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Review

Application of Analytical Probabilistic Models in Urban Runoff Control Systems' Planning and Design: A Review

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Abstract: Urban stormwater is known to cause a myriad of problems, ranging from flooding to water quality degradations. This paper provides an extensive review of analytical probabilistic model (APMs) used in the design of urban runoff control systems. APMs are closed-form mathematical expressions representing a long-term system's output performance derived from the probability distribution of the system's input variables. Once derived, the APMs are easy to handle, allow for sensitive analysis, and can be co-opted into optimization frameworks. The implementation of APM in the planning and design of runoff control systems will not only help address the runoff quantity and quality problems of urban stormwater, but will also go a long way in optimizing the benefits derived from the systems. This paper reviews studies that document the negative impacts of urbanization on runoff quantity and quality, and the best management practices (BMPs) used to mitigate the impacts. Three design methodologies used in urban stormwater control systems were reviewed. A detailed review of research on the development and use of APMs in urban stormwater management in various runoff control systems is presented, and recommendations are proffered.

Keywords: best management practices; low-impact development; water-sensitive urban design; blue-green infrastructure; sponge cities

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1. Introduction

Urbanization causes a disruption of the natural water cycle. The clearing of land surfaces reduces evapo-transpiration processes that intercept and return rainfall to the atmosphere. Grading the land involves filling depression storage and the removal of topsoil while subsoil is compacted. The construction of impervious surfaces such as roads, roofs and paved walkways reduces infiltration and increases surface runoff. Rainfall that used to infiltrate the ground now runs off the surface at an increased rate, depending on the level of changes made to the land surface. These changes cause an increase in the peak runoff and total volume of runoff. Conversely, the time of concentration of the catchment is decreased, which causes flows across the land surfaces to occur at faster rates. This effect is further aggravated by artificial drainage systems that are designed to convey runoff to rivers as quickly as possible. With the development of impervious surfaces, infiltration into the soil is reduced, thus reducing the quantity of water available to recharge aquifers and feed-in the base-flow during dry weather periods [1–5].

In addition to increasing the runoff quantity, urbanization also affects the runoff quality negatively, by increasing the concentration of pollutants carried by stormwater [6]. As runoff runs over roads, parking lots, rooftops of urbanized areas, it picks up a variety of pollutants and transports them to downstream water bodies. The receiving water body is affected by the cumulative impact of urban activities from the entire watershed, and the resultant changes in stormwater quantity and quality are felt in the downstream waters [7,8].

Several attempts have been made by researchers to quantify the effects of urbanization on stormwater runoff entering receiving water bodies. Todeschini (2016) [9] studied the modifications of stormwater runoff and water quality caused by increased imperviousness in the Bivio Vela industrial catchment in northern Italy. Runoff flows generated from 51 rainfall events of smaller and higher intensities were monitored. The results showed that the conversion of 33% of pervious to impervious surfaces had resulted in a great increase in peak flows, the volume of flows, the number and duration of combined sewer overflows and the pollutants mass discharge. Likewise, Schütte and Schulze (2017) [10] studied the effects of land-use changes due to the proposed urbanization of two sub-catchments of uMngeni catchment (South Africa) on hydrological flows. The study used ACRU software to model the current (2017) and future flows that may arise as a result of the conversion of agricultural lands to impervious surfaces. The results show that increases in impervious surfaces would result in a significant increase in stormflows due to a change in rainfall–runoff conversion caused by the reduced evapotranspiration. Urbanization within a watershed has a number of negative impacts on downstream waters. These impacts include: changes to stream flow and stream geomorphology, degradation of water quality and impact on aquatic habitat [1,4].

Stormwater best management practices (BMPs) are techniques, measures, or structural controls that are used in a given set of conditions to manage the quantity and/or improve the quality of stormwater runoff in the most cost-effective manner [11]. They are designed facilities or modified natural environments that help control the quantity as well as improve the quality of urban stormwater. Urban runoff control systems can be classified in to two BMPs: (1) Methods that are used to reduce the volume of stormwater runoff that will otherwise flow into the receiving water bodies. These methods allow the infiltration of the stormwater into the ground, thereby aiding in groundwater recharge. (2) Methods that remove pollutants from the stormwater through filtration, sedimentation, absorption, biological uptake, etc. [12,13]. However, most stormwater BMPs serve both purposes. The most commonly employed stormwater BMPs include various types of stormwater ponds, filtration practices, vegetated channel practices, wetlands, pervious pavements and rainwater tanks [9]. Green roofs iare also among the most commonly used stormwater BMPs [14–16].

Due to their importance, sustainable stormwater management concepts have been given different names all over the world. Qi et al. (2021) [17] presented a comprehensive list of them, as synonymously adopted all over the world. The list ranges from BMPs to low-impact development, integrated urban drainage systems, sustainable urban drainage systems, stormwater control measures, water-sensitive urban design, resilience cities and sponge cities. Nature-based solutions, green infrastructure, or blue–green infrastructure are other terminologies also used to refer to methods used in mitigating the impact of flood risk related to urban stormwater [18–20]. In its quest to restore its cities following the negative consequences of stormwater runoff due to urbanization, China developed the sponge cities plan in 2013, aimed at promoting source control. The concept uses natural methods to retain rainwater, thereby recharging groundwater, reducing flooding and water pollution problems, and gradually restoring the natural hydrology of the cities. The sponge cities pilot scheme started in 30 cities, and following the successes recorded, the concept is now being adopted at the national level [21,22].

This review paper compiles research on analytical probabilistic models' (APMs') applications to urban stormwater management over the last 35 years, when the models began to debut. A search of the literature was carried out in the SCOPUS and Google Scholar databases using different combinations of the terms: "analytical probabilistic models" AND "stormwater management" OR other BMPs such as "detention ponds, bioretention cells, green roofs, pervious pavements, rain garden, etc." A total of 183 entries were returned by Google Scholar, while SCOPUS returned 45 entries in the first search. The search was repeated and different entries were returned. The entries were filtered and a total of 126 published articles found to be relevant were reviewed. This attempt to compile and review studies on the use of APMs in urban stormwater management is, to the best knowledge of the authors, the first of its kind in this area of research. Figure 1 shows the chronological order of the articles.

