tfrac{1}{2}, \tfrac{1}{2}) \\ &G(s_p) = (s_p, 0), \ s_p \in S \end{aligned} \tag{1}$$

**Definition 4.** *[43] Let $S = \{s_p | p = 0, 1, \ldots, l\}$ be a linguistic term set and $(s_p, a_p)$ be a 2-tuple linguistic variable in S, such that $(s_p, a_p) < (s_q, a_q)$ for all $p < q$; then $H_T = \{x_i | i = 1, 2, \ldots, l(H_T)\}$ is called a hesitant 2-tuple linguistic set (H2TLS), where $l(H_T)$ is a granularity and $x_i$ is a 2-tuple linguistic variable.*

Clearly, any HFLTS can be transformed into a H2TLS through Equation (1). For example, we call $H_s = \{s_3, s_4, s_5\}$ a HFLTS on $S = \{s_1, s_2, \ldots, s_6\}$; through Equation (1), the HFLTS can be transformed into a H2TLS: $H_T = \{(s_3, 0), (s_4, 0), (s_5, 0)\}$.

**Definition 5.** *[43,51] Let $H_T = \{x_i | i = 1, 2, \ldots, l(H_T)\}$ be a H2TLS on S, $l(H_T)$ be a granularity, and $x_i$ be a 2-tuple linguistic variable; then the mean function of $H_T$ can be as shown below:*

$$S(H_T) = \frac{1}{l(H_T)} \sum_{i=1}^{l(H_T)} \triangle^{-1}(x_i) \tag{2}$$

**Definition 6.** *[43,51] Let $H_T = \{x_i | i = 1, 2, \ldots, l(H_T)\}$ be a H2TLS on S, $l(H_T)$ be a granularity, and $x_i$ be a 2-tuple linguistic variable; then the variance function of $H_T$ can be as shown below:*

$$V(H_T) = \frac{1}{l(H_T)} \left( \sum_{i=1}^{l(H_T)} \left| \triangle^{-1}(x_i) - S(H_T) \right|^2 \right)^{\frac{1}{2}} \tag{3}$$

**Definition 7.** *[43,51] Let $H_1 = \{x_i | i = 1, 2, \ldots, l(H_1)\}$ and $H_2 = \{x_{i'} | i = 1, 2, \ldots, l(H_2)\}$ be two H2TLSs on S. $S(H_a)$ $(a = 1, 2)$ and $V(H_a)$ $(a = 1, 2)$ are the mean function and the variance function, respectively. The following order relationships are introduced:*

(1) *If $S(H_1) < S(H_2)$, then $H_1 < H_2$.*
(2) *If $S(H_1) > S(H_2)$, then $H_1 > H_2$.*
(3) *If $S(H_1) = S(H_2)$, then*
    (a) *if $V(H_1) = V(H_2)$, then $H_1 \sim H_2$;*
    (b) *if $V(H_1) < V(H_2)$, then $H_1 > H_2$;*
    (c) *if $V(H_1) > V(H_2)$, then $H_1 < H_2$.*

Fuzzy integral-based aggregation operators can better reflect interaction information among criteria and are attracting more and more attention in MCDM [49,52]. We let $C = \{c_1, c_2, \ldots, c_n\}$ be the set of the criteria, $P(C)$ be the power set of $C$ and $\mu(c_i)$ $(i = 1, 2, \ldots, n)$ be the weights of criteria $c_i \in C(i = 1, 2, \ldots, n)$, where $\mu$ is a fuzzy measure, which is shown as follows.

**Definition 8.** *[53] A fuzzy measure $\mu$ on a set C is a function from $P(C)$ to $[0, 1]$, which meets the following three axioms:*

1. $\mu(\emptyset) = 0, \ \mu(C) = 1;$

2. $E \subseteq F$ *implies* $\mu(E) \le \mu(F)$, *for all* $E, F \subseteq C$;
3. $\mu(E \cup F) = \mu(E) + \mu(F) + \alpha\mu(E)\mu(F)$ *for all* $E, F \subseteq C$ *and* $E \cap F = \emptyset$, *where* $\alpha \in (-1, +\infty)$.

By adjusting the parameter $\alpha$, the different interaction influences between the criteria can be characterized. If $\alpha > 0$, this implies that the set $\{E, F\}$ has a multiplicative effect; if $\alpha < 0$, this implies that the set $\{E, F\}$ has a substitutive effect [53]. If $\alpha = 0$, then the third condition of the axioms is reduced to the additive measure.

