

## Supplementary File

### The Relationship between L-Moments and LH-Moments

Relations for LH-moments [16]:

$$L_1^\eta = E \left[ X_{(\eta+1):(\eta+1)} \right]$$

$$L_2^\eta = \frac{1}{2} \cdot E \left[ X_{(\eta+2):(\eta+2)} - X_{(\eta+1):(\eta+2)} \right]$$

$$L_3^\eta = \frac{1}{3} \cdot E \left[ X_{(\eta+3):(\eta+3)} - 2 \cdot X_{(\eta+2):(\eta+3)} + X_{(\eta+1):(\eta+3)} \right]$$

$$L_4^\eta = \frac{1}{4} \cdot E \left[ X_{(\eta+4):(\eta+4)} - 3 \cdot X_{(\eta+3):(\eta+4)} + 3 \cdot X_{(\eta+2):(\eta+4)} - X_{(\eta+1):(\eta+4)} \right]$$

where  $\eta = 0, 1, 2, 3, \dots$ . For  $\eta = 0$ , the method is identical to the L-moments method.

Based on these LH-moments, the statistical indicators are:

$$\tau_{H2} = \frac{L_2^\eta}{L_1^\eta} - \text{represents coefficient of LH-variation}$$

$$\tau_{H3} = \frac{L_3^\eta}{L_2^\eta} - \text{represents LH-skewness}$$

$$\tau_{H4} = \frac{L_4^\eta}{L_2^\eta} - \text{represents LH-kurtosis}$$

The relations between the high-order linear moments and the normalized weighted moments are the following:

$$L_1^\eta = B_\eta$$

$$L_2^\eta = \frac{(\eta+2)}{2} \cdot (B_{\eta+1} - B_\eta)$$

$$L_3^\eta = \frac{(\eta+3)}{6} \cdot ((\eta+4) \cdot B_{\eta+2} - 2 \cdot (\eta+3) \cdot B_{\eta+1} + (\eta+2) \cdot B_\eta)$$

$$L_4^\eta = \frac{(\eta+4)}{24} \cdot \left( (\eta+6) \cdot (\eta+5) \cdot B_{\eta+3} - 3 \cdot (\eta+5) \cdot (\eta+4) \cdot B_{\eta+2} + \right. \\ \left. 3 \cdot (\eta+4) \cdot (\eta+3) \cdot B_{\eta+1} - (\eta+3) \cdot (\eta+2) \cdot B_\eta \right)$$

where  $B_\eta = (\eta+1) \cdot b_\eta$  and  $b_\eta$  represent the natural estimators expressed by the L-moments method. For  $\eta = 0$ , the method become the L-moments method.

### The second LH-moments for the GEV, W3, PG, LL3 and RY distributions

#### GEV distribution

$$L_{H1} = \gamma + \frac{\beta}{\alpha} \cdot (1 - \Gamma(1 + \alpha) \cdot 3^{-\alpha})$$

$$L_{H2} = 2 \cdot \beta \cdot \Gamma(\alpha) \cdot (3^{-\alpha} - 4^{-\alpha})$$

$$L_{H3} = \frac{5}{3} \cdot \beta \cdot \Gamma(\alpha) \cdot (5 \cdot 4^{-\alpha} - 2 \cdot 3^{-\alpha})$$

Parameter  $\alpha$  can be approximate using the next relation depending on  $\tau_{H3}$ :

$$\alpha = \frac{0.590985391 - 2.285515331 \cdot |\tau_{H3}| + 0.427911532 \cdot \tau_{H3}^2}{1 + 0.082080011 \cdot |\tau_{H3}| - 0.081733267 \cdot \tau_{H3}^2 + 0.008299996 \cdot |\tau_{H3}|^3}$$

$$\beta = \frac{L_{H2}}{2 \cdot \Gamma(\alpha) \cdot (3^{-\alpha} - 4^{-\alpha})}$$

$$\gamma = L_{H1} - \frac{\beta}{\alpha} \cdot (1 - \Gamma(1 + \alpha) \cdot 3^{-\alpha})$$

