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Acceptance Sampling Plans for Finite and Infinite Lot Size under Power Lindley Distribution

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Abstract: In this paper, we have developed single and double acceptance sampling plans when the product life length follows the power Lindley distribution. The sampling plans have been developed by assuming infinite and finite lot sizes. We have obtained the operating characteristic curves for the resultant sampling plans. The sampling plans have been obtained for various values of the parameters. It has been found that for a finite lot size, the sampling plans provide smaller values of the parameters to achieve the specified acceptance probabilities.

Keywords: sampling plans; power Lindley distribution; lot acceptance probability

1. Introduction

Product quality is a key ingredient for its acceptability. Products with high quality have higher acceptability compared with products with low quality. Quality control engineers make sure that the products supplied to the customers are of high quality and ensure that the market receives shipments that do not contain defective products. The quality supervisors are of the view that the manufactured products meet specific requirements during the manufacturing process. To meet the requirements, quality control engineers continuously monitor the production process and ultimately the product quality. To streamline the process, quality control engineers set some rules for acceptance or rejection of the lot containing the products. In doing so, these engineers observe a specific number of items and fix a specific number of defective items for rejection of the lot; that is, if the number of defectives exceeds that fixed value, the lot is rejected. This is precisely the use of acceptance sampling in quality control. Acceptance sampling plans have a long history. The basis for the acceptance sampling plans has been provided by [1]. Acceptance sampling plans have been discussed by several authors, such as [2]. The use of sampling plans in quality control has been discussed in detail by [3].

Notably, products are classified on the basis of their life length, which is a random phenomenon and follows some probability model. Acceptance sampling plans have been studied by several authors, assuming that the life length of a product follows a specific probability distribution. Reference [4] provided sampling plans that followed the assumption that the life length of a product follows the Weibull distribution. Acceptance sampling plans for the Gamma distribution has been constructed by [5]. Work on the construction of an acceptance sampling plan continued with the development of new probability models. In [6] sampling plans for the log-logistic distribution has been discussed. Reference [7] provided sampling plans that followed the assumption that the life length follows a generalized Rayleigh distribution. Reference [8] have constructed acceptance sampling plans for when the life length of the product follows the Frechet distribution. The usefulness of sampling plans under various probability distributions depends upon the life length behavior of a product. The sampling plans discussed in this paper are useful when the life length of product follows the power Lindley distribution.

The double acceptance sampling plans are extensions of the single acceptance sampling plans; in these sampling plans, two samples are drawn for making decisions about acceptance or rejection of the lot. The double acceptance sampling plans have been discussed by [9].

This paper discusses single and double acceptance sampling plans when the life length of the component follows the power Lindley distribution. The organization of the paper is as follows. In Section 2, a brief description of the power Lindley distribution is given. Section 3 comprises a brief description of acceptance sampling plans. In Section 4, the single acceptance sampling plans using the power Lindley distribution are discussed, and Section 5 contains double-acceptance sampling plans for power Lindley distribution. Conclusions and recommendations are given in Section 6.

2. The Power Lindley Distribution

The power Lindley distribution is a useful distribution to model lifetime data. The distribution was proposed and extensively studied by [10]. The density and cumulative distribution function (CDF) of a random variable following a power Lindley distribution are

$$f(x;\theta,\lambda) = \frac{\lambda\theta^2}{\theta+1}\left(1+x^\lambda\right)x^{\lambda-1}\exp\left(-\theta x^\lambda\right), \tag{1}$$

and

$$F(x;\theta,\lambda) = 1-\left(1+\frac{\theta}{\theta+1}x^\lambda\right)\exp\left(-\theta x^\lambda\right) \tag{2}$$

such that; $x > 0, \theta > 0, \lambda > 0$. Additionally, θ is scale parameter and λ is shape parameter. The expression for rth moment and quantile function for the distribution, as given by [10], are

$$\mu_r^{/} = E(X^r) = \frac{r\,\Gamma(r/\lambda)[\lambda(\theta+1)+r]}{\lambda^2\theta^{r/\lambda}(\theta+1)} \tag{3}$$

and

$$Q(u) = -1-\frac{1}{\theta}-\frac{1}{\theta}W_{-1}\left\{-\frac{\theta+1}{e^{\theta+1}}(1-u)\right\} \tag{4}$$

where $W_{-1}(.)$ is a negative branch of the Lambert function, as given in [11]. The mean of distribution is immediately written from Equation (3) and is given as

$$\mu = E(X) = \frac{\Gamma(1/\lambda)[\lambda(\theta+1)+1]}{\lambda^2\theta^{1/\lambda}(\theta+1)}$$

The mean and quantile function are useful for constructing the acceptance sampling plans. In this paper, we have constructed the acceptance sampling plans when the life of a component follows the power Lindley distribution.

In the following section, a brief about single and double acceptance sampling plans is given.

3. Acceptance Sampling Plans

Acceptance sampling plans are useful in quality control, see, for example [3]. Acceptance sampling plans provide a basis for deciding about acceptance or rejection of lots in manufacturing products. Various methods to construct the sampling plans are available. The popular methods are single and double acceptance sampling plans. A single sampling plan involves determining the number of items to be inspected (n) and the maximum number of defective items among the inspected items (c) for acceptance of the lot. The single acceptance sampling plan is discussed by [2] and the acceptance probability of the lot is given as

$$L(p) = \sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-i} \tag{5}$$

where x is number of defectives in the lot and p is some pre-assigned probability. Single acceptance sampling plans for infinite lot size use Binomial distribution as the items can always be classified as good and defective.