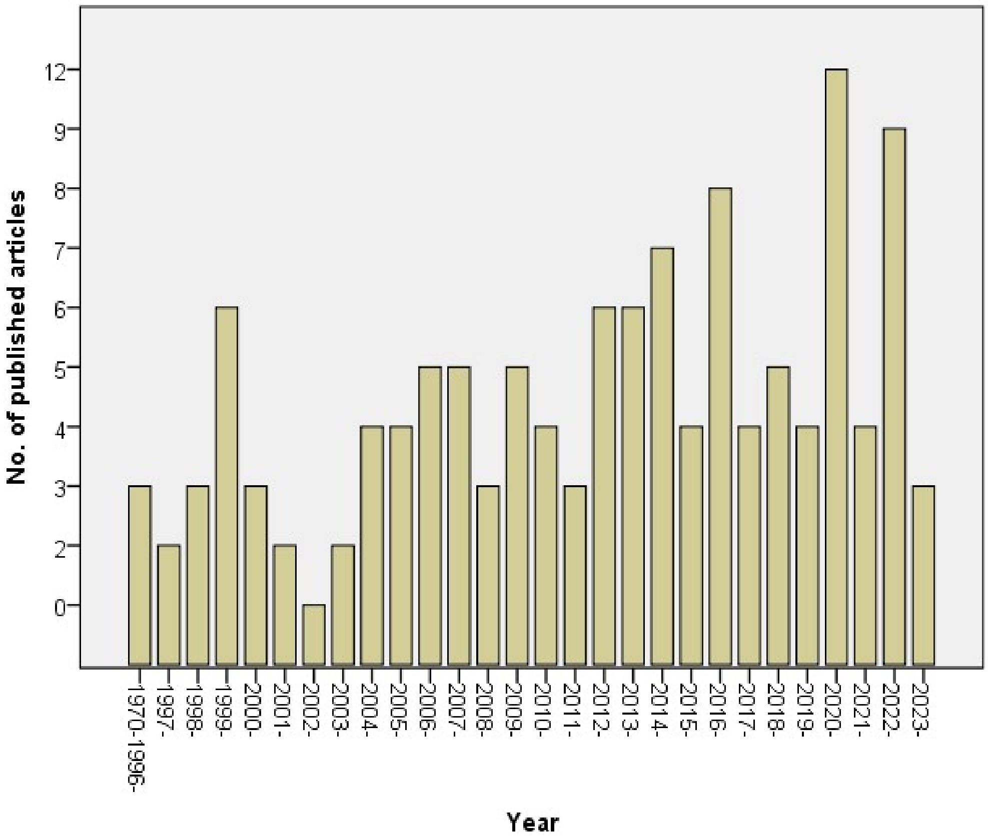

Figure 1. Number of published articles on APMs of stormwater management.

2. Design of Urban Runoff Control Systems

Runoff control systems can be designed based on three approaches: a design storm, continuous simulation and analytical probabilistic models.

The design storm approach uses the statistical analysis of rainfall to establish the IDF curves of an area. A design storm consists of rainfalls of various durations, developed to serve as the input that the runoff control system may experience during its life time [5]. The peak runoff, generated from the design storm, is routed through the runoff control facility in order to estimate the facility's capacity. Many authorities specify design storms in their stormwater regulations. For instance, commonly used regulations specify that post-development peak runoff be less than pre-development peaks for storms having recurrence intervals of 2, 5, and 10 years, with an emergency overflow spillway of 100-year recurrence interval capacity [23]. The duration of the design storm is chosen to be the same as the critical duration or the time of concentration of the catchment. The design storm used for the control system is selected based on risk factors, such as the risk of overflow to downstream conveyance structures and land-use. The assumption being used in the development of the design storm concept is that the recurrence interval of the resulting runoff is the same as the recurrence interval of the rainfall producing the event [24]. This assumption ignores the effect of storm separation time on runoff generation and the temporal variations in storm hyetograph pattern, which are known to affect the behavior of runoff. Due to this, the design storm approach suffers from severe criticisms voiced by many researchers, particularly in the design of stormwater storage systems [24]. Detailed shortcomings of the design approach can be found in [25].

Instead of analyzing the rainfall history to extract concise information, continuous simulation involves the conceptual representation of the catchment and a meteorological input over a longer period. In this case, the effects of storm separation time and the temporal variation of the storm event are captured. The long-term rainfall data are input directly into the continuous simulation software [5]. The result of continuous simulation is the response of the catchment to the rainfall input. However, despite the numerous advantages of the continuous simulation approach, its major problem is its computational burden, requiring a large number of simulations to evaluate system configuration [26]. The models are also extensive, requiring a large computer memory and time for the system analysis [25]. Many types of computer software have been developed for the continuous simulation of a catchment's response to rainfall input, but SWMM is by far the most widely used software. Todeschini et al. (2012) [27] used SWMM to investigate the effectiveness of design configurations and operating conditions of stormwater detention tanks in combination with flow regulators upstream. The results suggest that the optimum performances were obtained by regulating flows into the detention tanks to values between 0.5 to 1 liters per hectare, and tank volumes of 35 to 50 m^3 per hectare of impervious area, for catchments ranging from 4.8 to 48 hectors. These values gave the maximum annual pollutant mass entrapped by the tank and minimum volume of stormwater sent to downstream treatment plants. Mobley and Culver (2014) [28] used the SWMM model of a 12 ha residential site, near Fort Collins, Colorado, to modify the design of a detention pond such that the ecological flows downstream of the catchment were preserved, while maintaining regulatory flow requirements. Pereira et al. (2019) [29] used PC-SWMM to simulate surface runoff, and predict the impacts of urbanization and the use of detention ponds in an urban sub-catchment in Brasilia. Thirteen different configurations of ponds were tested and the best configuration of ponds was observed to be located near the outlet of the catchment. This configuration was found to reduce flood peak discharge by 10 to 30%, and reduce nutrient load by 40 to 60% and COD by 46%. Continuous simulation, though data-intensive and time-consuming, is most commonly used to design or modify the design of existing runoff control systems.

3. Analytical Probabilistic Models (APMs)

APMs are closed-form analytical expressions of a system's output performance derived from the probability distribution of the system's input variables. The approach was initially proposed by Benjamin and Connel (1970) [30]. The approach was first applied to water resources by Eagleson (1972) [31] in water resources engineering, and later extended to the area of stormwater management by many researchers [25,32]. In APMs of urban storm water management, continuous rainfall data are divided into individual events using a pre-defined minimum storm separation time, and the APM parameters are developed using the rainfall statistics [32–34]. The input variable is rainfall characteristics (i.e., rainfall depth, duration, inter-event time), and the output is the catchment's response to the rainfall input (i.e., runoff event volume, peak discharge, etc.). Using derived probability theory, the probability distribution of output variables can be determined analytically if the probability distributions of the input variables are known and the functional relationship between them is either monotonically increasing or decreasing [35].

The application of APMs in urban stormwater management involves the following stages, as described in [24–26,36–46]:

i. Selection of a case study of urban catchment and determination of its physiographic information (catchment area, slope, proportion of imperviousness, drainage length, depression storage, runoff coefficient);
ii. Discretization of the long-term continuous rainfall data into individual events using the minimum inter-event time definition;
iii. Use of probability distributions, such as Exponential, Weibull, etc., to fit rainfall depth, duration and intervention time;
iv. Development of APM parameters for the rainfall station;

v. Development of APMs for stormwater characteristics such as runoff volume and peak runoff captured by the facility, total runoff, pollutants captured and treated by the facility, volume of spilled runoff, etc.

The APMs, once derived, were validated with other known approaches such as design storm and continuous simulation, and the results were generally found to be in good agreement [5,24,26,36–45]. Guo (2001) [5] assessed the suitability of using APMs in the design of urban flood control detention ponds, alongside other approaches, namely, design storm and continuous simulation, using a hypothetical catchment in Chicago. The results show that all the three approaches generated similar results for the prediction of peak flow of various durations from small urban catchments. Quader and Guo (2006) [46] studied the discrepancy in peak flood estimations between design storm and the APM approach. The effects of sub-catchment aggregation and time of concentration, as represented in the two approaches, were examined. A case study catchment of Cataraqui North in Ontario (Canada) was selected, and MIDUSS software was used for the design storm modeling. The results were found to be in good agreement, with only 25% discrepancy between the two peaks generated using the two approaches, which were attributed to sub-catchment aggregation and the choice of time of concentration.