If $X$ is a finite set, then $\bigcup_{i=1}^{n} c_i = C$. The $\alpha$-fuzzy measure $\mu$ meets the following conditions:

$$\mu(C) = \mu(\bigcup_{i=1}^{n} c_i) = \begin{cases} \frac{1}{\alpha}\{\prod_{i=1}^{n}[1 + \alpha\mu(c_i)] - 1\} & if\ \alpha \neq 0 \\ \sum_{i=1}^{n}\mu(c_i) & if\ \ \alpha = 0 \end{cases} \tag{4}$$

where $c_i \cap c_j = \emptyset$ for all $(i, j = 1, 2, \ldots, n)$ and $i \neq j$. The element $\mu(c_i)$ is called a fuzzy density of a single element $\{c_i\}$. On the basis of Equation (4), the value of $\alpha$ can be calculated from $\mu(C) = 1$; we can obtain:

$$1 = \frac{1}{\alpha}\{\prod_{i=1}^{n}[1 + \alpha\mu(c_i)] - 1\} \tag{5}$$

## 3. New Hesitant 2-Tuple Linguistic Aggregation Operators

In order to facilitate a beneficial strategy for the individuals and agents, some dominant factors are required to be identified to form a database with dominant degrees [1,54]. In fact, experts or criteria have a different priority level in real decision environments. For example, when buying a house, we usually consider the location and the price of the house, and the price of the house has a higher priority than the location of the house. Similarity, the relative importance of the expert is also determined with a different prioritization in group decision-making. To settle such a question, Yager proposed an operator called the prioritized averaging (PA) aggregation operator [55].

**Definition 9.** *[55] Let* $C = \{c_1, c_2, ..., c_n\}$ *be the set of criteria with a prioritization relationship* $c_1 > c_2 > ... > c_n$ *(> represents "better than"). The value* $c_{ij}$ *represents the criterion value of the alternative i under criterion j and satisfies* $c_{ij} \in [0, 1]$. *Then*

$$PA(c_j) = \sum_{j=1}^{n} w_j c_{ij}$$

*where* $w_j = T_j / \sum_{j=1}^{n} T_j$, $T_j = \prod_{t=1}^{j-1} c_{it}$ $(j = 2, 3, 4, ..., n)$ *and* $T_1 = 1$.

### *3.1. Hesitant 2-Tuple Linguistic Prioritized Weighted Aggregation Operator*

In this part, we generalize the PA aggregation operator to the hesitant 2-tuple linguistic variable and develop the H2TLPWA aggregation operator.

**Definition 10.** *Let* $H = \{H_1, H_2, \ldots, H_n\}$ *be n H2TLSs; then the H2TLPWA aggregation operator is defined as follows:*

$$\begin{aligned} & H2TLPWA(H_1, H_2, \ldots, H_n) \\ & = \bigcup_{x_j \in H_j, (j=1,2,\ldots,n)} \triangle\left(\sum_{j=1}^{n} w_j * \triangle^{-1}(x_j)\right) \end{aligned}$$

*where* $w_j = T_j / \sum_{j=1}^{n} T_j$, $T_j = \prod_{t=1}^{j-1} S(H_t)$ $(j = 2, 3, \ldots, n)$, $T_1 = 1$ *and* $S(\cdot)$ *is the mean function.*

**Theorem 1.** *Let $H = \{H_1, H_2, \ldots, H_n\}$ be $n$ H2TLSs; then satisfactory properties of the H2TLPWA aggregation operator are shown as follows:*

(1) *(Idempotency): Let $H = \{H_1, H_2, \ldots, H_n\}$ be $n$ H2TLSs; if all $H_j$ $(j = 1, 2, ..., n)$ are equal to $H_j = \{x_1, x_2, \ldots, x_n\}$, then*

$$H2TLPWA(H_1, H_2, \ldots, H_n) = (x_1, x_2, \ldots, x_n)$$

(2) *(Boundedness): Let $H = \{H_1, H_2, \ldots, H_n\}$ be $n$ H2TLSs; then*

$$\min_{1 \le j \le n} \{H_j\} \le H2TLPWA(H_1, H_2, \ldots, H_n) \le \max_{1 \le j \le n} \{H_j\}$$

(3) *(Monotonicity): Let $H = \{H_1, H_2, \ldots, H_n\}$ and $H' = \{H_{1'}, H_{2'}, \ldots, H_{n'}\}$ be two collections of H2TLSs; if $H_j \le H_{j'}$ $(j = 1, 2, \ldots, n)$ for all $j$, then*

$$H2TLPWA(H_1, H_2, \ldots, H_n) \le H2TLPWA(H_{1'}, H_{2'}, \ldots, H_{n'})$$

The H2TLPWG aggregation operator, similarly to the H2TLPWA aggregation operator, has idempotency, boundedness and monotonicity.