Acceptance sampling plans are also characterized by the life length of components being tested. Items are inspected over a specific time, and the experiment is terminated at a pre-assigned time point t_0. The lot is accepted if fewer than c defective items appear in the time interval $[0, t_0]$. The acceptance or rejection of the lot is equivalent to testing of the hypothesis $H_0 : \mu > \mu_0$, where μ is life of the component and μ_0 is a pre-specified test value. During the construction of acceptance sampling plans, we do consider two important probabilities, namely, consumer risk (β) and producer risk (α). During the construction of single acceptance sampling plans, the values of n and c are obtained by solving the following two equations simultaneously for n and c

$$\sum_{i=0}^{c} \binom{n}{i} (AQL)^i (1 - AQL)^{n-i} \geq 1 - \alpha \tag{6}$$

and

$$\sum_{i=0}^{c} \binom{n}{i} (LTPD)^i (1 - LTPD)^{n-i} \leq \beta \tag{7}$$

where AQL is acceptable quality level, and $LTPD$ is lot tolerance percent defective. Equations (6) and (7) use binomial distribution as it is assumed that the lot size is infinite or when $N >> c * n$, where N is lot size and c is a sufficiently large number, say 500 or more. When the lot size is finite, then the binomial distribution is replaced with the hypergeometric distribution, as discussed in Section 4.2.

The double acceptance sampling plan is an extension of the single sampling plan and entails drawing two samples. The double acceptance sampling plan is described by [9] as below:

(1) Step 1. Draw the first sample of size n_1 from a lot and put them on test until time t_0.
(2) Step 2. Accept the lot if there are c_1 or smaller number of failures. Reject the lot and terminate the test as soon as more than c_2 failures are observed. If the number of failures is between c_1 and c_2, then draw the second sample of size n_2 from the lot and put them on test until time t_0.
(3) Step 3. Accept the lot if the total number of failures from the first and second samples is not greater than c_2. Otherwise, terminate the test and reject the lot.

The acceptance probability for a double sampling plan is given as

$$L(p) = \sum_{i=0}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i} + \sum_{j=c_1+1}^{c_2} \binom{n_1}{j} p^j (1-p)^{n_1-j} \sum_{i=0}^{c_2-j} \binom{n_2}{i} p^i (1-p)^{n_2-i} \tag{8}$$

The parameters of the double acceptance sampling plan are determined by solving the following linear programing problem

$$\begin{array}{lll} minimize & ASN(LTPD) & (8a) \\ subject\ to & L(AQL) \geq 1-\alpha & (8b) \\ & L(LTPD) \leq \beta & (8c) \\ & 1 \leq n_2 \leq n_1 & (8d) \\ & n_1, n_2 : \ Integers & (8e) \end{array}$$

where

$$ASN(p) = n_1 P_1 + (n_1 + n_2)(1 - P_1) \tag{9}$$

and P_1 is the probability that the lot is accepted on the basis of first sample, and is given as

$$P_1 = 1 - \sum_{i=c_1+1}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1 - 1} \tag{10}$$

We now give single and double sampling plans on the basis of the power Lindley distribution.

4. Single Acceptance Sampling Plans

In this section, we have constructed single sampling plans for life length of the components follows the power Lindley distribution given in (1). The sampling plans have been obtained by considering two situations, namely, finite lot size and infinite lot size. The sampling plans are given in the following sub-sections.

4.1. Acceptance Sampling Plans for Infinite Lot Size

The single acceptance sampling plan, in the case of an infinite lot size, is based upon obtaining the values of n and c, which satisfy Equations (6) and (7), where *AQL* and *LTPD* are probabilities obtained from the *CDF* of a power Lindley distribution given in Equation (2). To obtain the sampling plans, first consider the *CDF* and quantile function of power Lindley distribution as

$$p = F(x;\theta,\lambda) = 1 - \left(1 + \frac{\theta}{\theta+1} x^{\lambda}\right) \exp\left(-\theta x^{\lambda}\right) \tag{11}$$

and

$$p = 1 - \left[1 + \frac{\theta}{\theta+1} a_0^{\lambda} Q^{\lambda}(u) \left(\frac{\mu}{\mu_0}\right)^{-\lambda}\right] \exp\left[-\theta\, a_0^{\lambda} Q^{\lambda}(u) \left(\frac{\mu}{\mu_0}\right)^{-\lambda}\right]. \tag{12}$$

Assuming the test life by $t_0 = a_0\mu_0$, where μ_0 is acceptable average life. Then, using $x = a_0\mu = a_0\mu(\mu/\mu_0)^{-1}$ and $\mu = Q(u)$, we can write (12) as

$$p = 1 - \left[1 + \frac{\theta}{\theta+1} a_0^{\lambda} Q^{\lambda}(u) \left(\frac{\mu}{\mu_0}\right)^{-\lambda}\right] \exp\left[-\theta\, a_0^{\lambda} Q^{\lambda}(u) \left(\frac{\mu}{\mu_0}\right)^{-\lambda}\right]. \tag{13}$$

The acceptance sampling plans are constructed for various ratios (μ/μ_0), u, θ, and λ. The values of n and c that satisfy Equations (6) and (7), for different values of α, are given in Tables A1 and A2, respectively, in Appendix A. To obtain the values, we have assumed for *LTPD* that $(\mu/\mu_0) = 1$. The values of n and c in these tables provide information about number of items to be put on test and number of defective items observed for rejection of the lot. For example, in Table A1, the values of n and c for $\theta = 2.5$, $\lambda = 2.0$, p = 0.95, $\beta = 0.05$, $\alpha = 0.01$, a_0 = 0.5, and $(\mu/\mu_0) = 3$ are 11 and 2, respectively. These values indicate that if the quality control engineer is interested in testing the hypothesis that the life length of a component is 1000 h and true average life is thrice this value, then the engineer can test 11 items; if fewer than 2 items fail in 500 h; as a_0 = 0.5 and life length is in thousands of hours, then the engineer can conclude with 95% confidence that the life is more than 3000 h.