Exponential PDF is the most widely used distribution to model rainfall characteristics, particularly in Canada and the USA, where the APMs of stormwater management were developed. Hassini and Guo (2016) [47] used long-term rainfall data from seven rain-gauge stations in northern USA to test the validity of using one-parameter exponential distribution in modeling rainfall characteristics (depth, duration and inter-event time). Poisson's and Chi-square tests were used. It was found that exponential distribution fits rainfall characteristics well, and was therefore recommended for APM. The exponential PDF is used in more than 80% of the research papers reviewed herein. However, other distributions were tested to determine their fit to rainfall characteristics in other regions of the world. In this regard, Bacchi et al. (2008) [33] compared the use of one-parameter exponential distribution and two-parameter Weibull distribution to model rainfall characteristics for three stations in Italy. The results indicate that the Weibull distribution fits the Italian climate better. The distribution was combined with rainfall–runoff transformation to derive the PDF for the runoff volume and overflow volume of a storage facility, from which the design failure probability can be obtained. Balistrocchi et al. (2009) [48] also used Weibull distribution to model rainfall characteristics for Milano rainfall stations in Italy, while Generalized Pareto Type 3 distribution was used to model rain storm depth with long durations in Toronto, Canada [49]. Pareto and Gamma-2 PDF were found to work well for rainfall depth and duration in Spain [50]. Weibull distribution was also found to fit rainfall characteristics in Poland [51]. Log-normal distribution was found to fit rainfall characteristics for some stations in Korea [52], while three-parameter exponential distribution was used to model rainfall characteristics in Busan (Korea) as an improvement to single-parameter exponential distribution [53].

In rainfall–runoff process modelling from urban catchments, Guo and Adams (1998a) [36] used the exponential distribution to model the frequency distribution of rainfall depth, duration, and inter-event time, from which a closed-form expression for the average annual runoff event volume and the runoff event volume return period was derived. A hypothetical catchment with a different runoff coefficient and various soils was used to test the model against similar results obtained from a numerical simulation model (SWMM). A close agreement between the analytical model and the simulation model was obtained. Similarly, a close agreement was obtained between the results of runoff event volumes and average annual runoff volume with a specified recurrence interval. Guo and Adams (1998b) [37] also used the expression for runoff event volume and its duration together, with the catchment's average time of concentration, to derive an expression of peak discharge rate, using the assumption of a triangular inflow hydrograph and exponential distribution for rainfall depth. A closed-form analytical expression for the exceedance probability of peak discharge per rainfall event and its return period was determined. The results from the analytical model compared favorably with those obtained using a continuous simulation model, SWMM.

Guo et al. (2012) [54] further improved the APM by incorporating both Hortonian and saturated overland flow mechanisms into the model to cater for the increasing use of low-impact development (LID) techniques in urban watersheds, whose infiltration is always below the natural infiltration. The PDFs of runoff event volume under the two scenarios of infiltration excess and saturation excess runoff were derived separately, and combined to obtain APM expressions for the expected value of the runoff event and its recurrence interval, as well as the average annual runoff volume. The results were compared with continuous simulation from HEC-HMS and there was very good agreement. Hassini and Guo (2017) [55] derived APM expressions for the exceedance probability of peak discharge in a small urban catchment considering triangular and trapezoidal hydrographs. Rainfall data from Sherburne, Minnesota in the USA were used and a hypothetical catchment with different times of concentration, imperviousness and soil types was assumed. Design storm (using HEC-HMS model) was used to predict the peak discharges at different return periods. The peak discharges generated from the developed APM were found to be comparable with those of the design storm. Hassini and Guo (2020) [56] further added the effect of saturation excess and infiltration excess runoff to their previous work [24] to develop APMs for runoff event volume and exceedance probability of peak discharge in a small urban catchment. Rainfall data from seven stations in the USA were used and a test catchment in Hamilton (Canada) was used. The results of the APMs were found to be comparable to those of design storm, with a percent difference ranging from 0.1% to 18%. Hassini and Guo (2021, 2022) [57,58] recently developed a new and more accurate APM that can be used for the design of runoff control systems. An APM for the frequency distribution of runoff event volume was developed considering infiltration and saturation excess runoff generation. The new model can effectively estimate runoff volume with different recurrence intervals, and was found to be very sensitive to changes in soil saturation.

APMs have the ability to model flood routing. Guo and Zhuge (2008) [59] developed analytical probabilistic expressions of flood routing to determine the probability distribution of peak outflow from a channel reach with and without a detention pond in between. The outflow hydrographs were obtained, and the results of the analytical models were compared with those of a single-event design storm using stormwater modeling software—the MIDUSS and SWMM surface runoff routing algorithms. It was shown that the analytical models compared well with the design storm. However, the use of different surface routing models gave variations in the results of about 20%. Guo et al. (2009) [60] developed closed-form analytical probabilistic channel routing equations for determining the flood frequency distribution downstream of a catchment, given the catchments' characteristics and APM parameters derived from rainfall data. The model was verified by comparing its results with those of HEC-HMS continuous simulation using 25 reaches and rainfall data from Halifax, Canada. The results of the flood peak attenuations were presented as a function of storage-delay time and return period, which can be used for watershed and stormwater management purposes. Guo and Markus (2011) [61] enhanced the versatility of APMs applied to small watersheds by incorporating SCS-CN for the determination of rainfall excess from the catchments, and Clark's unit hydrograph for runoff routing. The results of the APM were compared with those of design storm using the HEC-HMS model. Twelve watersheds were used in Chicago under urbanizing conditions, and the results show that the analytical model can be used in stormwater management.

Flood peak estimation is another area wherein APMs were also found to be useful. Guo and Dai (2009) [62] expanded the ability of the analytical model to cater for a larger catchment and the master planning of a drainage system. A probabilistic rainfall areal reduction method was used. Both the APM and design storm approaches were used to obtain rainfall frequencies and flood peaks. A case study catchment of the Ganaraska river watershed, Canada was simulated using the OTTHYMO model, and rainfall data from Toronto Pearson International Airport station were used. The results show the capability of APMs in accurately representing the effects of rainfall characteristics across different geographical regions, and their effects on flood frequency.

In the case of modeling the pollutants build-up and wash-off from urban catchments, Behera et al. (2006) [45] assumed that: (1) rainfall duration, inter-event time, pollutant build-up and wash-off follow an exponential distribution; (2) the wash-off load is uniform over the entire catchment and depends on the runoff volume generated, to derive analytical expressions for the PDF of wash-off load, expected value of pollutant event wash-off load, annual average wash-off load and the long-term average pollutant event mean concentration (EMC). The analytical models were calibrated and verified against values observed in a test catchment, and a good agreement was obtained.

The APMs can be used to screen runoff control alternatives in order to determine additional data requirements. Similarly, the APMs can be used in sensitivity analysis to determine the most important parameters, which makes long-term meteorological computation simpler and more economic, supports decision-making and eases stormwater system design [25]. The APMs are computationally efficient and can be easily implemented in a spreadsheet or computer program, as compared to design storm or continuous simulation [54]. Therefore, analytical models can be used as an alternative to time-consuming continuous simulation.

One of the greatest advantages of APMs over design storm and continuous simulation is the option of co-opting them into an optimization framework. The optimization of runoff control systems can be classified based on the objective function (i.e., runoff quantity, quality and/or cost), uncertainty (deterministic or stochastic), and control approach (static or dynamic) [63]. Genetic algorithm, particle swarm optimization, ant colony optimization, artificial bee colony, simulated annealing, harmony search and cuckoo search are some of the optimization techniques that can be applied to optimize flood control systems [17].

4. APM Application to Urban Runoff Control Systems

A schematic diagram showing various stormwater BMPs is shown in Figure 2. APMs have been applied to a wide variety of stormwater BMPs. The application of APMs to these systems in discussed under this section.