### Weibull distribution

$$L_{H1} = \gamma - \frac{\beta}{\alpha} \cdot 6^{-\frac{1}{\alpha}} \cdot \Gamma\left(\frac{1}{\alpha}\right) \cdot \left(3^{\frac{1}{\alpha}+1} - 3 \cdot 6^{\frac{1}{\alpha}} - 2^{\frac{1}{\alpha}}\right)$$

$$L_{H2} = \frac{2 \cdot \beta}{\alpha} \cdot 6^{-\frac{1}{\alpha}} \cdot \Gamma\left(\frac{1}{\alpha}\right) \cdot \left(6^{\frac{1}{\alpha}} + 3 \cdot 2^{\frac{1}{\alpha}} - 3^{\frac{1}{\alpha}} \cdot \left(2^{\frac{1}{\alpha}} + 3\right)\right)$$

$$L_{H3} = \frac{5 \cdot \beta}{3 \cdot \alpha} \cdot 6^{-\frac{1}{\alpha}} \cdot \Gamma\left(\frac{1}{\alpha}\right) \cdot \left(6^{\frac{1}{\alpha}} \cdot \left(3 \cdot 5^{\frac{1}{\alpha}} + 1\right) + 12 \cdot 2^{\frac{1}{\alpha}} - 3^{\frac{1}{\alpha}} \cdot \left(10 \cdot 2^{\frac{1}{\alpha}} + 6\right)\right)$$

Parameter  $\alpha$  can be approximate using the next relation depending on  $\tau_{H3}$ :

if  $0.1 < |\tau_{H3}| \leq 0.40$ :

$$\alpha = \frac{103.6860892 + 2130.06511762 \cdot |\tau_{H3}| + 11439.55874939 \cdot \tau_{H3}^2 - 62374.50744931 \cdot |\tau_{H3}|^3 + 173468.49718504 \cdot \tau_{H3}^4 - 267472.87702451 \cdot |\tau_{H3}|^5 + 166118.74442011 \cdot \tau_{H3}^6}{1 - 15.29830556 \cdot |\tau_{H3}| + 10893.19365376 \cdot \tau_{H3}^2}$$

If  $0.40 < |\tau_{H3}| < 0.83$ :

$$\alpha = \frac{7.52721198 - 51.23777768 \cdot |\tau_{H3}| + 166.95004129 \cdot \tau_{H3}^2 - 297.93483838 \cdot |\tau_{H3}|^3 + 288.77916443 \cdot \tau_{H3}^4 - 135.39865923 \cdot |\tau_{H3}|^5 + 20.62550348 \cdot \tau_{H3}^6}{1 - 15.29830556 \cdot |\tau_{H3}| + 10893.19365376 \cdot \tau_{H3}^2}$$

$$\beta = \frac{L_{H2} \cdot \alpha}{2 \cdot 6^{-\frac{1}{\alpha}} \cdot \Gamma\left(\frac{1}{\alpha}\right) \cdot \left(6^{\frac{1}{\alpha}} + 3 \cdot 2^{\frac{1}{\alpha}} - 3^{\frac{1}{\alpha}} \cdot \left(2^{\frac{1}{\alpha}} + 3\right)\right)}$$

$$\gamma = L_{H1} + \frac{\beta}{\alpha} \cdot 6^{-\frac{1}{\alpha}} \cdot \Gamma\left(\frac{1}{\alpha}\right) \cdot \left(3^{\frac{1}{\alpha}+1} - 3 \cdot 6^{\frac{1}{\alpha}} - 2^{\frac{1}{\alpha}}\right)$$

### Generalized Pareto

For the second order LH-moments, the parameters have the following expressions:

$$\alpha = \frac{5 - 15 \cdot |\tau_{H3}|}{3 \cdot |\tau_{H3}| + 5}$$

$$\beta = \frac{L_{H2}}{12} \cdot (\alpha + 1) \cdot (\alpha + 2) \cdot (\alpha + 3) \cdot (\alpha + 4)$$

$$\gamma = L_{H1} - \frac{\beta \cdot (\alpha^2 + 6 \cdot \alpha + 11)}{(\alpha + 1) \cdot (\alpha + 2) \cdot (\alpha + 3)}$$

### Log-logistic

For the second order LH-moments, the parameters have the following expressions:

$$\alpha = \frac{15}{24 \cdot \tau_{H3} - 5}$$

$$\beta = \frac{3 \cdot L_{H2} \cdot \alpha^4 \cdot \sin\left(\frac{\pi}{\alpha}\right)}{\pi \cdot (\alpha + 1) \cdot (2 \cdot \alpha + 1)}$$

$$\gamma = L_{H1} - \frac{\beta \cdot \pi \cdot (\alpha + 1) \cdot (2 \cdot \alpha + 1)}{2 \cdot \alpha^3 \cdot \sin\left(\frac{\pi}{\alpha}\right)}$$

### Rayleigh

For the second order LH-moments, the parameters have the following expressions:

$$\alpha = \frac{\sqrt{\pi} \cdot (6 \cdot L_{H3} \cdot |\tau_{H3}| - 2 \cdot L_{H2})}{\pi \cdot (3 \cdot \sqrt{10} + 18 \cdot \sqrt{6} - 21 \cdot \sqrt{2} - 24)}$$

$$\beta = L_{H2} - \frac{\alpha \cdot \pi \cdot (2 \cdot \sqrt{6} + \sqrt{2} - 6)}{2 \cdot \sqrt{\pi}}$$

$$\gamma = \frac{\sqrt{\pi} \cdot (6 \cdot L_{H1} - 22 \cdot \beta) + \alpha \cdot \pi \cdot (9 - 9 \cdot \sqrt{2} - \sqrt{6})}{6 \cdot \sqrt{\pi}}$$

**The frequency factors for the PE3, GEV, W3, GP, RY, LN3 and LL3 distributions.**

Table S1 shows the frequency factors for the analyzed distributions.