4.2. Acceptance Sampling Plans for Finite Lot Size

Often, it happens that the lot size from where the inspection is made is of finite size, say N; in this case, the acceptance plans given in the previous section do not work. In fact, Equations (6) and (7) need suitable modification in this regard. It is a well-known fact that for a finite population size and under sampling without replacement, the hypergeometric distribution is an appropriate model to compute probabilities for specific characteristics of interest. Thus, we have to use the hypergeometric probabilities instead of binomial probabilities in Equations (6) and (7). Equations (6) and (7) for a finite lot size becomes

$$\sum_{i=0}^{c} \binom{N \times AQL}{i} \binom{N - N \times AQL}{n-i} \Big/ \binom{N}{n} \geq 1 - \alpha \tag{14}$$

and

$$\sum_{i=0}^{c} \binom{N \times LTPD}{i} \binom{N - N \times LTPD}{n-i} \Big/ \binom{N}{n} \leq \beta \tag{15}$$

where N is lot size. Then, using (14), the values of AQL and $LTPD$ can be obtained for various choices of (μ/μ_0) and various choices of parameters θ and λ. The values of n and c that satisfy (15) and (16) are given in Tables A3–A5. The values of n and c in these tables are the number of items to be put on test and the number of defective items observed for rejection of the lot, respectively. For example, in Table A3, the values of n and c for N = 100, $\theta = 2.5$, $\lambda = 2.0$, p = 0.95, $\beta = 0.05$, $\alpha = 0.01$, a_0 = 0.5, and $(\mu/\mu_0) = 3$ are 10 and 2, respectively. These values indicate that if the quality control engineer is interested in testing the hypothesis that the life length of a component is 1000 h and true average life is thrice this value, then the engineer can test 10 out of 100 items; if fewer than 2 items fail in 500 h; as a_0 = 0.5 and life length is in thousands of hours, then the engineer can conclude with 95% confidence that the life is more than 3000 h.

4.3. Operating Characteristic Curves

The operating characteristic curve is a useful way to judge the performance of an acceptance sampling plan. The operating characteristic values for a sampling plan provide the probability of acceptance of the lot under a given sampling plan when actual lot contains a specified percentage of defective items and is given in Equation (5). We have computed the operating characteristic values for the given sampling plan under the power Lindley distribution with specific values of the parameters given in Table A6 in Appendix A. We see that the probability of acceptance decreases as the value of "a_0" increases for fixed ratio (μ/μ_0). Additionally, we can see that for fixed value of "a_0", the acceptance probability increases as the ratio (μ/μ_0) increases.

5. Double Acceptance Sampling Plans

The double-acceptance sampling plan is an extension of the single-acceptance sampling plan and provides a way for acceptance or rejection of the lot by selecting two samples. The double-acceptance sampling plan is discussed in Section 3. The plan is based upon identifying values n_1, n_2, c_1, and c_2, which satisfy (3.4–3.7). In this section, we present a double-acceptance sampling plan when life length of the component follows the power Lindley distribution. The plans have been constructed for various choices of parameters and for various ratios (μ/μ_0). The values of n_1, n_2, c_1, and c_2 for various values of parameters θ and λ of the power Lindley distribution are given in Table A7 in Appendix A.

6. Conclusions and Recommendations

In this paper, we have discussed the acceptance sampling plans for when the life length of the component follows the power Lindley distribution. The sampling plans have been constructed for various choices of the distribution parameters. We have constructed single- and double-acceptance sampling plans. The single-acceptance sampling plans have been constructed for finite and infinite lot sizes. We have seen that the acceptance number decreases with an increase in the design parameters. We have also observed that with an increase in the ratio (μ/μ_0), the number of defective items required for acceptance of the lot decreases. We have also observed that the total number of items to be inspected is smaller for a finite lot size compared with the infinite lot size, and this difference decreases with an increase in the lot size. The sampling plans discussed in this paper are useful when the life length of components follow the power Lindley distribution and the quality control engineer wants to decide about the acceptable life of the components. In such cases, the quality control engineer can use the plan parameters obtained in this paper for efficient decision making. The same phenomenon has been observed in the double-acceptance sampling plans.

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Appendix A

Table A1. Single sampling plan values of (n,c) for infinite lot size and for various choices of parameters at $\alpha = 0.05$.

θ	λ	a_0	p	β	μ/μ_0			
					2	**3**	**4**	**5**
1.5	1.5	0.5	0.75	0.01	(61,13)	(32,5)	(24,3)	(20,2)
				0.05	(40,9)	(23,4)	(18,2)	(16,2)
				0.10	(34,8)	(17,3)	(13,2)	(13,2)
				0.25	(23,6)	(10,2)	(7,1)	(7,1)
			0.95	0.01	(31,12)	(17,5)	(13,3)	(10,2)
				0.05	(22,9)	(12,4)	(8,2)	(8,2)
				0.10	(19,8)	(9,3)	(7,2)	(5,1)
				0.25	(11,5)	(8,3)	(6,1)	(4,1)
		1.0	0.75	0.01	(27,14)	(13,5)	(9,3)	(8,2)
				0.05	(17,9)	(9,4)	(6,3)	(6,2)
				0.10	(16,9)	(7,3)	(6,2)	(6,2)
				0.25	(10,6)	(6,3)	(5,2)	(3,1)
			0.95	0.01	(21,16)	(8,5)	(6,3)	(6,3)
				0.05	(14,11)	(6,4)	(5,3)	(4,2)
				0.10	(14,11)	(6,3)	(5,2)	(2,1)
				0.25	(11,9)	(4,3)	(3,1)	(2,1)
2.5	2.0	0.5	0.75	0.01	(58,8)	(33,3)	(27,2)	(21,1)
				0.05	(40,6)	(21,2)	(15,1)	(15,1)
				0.10	(31,5)	(18,2)	(13,1)	(13,1)
				0.25	(18,3)	(9,1)	(9,1)	(9,1)
			0.95	0.01	(27,7)	(17,3)	(14,2)	(11,1)
				0.05	(18,5)	(11,2)	(8,1)	(8,1)
				0.10	(17,5)	(9,2)	(7,1)	(7,1)
				0.25	(12,4)	(5,1)	(5,1)	(5,1)
		1.0	0.75	0.01	(18,8)	(9,3)	(8,2)	(6,1)
				0.05	(13,6)	(6,2)	(6,2)	(5,1)
				0.10	(10,5)	(6,2)	(4,1)	(4,1)
				0.25	(7,4)	(3,1)	(3,1)	(3,1)
			0.95	0.01	(13,9)	(6,3)	(5,2)	(3,1)
				0.05	(11,8)	(5,3)	(4,2)	(3,1)
				0.10	(8,6)	(5,3)	(2,1)	(2,1)
				0.25	(5,4)	(3,2)	(2,1)	(2,1)