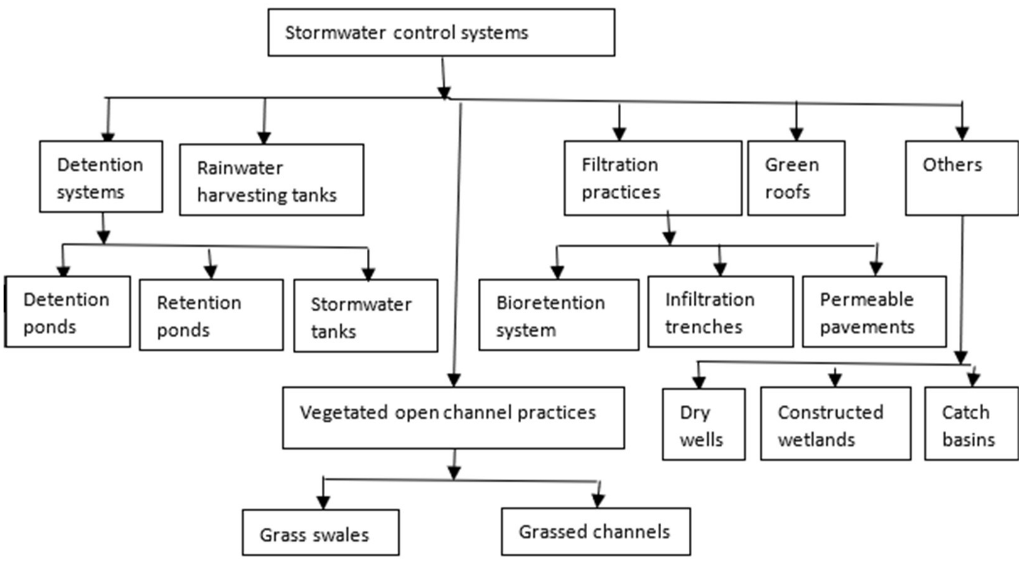

Figure 2. Stormwater BMPs.

4.1. Detention Ponds/Stormwater Tanks

Detention ponds involve the temporary storage of runoff in ponds, basins or even underground containers, and are meant to control the quantity as well as quality of urban runoff downstream of a catchment [64,65]. The purpose of stormwater detention is to reduce the flood damages caused by increased runoff due to imperviousness by limiting post-development peak discharges to be less than or equal to pre-development runoff [66], or to a rate based on other criteria specified by the stormwater authorities in charge [67]. Furthermore, stormwater detention improves the quality of stormwater runoff in addition to reducing the peak discharge [68]. The residence time resulting from stormwater detention allows for the suspended particulate matter and adsorbed contaminants to settle [69,70]. As a BMP, detention ponds can help limit the pollutants loaded into receiving water bodies.

Many researchers have dedicated much attention to the application of APMs in detention basins. Papa et al. (1997) [71] derived APM expressions for the pollution control performance of detention ponds for different combinations of active to permanent pool volumes. The results of the study have been compared to those simulated using SWMM software. It was found that the degree of suspended solid removal in both cases was comparable, with a difference of only 5 to 10% in extended dry ponds and 10 to 30% in wet ponds. Guo and Adams (1999a) [42] derived analytical expressions for the probability distribution of peak outflow rate from flood control detention ponds. The derived analytical expressions were used to determine the storage–discharge relationship required to achieve the specified level of flood control at the facility. Using the runoff volume and peak outflow rate presented in [36,37], the runoff rate exceedance probability per rainfall event was derived based on different combinations of storage and outflow. Comparisons were made between the results obtained from the analytical probabilistic model and similar results obtained from SWMM software, and the results were found to be in good agreement. Guo and Adams (1999b) [43] also used the expressions previously developed in [36,37] to derive APM expressions for the long-term performance of a stormwater quality control pond. The expression of flow capture efficiency was derived from the total spill volume, while the volume-weighted average detention time of the basin was derived by taking into account the variable inflow and outflow rates and the inter-runoff event time. The APM expressions describing the detention time and the statistical solution of flow capture efficiency were compared with similar values obtained from SWMM, and the results were found to be in close agreement, thus confirming the validity of the assumptions made in deriving the models.

Li and Adams (2000) [44] used an analytical probabilistic approach to derive runoff quantity and quality control performances for urban runoff control systems. Rainfall was first transformed to runoff, and the runoff transformed to overflow using the derived analytical expressions. The runoff volume was also transformed to runoff pollution mass load using the EMC concept, and was later transformed to total pollution mass discharge load. The APM expressions for fraction of runoff overflow and total pollution mass discharge load were used to derive closed-form APM expressions for the long-term runoff control and long-term pollution control performances of the stormwater storage and treatment systems. Comparisons between the runoff control performances predicted with the analytical model (coded in computer programs called SUDS and EXSUDS) and those obtained using a continuous simulation model STORM were conducted, and the results were in reasonably good agreement.

Analytical expressions for runoff control performances using different forms of rainfall–runoff transformations were developed [24,26,38–41]. Chen and Adams (2005a) [38] modified the rainfall–runoff transformation to consider infiltration rather than a common runoff coefficient, and developed closed-form analytical expressions for runoff control performances, including exceedance probability of a spill volume, expected value of a spill volume, average annual volume and number of spills, and runoff capture efficiency. The performance of the modified analytical model developed was tested against values obtained from continuous simulation using SWMM and the analytical models developed earlier by [25] for rainfall–runoff transformation (called ASTORM models), and good similarities between the three results were obtained. Chen and Adams (2005b) [39] also used the extended version of rainfall–runoff transformation, which divides the catchment into pervious and impervious areas with different depression storages and runoff coefficients, to develop APM expressions for the average annual number and volume of spills and the runoff control efficiencies. The results of the extended analytical model were compared with those from ASTORM and SWMM, and the results were in good agreement, with the extended model outperforming the ASTORM rainfall–runoff conversion model.

Chen and Adams (2006a) [40] used two types of rainfall–runoff transformations, ASTORM and the extended ASTORM, to derive analytical expressions for stormwater quality control based on build-up and wash-off functions. The appropriate models for pollutant build-up and wash-off (designated as Type 1 and Type 2) were chosen, and were combined to formulate the pollutant load model. Finally, the system quality control measures were derived, which are closed-form expressions that can be used to evaluate the long-term system behavior. Comparisons were made between the quality control models developed with observed values, and the values predicted using SWMM gave good estimates of system performance. Chen and Adams (2006b) [41] also used the derived analytical expressions based on three different rainfall–runoff transformations (i.e., ASTORM, Type 1 and Type 2) to derive APM expressions for stormwater quality control measures. In this case, pollutant removal via the extended detention dry pond was assumed to take place primarily through sedimentation, and TSS control was considered as a surrogate measure of other pollutants' removal. Closed-form APM expressions for average annual volume of runoff, average annual number of spills, average annual runoff control and pollutant removal efficiencies were derived. A comparison of the results from Type 1, Type 2 and SWMM was conducted, and the results were in good agreement.

Chen and Adams (2007a) [26] used the ASTORM rainfall–runoff transformation, extended ASTORM and the modified rainfall–runoff transformation to develop analytical expressions for average annual runoff volume from an urban catchment. In the second case, the Horton's infiltration equation was slightly modified, in that the rainfall duration was assumed to be a temporarily averaged constant. Model verification showed that both of the two analytical models compared favorably with results obtained from SWMM. Chen and Adams (2007b) [24] also used rainfall–runoff transformations, and pollutant build-up and wash-off functions, to derive analytical expressions of the cumulative density function (CDF) of pollutants load, as well as the expected value of pollutant EMC and average annual pollutant EMC. In the rainfall–runoff transformation, two types of models were proposed: the lumped parameter rainfall–runoff, and its extended form [39,44]. Two forms of pollutant load model (Type 1 and Type II) were obtained, and the expected pollutants' EMC and average annual pollutants' EMC values were derived. The pollutant load models were compared with observed values, and a good agreement was obtained. However, the Type II load model was found to outperform Type I in the estimation of average annual pollutants' EMC.