**Table S1.** Frequency factors.

Distr.	Frequency factor, $K_p(p)$	
	Quantile function	
	L-moments	LH-moments
	$x(p) = L_1 + L_2 \cdot K_p(p)$	$x(p) = L_{H1} + L_{H2} \cdot K_p(p)$
PE3	$\frac{\Gamma(\alpha) \cdot \sqrt{\pi} \cdot (qgamma(1-p, \alpha) - \alpha)}{\Gamma(\alpha + 0.5)}$	$\frac{\frac{2}{3} \cdot (qgamma(1-p, \alpha) - 2 \cdot z_2)}{z_1}$ the expressions for $z_1$ and $z_2$ can be found in Appendix
GEV	$\frac{1}{1-2^{-\alpha}} \cdot \left( 1 - \frac{(-\ln(1-p))^\alpha}{\Gamma(\alpha+1)} \right)$	$\frac{\Gamma(\alpha+1) \cdot 2^{1-\alpha} - 2 \cdot (-\ln(1-p))^\alpha}{\Gamma(\alpha+1) \cdot (3 \cdot 2^{-\alpha} - 3^{1-\alpha})}$
W3	$\frac{1}{1-2^{-\frac{1}{\alpha}}} \cdot \left( \frac{\ln\left(\frac{1}{p}\right)^{\frac{1}{\alpha}}}{\frac{1}{\alpha} \cdot \Gamma\left(\frac{1}{\alpha}\right)} - 1 \right)$	$\frac{2 \cdot \alpha \cdot (-\ln(p))^{\frac{1}{\alpha}} + \Gamma\left(\frac{1}{\alpha}\right) \cdot \left( 2^{\frac{1}{\alpha}} - 4 \right)}{\Gamma\left(\frac{1}{\alpha}\right) \cdot \left( 3^{\frac{1}{\alpha}} + 3 - 3 \cdot 2^{\frac{1}{\alpha}} \right)}$
GP	$1 + \frac{2}{\alpha} \cdot (1 - p^\alpha) - p^\alpha \cdot (\alpha + 3)$	$\frac{(\alpha + 3) \cdot (2 - p^\alpha \cdot (\alpha^2 + 3 \cdot \alpha + 2))}{3 \cdot \alpha}$
RY	$\frac{\alpha \cdot (2 \cdot \sqrt{-2 \cdot \ln(p)} - \sqrt{\pi}) - \beta \cdot (4 \cdot \ln(p) + 2)}{\alpha \cdot (\sqrt{2 \cdot \pi} - \sqrt{\pi}) + 2 \cdot \beta} - 1$	$\frac{4 \cdot \alpha \cdot \sqrt{-2 \cdot \pi \cdot \ln(p)} + 2 \cdot \alpha \cdot \pi \cdot (1 - 2 \cdot \sqrt{2}) - \sqrt{\pi} \cdot 4 \cdot \beta \cdot (1 \cdot \ln(p) + 3)}{\alpha \cdot \pi \cdot (\sqrt{6} + 3 \cdot \sqrt{2} - 6) + 4 \cdot \beta \cdot \sqrt{\pi}}$
LN3	$\frac{\exp(\alpha + \beta \cdot qnorm(1-p, 0, 1)) - \exp(\alpha + 0.5 \cdot \beta^2)}{erf(0.5 \cdot \beta) \cdot \exp(0.5 \cdot \beta^2 + \alpha)}$	$\frac{\exp(\beta \cdot qnorm(1-p, 0, 1)) - 2 \cdot z_2}{\frac{3}{2} \cdot z_1}$ the expressions for $z_1$ and $z_2$ can be found in Appendix A
LL3	$\frac{\alpha^2 \cdot \sin\left(\frac{\pi}{\alpha}\right) \cdot \left(\frac{1-p}{p}\right)^{\frac{1}{\alpha}}}{\pi}$	$\frac{4 \cdot \alpha}{3} \cdot \left( \frac{\alpha^2 \cdot \sin\left(\frac{\pi}{\alpha}\right) \cdot \left(\frac{1}{p} - 1\right)^{\frac{1}{\alpha}}}{\pi \cdot (\alpha + 1)} - 1 \right)$

### Frequency Factor approximation for PE3

The frequency factor for L-moments was presented in previous materials [18].

