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Probabilistic Models for the Peak Residential Water Demand

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Abstract: Peak water demand is one of the most stringent operative conditions for a Water Distribution System (WDS), not only for the intensity of the event itself, but also for its recurring nature. The estimation of the maximum water demand is a crucial aspect in both the design and management processes. Studies in the past have tackled this issue with deterministic approaches, even if peak phenomena are distinctly random. In this work, probabilistic models have been developed to study and forecast the daily maximum residential water demand. Some probability distributions have been tested by means of statistical inferences on different data samples related to three monitored WDS. The parameter estimations of the proposed equations have been related to the number of supplied users. Furthermore, this work investigates time scaling effects on the effectiveness of the proposed distributions and relations. Corrective factors that take into account the effect of time averaging step on the above-mentioned parameters have been proposed.

Keywords: residential water demand; peak water demand; probabilistic approach; logistic distribution; monitoring system; WDS

1. Introduction

The peak water demand for residential users implies one of the most onerous operative conditions for an urban Water Distribution System (WDS). The maximum water demand during the day, along with mechanical failure of one or more components of the system (e.g., pipes, pumps, and valves) or fire-fighting flow conditions, is one of the scenarios for which the WDSs may experience some deficiency.

Hence, the peak water demand is a working condition that is usually taken into account for the design and management of WDSs (e.g., [1,2]).

In the scientific literature, few papers define completely the daily maximum water demand for a considerable number of users (around one thousand inhabitants) following a probabilistic approach. This issue is often tackled by means of the more practical deterministic equations (e.g., [3–5]), although the random nature of the water demand is generally recognized.

On the other hand, several stochastic models describing the water demand for few end users (a dozen of dwellings) were proposed (e.g., [6–9]). These approaches model the residential water demand as it happens at the hydro-sanitary devices of the dwellings. Therefore, the demanded flow is generally composed by the overlapping of several rectangular pulses, which are generated by turning on the hydro-sanitary devices.

The rectangular pulses approaches need to model at least three random phenomena [6]: arrival, duration and intensity of the demand pulses. Hence, these models induce significant computational

efforts and require a detailed knowledge of the lifestyle related to the users on which the parameters of the component distributions depend.

However, the rectangular pulse models could be overwhelming when the target of the study is the estimation of the water requirement in a specific condition, such as the maximum water demand during the day. Examples of these approaches can be found in [10,11] where a very detailed knowledge of the dwelling composition and user habits is required.

On the other hand, when the number of the users is greater than 200, the trend of the water demand during the day becomes smooth and the single pulses are no longer distinguishable [12,13]. In these conditions, the water demand can be represented by means of a single random variable that is continuous and positive.

The aim of this work is to obtain a reliable probabilistic approach that is capable of providing accurate estimations of the maximum residential water demand. For this goal, a specific field laboratory, which monitors the actual water demand of a small town in Southern Italy, was set up by the *Laboratorio di Ingegneria delle Acque* (LIA) of the *University of Cassino and Southern Lazio*. This monitoring system allowed obtaining copious data samples, where the number of the served inhabitants is known. The data sample was increased considering two further case studies whose users have very different lifestyles in respect to the monitored inhabitants of the Southern Lazio. In particular, the peak water demand for two further towns was also taken into account, one in the Netherlands (the case study of Franeker [14]) and one in Northern Italy (the case study of Castelfranco Emilia [7]).

The enlargement of the data sample showed the independence of the daily maximum demand from the local habits. Hence, the results of this study could be considered of great practical application, allowing the use of the hereby proposed probabilistic models to obtain a good estimation of the residential peak water demand.

Some effective distributions, able to model the daily maximum demand for residential users, have been proposed by means of statistical inferences on the above-mentioned observed data. In addition, practical and reliable equations have been suggested which allow estimating the parameters of the suggested *Cumulative Distribution Functions* (CDFs) in relation to the number of users, N_{us} , where N_{us} represents the total number of the dwelling occupants.

The temporal resolution is a fundamental choice when modeling water demand (e.g., [15–19]), leading to significant reductions of the peak demand when the time step increases [20,21]. Therefore, time scaling effects were also investigated and further equations that take into account such phenomenon were presented. The proposed equations allow estimating the CDFs' parameters when the time step (Δt) varies in the interval $1 \text{ min} \leq \Delta t \leq 1 \text{ h}$.

2. The monitoring Systems and the Data Samples

In real WDSs monitoring systems, flow measures do not often represent the actual users demand, generally for two main issues.

The hydraulic shortages (first issue), which could be caused in different points of a WDS for under-dimensioned pipes or failure of electro-mechanical components, do not allow considering the flow measurements as an expression of the water demand. Indeed, in these working conditions, the measurements represent only the water delivery capability of the WDS under specific user requests; hence, the obtained data underestimate the actual water demand. In addition, these shortage conditions are likely when the daily maximum water demand occurs.

The monitored WDSs have to be redundant in order to avoid that the collected data are compromised by such operating conditions. In this way, the WDSs are able to totally satisfy the users' water demand, even when this is particularly high. Hence, the monitoring system must be composed by both flow meters and pressure probes, to check that demand data obtained is reliable.

A fundamental parameter to describe the peak phenomenon is N_{us} (e.g., [5,22]), but for looped WDSs, when the measurements are performed on the links, it is not possible to know univocally the

number of users (second issue) related to each measurement point. The served users can vary with the different working conditions.

In order to solve this issue, the measuring points have to be set on the pipes which connect the Demand Monitoring Areas (DMAs) to the rest of the WDS. A DMA is a part of the entire distribution network which is linked to the rest of the WDS by means of one or a limited number of pipes. In this way, N_{us} for each DMA is not dependent on the working conditions.

An accurate census must define the number of the resident population in the considered DMA, and it must verify that the users are only residential.