**Definition 11.** *Let $H = \{H_1, H_2, \ldots, H_n\}$ be $n$ H2TLSs; then the H2TLPWG aggregation operator is defined as follows:*

$$\begin{aligned} & H2TLPWG(H_1, H_2, \ldots, H_n) \\ & = \bigcup_{x_j \in H_j, (j=1,2,\ldots,n)} \triangle \left( \prod_{j=1}^{n} (\triangle^{-1}(x_j))^{w_j} \right) \end{aligned}$$

*where $w_j = T_j / \sum_{j=1}^{n} T_j$, $T_j = \prod_{t=1}^{j-1} S(H_t)$ $(j = 2, 3, \ldots, n)$, $T_1 = 1$ and $S(\cdot)$ is the mean function.*

### *3.2. Hesitant 2-Tuple Linguistic Correlated Aggregation Operator*

In real MCDM problems, interrelationships between different criteria usually exist and are required to be further explored; for example, wishing to buy a high performance car at the proper price. It is well known that high performance will correspond to high prices; thus the two criteria are not fully independent. To address this problem, Grabisch introduced the discrete Choquet integral [56], which is shown below.

**Definition 12.** *[56] Let $f$ be a positive real-valued function on $X = \{x_1, x_2, \ldots, x_n\}$ and $\mu$ be a fuzzy measure on $X$. The discrete Choquet integral of $f$ with respect to $\mu$ is defined by*

$$C_\mu(f) = \sum_{j=1}^{n} f_{(j)} [\mu(A_{(j)}) - \mu(A_{(j+1)})] \tag{6}$$

where $A_{(j)} = \{x_{(j)}, \ldots, x_{(n)}\}$, $A_{(n+1)} = \phi$, and $(\cdot)$ indicates a permutation on $X$ such that $f_{(1)} \le f_{(2)} \le \cdots \le f_{(n)}$.

In this section, we combine the Choquet integral and hesitant 2-tuple linguistic variables to introduce an operator called the H2TLCA aggregation operator.

**Definition 13.** *Let $H = \{H_1, H_2, \ldots, H_n\}$ be n H2TLSs, $C = \{c_1, c_2, \ldots, c_n\}$ be the set of criteria and $\mu$ be a fuzzy measure on C. Then the H2TLCA aggregation operator is defined as shown below:*

$$\begin{aligned} &H2TLCA(H_1, H_2, \ldots, H_n) \\ &= \bigcup_{x_{\sigma(j)} \in H_{\sigma(j)}, (j=1,2,\ldots,n)} \triangle \left( \sum_{j=1}^{n} (\mu(A_{\sigma(j)}) - \mu(A_{\sigma(j-1)})) * \triangle^{-1}(x_{\sigma(j)}) \right) \end{aligned}$$

*where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$ such that $H_{\sigma(1)} \geq H_{\sigma(2)} \geq \ldots \geq H_{\sigma(n)}$, $c_{\sigma(j)}$ is a criterion corresponding to $H_{\sigma(j)}$. $A_{\sigma(j)} = \{c_{\sigma(k)} | k \leq j\}$ for $j \geq 1$, $A_{\sigma(0)} = \varnothing$.*

Some special cases of the H2TLCA aggregation operator are given as follows:

1. If $\mu(A) = 1$, for any $A \in P(C)$, then

$$H2TLCA_\mu(H_1, H_2, \ldots, H_n) = \max\{H_1, H_2, \ldots, H_n\} = H_{\sigma(1)}$$

2. If $\mu(A) = 0$, for any $A \in P(C)$ and $A \neq C$, then

$$H2TLCA_\mu(H_1, H_2, \ldots, H_n) = \min\{H_1, H_2, \ldots, H_n\} = H_{\sigma(n)}$$

3. For any $E, F \in P(C)$ such that $|E| = |F|$, where $|E|$ and $|F|$ are the number of elements in $E$ and $F$, respectively, if $\mu(E) = \mu(F)$ and $\mu(A_{\sigma(j)}) = \frac{j}{n}$, $1 \leq j \leq n$, then

$$H2TLCA_\mu(H_1, H_2, \ldots, H_n) = \bigcup_{x_j \in H_j, (j=1,2,\ldots,n)} \triangle \left( \sum_{j=1}^{n} \frac{1}{n} * \triangle^{-1}(x_j) \right)$$