The frequency factor for LH-moments can be estimated using a polynomial function:

$$K_p(p) = a + b \cdot \tau_{H3} + c \cdot \tau_{H3}^2 + d \cdot \tau_{H3}^3 + e \cdot \tau_{H3}^4 + f \cdot \tau_{H3}^5 + g \cdot \tau_{H3}^6$$

**Table S2.** The frequency factor with LH-moments for Pearson III.

P (%)	a	b	c	d	e	f	g
0.01	138.44539	-2465.14034	17892.31211	-63752.69124	120214.51108	-114563.99245	43558.796
0.1	52.68673	-889.81398	6536.44619	-23385.63059	44343.20937	-42529.19337	16305.20832
0.5	8.52826	-84.40503	710.04423	-2635.33211	5249.29588	-5311.16131	2175.4185
1	-2.62796	115.73984	-752.64855	2601.15231	-4681.31813	4213.76135	-1475.06545
2	-6.49475	179.9173	-1244.96492	4403.95283	-8196.90419	7690.84016	-2858.81404
3	-4.98987	146.97555	-1028.60893	3668.77984	-6898.55322	6550.0621	-2473.13407
5	-0.39881	54.15976	-384.55056	1408.41202	-2727.0609	2675.45124	-1052.85277
10	3.52157	-33.04936	230.56841	-804.89751	1478.44535	-1366.32017	491.25432
20	0.60222	2.42114	-26.15596	73.87246	-93.86151	27.90766	16.29371
40	-0.52763	-1.56858	-6.0962	38.24575	-109.57418	142.81235	-65.69078
50	-0.86623	-7.33884	41.89976	-140.83664	240.56122	-191.21672	56.246
80	-3.96106	8.18313	-33.00426	110.27596	-184.78442	147.26491	-45.41378
90	-5.11846	6.75273	1.21902	17.21686	-89.74482	117.26047	-49.41254

**Frequency Factor approximation for LN3**

The frequency factor for L-moments can be estimated using a rational function:

$$K_p(p) = \frac{a + b \cdot \tau_3 + c \cdot \tau_3^2 + d \cdot \tau_3^3 + e \cdot \tau_3^4 + f \cdot \tau_3^5 + g \cdot \tau_3^6}{1 + h \cdot \tau_3 + i \cdot \tau_3^2 + j \cdot \tau_3^3}$$

**Table S3.** The frequency factor for Log-normal.

P (%)	a	b	c	d	e	f	g	h	i
0.01	7.34577	-11.93543	716.47126	-4366.76433	13233.45376	-17502.29426	9087.37090	0.13890	-0.31483
0.1	-182.88539	80948.57367	69705.91443	29162.82665	-47509.61155	-40863.74927	-86394.58129	13007.97382	-22817.93119
0.5	4.46079	768.03648	392.38804	2614.73022	-5453.94909	4471.71101	-2769.46863	161.69088	-188.52368
1	5.05392	3381.94490	4190.83018	1683.11151	-4342.06310	-1032.28092	-3834.14862	828.61007	-638.47665
2	3.72540	559.11509	678.67334	283.47264	-1231.65344	218.68686	-530.59615	152.94462	-60.42997
3	3.34676	268.95983	249.09931	62.99081	-578.86038	94.48113	-119.72659	79.34274	-32.25046
5	2.90555	3.51178	-4.44349	21.53572	-56.29603	59.07715	-27.31915	-0.00055	0.00201
10	2.27392	10.31129	0.12209	-25.46670	15.97668	-30.11238	26.69439	4.08106	-2.17704
20	1.47852	1457.84690	-918.40881	-2278.41631	-529.77821	2130.23329	-102.47085	977.02214	-257.24175
40	0.44943	-1.71341	-0.58245	-0.17569	2.15788	-1.99608	0.85851	-	-
50	-0.00005	-1.81172	-0.02237	0.94326	-0.20718	0.17417	-0.07598	-	-
80	-1.49191	-0.52177	1.78568	-0.41759	-0.93231	0.93656	-0.35804	-	-
90	-2.27165	1.17186	1.28540	-1.61220	0.18629	0.49903	-0.25817	-	-

The frequency factor for LH-moments can be estimated using a polynomial function:

$$K_p(p) = a + b \cdot \tau_{H3} + c \cdot \tau_{H3}^2 + d \cdot \tau_{H3}^3 + e \cdot \tau_{H3}^4$$