Apart from the tremendous contributions made to the development of APM in relation to detention pond's analysis and design, coming from Canada and USA, some important contributions coming from Italy are noticeable. Becciu and Raimondi (2014) [72] derived APM expressions for the overflow spill of stormwater detention ponds. Two management rules regarding the emptying of the pond were considered. Likewise, the probabilities for spilled volumes varied from zero to one, corresponding to no spill and a spill volume equal to the storage capacity of the pond, respectively. Data of rainfall series from Milano-Monviso, Italy, were used. The resulting analytical expressions can act as very valuable tools that can be used to estimate the overflow probability and the probability of a specific spilled volume. Raimondi and Becciu (2015) [73] used rainfall statistics, detention pond outlet operation rules, storage volume and maximum outflow to derive APM expressions for the pre-filling probability of detention ponds. As in their previous paper, the same management rules regarding the pond's emptying were considered. The results can be used to estimate the pond's volume and out flow rate as a function of pre-filling probability. A comparison of the analytical results with continuous simulation, using case study rainfall data from Monviso, Milano (Italy), showed a very good agreement, thus confirming the applicability of the method in the design and performance assessment of stormwater detention basins. Becciu and Raimondi (2015) [74] derived a similar expression for the PDF of a detention pond's spilled volume in order to evaluate its efficiency. Becciu et al. (2015) [75] also derived APMs of retention time in stormwater detention ponds. The analytical formulae developed can be used for the design of pond storage corresponding to a specified retention time that ensures some pollutants are removed from the pond. The APM expressions were validated against results from a continuous simulation using the case study in Monviso, Milano (Italy), and were found to fit very well. Raimondi et al. (2022) [76] derived APM expressions for the probability of runoff volume and residual storage in sustainable urban drainage systems. The models were applied to two catchments in Genova and Milano (Italy) using rainfall data from Monviso station. In both cases, the results were compared with those from continuous simulation, and were found to be accurate.

Due to the shorter rainfall durations compared to the corresponding dry spell between the rainfall events, some researchers have considered rainfall arrival as a marked Poisson's process, and modeled rainfall characteristics stochastically [77–80]. Wang and Guo (2019) [81] used analytical stochastic models (ASM) to describe the runoff capture efficiency of detention ponds as a power function, rather than linear. The ASM results were compared with the results of an SWMM continuous simulation using a case study catchment area located in Jackson, Mississippi. The values of the root mean square error (RMSE), Nash–Sutcliff efficiency (NSE) and correlation coefficient (R) for runoff capture efficiency were 0.021, 0.994 and 0.9983, while these values for average pond fullness level were 0.012, 0.998 and 0.9997, respectively. This indicates the applicability of ASMs.

Stormwater retention basins can also be analyzed by using stochastic water balance to develop analytical models. Parolari et al. (2018) [82] developed a stochastic water balance model of stormwater retention ponds under passive and active outlet conditions. Analytical expressions of the steady-state and joint PDF of water level and valve closure time, which can be used to define the water level and flow duration curves of the basin, were derived. The model's performance was tested by taking observations of water levels from a retention pond located in Ann Arbor, MI, USA. He results show that the model accurately predicts the water level PDF, which can be used to form a basis for evaluating the changes in rainfall–runoff due to climate change and land-use.

Stormwater detention tanks are used to mitigate the impact of sewer overflow. Balistrocchi et al. (2009) [48] applied APMs to develop a CDF of the overflow volume and pollutant load distribution of a sewer tank. Weibull distribution was used to model rainfall characteristics. Analytical expressions of performance indices such as the decrease in the annual runoff volume and ratio of pollutant load captured by the tank were derived. The model was verified with SWMM continuous simulation, using the urban catchment of Brescia, Italy, and the results were found to be satisfactory. Andres-Domenech et al. (2010) [50] derived analytical PDFs of the number of overflows, volume of overflows and overflow reduction efficiency of a stormwater tank. Rainfall data from Valencia and Santander, Spain using different probability distributions were tested. Pareto and Gamma-2 PDFs were found to fit well. The analytical results regarding long-term volumetric flow and overflow reduction efficiencies were compared with those of IW continuous simulation, and were found to be similar. Becciu and Raimondi (2012) [83] developed APM expressions for the pre-filling probability of stormwater tanks. The effects of minimum inter-event time definition on outflow rate and storage volume were investigated using rainfall data from Monviso, Milano, Italy. The results of the APM were compared to the results of continuous simulation, and it was shown that the APM underestimated the pre-filling probability due to some assumptions made in the development of the model. Thus, the model needs to be refined further. Stormwater tanks, designed using APM, have also been found to be capable of improving the quality of sewer discharges from catchments along the Tyrrhenian coast of Italy [84].

Detaining runoff in stormwater detention ponds for a longer period improves the quality of the treated runoff, but this poses the risk of overflow from subsequent rainfall, which may generate runoff. There is an optimal detention time in the facilities such that the trade-off between runoff and pollution control is addressed [18]. There is also a need to minimize the cost of building the facility, while at the same time achieving the objectives. Papa and Adams (1997) [85] used APM expressions to develop a dynamic programming model for the optimization of the cost of building detention ponds in multiple parallel catchments, subject to meeting runoff quality control constraints. Shamsudin et al. (2014) [86] used long-term rainfall data to obtain the rainfall characteristics and develop APM parameters for a catchment in Malaysia. The APM was coded via particle swam optimization (PSO) to develop a methodology that addresses the trade-off between the runoff and pollution control performances of detention ponds. The detention pond's volume and outlet were appropriately sized such that a least-cost design was obtained.

Dan'azumi et al. (2013a) [35] developed APM parameters relevant to the rainfall characteristics of Malaysian cities, and used the parameters to develop an optimization algorithm via PSO that can be used to optimize detention time in wet detention ponds such that they give the best pollution control performance [87]. Behera and Teegavarapu (2015) [88] used the APM expressions of pollution control in extended wet detention ponds to compare the results of three optimization techniques: dynamic programming (DP), non-linear programming (NLP) and genetic algorithm (GA). They sought to obtain the optimal values of pollution control, pond depth, storage volume and release rate of ponds treating urban stormwater from multiple sub-catchments releasing their outflow into a common downstream point, such that the quality control target at the downstream river could be met at minimum cost. The results show that the NLP and GA provided an improved solution compared to the DP.

4.2. Rainwater Harvesting System/Rainwater Tanks

Rainwater tanks, consisting of rain-barrels and cisterns, are rainwater harvesting systems (RHS) that store rainwater for household use and reduce the volume of runoff generated from urban surfaces. The use of rainwater tanks reduces water consumption from municipal supply, and thus reduces the water bill. The water stored in the tank can be used for gardening and toilet flushing, thus reducing municipal water consumption. Some rainwater tanks have two compartments: the rainwater tank itself and an infiltration facility, which aids in groundwater recharge [89,90].

Raimondi and Becciu (2014a) [89] developed APM expressions to estimate the probability of meeting the water demand using rainwater tanks as a function of household population and number of storm events occurring, using long-term rainfall data from 35 years at the Milano-Monviso station. The results of the study can be used to determine whether it is efficient to use rainwater harvesting alone, or in combination with municipal water supply. Raimondi and Becciu (2014b) [90] developed APMs for the design of multi-use rainwater tanks. These rainwater tanks were designed to have two basins: a rainwater basin and an infiltration basin. A trade-off between the risk of water shortage in the basin and the risk of overflow was studied. The results of a case study in a catchment in Milan, Italy, show that the probability of complete rainwater use in a household depends on the period of regulation, with weekly regulation yielding a higher probability compared to daily regulation. Additionally, the probability of overflow was high for a small storage volume and low infiltration rate. Becciu et al. (2016) [91] improved on their previous models by considering the effect of re-filling during the regulation period, and developed an analytical expression to estimate the CDF of active storage in the rainwater tank. The results were compared with those of a continuous simulation model using data from Milano, Italy, and there was a good agreement.