Based on these considerations a specific monitoring system, which involves the real WDS of a small town, Piedimonte San Germano (PSG), was realized.

In detail, the monitoring system is composed by four measurement points, each with an electromagnetic flow meter and a pressure cell, where all probes are connected to a data logger. Flows and pressures were recorded continuously, where a variable frequency data logger allowed an acquisition frequency up to 1 Hz. The location of the probes allowed the measurement of the water demand for four DMAs, where the number of users was respectively equal to: 239, 777, 981 and 1220. More details of the PSG laboratory are provided in [21,23].

In order to expand the data set, the time series of two further case studies, with similar characteristics to PSG in terms of data reliability, but characterized by a different users lifestyle, were also used: Castelfranco Emilia (CE) in Italy and Franeker (Fr) in the Netherlands.

CE monitoring system covering 596 users is part of a wider WDS. Data are collected by means of volumetric measures (Woltmann volume meter), and they are averaged over one-minute intervals. For Fr network, continuous per minute flow measurements are taken on the overall network, which supplies about 1150 users.

Alvisi et al. [7] and Blokker et al. [14] provide respectively detailed information of the CE and Fr field laboratories.

Table 1 summarizes the principal characteristic of the three in-situ laboratories.

Table 1. Principal characteristics of the monitoring systems.

	PSG (I)	Fr (NL)	CE (I)
N_{us}	239, 777, 981, 1220	1150	596
Type of counter	Flow meter	Flow meter	Volumetric meter
Frequency of sampling	1 Hz	1/60 Hz	1/60 Hz
Period of observation	24 November 2008–13 April 2009	21 January 2006–3 August 2006	6 January 2009–14 March 2009
Number of used days for the statistics	85	113	46

Initially, the analysis assumed a time resolution of one minute, which was increased up to $\Delta t = 1$ h in the following elaborations in order to investigate the effects of the temporal aggregation.

The water demand presents different trends between working days and weekends which affect also the maximum water demand during the day. Therefore, only the working days have been considered in which the peak water demand is slightly more pronounced.

No significant seasonal trend of the water demands was observed for the field data examined, contrary to what happens for residential users in some countries [24,25].

On this basis, in order to obtain a more reliable data set, the time series of the three field laboratories were filtered. The daily series were removed when they presented one of the following conditions: holidays/weekends; days with missing data; days with anomalous data (e.g., fire flow, corrupted signals of the probes, and big leakages). The number of days used during the following elaborations is indicated in the last row of Table 1.

In the following sections, the maximum demand of flow during the day has been made dimensionless by means of the ratio:

$$C_p = \frac{Q_p}{\mu_Q} \quad (1)$$

where μ_Q is the daily mean water demand, and Q_p the peak flow during the day. Equation (1) gives the *Peak Demand Coefficient*, C_p . This work focuses on the dimensionless variable C_p .

3. Probabilistic Distributions for the C_p

The extreme events are effectively described by means of the Log-Normal distribution (*LN*) (e.g., [26])

$$f_x^{LN} = \frac{1}{x\sigma_y\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu_y}{\sigma_y}\right)^2\right] \quad (2)$$

and Gumbel distribution (*Gu*) (e.g., [26])

$$f_x^{Gu} = \alpha \exp[-\alpha(x - \varepsilon) - \exp[-\alpha(x - \varepsilon)]] \quad (3)$$

where f_x is the probability density function; x is the random variable and $y = \ln x$; and μ and σ represent, respectively, the mean and the standard deviation. The parameters α and ε of *Gu* are given by the relations:

$$\begin{aligned} \frac{1}{\alpha} &= \frac{\sqrt{6}}{\pi}\sigma_x \\ \varepsilon &= \mu_x - 0.45\sigma_x \end{aligned} \quad (4)$$

In the scientific literature, the peak water demand for residential users was described by means of these probabilistic models (e.g., [20,21,27]).

The *LN* and *Gu* have been tested by means of the observed data for the above described case studies, assuming a time aggregation $\Delta t = 1$ min. In addition, the Log-Logistic (*LL*) model was investigated to describe the daily maximum water demand. The *LL* distribution presents a trend very similar to the *LN*, but, contrary to Gauss model, the probability density function can be analytically integrated [28,29]. The CDF of the *LL* distribution has the following expression:

$$F_x^{LL} = \left[\left(\frac{e^{\mu_y}}{x} \right)^{\frac{\pi}{\sigma_y\sqrt{3}}} + 1 \right]^{-1} \quad (5)$$

If $x = C_p$ (dimensionless peak demand Equation (1)), y is $\ln C_p$.

The parameters of Equation (5) can be estimated by means of statistics on the logarithmic of the original sample, or by means of μ and σ of the pristine random variable by means of the following equations [28]:

$$\begin{aligned} \mu_y &= \ln \left[\frac{\mu_x}{\sigma_y\sqrt{3}} \sin(\sigma_y\sqrt{3}) \right] \\ CV_x &= \sqrt{\frac{1}{\sigma_y\sqrt{3}} \tan(\sigma_y\sqrt{3})} - 1 \end{aligned} \quad (6)$$

where $CV_x = \sigma_x/\mu_x$ is the variation coefficient.

However, an early and rough estimation of the standard deviation of y variable can be obtained by means of the assumption $\sigma_y \approx CV_x$ (this assumption becomes absolutely robust when CV_x is less than 0.15).

The diagrams in Figure 1 are the Quantile-Quantile (Q-Q) plots for three monitored number of users (for the sake of synthesis, the plots of PSG $N_{us} = 239, 777$ and 1220 were not here reported, but they are similar to diagrams of Figure 1). More precisely, the Q-Q plots show the comparison between the observed and the theoretical C_p quantiles with the same probability of occurrence. The theoretical values were estimated by means of the integral of Equations (2) and (3)—numerical integration for *LN* distribution—and Equation (5). The quantiles of the peak demand of Figure 1 fit well the bisector line, which represents the condition where the observed quantiles are equal to the theoretical quantiles.