**Table S4.** The frequency factor for estimation with LH-moments for Log-normal.

P (%)	a	b	c	d
0.01	20.4560566	0.0157214	7.6751191	0.1690430
0.1	19.9223136	0.0157565	7.7306869	0.1694185
0.5	19.4863608	0.0157853	7.7760593	0.1697253
1	19.2751529	0.0157992	7.7980364	0.1698740
2	19.0445424	0.0158144	7.8220289	0.1700363
3	18.8983187	0.0158240	7.8372399	0.1701393
5	18.6989591	0.0158372	7.8579761	0.1702796
10	18.3922410	0.0158574	7.8898737	0.1704956
20	18.0212465	0.0158819	7.9284472	0.1707569
40	17.5258663	0.0159145	7.9799386	0.1711059
50	17.3127762	0.0159286	8.0020827	0.1712560
80	16.6059721	0.0159752	8.0755103	0.1717541
90	16.2371749	0.0159996	8.1138096	0.1720141

**Frequency Factor approximation for the Log-logistic**

The frequency factor for L-moments was presented in previous materials [43].

The frequency factor for LH-moments can be estimated using a polynomial function:

$$K_p(p) = a + b \cdot \tau_{H3} + c \cdot \tau_{H3}^2 + d \cdot \tau_{H3}^3 + e \cdot \tau_{H3}^4 + f \cdot \tau_{H3}^5 + g \cdot \tau_{H3}^6 + h \cdot \tau_{H3}^7$$

**Table S5.** The frequency factor for estimation with LH-moments for Log-logistic.

P (%)	a	b	c	d	e	f	g	h
0.01	103.44298	-2247.27189	21184.43377	-102767.75773	286457.93986	-457461.25129	395262.37012	-142770.31432
0.1	6.05545	-17.08849	343.77445	-1533.35095	4753.01020	-8113.84196	7880.40650	-3509.19037
0.5	3.87086	11.61429	-5.07346	105.17812	-250.39810	371.84638	-323.25044	54.78038
1	3.51568	8.09347	-1.16064	48.37422	-135.48438	202.21526	-205.45954	65.24658
2	3.07785	5.02904	0.91937	8.87123	-39.66855	50.29374	-61.12793	25.47253
3	2.77481	3.55471	0.40510	-0.82946	-14.11642	11.57912	-18.50701	10.28746
5	2.34112	1.93904	-0.92188	-4.44877	-1.80985	-3.58268	2.10840	1.21556
10	1.64347	0.07494	-2.24613	-2.84378	0.70228	-1.42733	3.02749	-0.93766
20	0.75853	-1.35636	-1.94317	-0.08669	0.71560	0.88025	-0.41732	-0.04417
40	-0.52183	-1.88716	0.21358	1.22206	0.14678	-0.53137	0.04491	0.04564
50	-1.12906	-1.58048	1.23734	0.91658	-0.49333	-0.45291	0.32981	-0.05510
80	-3.61986	2.78806	1.59348	-3.21610	1.10363	0.63340	-0.59631	0.14058
90	-5.43104	8.59806	-4.49534	-2.96004	5.88869	-3.80245	1.18164	-0.14427

### Frequency Factor approximation for the generalized Pareto

The frequency factor for L-moments can be estimated using a polynomial function:

$$K_p(p) = a + b \cdot \tau_3 + c \cdot \tau_3^2 + d \cdot \tau_3^3 + e \cdot \tau_3^4 + f \cdot \tau_3^5 + g \cdot \tau_3^6 + h \cdot \tau_3^7 + i \cdot \tau_3^8 + j \cdot \tau_3^9$$

**Table S6.** The frequency factor for estimation with L-moments for generalized Pareto.

P [%]	a	b	c	d	e	f	g	h	i	j
0.01	2.8976	16.049	-166.33	2107.2	-11711	41260	-84364	$1.0671 \times 10^5$	-68465	14604
0.1	2.9921	7.9122	21.160	46.340	207.03	-394.80	1467.1	-2026.5	667.72	-
0.5	2.9751	7.0733	21.320	5.3452	97.924	-99.393	-95.564	-59.326	-	-
1	2.9389	6.8741	14.223	10.895	33.199	-116.24	47.113	-	-	-
2	2.8667	6.5598	3.8790	28.740	-66.614	23.564	-	-	-	-
3	2.8214	5.3368	6.0252	1.9543	-31.305	14.174	-	-	-	-
5	2.7060	3.9951	4.1471	-13.220	-2.6956	4.0738	-	-	-	-
10	2.4017	2.0082	-1.6983	-9.9252	7.8868	-1.6725	-	-	-	-
20	1.7989	-0.48676	-4.2616	1.0072	1.7490	-0.80770	-	-	-	-
40	0.60025	-2.4049	-0.31267	2.3727	-1.6966	0.44147	-	-	-	-
50	$3.8891 \times 10^{-4}$	-2.3323	1.3905	0.48092	-0.81390	0.27475	-	-	-	-
80	-1.8003	0.52569	1.0120	-1.3741	0.86642	-0.22985	-	-	-	-
90	-2.4001	2.1286	-0.96924	0.23252	0.041754	-0.033573	-	-	-	-