Table A1. *Cont.*

θ	λ	a_0	p	β	μ/μ_0 2	3	4	5
1.5	2.5	0.75	0.75	0.01	(25,7)	(15,3)	(13,2)	(10,1)
				0.05	(19,6)	(10,2)	(7,1)	(7,1)
				0.10	(15,5)	(9,2)	(6,1)	(6,1)
				0.25	(9,3)	(7,2)	(5,1)	(5,1)
			0.95	0.01	(16,8)	(8,3)	(7,2)	(5,1)
				0.05	(11,6)	(7,3)	(4,1)	(4,1)
				0.10	(9,5)	(5,2)	(4,1)	(4,1)
				0.25	(7,4)	(4,2)	(3,1)	(3,1)
		1.0	0.75	0.01	(18,8)	(9,3)	(8,2)	(6,1)
				0.05	(13,6)	(6,2)	(6,2)	(5,1)
				0.10	(10,5)	(6,2)	(4,1)	(4,1)
				0.25	(7,1)	(3,1)	(3,1)	(3,1)
			0.95	0.01	(13,9)	(6,3)	(5,2)	(3,1)
				0.05	(11,8)	(5,3)	(4,2)	(3,1)
				0.10	(8,6)	(5,3)	(2,1)	(2,1)
				0.25	(5,4)	(3,2)	(2,1)	(2,1)

Table A2. Single sampling plan values of (*n*,*c*) for infinite lot size and for various choices of parameters at $\alpha = 0.01$.

θ	λ	a_0	p	β	μ/μ_0 2	3	4	5
1.5	1.5	0.5	0.75	0.01	(81,19)	(43,8)	(32,5)	(28,4)
				0.05	(60,15)	(30,6)	(23,4)	(19,3)
				0.10	(46,12)	(24,5)	(17,3)	(17,3)
				0.25	(35,10)	(17,4)	(13,3)	(10,2)
			0.95	0.01	(45,19)	(21,7)	(17,5)	(15,4)
				0.05	(31,14)	(16,6)	(12,4)	(10,3)
				0.10	(26,12)	(13,5)	(9,3)	(9,3)
				0.25	(18,9)	(9,4)	(8,2)	(6,1)
		1.0	0.75	0.01	(35,19)	(16,7)	(13,5)	(9,3)
				0.05	(26,15)	(13,6)	(9,4)	(8,3)
				0.10	(22,13)	(10,5)	(7,3)	(7,3)
				0.25	(16,10)	(7,4)	(6,3)	(5,2)
			0.95	0.01	(26,21)	(13,9)	(8,5)	(7,4)
				0.05	(23,19)	(11,8)	(6,4)	(5,3)
				0.10	(19,16)	(8,6)	(6,4)	(5,3)
				0.25	(15,13)	(5,4)	(4,3)	(3,2)
2.5	2.0	0.5	0.75	0.01	(71,11)	(43,5)	(33,3)	(27,2)
				0.05	(53,9)	(30,4)	(21,2)	(21,2)
				0.10	(44,8)	(22,3)	(18,2)	(18,2)
				0.25	(30,6)	(13,2)	(13,2)	(9,1)
			0.95	0.01	(37,11)	(19,4)	(17,3)	(14,2)
				0.05	(28,9)	(16,4)	(11,2)	(11,2)
				0.10	(24,8)	(12,3)	(9,2)	(9,2)
				0.25	(16,6)	(10,3)	(7,2)	(5,1)
		1.0	0.75	0.01	(22,11)	(13,5)	(9,3)	(8,2)
				0.05	(17,9)	(9,4)	(6,2)	(6,2)
				0.10	(15,8)	(7,3)	(6,2)	(6,2)
				0.25	(10,6)	(6,3)	(5,2)	(3,1)
			0.95	0.01	(17,13)	(8,5)	(6,3)	(5,2)
				0.05	(14,11)	(6,4)	(5,3)	(4,2)
				0.10	(14,11)	(6,4)	(5,3)	(4,2)
				0.25	(11,9)	(4,3)	(3,2)	(3,2)