4. If $\mu(E) = \sum_{c_i \in E} \mu(c_i)$, for all $E \subseteq C$ holds, then

$$\mu(c_{\sigma(j)}) = \mu(A_{\sigma(j)}) - \mu(A_{\sigma(j-1)}),\ j = 1, 2, \ldots, n$$

In this situation, the H2TLCA aggregation operator is reduced to the hesitant 2-tuple linguistic weighted averaging (H2TLWA) aggregation operator:

$$H2TLWA_\mu(H_1, H_2, \ldots, H_n) = \bigcup_{x_j \in H_j, (j=1,2,\ldots,n)} \triangle \left( \sum_{j=1}^{n} \mu(c_j) * \triangle^{-1}(x_j) \right)$$

5. For $A \in P(C)$, let $\omega_j = \mu(A_{\sigma(j)}) - \mu(A_{\sigma(j-1)})$, $j = 1, 2, \ldots, n$. $\sum_{j=1}^{n} \omega_j = 1$ and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$; if $\mu(A) = \sum_{j=1}^{|A|} \omega_j$, where $|A|$ is the number of the elements in $A$, then the H2TLCA aggregation operator is reduced to the hesitant 2-tuple linguistic ordered weighted averaging (H2TLOWA) aggregation operator:

$$H2TLOWA_\mu(H_1, H_2, \ldots, H_n) = \bigcup_{x_{\sigma(j)} \in H_{\sigma(j)}, (j=1,2,\ldots,n)} \triangle \left( \sum_{j=1}^{n} \omega_j * \triangle^{-1}(x_{\sigma(j)}) \right)$$

**Theorem 2.** *Let $H = \{H_1, H_2, \ldots, H_n\}$ be n H2TLSs; then the properties of the H2TLCA operator are explored as shown below:*

(1) *(Idempotency): Let $H = \{H_1, H_2, \ldots, H_n\}$ be n H2TLSs; if all $H_j$ $(j = 1, 2, ..., n)$ are equal with $H_j = (x_1, x_2, \ldots, x_n)$, then*

$$H2TLCA(H_1, H_2, \ldots, H_n) = (x_1, x_2, \ldots, x_n)$$

(2) *(Boundedness): Let* $H = \{H_1, H_2, \ldots, H_n\}$ *be n H2TLSs; then*

$$\min_{1 \le j \le n} \{H_j\} \le H2TLCA(H_1, H_2, \ldots, H_n) \le \max_{1 \le j \le n} \{H_j\}$$

(3) *(Monotonicity): Let* $H = \{H_1, H_2, \ldots, H_n\}$ *and* $H' = \{H_{1'}, H_{2'}, \ldots, H_{n'}\}$ *be two collections of H2TLSs; if* $H_j \le H_{j'}$ $(j = 1, 2, \ldots, n)$ *for all j, then*

$$H2TLCA(H_1, H_2, \ldots, H_n) \le H2TLCA(H_{1'}, H_{2'}, \ldots, H_{n'})$$

(4) *(Commutativity): If* $\{H_{1'}, H_{2'}, \ldots, H_{n'}\}$ *is a permutation of* $\{H_1, H_2, \ldots, H_n\}$*, then*

$$H2TLCA(H_1, H_2, \ldots, H_n) = H2TLCA(H_{1'}, H_{2'}, \ldots, H_{n'})$$

In fact, the H2TLCA aggregation operator and H2TLCG aggregation operator are symmetric functions, which leave aggregation values unchanged when the locations of entered variables are changed. In comparison, the H2TLPWG aggregation operator and H2TLPWA aggregation operator are not symmetric functions.

**Definition 14.** *Let* $H = \{H_1, H_2, \ldots, H_n\}$ *be n H2TLSs, C be the sets of criteria and* $\mu$ *be a fuzzy measure on C. Then we propose the H2TLCG operator, which is defined below:*

$$\begin{aligned} & H2TLCG(H_1, H_2, \ldots, H_n) \\ & = \bigcup_{x_{\sigma(j)} \in H_{\sigma(j)}, (j=1,2,\ldots,n)} \triangle \left( \prod_{j=1}^{n} (\triangle^{-1}(x_{\sigma(j)}))^{(\mu(A_{\sigma(j)}) - \mu(A_{\sigma(j-1)}))} \right) \end{aligned}$$

*where* $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ *is a permutation of* $(1, 2, \ldots, n)$ *such that* $H_{\sigma(1)} \ge H_{\sigma(2)} \ge \ldots \ge H_{\sigma(n)}$*,* $c_{\sigma(j)}$ *is the criterion corresponding to* $H_{\sigma(j)}$*.* $A_{\sigma(j)} = \{c_{\sigma(k)} | k \le j\}$ *for* $j \ge 1$*,* $A_{\sigma(0)} = \emptyset$*.*

The H2TLCG aggregation operator, as for the H2TLCA aggregation operator, has idempotency, boundedness, monotonicity and commutativity. The special examples of the H2TLCG aggregation operator are similar to those of the H2TLCA aggregation operator.