The frequency factor for LH-moments can be estimated using a polynomial function:

$$K_p(p) = a + b \cdot \tau_{H3} + c \cdot \tau_{H3}^2 + d \cdot \tau_{H3}^3 + e \cdot \tau_{H3}^4 + f \cdot \tau_{H3}^5 + g \cdot \tau_{H3}^6 + h \cdot \tau_{H3}^7$$

**Table S7.** The frequency factor for generalized Pareto.

P [%]	a	b	c	d	e	f	g	h
0.01	127.52653	-3133.26097	31256.43005	-160882.76618	469519.36329	-784121.48421	705520.62434	-264098.50757
0.1	0.55226	54.02709	-427.81690	2197.50726	-5596.20281	8171.75165	-5057.70040	311.34905
0.5	2.34460	9.04692	-14.93090	179.80355	-485.14463	927.92082	-1044.44624	378.98343
1	2.41429	5.30752	13.10852	-3.39848	23.00815	-41.46804	-82.23960	63.19792
2	2.31029	4.32307	8.11442	-15.24676	30.86405	-97.92152	70.31997	-11.84745
3	2.20245	3.65301	3.16849	-9.16777	1.48768	-38.28013	42.42756	-11.40870
5	1.99879	2.34790	-1.34216	-5.66879	-11.06583	8.20383	4.87852	-2.99898
10	1.49986	-0.06541	-3.98153	-3.59998	1.16450	6.91929	-5.25572	1.14932
20	0.50011	-2.35085	-2.38207	2.08504	2.85979	-3.34721	1.24717	-0.16166
40	-1.50002	-1.91397	2.77403	0.48191	-2.27210	1.60762	-0.54501	0.07843
50	-2.49995	-0.17790	3.21866	-2.18282	-0.07344	0.89937	-0.54893	0.12201
80	-5.50006	8.93496	-7.45062	3.83533	-1.00503	-0.18694	0.26476	-0.07197
90	-6.49988	12.96321	-14.95610	12.89808	-8.97451	4.87628	-1.80348	0.32790

### Estimation of the Frequency Factor for the GEV Distribution

The frequency factor for L-moments can be estimated using a rational function:

$$K_p(p) = \frac{a + b \cdot \tau_3}{1 + c \cdot \tau_3 + d \cdot \tau_3^2 + e \cdot \tau_3^3 + f \cdot \tau_3^4 + g \cdot \tau_3^5 + h \cdot \tau_3^6}$$

**Table S8.** The frequency factor for GEV.

P [%]	a	b	c	d	e	f	g	h
0.01	43.997278	479.223151	112.991079	-478.513693	606.212296	38.131031	-574.431973	297.600956
0.1	4.593731	-4.599738	-4.914425	10.573881	-12.418413	8.168773	-2.751896	0.346790
0.5	4.231408	-4.256125	-3.710814	5.637548	-3.209217	-1.415680	2.689191	-0.969345
1	3.920710	-3.964687	-3.310486	4.702659	-2.639564	-0.980350	2.004350	-0.734317
2	3.547981	-3.630077	-2.865169	3.767118	-2.103467	-0.588628	1.400705	-0.529414
3	3.292962	-3.411972	-2.581338	3.224279	-1.787497	-0.424176	1.119925	-0.432908
5	2.923725	-3.114144	-2.195436	2.551143	-1.361430	-0.354604	0.909776	-0.359544
10	2.315824	-2.678716	-1.614374	1.717971	-0.883351	-0.220842	0.613879	-0.250675
20	1.535444	-2.240652	-0.939149	1.018412	-0.488547	-0.071811	0.312582	-0.126376
40	0.458641	-1.897210	-0.076185	0.410230	0.142966	-0.058015	-0.064798	0.084806
50	-0.006962	-1.848988	0.259172	0.403972	0.162322	0.002282	-0.021729	0.050165
80	-1.522088	0.021531	-0.146382	0.914093	-0.434557	-0.009206	0.338580	-0.162479
90	-2.283526	1.292147	0.047707	0.243670	-0.278656	-0.068485	0.103177	-0.056225