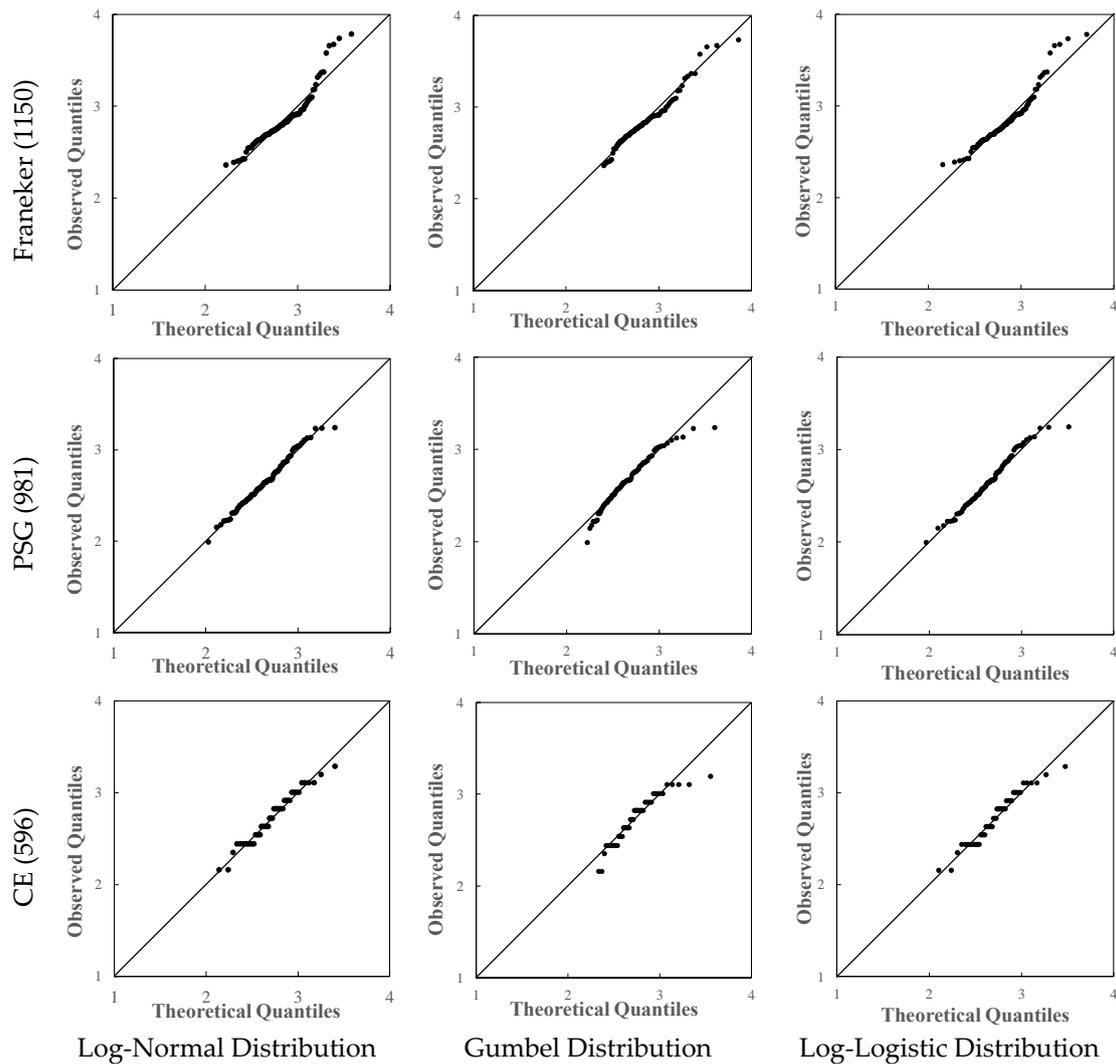


Figure 1. Q-Q plots of the peak demand coefficient for: $\Delta t = 1$ min; three distributions (*LN*, *Gu* and *LL*); and different users ($N_{us} = 1150, 981$ and 596).

The effectiveness of the three distributions was checked by means of the Kolmogorof–Smirnov (KS) test. Table 2 summarizes the threshold values for a 2% confidence level ($D_{2\%}$ —column 3) and the relative KS parameters (D_{KS}) for the *LN*, *Gu*, and *LL* CDFs (columns 4–6). The goodness of fit tests were satisfied ($D_{KS} < D_{2\%}$) for all monitored users. In addition, Table 2 shows the statistics of the data sample (μ_{Cp} and σ_{Cp} —columns 7–8). These probability distributions proved to be equally effective to model the peak phenomena, thus the choice is left to the decision maker.

Table 2. Goodness of fit and statistics for the analyzed samples.

Case study	N_{us}	$D_{2\%}$	D_{KS}			μ_{Cp}	σ_{Cp}
			<i>LN</i>	<i>Gu</i>	<i>LL</i>		
PSG	1220	0.164	0.1152	0.0642	0.1140	2.50	0.22
	981	0.164	0.0616	0.0536	0.0526	2.64	0.27
	777	0.164	0.0741	0.0498	0.0893	2.89	0.28
	239	0.164	0.1358	0.1256	0.1238	4.04	0.64
Fr	1150	0.142	0.1322	0.0886	0.1158	2.84	0.27
CE	596	0.223	0.0432	0.0673	0.0536	2.72	0.27

Equations (2) and (3) allow to explicate the *Peak Demand Coefficient* $[C_p]_F$ for a predefined probability $\text{Pr}[S]$ of not exceedance. Hence the peak coefficient with a predefined probability of not exceedance following *LL* is given by:

$$[C_p]_F = e^{\mu_y} \left[\frac{\text{Pr}[S]}{1 - \text{Pr}[S]} \right]^{\frac{\sigma_y \sqrt{3}}{\pi}} \tag{7}$$

while, for *Gu* distribution, $[C_p]_F$ presents the following relation:

$$[C_p]_F = \sigma_x \left[\frac{1}{\text{CV}_x} - 0.45 - \frac{\sqrt{6}}{\pi} \ln \left(\ln \frac{1}{\text{Pr}[S]} \right) \right] \tag{8}$$

The quantile $[C_p]_F$ for *LN* model can be estimated only by means of a numerical approximation.