Table A2. *Cont.*

θ	λ	a_0	p	β	μ/μ_0 2	3	4	5
1.5	2.5	0.75	0.75	0.01	(30,7)	(18,3)	(15,2)	(12,1)
				0.05	(22,6)	(12,2)	(12,2)	(9,1)
				0.10	(18,5)	(10,2)	(7,1)	(7,1)
				0.25	(13,4)	(8,2)	(5,1)	(5,1)
			0.95	0.01	(16,7)	(10,3)	(8,2)	(6,1)
				0.05	(11,5)	(6,2)	(6,2)	(5,1)
				0.10	(10,5)	(6,2)	(4,1)	(4,1)
				0.25	(8,4)	(5,2)	(3,1)	(3,1)
		1.0	0.75	0.01	(16,7)	(9,3)	(8,2)	(6,1)
				0.05	(13,6)	(6,2)	(6,2)	(5,1)
				0.10	(10,5)	(6,2)	(4,1)	(4,1)
				0.25	(7,4)	(5,2)	(3,1)	(3,1)
			0.95	0.01	(12,8)	(6,3)	(5,2)	(3,1)
				0.05	(10,7)	(5,3)	(4,2)	(3,1)
				0.10	(8,6)	(5,3)	(2,1)	(2,1)
				0.25	(5,4)	(3,2)	(2,1)	(2,1)

Table A3. Single sampling plan values of (n,c) for finite lot of size 100 using various choices of parameters and at $\alpha = 0.05$.

θ	λ	a_0	p	β	μ/μ_0 2	3	4	5
1.5	1.5	0.5	0.75	0.01	(39,8)	(26,4)	(19,2)	(19,2)
				0.05	(31,7)	(18,3)	(15,2)	(15,2)
				0.10	(26,6)	(16,3)	(13,2)	(9,1)
				0.25	(16,4)	(10,2)	(7,1)	(7,1)
			0.95	0.01	(26,10)	(14,4)	(12,3)	(10,2)
				0.05	(20,8)	(10,3)	(8,2)	(8,2)
				0.10	(17,7)	(9,3)	(7,2)	(5,1)
				0.25	(11,5)	(6,2)	(4,1)	(4,1)
		1.0	0.75	0.01	(20,10)	(11,4)	(9,3)	(8,2)
				0.05	(15,8)	(9,4)	(6,2)	(4,1)
				0.10	(13,7)	(7,3)	(5,2)	(4,1)
				0.25	(10,6)	(6,3)	(5,2)	(3,1)
			0.95	0.01	(17,13)	(8,5)	(6,3)	(6,3)
				0.05	(14,11)	(6,4)	(5,3)	(4,2)
				0.10	(10,8)	(6,4)	(5,3)	(2,1)
				0.25	(10,8)	(4,3)	(3,2)	(2,1)
2.5	2.0	0.5	0.75	0.01	(38,5)	(24,2)	(19,1)	(19,1)
				0.05	(28,4)	(19,2)	(15,1)	(15,1)
				0.10	(25,4)	(17,2)	(12,1)	(12,1)
				0.25	(17,3)	(9,1)	(9,1)	(5,1)
			0.95	0.01	(23,6)	(13,2)	(10,1)	(10,1)
				0.05	(15,4)	(10,2)	(8,2)	(8,2)
				0.10	(14,4)	(9,2)	(7,2)	(7,2)
				0.25	(9,3)	(5,1)	(5,1)	(5,1)
		1.0	0.75	0.01	(16,7)	(9,3)	(8,2)	(6,1)
				0.05	(11,5)	(6,2)	(4,1)	(4,1)
				0.10	(10,5)	(5,2)	(4,1)	(4,1)
				0.25	(7,4)	(3,1)	(3,1)	(2,1)
			0.95	0.01	(10,7)	(6,3)	(5,2)	(3,1)
				0.05	(8,6)	(5,3)	(4,2)	(3,1)
				0.10	(8,6)	(5,3)	(2,1)	(2,1)
				0.25	(5,4)	(3,2)	(2,1)	(2,1)

Table A3. *Cont.*

θ	λ	a_0	p	β	μ/μ_0 2	3	4	5
1.5	2.5	0.75	0.75	0.01	(20,4)	(14,2)	(11,1)	(11,1)
				0.05	(14,3)	(8,1)	(8,1)	(5,1)
				0.10	(12,3)	(7,1)	(7,1)	(4,1)
				0.25	(8,2)	(5,1)	(5,1)	(3,1)
			0.95	0.01	(11,4)	(8,2)	(6,1)	(6,1)
				0.05	(8,3)	(5,1)	(5,1)	(5,1)
				0.10	(7,3)	(4,1)	(4,1)	(2,1)
				0.25	(5,2)	(3,1)	(3,1)	(2,1)
		1.0	0.75	0.01	(11,4)	(8,2)	(6,1)	(6,1)
				0.05	(8,3)	(4,1)	(4,1)	(4,1)
				0.10	(7,3)	(4,1)	(4,1)	(2,1)
				0.25	(6,3)	(3,1)	(2,1)	(2,1)
			0.95	0.01	(8,5)	(5,2)	(3,1)	(3,1)
				0.05	(6,4)	(4,2)	(3,1)	(3,1)
				0.10	(6,4)	(2,1)	(2,1)	(2,1)
				0.25	(3,2)	(2,1)	(2,1)	(2,1)

Table A4. Single sampling plan values of (n,c) for finite lot of size 300 using various choices of parameters and at $\alpha = 0.05$.