Guo and Baetz (2007) [78] derived an analytical expression that could be used to design rainwater storage units in green buildings, focusing on the rate of water use in the building, the climate characteristics of the area and the reliability of the system. The APM was applied to a hypothetical catchment in Chicago and Montana, USA, and it was shown that the APM provided an efficient approach to designing the system. De Paola and De Martino (2013) [92] studied the efficiency of four stormwater tank configurations using SWMM, and applied the semi-probabilistic approach to determine the qualitative and quantitative stormwater capture efficiencies of the most efficient tank configuration. It was concluded that the analytical approach provided similar results to continuous simulation. Kim et al. (2012) [93] used mass balance equations for each component of a rainwater tank to develop APM expressions for the rainfall–runoff reduction in an RHS. The PDF and CDF of runoff from the catchment and the RHS were derived, and the expected value of runoff volume was determined. The model was applied to a dormitory building in Seoul (Korea) to design an RHS and to estimate the runoff reduction achieved as a result of it. Di Chiano et al. (2023) [94] used APM expressions to derive the CDF of active storage in RHS. Active storage was considered as a function of rainfall moments, water demand and mean number of chained events under deficit conditions. The results of the model were compared with those of continuous simulation, using rainfall data from Monviso, Milano (Italy), focusing on a case study of RHS in Milan. An average normalized RMSE of 0.033, under three demand conditions, was obtained between the APM and the continuous simulation, suggesting a very good prediction.

Stochastic mass balance equations of RHS have been used to develop analytical models for RHS systems. Guo and Guo (2018a) [95] derived an ASM that could be used to determine the size of an RHS using a differential mass balance equation. Analytical expressions of a rainwater tank's efficiency in terms of water supply reliability, required storage volume and its runoff reduction benefits were derived. The stochastic models, developed using rainfall data from five different climates (Atlanta, Concord, Detroit, Flagstaff and Billings) in the USA, were validated against the results obtained from SWMM continuous simulation and also those of Guo and Baetz (2007) [78]. The values of mean Nash–Sutcliffe efficiency (NSE), root mean square error (RMSE) and correlation coefficient of 0.98, 0.035 and 0.99, respectively, were obtained, indicating a good result. Pelak and Porporato (2016) [96] modeled rainfall as a marked Poisson's process, and developed an analytical expression that optimizes the volume of a rainwater harvesting system at minimum cost. The volume was expressed as a function of rainfall parameters, roof area, water use rate, and the cost of the cistern and that of the external water source. The cost consists of fixed and distributed costs. The result of the study can be used to size an RHS in any climate. This will help reduce urban stormwater runoff and water consumption from public mains. Sim and Kim (2020) [97] used stochastic mass balance to develop an analytical model for the quantification of the water supply and stormwater interception efficiency of an RHS. In the study, the sensitivity of the RHS to climate change was evaluated, and the model was assessed using rainfall data from Busan (Korea). The results of the analytical model were compared with those derived using multiple regression. The R^2 and RMSE values for water supply and stormwater interception efficiency ranged from 0.91 to 0.96 and 0.026 to 0.033, respectively. Cheng et al. (2021) [98] also used water balance to develop a stochastic model of an RHS. Due to the random occurrence of rainfall, the reliability of the model was expressed in terms of the fraction of time for which the RHS satisfies water demand. The model was applied to three RHSs in Toronto, Canada, and was found to have high accuracy.

4.3. Green Roofs

A green roof is a rooftop garden. These are used to provide shade, reduce the temperatures of the roof surface and surrounding air, and to moderate the heat island effect [99]. Green roofs comprise four layers: a vegetation layer, a substrate layer, a drainage layer and a waterproof layer. Some green roofs have a water storage layer combined with the drainage layer for holding more rainwater. Vegetation is planted on top of the substrate layer, where rainwater is retained. Excess rainwater from the roof is drained through the drainage layer [100].

Researchers also explored the application of APMs to green roof design and analysis. Zhang and Guo (2013a) [79] derived analytical expressions for runoff generation from green roofs. The results obtained from the analytical models were compared with those of continuous simulation using the LID module of SWMM, and also from the field results derived from a real case study in Portland, USA. The results of the APM were found to be in good agreement with both. Additionally, Guo et al. (2014) [101] derived analytical expressions for long-term average runoff reduction rates (defined as the ratio of total runoff captured to that of total runoff generated) and the irrigation water requirement of green roofs. The performance values of the APM in terms of runoff reduction rates and irrigation time fraction at different growing medium depths under semi-arid climate (Atlanta, USA) and humid climate (Billings, USA) conditions were compared with those from continuous simulation using SWMM, and it was concluded that the APM can be used as an alternative to SWMM in the planning, design and management of green roofs.

Guo (2016) [102] further refined the work of [101] by considering rainfall occurrence as a stochastic process to derive a stochastic differential equation of green roofs. The stochastic water balance equation was formulated to determine the mean and PDF of the moisture contents of green roofs. The accuracy of the model, in terms of runoff reduction rates and irrigation time fraction, was evaluated by testing the results against those of SWMM continuous simulation using four sets of rainfall data from Billings, Phoenix, Atlanta and Boston (USA), and using sandy and loamy soils as growing media. The comparison of results between SWMM and the stochastic model implied the good correlation coefficients of 0.993 and 0.995, respectively, for runoff reduction rates and irrigation time fraction. Most studies assume that the RHS is dry at the beginning of the rainfall event. However, some moisture retention is possible in the roof at the beginning of the next rainfall period. Raimondi and Becciu (2020) [100] considered the possibility of pre-filling from previous rainfall events to develop an APM for the design of green roofs. The APM was tested against the results of continuous simulation, using rainfall data from the Milano Monviso (Italy) station. The results show that the model compared well with continuous simulation. Thus, the APM can be used for the optimization of the design of green roofs. Raimondi et al. (2022a) [103] extended their work from 2020 to develop an APM that could be used to determine the thickness of the substrate layer of green roofs as a function of runoff reduction. The results of the study compare well with those obtained from continuous simulation.

Raimondi et al. (2022b) [104] used APM to develop a model for the survival of vegetation on green roofs without the need for irrigation. The thickness of substrate medium and risk of vegetation withering were combined to design the green roof. The APM was tested using two green roofs in Milano and Calabria (Italy). The results from the analytical model compared excellently with those of continuous simulation. Guo et al. (2022) [105] used a stochastic model of rainfall to model the hydrologic and hydraulic process occurring on green roofs. Both the saturation excess runoff and infiltration excess runoff were considered, and analytical equations that can be used for the quantification of the performance of green roofs were derived. The results of the analytical model were compared with those of continuous simulation, and were found to be accurate. Raimondi et al. (2023) [106] used APMs to determine the probability that runoff from a green roof will exceed a certain threshold, given the substrate thickness, climatic variable and moisture content of the roof. The reduced retention capacity of the system due to previous rainfall was also considered. The analytical model was tested using a case study in Milano (Italy), and the results were similar between the model and the continuous simulation.

4.4. Filtration Practices

Filtration practices are surface or underground practices that reduce the volume of runoff by infiltration through the soil. They provide a performance that is independent of local conditions, and their designs are applicable to roadside and congested urban conditions. According to [107], bioretention cells and sand filters are amongst the filtration practices commonly used for small to medium catchment basins, because they usually occupy only 2 to 3% of the drainage area, and hence are suitable in dense urban settings. Sand and gravel filters are also commonly used as filtration practices for the management of urban stormwater [108]. Other filtration practices include pervious pavements, etc.