Figure 2 shows—as an example—the comparison between the observed data (in this case the peak demand of PSG for $N_{us} = 1220$ and $\Delta t = 1$ min) and the values of $[C_p]_F$ estimated by means of Equations (7) and (8), assuming $\mu_x = 2.50$ and $\sigma_x = 0.22$. The plots confirm that both the *LL* and *Gu* distributions fit well the experimental points.

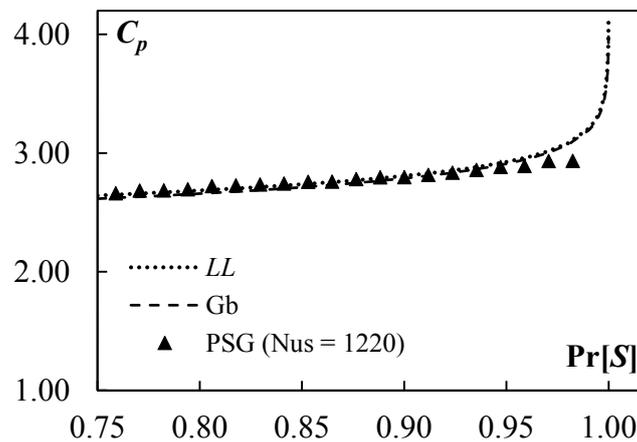


Figure 2. Peak water demand coefficient $[C_p]_F$ versus $\text{Pr}[S]$.

4. Estimation of Parameters

All the proposed distributions (*LN*, *Gu* and *LL*) are bi-parametric models; hence, they require the estimation of two parameters, for instance, the mean and the variation coefficient.

The PSG demand data, jointly with data from the other monitored users (*CE* and *Fr*), shows that the dimensionless average of the peak water demand varies in relation to the number of users according to the following equation (Figure 3a):

$$\mu_{C_p} = 10 N_{us}^{-0.2} \tag{9}$$

Equation (9) resembles the relation proposed by Babbitt [30], but the coefficient 10 of the power function is significantly less than the one of the Babbitt equation, as already shown in Tricarico et al. [21]. The deterministic approaches, of which the Babbitt equation represents a classic example, imply an overestimation of the peak demand.

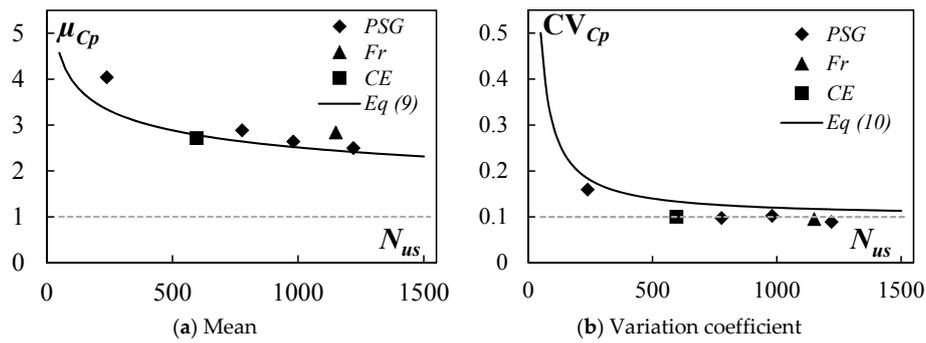


Figure 3. Comparison of the proposed equations and of the observed data for the estimation of the: mean (a); and variation coefficients (b) of C_p versus N_{us} .

CV_{C_p} for peak demand can also be estimated in relation to the number of users.

$$CV_{C_p} = 0.1 + \frac{20}{N_{us}} \tag{10}$$

Equation (10) fits the experimental points (Figure 3b), and it highlights a decreasing trend of the CV_{C_p} which tends asymptotically towards 0.1 when the number of the users increases. Therefore it is reasonable to assume $CV_{C_p} = 0.1$ when the number of residential inhabitants is greater than 1000 [21,31].

It is worth noting that the investigated distributions and the suggested relations—Equations (9) and (10)—are effective for users with completely different habits. Gargano et al. [13] give more details about the daily pattern of the water demand for Ce, Fr and PSG users.

Equations (9) and (10) adjust the relations introduced by Tricarico et al. [21], because the hereby proposed equations have been obtained on the basis of more copious and detailed data sample. In addition, the data of water demand recorded in five suburbs of Melbourne in Australia [32] show similar trends to the ones of Equations (9) and (10), respectively, for the mean and coefficient of variation.

For a predefined number of users, the parameters (μ_{C_p} and CV_{C_p}) can be estimated by means of the Equations (9) and (10), and hence Equations (7) and (8) allow defining the Peak Demand Coefficient with assigned probabilities of not exceedance.

The diagrams of Figure 4 show that higher values of the probability of not exceeding of C_p lead to greater peak demand coefficients, and this effect is more evident when the number of users decreases. Moreover, the comparison of Figure 4a,b denotes a close trend between Gu and LL distributions for equal values of N_{us} and fixed probability.