θ	λ	a_0	p	β	μ/μ_0 2	3	4	5
1.5	1.5	0.5	0.75	0.01	(48,10)	(31,5)	(24,3)	(20,2)
				0.05	(36,8)	(22,4)	(15,2)	(15,2)
				0.10	(30,7)	(17,3)	(13,2)	(10,1)
				0.25	(19,5)	(10,2)	(7,1)	(7,1)
			0.95	0.01	(31,12)	(17,5)	(13,3)	(10,2)
				0.05	(22,9)	(12,4)	(8,2)	(8,2)
				0.10	(19,8)	(9,3)	(7,2)	(5,1)
				0.25	(11,5)	(6,2)	(6,2)	(4,1)
		1.0	0.75	0.01	(24,12)	(13,5)	(9,3)	(8,2)
				0.05	(17,9)	(9,4)	(6,2)	(6,2)
				0.10	(15,8)	(7,3)	(6,2)	(6,2)
				0.25	(10,6)	(6,3)	(5,2)	(3,1)
			0.95	0.01	(18,14)	(8,5)	(6,3)	(6,3)
				0.05	(14,11)	(6,4)	(5,3)	(4,2)
				0.10	(11,9)	(6,4)	(5,3)	(2,1)
				0.25	(11,9)	(4,3)	(3,2)	(2,1)
2.5	2.0	0.5	0.75	0.01	(46,6)	(31,3)	(26,2)	(21,1)
				0.05	(34,5)	(20,2)	(15,1)	(15,1)
				0.10	(26,4)	(17,2)	(13,1)	(13,1)
				0.25	(17,3)	(9,1)	(9,1)	(9,1)
			0.95	0.01	(27,7)	(16,3)	(13,2)	(10,1)
				0.05	(18,5)	(11,2)	(8,1)	(8,1)
				0.10	(14,4)	(9,2)	(7,1)	(7,1)
				0.25	(12,4)	(5,1)	(5,1)	(5,1)
		1.0	0.75	0.01	(16,7)	(9,3)	(8,2)	(6,1)
				0.05	(13,6)	(6,2)	(4,1)	(4,1)
				0.10	(10,5)	(6,2)	(4,1)	(4,1)
				0.25	(7,4)	(3,1)	(3,1)	(2,1)
			0.95	0.01	(13,9)	(6,3)	(5,2)	(3,1)
				0.05	(11,8)	(5,3)	(4,2)	(3,1)
				0.10	(8,6)	(5,3)	(2,1)	(2,1)
				0.25	(5,4)	(3,2)	(2,1)	(2,1)

Table A4. *Cont.*

θ	λ	a_0	p	β	μ/μ_0 2	3	4	5
1.5	2.5	0.75	0.75	0.01	(20,4)	(15,2)	(11,1)	(11,1)
				0.05	(14,3)	(8,1)	(8,1)	(5,1)
				0.10	(13,3)	(7,1)	(7,1)	(4,1)
				0.25	(8,2)	(5,1)	(3,1)	(3,1)
			0.95	0.01	(11,4)	(8,2)	(6,2)	(6,1)
				0.05	(8,3)	(5,1)	(5,1)	(5,1)
				0.10	(7,3)	(4,1)	(4,1)	(2,1)
				0.25	(5,2)	(3,1)	(3,1)	(2,1)
		1.0	0.75	0.01	(11,4)	(8,2)	(6,1)	(6,1)
				0.05	(9,4)	(4,1)	(4,1)	(4,1)
				0.10	(7,3)	(4,1)	(4,1)	(2,1)
				0.25	(6,3)	(3,1)	(2,1)	(2,1)
			0.95	0.01	(8,5)	(5,2)	(3,1)	(3,1)
				0.05	(6,4)	(4,2)	(3,1)	(3,1)
				0.10	(6,2)	(2,1)	(2,1)	(2,1)
				0.25	(3,2)	(2,1)	(2,1)	(2,1)

Table A5. Single sampling plan values of (n,c) for finite lot of size 500 using various choices of parameters and at $\alpha = 0.05$.

θ	λ	a_0	p	β	μ/μ_0 2	3	4	5
1.5	1.5	0.5	0.75	0.01	(56,12)	(32,5)	(24,3)	(20,2)
				0.05	(40,9)	(23,4)	(15,2)	(15,2)
				0.10	(30,7)	(17,3)	(13,2)	(10,1)
				0.25	(23,6)	(10,2)	(7,1)	(7,1)
			0.95	0.01	(31,12)	(17,5)	(13,3)	(10,2)
				0.05	(22,9)	(12,4)	(8,2)	(8,2)
				0.10	(19,8)	(9,3)	(7,2)	(5,1)
				0.25	(11,5)	(6,2)	(6,2)	(4,1)
		1.0	0.75	0.01	(25,13)	(13,5)	(9,3)	(8,2)
				0.05	(17,9)	(9,4)	(6,2)	(6,2)
				0.10	(15,8)	(7,3)	(6,2)	(6,2)
				0.25	(10,6)	(6,3)	(5,2)	(3,1)
			0.95	0.01	(18,14)	(8,5)	(6,3)	(6,3)
				0.05	(14,11)	(6,4)	(5,3)	(4,2)
				0.10	(14,11)	(6,4)	(5,3)	(2,1)
				0.25	(11,9)	(4,3)	(3,2)	(2,1)
2.5	2.0	0.5	0.75	0.01	(51,7)	(32,3)	(27,2)	(21,1)
				0.05	(35,5)	(20,2)	(15,1)	(15,1)
				0.10	(27,4)	(18,2)	(13,1)	(13,1)
				0.25	(18,3)	(9,1)	(9,1)	(9,1)
			0.95	0.01	(27,7)	(16,3)	(14,2)	(11,1)
				0.05	(18,5)	(11,2)	(8,1)	(8,1)
				0.10	(14,4)	(9,2)	(7,1)	(7,1)
				0.25	(12,4)	(5,1)	(5,1)	(5,1)
		1.0	0.75	0.01	(16,7)	(9,3)	(8,2)	(6,1)
				0.05	(13,6)	(6,2)	(6,2)	(5,1)
				0.10	(10,5)	(6,2)	(4,1)	(4,1)
				0.25	(7,4)	(3,1)	(3,1)	(2,1)
			0.95	0.01	(13,9)	(6,3)	(5,2)	(3,1)
				0.05	(11,8)	(5,3)	(4,2)	(3,1)
				0.10	(8,6)	(5,3)	(2,1)	(2,1)
				0.25	(5,4)	(3,2)	(2,1)	(2,1)