4.4.1. Bioretention Cells/Biofilters/Rain Gardens/Impervious Area Disconnection

Bioretention systems are shallow landscaped depressions, commonly located in parking lots or within residential land-use areas, that are designed to incorporate many of the pollutant removal mechanisms that are operated in forested ecosystems. They are also known as biofilters or rain gardens [13]. Stormwater treatment in a bioretention cell is achieved through sedimentation, filtration, soil adsorption, micro-biological decay processes and the uptake of pollutants by plants [109]. The components of a bioretention area include a grass buffer strip, planting soil, plant material, a ponding area with surface mulch, an underground sand bed, an organic layer and infiltration chambers [110].

Daly et al. (2011) [13] tracked the water balance of a biofilter by considering its inflow variability, filter media and vegetation type. An analytical model for the long-term PDF of soil moisture content of the filter, and the statistics of outflow, evapotranspiration and overflow, were derived. The total nitrogen removal performance was also estimated from the model. The results of the analytical model were tested with real data collected from a biofilter in Malborne, Australia, and it was shown that the model could be used to assess the performance of biofilters across different climates.

Zhang and Guo (2014) [111] modeled runoff from both pervious and impervious urban surfaces to develop closed-form APM expressions for the stormwater runoff capture efficiency of bioretention cells. The results obtained were compared to those of an SWMM continuous simulation model, and close agreement was observed, thus validating the APM expression. However, some assumptions were made regarding the amount of water present in the cells before any rainfall event, which need to be considered in extreme cases. Accordingly, Guo et al. (2020) [112] refined the work of [111] to consider wider ranges of application. A dynamic water balance was considered to stochastically model the hydrology of bioretention cells. Analytical expressions for the long-term runoff capture efficiency, the fraction of time for which the cell processes runoff, the average water stored inside a cell, and the storage capacity required to achieve capture efficiency were derived. These four performance indices generated by the ASM were compared with the results of continuous simulation, and close agreements were obtained, thus verifying the applicability of the ASM.

The resilience and reliability of using bioretention cells as runoff control systems was studied by [113]. APM expressions were used to evaluate the resilience indices related to the system's robustness, rapidity and serviceability in the context of extreme runoff events. The results of the APM were compared with those generated using the continuous simulation SWMM. Resilience indices of 0.66 to 1.0 and 0.73 to 1.0, respectively, were observed for the APM and SWMM. The reliability index found ranged from 56% to 100% and 60% to 100% for the APM and SWMM, respectively.

Impervious area disconnection is a system that works in a similar way to bioretention cells. Runoff from urban surfaces (roof tops, pavements) is made to pass through pervious surfaces (grassed area), where processes such as infiltration and pollutant removal occur, thus reducing the volume of surface runoff. The time of concentration in the catchment is also reduced, thereby reducing the peak discharge from the catchment. Wang et al. (2019) [81] determined the effect of impervious area disconnection on runoff reduction from two urban catchments in the USA. Two different catchments' soils (sandy and loamy) were used. The runoff reduction due to impervious area disconnection was examined using different imperviousness ratios. The results of the APM and SWMM were compared, and in all cases, impervious area disconnection was found to significantly reduce the volume of runoff to the sewer system, and the APM results compared very well with those of the SWMM. Zhang and Guo (2013b) [80] studied the hydrologic operations of a rain garden to derive analytical expressions for its long-term runoff capture efficiency. The results from the APM model were compared with those of SWMM simulations, and a very good agreement between the APM and continuous simulation results was observed. The APM was applied to rain gardens in Atlanta and Flagstaff in USA to demonstrate the sensitivity of runoff capture efficiency to specific model parameters.

4.4.2. Infiltration Trenches/Basins

Infiltration trenches are rectangular excavations with void-forming materials, such as gravel aggregates, which are designed to receive, filter, store and infiltrate urban stormwater. They aid in reducing urban runoff and improving groundwater recharge. They also assist in sediment and heavy metal removal from stormwater [114]. Guo and Gao (2016) [115] derived analytical expression for the total annual overflow volume and total runoff reduction rate of infiltration basins. The results of the APM were compared with those of SWMM continuous simulation using rainfall data from Atlanta and Billings (USA), and the results were found to be consistent, with a relative difference of less than 10%. Guo and Guo (2018b) [116] derived APM expressions for the overflow frequency and stormwater capture efficiency of non-vegetated infiltration facilities, such as infiltration trenches, infiltration chambers, dry wells, etc. In deriving the expressions, infiltration was assumed to occur at the bottom only. The results from the analytical expressions were compared with those of SWMM simulations in relation to a case study of a catchment in Concord, New Hampshire (USA), using sandy and loamy soils, and the two sets were found to be in good agreement. The average absolute difference and average relative difference between the APM and SWMM were found to be 0.04% and 5%, respectively. Wang and Guo (2020a) [117] analyzed the water balance of infiltration-based BMPs by considering infiltration through their sides and bottom, in an attempt to overcome the shortcomings of [108]. The mean degree of saturation and mean runoff capture efficiency were derived, and the results of the analytical model were compared with those of SWMM. Two soils, sandy and loamy, were used, and the rainfall data from two climate conditions (Billings and Jackson) were used to develop the APM model. The results were found to be reasonably comparable, with the largest absolute relative difference being less than 10%.

Following the design guidelines released by the Atlanta and New Durham authorities, Wang and Guo (2020b) [118] applied the analytical models they had developed earlier in [117] to a practical design analysis of infiltration trenches. Runoff values, generated using rainfall data from hypothetical catchments, in the two locations were assumed, and the performance of the trench was assessed as a function of its soil type, footprint dimensions, drain time and infiltration conditions. The results of the runoff reduction ratio indicate that the conditions of infiltration through the sides, the bottom, and both combined have profound effects on the runoff reduction ratio, by up to 15%. The runoff reduction ratio was found to be most highly affected by changes in soil type and trench dimensions.

4.4.3. Permeable Pavements

Pervious pavements consist of pavements made with porous blocks or porous asphalt that permits water to infiltrate. Pervious pavements may also be made from impervious blocks that are fitted in such a way that water can pass between them. They can be used in road surfaces with light traffic or in car parks. The infiltration rate through the pavement may be as high as 1000 mm/h in new developments, although this value may reduce to 10% of the original value over the lifetime of the pavement [119]. Zhang and Guo (2015) [120] derived analytical expressions for the runoff capture efficiency of a permeable pavement as an LID system to mitigate the impact of urban stormwater. SWMM simulations were run on the modeled pavement in order to validate the APM expressions, and the results showed little discrepancy. It is recommended that the APM results be compared with those of a case study on real-life pavements.

Stochastic differential equations of permeable pavements were used by Guo et al. (2018) [121] to model the dynamic water balance of their system. Rainfall and the corresponding net inflow were represented as a marked Poisson process to develop the PDF of inflow volume, and to derive analytical expressions for the long-term stormwater capture efficiency and moisture content of permeable pavements. The results of the APM were compared with those simulated using SWMM using data from the four climates of Atlanta, New Durham, Charlotte and Flagstaff (USA), and were found to be very similar.

Three runoff control systems—bioretention cell, permeable pavement and green roofs—were compared to determine the most cost-effective. The runoff reduction efficiencies and licecycle costs of implementing each of them were considered. APM expressions were combined with a genetic algorithm for the optimization. The objective function was to maximize runoff reduction capacity and minimize the lifecycle cost. The results show that the bioretention cell had the greatest runoff reduction capability, but given the high land cost in urbanized areas, permeable pavements are the most reasonable option [122].

4.5. Vegetated Open Channel Practices

These are systems designed to treat stormwater runoff in a swale or channel formed by check dams or other processes. Usually, they do not allow for quantity control, and are therefore combined with other stormwater BMPs to meet regulations. These systems directly receive runoff from an impervious surface; they have a temporary ponding time of less than 48 h and feature a 6-inch drop onto a protected shelf to minimize the clogging potential of the inlet [4]. Up to the time of submitting this review, no publications have been found on the application of APMs to vegetated open channel practices. This issue can be explored by future researchers. Two of the common types of vegetated open channel practices include grassed swales (dry/wet) and grassed channels.