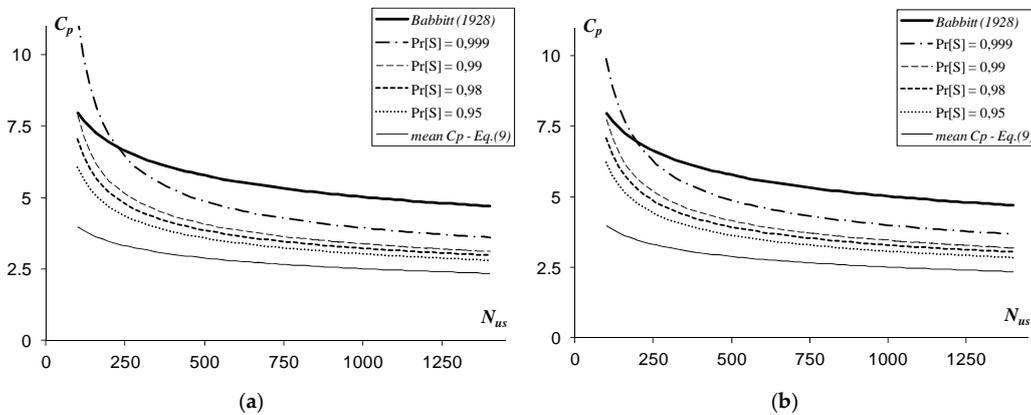


Figure 4. Peak Demand Coefficient versus user number for predefined success probability, and for CDFs: Log-Logistic Distribution (a); Gumbel Distribution (b).

Finally, for the investigated number of users (200–1250), Figure 4 shows that the old deterministic equations (i.e., [30]) overestimate the peak phenomena with respect to the probabilistic relations, also when these latter assume high values of the success probability (i.e., $\Pr[C_p \leq \bar{C}_p] = \Pr[S] = 0.999$).

It is worth noting that the classic relation of Babbitt [30] has been recalled here only as an example of deterministic relations, in fact all of them usually lead to an overestimation of the peak phenomenon of the water demand [31]. Several researchers [3,10,22] give an interesting overview of the ancient deterministic equations for the estimation of the peak phenomena.

The Peak Coefficient curve (Figure 4) for a defined probability of non-exceedance can also be expressed by means of the product:

$$[C_p]_F = K_F \cdot \mu_{C_p} \quad (11)$$

where μ_{C_p} is estimated by means of Equation (9) in function of the users number, and K_F represents a growth factor ($K_F > 1$), as used for flood frequency analysis (e.g., [33]); therefore, it increases when the $\Pr[S]$ increases. For *LL* and *Gu*, the growth factors are obtained, respectively, from Equations (7) and (8):

$$K_F = \left[\frac{\Pr[S]}{1 - \Pr[S]} \right]^{\frac{\sigma_y \sqrt{3}}{\pi}} \frac{\sin(\sigma_y \sqrt{3})}{\sigma_y \sqrt{3}} \text{LLdistribution} \quad (12)$$

$$K_F = 1 - 0.45 CV_x - \left(\frac{\sqrt{6}}{\pi} \ln \left(\ln \frac{1}{\Pr[S]} \right) \right) CV_x \text{Gu distribution} \quad (13)$$

K_F is estimated when the probability of not exceedance $\Pr[S]$ is fixed (Equations (12) and (13)) and the probabilistic parameters are defined in relation to the number of the inhabitants.

The probability of non-exceedance usually is an engineering choice made by the decision makers and should be a consequence of a cost–benefit analysis.

5. Time Aggregation Effects

The results shown in the previous sections are related to a time step equal to $\Delta t = 1$ min, while, in the numerical analysis of WDSs, a less refined time resolution is usually assumed ($1 \text{ min} < \Delta t < 60 \text{ min}$).

The time step presents noticeable implications on the estimation of the maximum water demand [3,27], in fact when Δt increases, the time averaging effect smooths the peak phenomena.

In the following sections, the effects of the time aggregations are investigated in order to extend the above proposed distributions and relations to other temporal averaging intervals.

5.1. Distributions in Relation to Time Step

The same data samples of the residential water demand, which were considered in previous sections for $\Delta t = 1$ min, were resampled to define the distributions with a different time aggregation ($\Delta t = 5, 10, 15, 30$ and 60 min). The effectiveness of the *LN*, *Gu* and *LL* distributions was demonstrated by means of statistical inferences for different time resolutions, from 1 min up to 1 h.

For the sake of the synthesis, in Figure 5, only the diagrams for the C_p of PSG 981 with different time resolutions ($\Delta t = 1, 10, 30$ and 60 min), are reported. However, similar results have been obtained for all other monitored users with a time resolution up to one hour.

The Q-Q plots of Figure 5 show the goodness of the three distributions for all investigated time aggregations, which has been also demonstrated more rigorously by means of KS tests.

The tests with significance level 2% (the value of $D_{2\%}$ is represented by the dashed line in Figure 6a,c,e) were all satisfied for *LN*, *Gu* and *LL* distributions for any value of Δt that falls into the interval 1–60 min.