## 4. An Approach to Multi-Criteria Decision-Making with Hesitant 2-Tuple Linguistic Information

In this part, we give the detailed steps of solving MCDM problems with hesitant 2-tuple linguistic information. Let $D = \{D_1, D_2, \ldots, D_m\}$ be a set of $m$ alternatives, $C = \{c_1, c_2, \ldots, c_n\}$ be a set of $n$ criteria and $e = \{e_1, e_2, ..., e_K\}$ be a set of $K$ experts. We suppose that $R^{(k)} = (r_{ij}^{(k)})_{m \times n}$ is the $k$th decision matrix, where $r_{ij}^{(k)} = \langle H_{ij}^{(k)} \rangle$ is the value given by $e_k$ for alternative $D_i$ with respect to the criterion $c_j$.

By virtue of the H2TLPWA aggregation operator and H2TLCA aggregation operator, an approach of settling MCDM problems is developed as follows.

**Step 1**: Obtain the decision information matrices and transform the linguistic expression into H2TLSs:

$$\mathbf{R}^{(k)} = (r_{ij}^{(k)})_{m \times n} \ \ (k = 1, 2, \ldots, K)$$

**Step 2**: Apply Equation (7) to calculate the value of $T_{ij}^{(k)}$:

$$\begin{aligned} T_{ij}^{(1)} &= 1 \\ T_{ij}^{(k)} &= \textstyle\prod_{t=1}^{k-1} S(H_{ij}^{(t)}) \ (k = 2, 3, \ldots, K) \end{aligned} \tag{7}$$

**Step 3**: Utilize the H2TLPWA operator to aggregate individual values to obtain H2TLSs, which are given below:

$$
\begin{aligned}
& H2TLPWA(H_{ij}^{(1)}, H_{ij}^{(2)}, \ldots, H_{ij}^{(K)}) \\
& = \bigcup_{x_{ij}^{(k)} \in H_{ij}^{(k)}, (k=1,2,\ldots,K)} \triangle \left( \sum_{k=1}^{K} w_{ij}^{(k)} * \triangle^{-1}(x_{ij}^{(k)}) \right)
\end{aligned} \tag{8}
$$

where $w_{ij}^{(k)} = T_{ij}^{(k)} / \sum_{k=1}^{K} T_{ij}^{(k)}$.

The collective decision matrix $\mathbf{R}$ is defined as $\mathbf{R} = (r_{ij})_{m \times n}$.

**Step 4**: Utilize the H2TLCA aggregation operator to aggregate all the criteria of the alternative and obtain the overall values of alternatives:

$$
\begin{aligned}
& H2TLCA(H_{i1}, H_{i2}, \ldots, H_{in}) \\
& = \bigcup_{x_{i\sigma(j)} \in H_{i\sigma(j)}, (j=1,2,\ldots,n)} \triangle \left( \sum_{j=1}^{n} (\mu(A_{i\sigma(j)}) - \mu(A_{i\sigma(j-1)})) * \triangle^{-1}(x_{i\sigma(j)}) \right)
\end{aligned}
$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$ such that $H_{i\sigma(1)} \geq H_{i\sigma(2)} \geq \ldots \geq H_{i\sigma(n)}$, and $c_{i\sigma(j)}$ is the set of the $j$th criterion corresponding to $H_{i\sigma(j)}$. $A_{i\sigma(j)} = \{c_{i\sigma(k)} | k \leq j\}$ for $j \geq 1$, $A_{i\sigma(0)} = \varnothing$.

**Step 5**: Sort the overall preference values in descending order by using Equations (5)–(7), and the best value can be identified.

## 5. An Illustrative Example

In this part, we employ an example [43] to illustrate the validity of the presented method. Good suppliers can reduce the supply chain uncertainty and risk and improve service levels, inventory levels and cycle times. In order to enhance the comprehensive competitiveness of a company and improve the earnings of that company, the company wishes to select the most powerful supplier from four candidates suppliers. We suppose that four suppliers are expressed by $D = \{D_1, D_2, D_3, D_4\}$, the experts are expressed by $e = \{e_1, e_2, e_3\}$, and that there is a prioritization relationship for experts, $e_1 > e_2 > e_3$. Three evaluation criteria are employed to evaluate four alternative suppliers, that is, $c_1$ quality, $c_2$ supply capacity and flexibility and $c_3$ price.