According to [123], grassed swales are broad, shallow earthen channels used to treat stormwater runoff using flood-tolerant and erosion-resistant grasses. Filtering via these practices occurs through the vegetation, a subsoil matrix, and infiltration into the underlying soils. Grassed swales feature gentle longitudinal slopes, with check dams perpendicular to the flow so as to slow down the flows and allow the particulates to settle. There are two types of grass swales—dry swales, which have a filter bed of prepared soil laid over an under-drain system, and wet swales, which are designed to sustain moisture conditions that support wetland vegetation [124].

Grassed channels are used in pretreatment practices that provide nominal treatment, because they lack the filter media present in grassed swales. They act by allowing the infiltration of some runoff from small storms into areas with pervious soils, and are therefore most highly applicable to other structural stormwater BMPs [123]. They help in reducing the effect of imperviousness, and provide aesthetic benefits. Grassed channels are designed for use on <4% flat slopes with infiltration rates greater than 0.27 inches per hour. The stormwater runoff takes an average of 5 min to flow from the top to the bottom of the channel. For efficient usage, the channel should be used to treat small drainage areas of less than 5 acres. For the effective removal of particles, the grass of the channel should be maintained at a height of 3 to 4 inches [4].

4.6. Other Stormwater BMPs

Other types of stormwater BMPs that are used to control urban stormwater runoff include: constructed wetlands, dry wells, artificial marshes, oil/greet separators, catch basins, etc. [119,125]. Their effectiveness can be represented via a decrease in the SCS curve number of the basins. Perez-Pedini et al. (2005) [126] determined the optimal number and location of infiltration facilities in a watershed for the purpose of peak flow reduction at the watershed outlet. The watershed was discretized into 4553 hydrologic response units, whereby each unit represents a 120×120 m plot of the watershed. Different types of infiltration-based BMPs were conceptualized as binary integers that decrease the curve number of hydrologic response unit by five. The results of the optimization show that the optimal number and location of infiltration-based BMPs depends on various factors, such as flow travel time, catchment network connectivity, land-use, contributing area, and distance to the channel. APMs of stormwater management can be applied to constructed wetlands, dry wells, artificial marshes, catch basins, etc., in future studies.

5. Recommendations for Future Direction

Urban stormwater raises flood and water pollution problems for many communities across the world, and the cost of the damage cannot be easily quantified. This paper has reviewed the literature on APM applications in urban stormwater management. Once derived, the APM models can easily be co-opted into any optimization frameworks, thereby giving the freedom to maximize benefits and minimize cost. The following recommendations are given:

(a) The APM parameters were obtained from analyses of the long-term data on rainfall depth, duration and inter-event time. To make them more applicable, it is necessary to develop a comprehensive database of APM parameters describing rainfall characteristics in cities across the world, for the purpose of runoff control systems design;
(b) Most rain-gauge stations, particularly in developing countries, record daily rainfall only. Urban catchments have shorter times of concentration, and studies in these parts of the world have to rely on rainfall disaggregation techniques, whereby daily rainfall is broken down to an hourly or even sub-hourly time scale, which may raise some reliability problems. There is a need for a database of finer-resolution rainfall data. The provision of a large network of hourly and sub-hourly rain-gauge data will not only be useful to urban hydrologists, but also to other professionals. It will also help in reducing the uncertainties caused by rainfall disaggregation. Another source of uncertainty is the spatial distribution of rain-gauge stations used to develop rainfall characteristics and APM parameters. Research is required into the effects of the spatial distribution of rain-gauge stations on the reliability of rainfall characteristics;
(c) The APM parameters were derived based on minimum inter-event times of 2 h, 6 h, 12 h and 24 h. In the case of small urban catchments, with faster concentration, it is recommended that a database of APM parameters based on a smaller discretized inter-event time, such as 5 min, 15 min, 30 min or 1 h, be developed. This requires the archiving of rainfall data at a sub-hourly resolution, which could then be used to develop its own database;
(d) There is uncertainty about the inter-event time value to be used in rainfall event aggregation from a continuous time series. This calls for further research on its reliability;
(e) The APMs are mostly based on the exponential distribution of rainfall characteristics. Rainfall characteristics were also found to follow other distributions, such as Gamma, Weibull, and log-normal. A distribution fit test for other PDFs needs to be undertaken in different climates;
(f) A decision support system that incorporates meteorological, catchment and runoff control systems' characteristics altogether needs to be developed, which can then eventually be used in the design and real-time control of these systems;
(g) The design of some systems, such as rainwater tanks, involves the consideration of rainfall variability vis-à-vis water demand and the cost of municipal water consumption. Likewise, designing detention ponds for runoff quantity and pollution control involves conflicting objectives. There is a need for studies that embed APMs into optimization techniques so as to derive optimum benefits from the runoff control systems at the least cost;
(h) Climate change is known to affect the design of stormwater conveyance and storage systems. There is a research gap regarding the effect of climate change on runoff control systems designed using APMs. The impact of climate change on the reliability of the systems needs to be investigated, so as to ensure their design functions are met;
(i) There is a research gap regarding the APMs related to the runoff reduction efficiency and pollution control performance of vegetated open channel technologies, such as swales, grass channel, etc.;
(j) Different runoff control systems have been reviewed in this paper. Some systems may be more suitable to specific climates. There is a research gap in the determination of the best system for each specific geographical area.

6. Conclusions

Urban stormwater runoff is detrimental to downstream drainage systems and to receiving water bodies. The risks range from flooding to water pollution. This paper has reviewed the literature on runoff control systems, such as detention basins, rain gardens, rainwater harvesting system, bioretention cells, pervious pavements, infiltration trenches, etc. The design of runoff control systems can be carried out using the traditional design storm approach, continuous simulation and APMs. The major flaw of the design storm approach is its inability to capture the effects of inter-event time. That is, the design storm assumes that the recurrence interval of runoff is the same as that of the rainfall that causes it. The continuous simulation approach, on the other hand, is laborious and time-consuming, thus making it unsuitable for use at the planning stage of a runoff control project. APMs, however, are more compact, easy to use, and offer a direct way to conduct sensitivity analyses in routine planning projects. Moreover, APMs are flexible and can be co-opted into an optimization framework. Despite their simplicity, the APMs provide results that are as accurate as those of continuous simulation. This paper offers an extensive review of the applications of APMs to urban stormwater management.

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Abbreviations

ACRU	Agricultural Catchments Research Unit
APM	Analytical Probabilistic Models
ASM	Analytical Stochastic Models
ASTORM	Analytical STORM
BMPs	Best Management Practices
CDF	Cumulative Distribution Function
COD	Chemical Oxygen Demand
DP	Dynamic Programming
EMC	Event Mean Concentration
EX-SUDS	Extended Sustainable Urban Drainage System
GA	Genetic Algorithm
HEC-HMS	Hydrologic Engineering Center's Hydrologic Modeling System
IDF	Intensity–Duration–Frequency
MIDUSS	Microcomputer Interactive Design of Urban Stormwater Drainage Systems
NLP	Non-Linear Programming
NSE	Nash–Sutcliff Efficiency
OTTHYMO	Ottawa Hydrological Model
PC-SWMM	Personal Computer–Storm Water Management Model
PDF	Probability Density Function
PSO	Particle Swam Optimization
RMSE	Root Mean Square Error
SCN-CN	Soil Conservation Service–Curve Number
STORM	Stormwater Management Software
SUDS	Sustainable Urban Drainage system
SWMM	Storm Water Management Model

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