Table A5. *Cont.*

θ	λ	a_0	p	β	μ/μ_0 2	3	4	5
1.5	2.5	0.75	0.75	0.01	(21,4)	(15,2)	(11,1)	(11,1)
				0.05	(14,3)	(8,1)	(8,1)	(5,1)
				0.10	(13,3)	(7,1)	(7,1)	(4,1)
				0.25	(8,2)	(5,1)	(5,1)	(3,1)
			0.95	0.01	(11,4)	(8,2)	(6,2)	(6,2)
				0.05	(8,3)	(5,1)	(5,1)	(5,1)
				0.10	(7,3)	(4,1)	(4,1)	(2,1)
				0.25	(6,3)	(3,1)	(3,1)	(2,1)
		1.0	0.75	0.01	(11,4)	(8,2)	(6,1)	(6,1)
				0.05	(9,4)	(5,1)	(5,1)	(5,1)
				0.10	(7,3)	(4,1)	(4,1)	(2,1)
				0.25	(6,3)	(3,1)	(2,1)	(2,1)
			0.95	0.01	(8,5)	(5,2)	(3,1)	(3,1)
				0.05	(6,4)	(4,2)	(3,1)	(3,1)
				0.10	(6,4)	(2,1)	(2,1)	(2,1)
				0.25	(3,2)	(2,1)	(2,1)	(2,1)

Table A6. Operating characteristic values for power Lindley sampling plan.

$\theta = 0.5; \lambda = 1.5; p = 0.85; c = 2$						**$\theta = 0.5; \lambda = 1.5; p = 0.90; c = 2$**					
n	a	μ/μ_0 2	3	4	5	n	a	μ/μ_0 2	3	4	5
16	0.4	0.7962	0.9534	0.9855	0.9944	16	0.4	0.7163	0.9299	0.9776	0.9912
12	0.6	0.6008	0.8933	0.9653	0.9863	12	0.6	0.4812	0.8438	0.9470	0.9787
10	0.8	0.3978	0.8046	0.9326	0.9728	10	0.8	0.2731	0.7242	0.8987	0.9579
9	1.0	0.2166	0.6784	0.8786	0.9492	9	1.0	0.1212	0.5677	0.8222	0.9226
7	1.2	0.1941	0.6597	0.8719	0.9470	7	1.2	0.1049	0.5444	0.8118	0.9187
6	1.4	0.1471	0.6062	0.8465	0.9359	6	1.4	0.0734	0.4833	0.7766	0.9020
5	1.6	0.1437	0.6000	0.8442	0.9355	5	1.6	0.0720	0.4763	0.7729	0.9009
$\theta = 0.5; \lambda = 1.5; p = 0.95; c = 2$						**$\theta = 0.5; \lambda = 1.5; p = 0.99; c = 2$**					
n	a	μ/μ_0 2	3	4	5	n	a	μ/μ_0 2	3	4	5
12	0.4	0.7467	0.9411	0.9818	0.9929	12	0.4	0.5057	0.8548	0.9512	0.9804
10	0.6	0.4433	0.8284	0.9419	0.9767	10	0.6	0.1729	0.6321	0.8551	0.9379
9	0.8	0.1895	0.6518	0.8659	0.9433	9	0.8	0.0347	0.3748	0.6978	0.8573
7	1.0	0.1332	0.5875	0.8354	0.9301	7	1.0	0.0186	0.2999	0.6390	0.8252
6	1.2	0.0788	0.4948	0.7838	0.9056	6	1.2	0.0077	0.2122	0.5514	0.7710
5	1.4	0.0663	0.4627	0.7642	0.8965	5	1.4	0.0061	0.1873	0.5203	0.7504
4	1.6	0.0921	0.5082	0.7922	0.9111	4	1.6	0.0117	0.2286	0.5638	0.7796
$\theta = 1.5; \lambda = 2.0; p = 0.85; c = 2$						**$\theta = 1.5; \lambda = 2.0; p = 0.90; c = 2$**					
n	a	μ/μ_0 2	3	4	5	n	a	μ/μ_0 2	3	4	5
16	0.4	0.9284	0.9910	0.9982	0.9995	16	0.4	0.8931	0.9856	0.9970	0.9992
12	0.6	0.7856	0.9662	0.9926	0.9979	12	0.6	0.7025	0.9479	0.9881	0.9965
10	0.8	0.5815	0.9157	0.9798	0.9939	10	0.8	0.4634	0.8748	0.9683	0.9902
9	1.0	0.3523	0.8254	0.9533	0.9852	9	1.0	0.2363	0.7529	0.9286	0.9766
7	1.2	0.2922	0.7937	0.9433	0.9819	7	1.2	0.1839	0.7115	0.9140	0.9714
6	1.4	0.2085	0.7349	0.9231	0.9748	6	1.4	0.1180	0.6380	0.8848	0.9606
5	1.6	0.1814	0.7115	0.9150	0.9720	5	1.6	0.0986	0.6093	0.8730	0.9562

Table A6. *Cont.*

$\theta = 1.5; \lambda = 2.0; p = 0.95; c = 2$						$\theta = 1.5; \lambda = 2.0; p = 0.99; c = 2$					
n	a	μ/μ_0				n	a	μ/μ_0			
		2	3	4	5			2	3	4	5
12	0.4	0.9087	0.9881	0.9976	0.9993	12	0.4	0.7907	0.9673	0.9929	0.9979
10	0.6	0.6803	0.9429	0.9869	0.9961	10	0.6	0.4293	0.8609	0.9642	0.9888
9	0.8	0.3767	0.8377	0.9572	0.9865	9	0.8	0.1381	0.6595	0.8923	0.9631
7	1.0	0.2613	0.7736	0.9365	0.9795	7	1.0	0.0702	0.5552	0.8462	0.9450
6	1.2	0.1517	0.6800	0.9021	0.9671	6	1.2	0.0268	0.4249	0.7751	0.9147
5	1.4	0.1109	0.6283	0.8813	0.9594	5	1.4	0.0158	0.3619	0.7342	0.8963
4	1.6	0.1244	0.6443	0.8887	0.9625	4	1.6	0.0202	0.3810	0.7474	0.9029

Table A7. Values of (n_1, n_2, c_1 and c_2) for double acceptance sample plan under the power Lindley distribution for various choices of parameters.