Usually, the linguistic label set should be predefined to serve as a reference scale in evaluating all the alternatives, which includes symmetrical distribution linguistic terms and unbalanced linguistic terms [57,58]. In this part, we take seven linguistic labels, for which the label $s_3$ denotes "medium" in semantic explanation, and other linguistic terms with an opposite semantics explanation are symmetrically located around $s_3$: $S = \{s_0 = \textit{extremely bad}, s_1 = \textit{very bad}, s_2 = \textit{bad}, s_3 = \textit{medium}, s_4 = \textit{good}, s_5 = \textit{very good}, s_6 = \textit{extremely good}\}$.

Decision matrices $\mathbf{R}^{(k)} = (r_{ij}^{(k)})_{(4 \times 3)}$ $(k = 1, 2, 3)$ are structured and are shown in Tables 1, 2 and 3, respectively.

**Table 1.** Hesitant 2-tuple linguistic decision matrix $R^1$ [43].

| | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|
| $D_1$ | Between good and very good | Medium | Good |
| $D_2$ | bad | Between very good and extremely good | Medium |
| $D_3$ | Very bad | Very good | Between medium and good |
| $D_4$ | Between very good and extremely good | Good | Medium |

**Table 2.** Hesitant 2-tuple linguistic decision matrix $R^2$ [43].

| | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|
| $D_1$ | Very good | Bad | Between medium and good |
| $D_2$ | Between medium and good | Between very good and very good | Bad |
| $D_3$ | Bad | Between medium and good | Between medium and good |
| $D_4$ | Very good | Between very good and extremely good | Bad |

**Table 3.** Hesitant 2-tuple linguistic decision matrix $R^3$ [43].

| | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|
| $D_1$ | Extremely good | Medium | Between medium and good |
| $D_2$ | Medium | Extremely good | Medium |
| $D_3$ | Medium | Very good | Between medium and good |
| $D_4$ | Between very good and extremely good | Very good | Medium |

**Step 1**: Establish the decision matrices $\mathbf{R}^{(k)} = (r_{ij}^{(k)})_{(4\times 3)}$ $(k = 1, 2, 3)$ and transform the linguistic expression into H2TLSs, which are shown in Tables 4, 5 and 6, respectively.

**Step2**: Apply Equation (7) to calculate the values of $T_{ij}^{(k)}$ $(k = 1, 2, 3)$, which are given below:

$$T_{ij}^{(1)} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad T_{ij}^{(2)} = \begin{pmatrix} 4.5 & 3 & 4 \\ 2 & 5.5 & 3 \\ 1 & 5 & 3.5 \\ 5.5 & 4 & 3 \end{pmatrix}, \quad T_{ij}^{(3)} = \begin{pmatrix} 22.5 & 6 & 14 \\ 7 & 24.75 & 6 \\ 2 & 17.5 & 12.25 \\ 27.5 & 22 & 6 \end{pmatrix}$$

**Step 3**: Utilize the H2TLPWA aggregation operator to aggregate individual values, which are given in Table 7.

**Table 4.** Hesitant 2-tuple linguistic decision matrix $R^1$.

| | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|
| $D_1$ | $\{(s_4, 0), (s_5, 0)\}$ | $\{(s_3, 0)\}$ | $\{(s_4, 0)\}$ |
| $D_2$ | $\{(s_2, 0)\}$ | $\{(s_5, 0), (s_6, 0)\}$ | $\{(s_3, 0)\}$ |
| $D_3$ | $\{(s_1, 0)\}$ | $\{(s_5, 0)\}$ | $\{(s_3, 0), (s_4, 0)\}$ |
| $D_4$ | $\{(s_5, 0), (s_6, 0)\}$ | $\{(s_4, 0)\}$ | $\{(s_3, 0)\}$ |