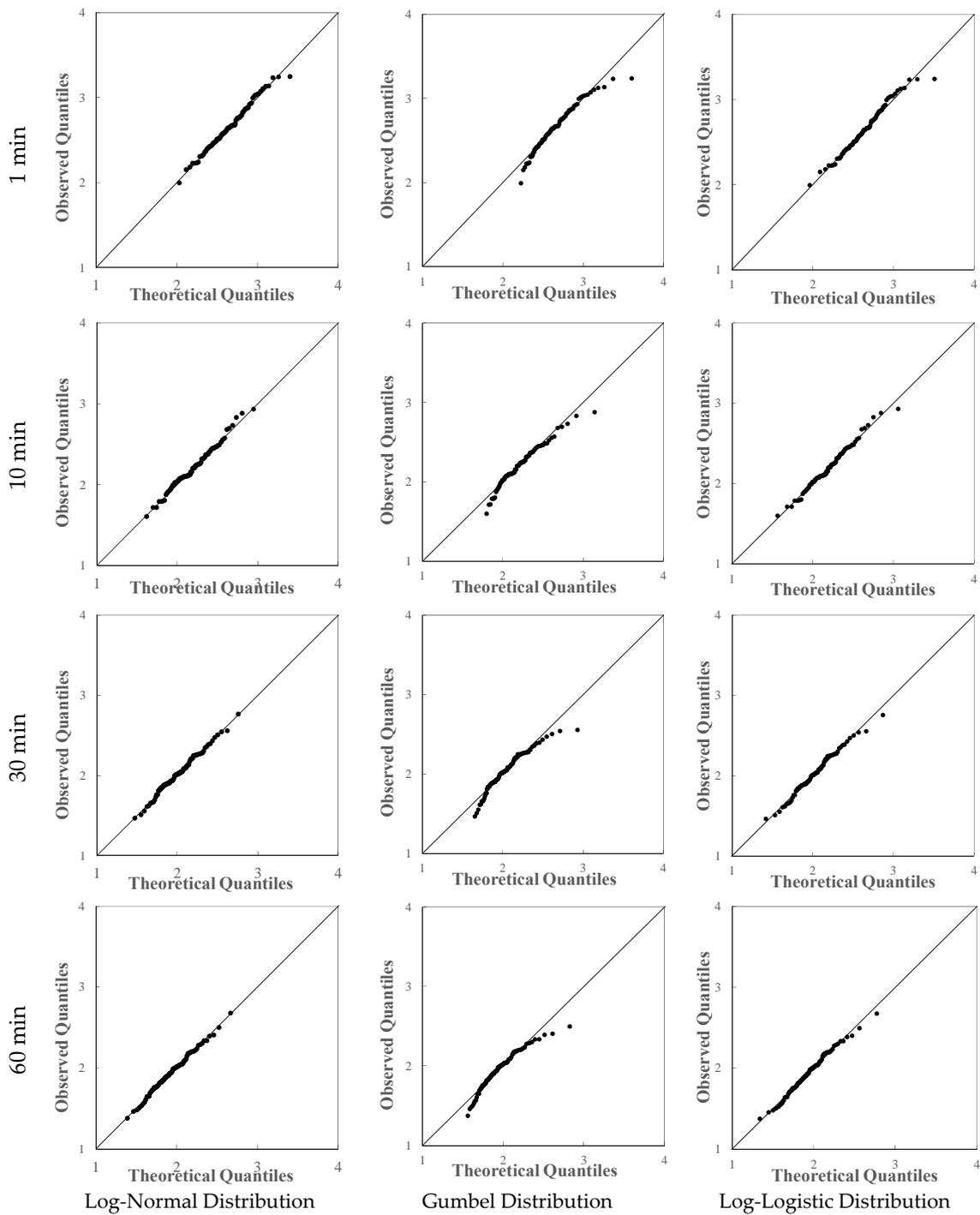


Figure 5. Q-Q plots of the peak demand coefficient for $N_{us} = 981$ (PSG) for various time aggregations and distributions (*LN*, *Gu* and *LL*).