$\theta = 1.5; \lambda = 2.5; \alpha = 0.05$								$\theta = 1.5; \lambda = 2.5; \alpha = 0.01$							
a_0	p	β	μ/μ_0				a_0	p	β	μ/μ_0			
			2		4					2		4	
			c_1,c_2	n_1,n_2	c_1,c_2	n_1,n_2				c_1,c_2	n_1,n_2	c_1,c_2	n_1,n_2
		0.01	2,5	15,13	1,3	14,10			0.01	2,6	18,15	2,3	17,14
	0.75	0.05	2,5	15,13	1,4	13,10		0.75	0.05	2,5	16,13	2,4	15,10
		0.10	2,5	12,8	1,3	11,9			0.10	2,5	12,9	1,3	12,7
		0.01	3,5	15,8	2,3	18,7			0.01	3,6	16,10	2,4	21,13
0.75	0.90	0.05	2,5	14,8	2,4	16,12	0.75	0.90	0.05	2,6	14,9	2,3	18,12
		0.10	2,6	12,10	1,3	14,10			0.10	3,6	13,10	1,4	15,11
		0.01	3,7	12,10	1,3	15,11			0.01	4,7	13,9	1,3	19,11
	0.95	0.05	2,6	10,9	1,3	13,10		0.95	0.05	2,5	11,9	1,3	16,10
		0.10	1,6	9,8	1,2	10,9			0.10	2,6	10,9	1,2	12,9
		0.01	1,7	12,10	1,3	11,8			0.01	1,7	12,10	1,3	14,9
	0.75	0.05	1,6	10,9	1,3	11,7		0.75	0.05	1,6	10,9	1,3	13,7
		0.10	1,5	9,6	1,3	9,8			0.10	1,5	9,6	1,3	11,8
		0.01	2,6	11,9	2,4	9,8			0.01	2,6	11,9	2,4	16,10
0.95	0.90	0.05	1,6	10,8	1,3	9,7	0.95	0.90	0.05	1,6	10,8	1,3	14,9
		0.10	1,5	9,8	1,3	9,7			0.10	1,5	9,8	1,3	13,8
		0.01	1,5	10,9	1,3	11,7			0.01	1,5	10,9	1,3	14,9
	0.95	0.05	1,4	9,7	1,3	10,7		0.95	0.05	1,4	9,7	1,3	13,7
		0.10	1,2	8,7	1,3	9,6			0.10	1,2	8,7	1,3	11,7

$\theta = 2.5; \lambda = 2.0; \alpha = 0.05$								$\theta = 2.5; \lambda = 2.0; \alpha = 0.01$							
a_0	p	β	μ/μ_0				a_0	p	β	μ/μ_0			
			2		4					2		4	
			c_1,c_2	n_1,n_2	c_1,c_2	n_1,n_2				c_1,c_2	n_1,n_2	c_1,c_2	n_1,n_2
		0.01	4,9	19,14	3,8	17,12			0.01	5,9	22,16	4,9	20,15
	0.75	0.05	4,8	18,13	3,6	17,11		0.75	0.05	5,9	21,16	3,8	20,14
		0.10	3,7	17,12	2,5	15,11			0.10	4,8	18,12	3,6	19,14
		0.01	3,8	19,11	2,7	17,10			0.01	3,9	22,12	2,7	20,14
0.75	0.90	0.05	3,7	18,13	1,5	16,12	0.75	0.90	0.05	3,9	21,11	2,6	19,14
		0.10	2,6	18,13	2,5	17,11			0.10	2,8	20,10	2,6	19,13
		0.01	2,5	18,9	1,5	16,9			0.01	2,6	22,11	2,5	21,13
	0.95	0.05	2,5	17,9	1,4	15,8		0.95	0.05	2,6	22,10	2,5	20,12
		0.10	2,4	17,8	1,4	14,8			0.10	2,5	21,9	1,5	19,13
		0.01	2,7	18,11	2,6	16,9			0.01	2,8	21,9	2,4	21,13
	0.75	0.05	2,7	17,11	2,6	14,9		0.75	0.05	1,7	21,9	2,4	19,11
		0.10	1,6	16,10	1,5	14,8			0.10	1,7	21,9	2,3	17,12
		0.01	2,6	17,10	2,5	14,9			0.01	1,8	19,10	2,4	14,9
0.95	0.90	0.05	1,6	17,10	1,5	15,9	0.95	0.90	0.05	1,7	19,9	2,4	13,9
		0.10	1,6	16,9	1,5	15,7			0.10	1,7	19,9	1,3	13,8
		0.01	1,5	17,11	1,4	14,8			0.01	1,6	18,9	1,3	10,8
	0.95	0.05	1,4	17,10	1,3	15,7		0.95	0.05	1,5	18,9	1,3	9,7
		0.10	1,4	16,9	1,3	12,7			0.10	1,5	17,9	1,3	9,7

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