**Table 5.** Hesitant 2-tuple linguistic decision matrix $R^2$.

| | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|
| $D_1$ | $\{(s_5, 0)\}$ | $\{(s_2, 0)\}$ | $\{(s_3, 0), (s_4, 0)\}$ |
| $D_2$ | $\{(s_3, 0), (s_4, 0)\}$ | $\{(s_4, 0), (s_5, 0)\}$ | $\{(s_2, 0)\}$ |
| $D_3$ | $\{(s_2, 0)\}$ | $\{(s_3, 0), (s_4, 0)\}$ | $\{(s_3, 0), (s_4, 0)\}$ |
| $D_4$ | $\{(s_5, 0)\}$ | $\{(s_5, 0), (s_6, 0)\}$ | $\{(s_2, 0)\}$ |

**Table 6.** Hesitant 2-tuple linguistic decision matrix $R^3$.

| | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|
| $D_1$ | $\{(s_6, 0)\}$ | $\{(s_3, 0)\}$ | $\{(s_3, 0), (s_4, 0)\}$ |
| $D_2$ | $\{(s_3, 0)\}$ | $\{(s_6, 0)\}$ | $\{(s_3, 0)\}$ |
| $D_3$ | $\{(s_3, 0)\}$ | $\{(s_5, 0)\}$ | $\{(s_3, 0), (s_4, 0)\}$ |
| $D_4$ | $\{(s_5, 0), (s_6, 0)\}$ | $\{(s_5, 0)\}$ | $\{(s_3, 0)\}$ |

**Table 7.** The collective opinion for all experts.

| | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|
| $D_1$ | $\{(s_6,-0.2321),(s_6,-0.1964)\}$ | $\{(s_3,-0.3)\}$ | $\{(s_3,0.0523),(s_3,0.2628),(s_4,-0.2109),(s_4,0)\}$ |
| $D_2$ | $\{(s_3,-0.1),(s_3,0.1)\}$ | $\{(s_6,-0.384),(s_6,-0.352),(s_6,-0.208),(s_6,-0.176)\}$ | $\{(s_3,-0.3)\}$ |
| $D_3$ | $\{(s_2,0.25)\}$ | $\{(s_5,-0.4251),(s_5,-0.2123)\}$ | $\{(s_3,0),(s_3,0.0597),(s_3,0.209),(s_3,0.2687),(s_3,0.403),(s_4,-0.2687),(s_4,-0.209),(s_4,0)\}$ |
| $D_4$ | $\{(s_5,0),(s_5,0.0294),(s_6,-0.1912),(s_6,-0.1618)\}$ | $\{(s_5,-0.0375),(s_5,0.1106)\}$ | $\{(s_3,-0.3)\}$ |

**Step 4**: Calculate fuzzy measures of criteria $c_1$, $c_2$ and $c_3$ and their $\alpha$ parameter. We suppose that $\mu(c_1)=0.3$, $\mu(c_2)=0.25$ and $\mu(c_3)=0.37$. Then $\alpha=0.2795$ is determined by using Equation (5). According to the Equation (4) fuzzy measures of criteria sets of $C=\{c_1,c_2,c_3\}$, we can obtain $\mu(c_1,c_2)=0.5710$, $\mu(c_1,c_3)=0.7010$, $\mu(c_2,c_3)=0.6459$, and $\mu(c_1,c_2,c_3)=1$.

Then we apply the H2TLCA aggregation operator to aggregate $H_{ij}$ $(j=1,2,3)$.

**a.** According to the score function, we can obtain the following:

$$\begin{aligned}
H_{1\sigma(1)} &= \{(s_6,-0.2321),(s_6,-0.1964)\}\\
H_{1\sigma(2)} &= \{(s_3,0.0523),(s_4,-0.2109),(s_3,0.2628),(s_4,0)\}\\
H_{1\sigma(3)} &= \{(s_3,-0.3)\}
\end{aligned}$$

By using $A_{1\sigma(1)}=\{c_1\}, A_{1\sigma(2)}=\{c_1,c_3\}$, and $A_{1\sigma(3)}=\{c_1,c_2,c_3\}$, we can first obtain aggregation-associated weights $\omega_{11}=0.3, \omega_{12}=0.401$, and $\omega_{13}=0.299$. Then, by using the H2TLCA aggregation operator, the following results are obtained:

$$\begin{aligned}
r_1 \;=\; &\{(s_4,-0.2384),(s_4,-0.2276),(s_4,-0.1539),(s_4,-0.1432),(s_4,0.0571),(s_4,0.0678),\\
&(s_4,0.1417),(s_4,0.1524)\}
\end{aligned}$$

The rest of the overall preference values of each alternative can be obtained in the same way.