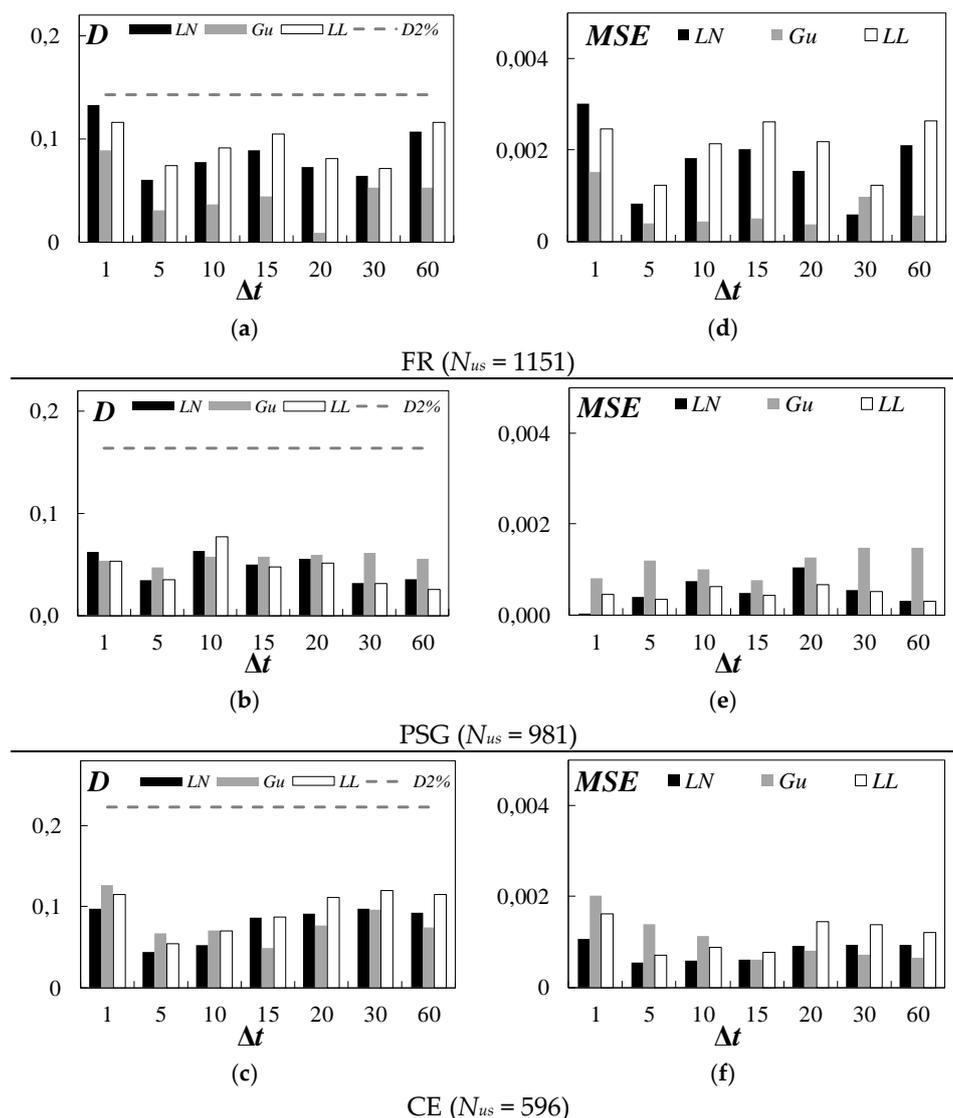


Figure 6. Effects of the time aggregation—KS parameter and MSE versus Δt .

The results of the KS tests did not highlight any trends in relation to the time step, hence the effectiveness the proposed distributions seem to be independent of the time resolution. In addition to KS test, the Mean Squared Error (MSE) was investigated, where MSE is given by:

$$MSE = \frac{1}{n} \sum_{i=1}^n (x_i - \hat{x}_i)^2 \tag{14}$$

where x_i and \hat{x}_i are, respectively, the observed and the predicted C_p values, and n is total number of observations.

The MSE plots (Figure 6b,d,f) show that the time aggregation for the investigated range seems not to trigger any tendency on the effectiveness of the three probability distributions. Therefore, the estimated values of MSE vary independently by Δt and N_{us} .

5.2. Time Dependence of the Parameters

If the time scale does not lead to significant effects on the probabilistic models, it has to be considered in order to estimate the parameters when Δt changes. Indeed, the statistical analysis

highlighted that the proposed equations to estimate the parameters of distributions need a reduction coefficient when the time step is greater than 1 min. Therefore, Equation (9) becomes:

$$\mu_{C_p}(\Delta t) = k_{\Delta} \mu_{C_p} \tag{15}$$

where $\mu_{C_p}(\Delta t)$ is the average value of C_p for $\Delta t \geq 1$ min and k_{Δ} represents the corrective coefficient of the mean peak demand coefficient for $\Delta t = 1$ min (μ_{C_p}), which decreases when Δt increases (Figure 7a).

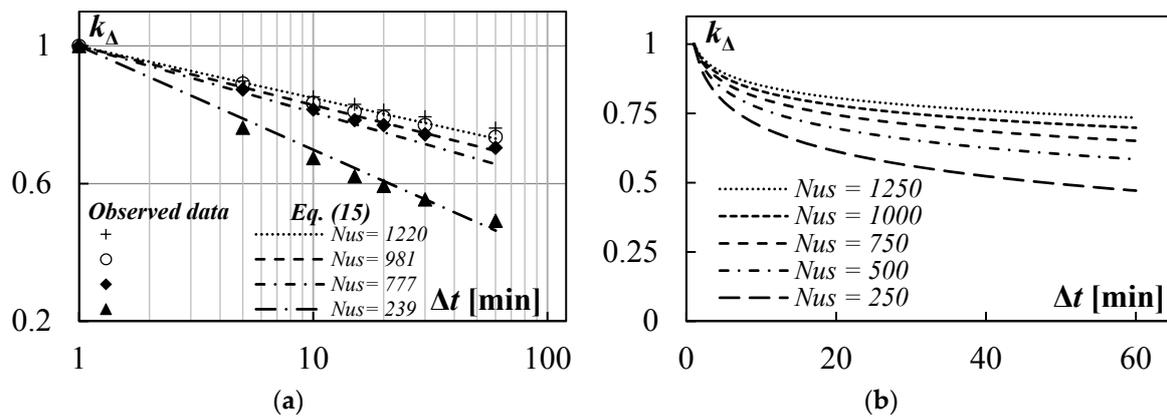


Figure 7. (a) Comparison of the observed data and Equations (15) and (16); and (b) abacus of the corrective coefficient k_{Δ} for different values of N_{us} .

Hence, k_{Δ} is estimated in relation to the users' number and Δt

$$k_{\Delta} = 1 + (0.04 \ln N_{us} - 0.35) \ln(\Delta t) \tag{16}$$

where the time is counted in min. Equations (17) and (18) are valid for $1 \text{ min} \leq \Delta t \leq 60 \text{ min}$ and $250 \text{ users} < N_{us} < 1250 \text{ users}$.

The abacus of Figure 7b shows that the peak demand coefficient decreases significantly when Δt increases from 1 min to 60 min. For instance, it decreases of about 25% for $N_{us} = 1250$, and about 50% for $N_{us} = 250$.

Likewise, Equation (10) requires a corrective coefficient β in order to estimate $CV_{C_p}(\Delta t)$ when the time aggregation is greater than 1 min:

$$CV_{C_p}(\Delta t) = 0.1 + \frac{20}{\beta N_{us}} \tag{17}$$

The coefficient depends on the N_{us} and the time step according to Equation (18)

$$\beta = \exp \left[(\Delta t - 1) \frac{N_{us}}{10^4} \right] \tag{18}$$

where the time is counted in minutes and Equations (17) and (18) are valid for $1 \text{ min} \leq \Delta t \leq 60 \text{ min}$ and $250 \text{ users} < N_{us} < 1250 \text{ users}$.

Figure 8 shows the effectiveness of Equations (17) and (18) to describe the effects of the time aggregation on the C_p variation coefficient. Moreover, Figure 8 confirms what has been shown in Figure 3b; in fact $CV_{C_p} = 0.1$ seems to be a good simplification when the number of users is greater than 250 [31,34].

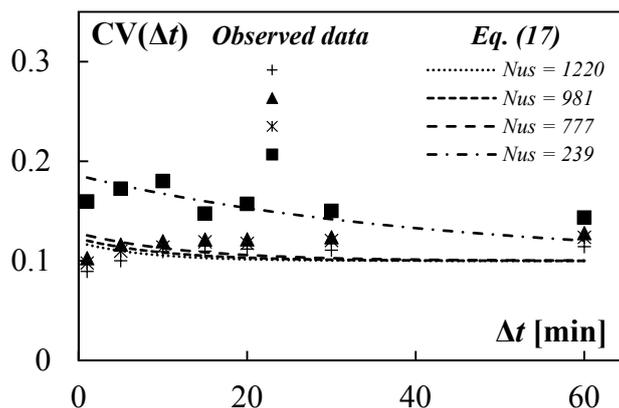


Figure 8. Effects of the time aggregation on the CV.

Equations (9) and (10), combined with Equations (15)–(18), allow estimating the parameters of the suggested distributions, for every value of the time aggregation and users number, when $1 \text{ min} \leq \Delta t \leq 60 \text{ min}$ and $200 \text{ users} < N_{us} < 1250 \text{ users}$.

6. Conclusions

The peak water demand is a very demanding working condition in WDSs and represents an issue of great interest for both design and management problems. The results of the present research give a practical contribution to the characterization of the daily maximum water demand according to a probabilistic approach. The proposed method allows estimating the peak demand through biparametric probability distributions and relations with practical applicability.

By means of statistical inferences which took into account copious data samples related to different residential users, the effectiveness of the Log-Normal and Gumbel distributions was verified to describe the peak water demand during the day. In addition, also the Log-Logistic distribution, whose CDF presents a closed form (as Gumbel distribution), is capable of modeling the maximum daily water demand.

Therefore, some practical equations were suggested, which allow estimating the daily peak demand for predefined values of the success probability.

The investigated case studies are very significant, not for the geographic location, but for the very different users' behavior. In fact, the three towns present daily demand patterns whose peaks are different in number, in time of occurrence and relative entity (e.g., the ratio of the morning peak in respect to the evening one). The probabilistic models proved to be independent of the daily demand pattern and of the peak occurrence time. In fact, although the lifestyles of the investigated users are completely different, the proposed models fitted very well all datasets.

In addition, statistical analysis allowed obtaining effective equations to estimate the parameters of the proposed distributions in relation to the users' number. Even if the investigated range of the aggregated users (200–1250) is limited, it is effective to model the water demand at the nodes of a WDS, because in numerical simulations the number of served users often falls in this interval.

The time resolution has been analyzed investigating the effects of the time step, in the range 1 min–1 h, on the peak phenomena. It was observed that an aggregation of $1 \text{ min} \leq \Delta t \leq 1 \text{ h}$ does not affect the choice of the distribution. In fact, the three distributions (*LN*, *Gu* and *LL*) proved to be all equally effective when modeling the peak water demand for all the investigated Δt . On the other hand, when estimating the parameters of the distributions, corrective coefficients are needed and some effective equations have been proposed for this purpose. These latter allow estimating the parameters of the CDFs in relation to the number of users and the value of Δt . The time resolution analysis considered the range 1 min–1 h, because it covers a wide spectrum of engineering problems (e.g., design and management problems).

In the future, it is desirable that the results herein obtained could be made more robust increasing the data sample, so that the validity field of the models could also be increased. For this end, new in-situ laboratories, whose characteristics are consistent with those described in the work, are necessary.

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List of the Symbols and Acronyms

C_p	[-] Peak demand coefficient
CDF	(acronym) Cumulative Distribution Function
CE	(acronym) Castelfranco Emilia (one of three case studies)
CV	[-] Variation coefficient
CV_{C_p}	[-] Variation coefficient for peak demand coefficient when $\Delta t = 1$ min
$CV_{C_p}(\Delta t)$	[-] Variation coefficient for $1 \text{ min} < \Delta t \leq 60 \text{ min}$
A	(acronym) Demand Monitoring Area
f_x	[depending on variable] Probability density function
F_x	[-] CDF of random variable
Fr	(acronym) Franeker (one of three case studies)
Gu	(acronym) Gumbel distribution
K_F	[-] growth factor
k_Δ	[-] corrective coefficient of $\mu_{C_p(\Delta t)}$, valid $1 \text{ min} < \Delta t \leq 60 \text{ min}$
KS	(acronym) Kolmogorov Smirnov test
LL	(acronym) Log-Logistic distribution
LN	(acronym) Log-Normal distribution
MSE	[depending on variable] Mean Squared Error
n	[-] total data number
N_{us}	[-] Number of users
PSG	(acronym) Piedimonte San Germano (one of three case studies)
Q_p	[L ³ /T] Peak flow of the demanded water during the day
WDS	(acronym) Water Distribution System
x	[depending on variable] Random variable
y	[depending on variable] $\ln x$
α	[depending on variable] Gumbel parameter
β	[-] valid $1 \text{ min} < \Delta t \leq 60 \text{ min}$, valid $1 \text{ min} < \Delta t \leq 60 \text{ min}$
Δt	[T] Time step
ε	[depending on variable] Gumbel parameter, mode
μ_{C_p}	[-] mean peak demand coefficient for $\Delta t = 1$ min
$\mu_{C_p}(\Delta t)$	[-] mean peak demand coefficient for $1 \text{ min} < \Delta t \leq 60 \text{ min}$
μ_Q	[L ³ /T] mean daily water demand
σ	[depending on variable] standard deviation

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