**b.** For the alternative $D_2$, the following information can be obtained:

$$\begin{aligned}
H_{2\sigma(1)} &= \{(s_6,-0.384),(s_6,-0.208),(s_6,-0.352),(s_6,-0.176)\}\\
H_{2\sigma(2)} &= \{(s_3,-0.1),(s_3,0.1)\}\\
H_{2\sigma(3)} &= \{(s_3,-0.3)\}
\end{aligned}$$

and aggregation-associated weights are determined as follows: $\omega_{21}=0.25, \omega_{22}=0.321$, and $\omega_{23}=0.429$. By using the H2TLCA operator, we can obtain

$$\begin{aligned}
r_2 \;=\; &\{(s_3,0.4932),(s_4,-0.4988),(s_4,-0.4628),(s_4,-0.4548),(s_4,-0.4426),(s_4,-0.4346),\\
&(s_4,-0.3986),(s_4,-0.3906)\}
\end{aligned}$$

**c.** For the alternative $D_3$, the following results are obtained:

$$\begin{aligned}
H_{3\sigma(1)} &= \{(s_5,-0.4251),(s_5,-0.2123)\}\\
H_{3\sigma(2)} &= \{(s_3,0),(s_4,-0.2687),(s_3,0.209),(s_3,0.403),\\
&\quad (s_3,0.0597),(s_4,-0.209),(s_3,0.2687),(s_4,0)\}\\
H_{3\sigma(3)} &= \{(s_2,0.25)\}
\end{aligned}$$

and aggregation-associated weights are determined as follows: $\omega_{31} = 0.25, \omega_{32} = 0.3959$, and $\omega_{33} = 0.3541$. By using the H2TLCA operator, we can obtain

$$\begin{aligned} r_3 \ = \ & \{(s_3, -0.4801), (s_3, 0.0633), (s_3, 0.1165), (s_3, 0.1530), (s_3, 0.1786), (s_3, 0.2062), (s_3, 0.2318), (s_3, 0.2426), \\ & (s_3, 0.2682), (s_3, 0.2958), (s_3, 0.3214), (s_3, 0.3259), (s_3, 0.3791), (s_3, 0.4667), (s_4, -0.3648), (s_4, -0.4180)\} \end{aligned}$$

**d.** For the alternative $D_4$, the following results are obtained:

$$\begin{aligned} H_{4\sigma(1)} &= \{(s_5, 0), (s_6, -0.1912), (s_5, 0.0294), (s_6, -0.1618)\} \\ H_{4\sigma(2)} &= \{(s_5, -0.0375), (s_5, 0.1106)\} \\ H_{4\sigma(3)} &= \{(s_3, -0.3)\} \end{aligned}$$

and aggregation-associated weights are determined as follows: $\omega_{41} = 0.3, \omega_{42} = 0.271$, and $\omega_{43} = 0.429$. By using the H2TLCA aggregation operator, we can obtain

$$r_4 \ = \ \{(s_4, 0.0031), (s_4, 0.0120), (s_4, 0.0433), (s_4, 0.0521), (s_4, 0.2458), (s_4, 0.2546), (s_4, 0.2859), (s_4, 0.2947)\}$$

**Step 5**: According to Definition 5, we can obtain the mean value of each alternative:

$$S(r_1) = 3.957, \ S(r_2) = 3.5513$$
$$S(r_3) = 3.249, \ S(r_4) = 4.1489$$

The order of the overall preference value of $H_i$ $(i = 1, 2, 3, 4)$ can be determined as follows:

$$r_4 > r_1 > r_2 > r_3$$

Hence the best alternative is $D_4$. Moreover, Ge [43] employed the hesitant 2-tuple-weighted averaging (H2TWA) aggregation operator and the hesitant 2-tuple-weighted ordered weighted averaging (H2TWOWA) aggregation operator to settle this problem, and the ranking result was also $r_4 > r_1 > r_2 > r_3$. This demonstrates that the presented method is effective and feasible.

## 6. Conclusions

On the basis of the PA operator, we present the H2TLPWA aggregation operator and H2TLPWG aggregation operator, which can consider the prioritization relationship of the experts and obtain decision results closer to the true results. On the basis of the dependencies between criteria in the decision-making process, we propose the H2TLCA aggregation operator and H2TLCG aggregation operator. The properties of four new operators are discussed in detail, and some special cases of the H2TLCA aggregation operator can be obtained by taking some special values of $\mu(A)$. Comparing the results acquired by the existing approach and proposed approach, we further demonstrate the validity and feasibility of our method.

In the future, the H2TLPWA aggregation operator, H2TLPWG aggregation operator, H2TLCA aggregation operator and H2TLCG aggregation operator can be used to solve some real problems because they are closer to the true decision-making process of a human being.

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