The frequency factor for LH-moments can be estimated using a polynomial function:

$$K_p(p) = a + b \cdot \tau_{H3} + c \cdot \tau_{H3}^2 + d \cdot \tau_{H3}^3 + e \cdot \tau_{H3}^4 + f \cdot \tau_{H3}^5 + g \cdot \tau_{H3}^6 + h \cdot \tau_{H3}^7 + i \cdot \tau_{H3}^8$$

**Table S9.** The frequency factor for estimation with LH-moments for GEV.

P [%]	a	b	c	d	e	f	g	h	i
0.01	2.84193	72.89009	-1208.45990	12033.50122	-59731.52949	172890.26456	-284810.22514	257879.13515	-99546.89774
0.1	3.55588	10.78712	43.28599	-63.74219	654.48792	-1426.75704	2111.74697	-1473.53626	-41.99786
0.5	3.29715	8.65198	17.23792	6.91346	61.74994	-188.09188	261.35329	-321.69357	121.00099
1	3.11233	7.10050	10.15730	5.05963	-3.86343	-30.89273	8.28187	-37.80693	24.91187
2	2.85379	5.24293	4.55201	-1.68345	-12.14255	-3.88219	-13.57036	12.13614	-0.39754
3	2.65592	4.04592	1.81490	-4.26422	-9.35750	-1.59873	-4.65570	8.57527	-1.94883
5	2.34127	2.46046	-0.91900	-5.21299	-4.65998	0.36208	1.67495	1.66347	-0.82582
10	1.75396	0.29628	-2.90218	-3.14928	0.27245	1.72803	0.53092	-0.60600	0.07431
20	0.88129	-1.54098	-2.35325	0.36569	1.53760	-0.04081	-0.50253	0.17109	-0.01739
40	-0.52878	-2.10947	0.67009	1.43795	-0.67473	-0.31022	0.35590	-0.13646	0.02264
50	-1.20755	-1.59319	1.78319	0.48349	-1.07574	0.39579	0.04032	-0.07752	0.02036
80	-3.69179	3.48265	-0.01656	-2.24346	2.05403	-0.97650	0.23017	0.00536	-0.01228
90	-5.09402	8.08189	-6.10375	1.78936	1.25678	-2.06800	1.46894	-0.59841	0.11185



### Frequency Factor approximation for the Weibull

The frequency factor for L-moments can be estimated using a rational function:

$$K_p(p) = \frac{a + b \cdot \tau_3}{1 + c \cdot \tau_3 + d \cdot \tau_3^2 + e \cdot \tau_3^3 + f \cdot \tau_3^4 + g \cdot \tau_3^5 + h \cdot \tau_3^6}$$

**Table S10.** The frequency factor for Weibull.

P [%]	a	b	c	d	e	f	g	h
0.01	3.881157	464.188509	58.471077	-100.174864	-6.313148	48.241257	37.515608	-38.451757
0.1	5.681663	107.734771	20.530306	-52.062588	39.704691	10.866413	-35.950666	16.615265
0.5	4.402203	-4.418888	-2.804339	2.750395	-0.417193	-1.647128	1.673333	-0.554708
1	4.017978	-4.034465	-2.569301	1.977775	1.173936	-3.951053	3.515693	-1.145162
2	3.583439	-3.605303	-2.312332	1.228444	2.780465	-6.579336	5.845979	-1.955337
3	3.300340	-3.332600	-2.155135	0.881847	3.445384	-7.817715	7.101007	-2.437017
5	2.905854	-2.974573	-1.956442	0.684947	3.395098	-7.883958	7.475747	-2.660926
10	2.282379	-2.510781	-1.716588	1.326785	-0.514585	-0.274260	0.595009	-0.203138
20	1.523802	-2.118413	-1.196587	0.505068	0.569479	-0.584168	-0.588348	0.876540
40	0.468701	-1.845141	-0.577517	0.879178	-0.785897	0.131841	0.981618	-0.254152
50	0.006103	-1.802922	-0.486914	1.766501	-2.057441	0.362834	2.720117	-1.511934
80	-1.537064	-0.149812	-0.375924	1.427798	-0.410872	0.163424	0.347573	-0.479485
90	-2.317176	1.317548	-0.064713	0.721028	-0.028389	-0.602600	-0.461070	0.429757

The frequency factor for LH-moments can be estimated using a polynomial function:

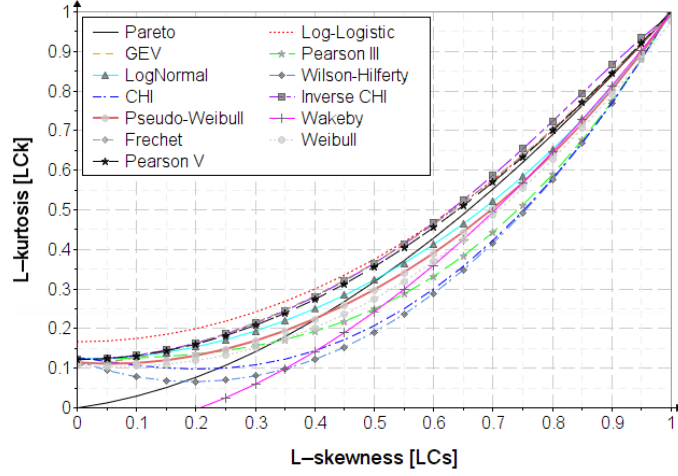
$$K_p(p) = a + b \cdot \tau_{H3} + c \cdot \tau_{H3}^2 + d \cdot \tau_{H3}^3 + e \cdot \tau_{H3}^4 + f \cdot \tau_{H3}^5 + g \cdot \tau_{H3}^6 + h \cdot \tau_{H3}^7 + i \cdot \tau_{H3}^8$$

**Table S11.** The frequency factor for estimation with LH-moments for Weibull.

P [%]	a	b	c	d	e	f	g	h	i
0.01	10.38815	-337.16546	7056.29489	-62596.82183	292870.50322	-769929.64519	1147108.66145	-904666.66322	294517.97591
0.1	2.96147	107.40737	-1912.50408	16999.23656	-78064.02343	200931.07591	-290664.39863	220564.61486	-68045.52714
0.5	3.37977	31.60723	-483.30612	4422.33779	-20715.47605	54607.16373	-81196.16570	63620.63386	-20418.09920
1	3.29504	12.11788	-124.71392	1204.55002	-5772.34918	15642.79909	-23975.41219	19419.50988	-6477.14385
2	2.99597	3.20856	22.71873	-152.83087	623.30827	-1331.53779	1434.29003	-631.13514	4.95780
3	2.73864	1.26748	42.95589	-358.62125	1632.50399	-4142.06642	5852.88538	-4299.90165	1258.75634
5	2.34263	0.53052	33.34481	-298.80902	1399.47710	-3691.49466	5450.72462	-4218.05402	1321.36770
10	1.67937	0.22396	5.31829	-68.36718	344.01600	-1000.07004	1611.60205	-1355.15527	461.98724
20	0.80909	-0.79607	-6.57644	37.09428	-158.96448	363.78771	-456.03659	293.23996	-73.22732
40	-0.51427	-1.95635	-1.62576	13.09331	-55.69512	149.55344	-221.15070	171.28892	-54.49616
50	-1.15696	-1.86177	1.02431	1.69090	-5.14088	25.34311	-54.53871	53.27819	-20.12289
80	-3.68218	2.88032	4.31108	-17.49187	53.53072	-116.81199	139.84852	-81.71878	17.72063
90	-5.27968	9.47245	-3.21088	-20.99956	68.28625	-134.06287	158.19358	-96.53724	22.73513

### The variation diagram for L and LH-moments

In Figure S1 are presented the curves for L-kurtosis and L-skewness.



**Figure S1.** The L-skewness and L-kurtosis variation.

Pearson III:

$$\tau_4 = 0.1217175 + 0.030285 \cdot \tau_3 + 0.0266125 \cdot \tau_3^2 + 0.8774691 \cdot \tau_3^3 - 0.0564795 \cdot \tau_3^4$$

Pearson V:

$$\tau_4 = 0.1089545 - 0.1542626 \cdot \tau_3 + 1.0657605 \cdot \tau_3^2 - 0.3521005 \cdot \tau_3^3 + 0.3269967 \cdot \tau_3^4$$

Wilson-Hilferty:

$$\tau_4 = 0.1177849 - 0.5367173 \cdot \tau_3 + 1.4180786 \cdot \tau_3^2 - 0.2084697 \cdot \tau_3^3 + 0.2098975 \cdot \tau_3^4$$

CHI:

$$\tau_4 = 0.1274475 - 0.2174617 \cdot \tau_3 - 0.0945508 \cdot \tau_3^2 + 2.6572905 \cdot \tau_3^3 - 2.369862 \cdot \tau_3^4 + 0.9016064 \cdot \tau_3^5$$

ICH:

$$\tau_4 = 0.1215494 + 0.0260015 \cdot \tau_3 + 0.6839989 \cdot \tau_3^2 + 2.3432188 \cdot \tau_3^3 - 7.9178585 \cdot \tau_3^4 + 11.9165941 \cdot \tau_3^5 - 8.06007 \cdot \tau_3^6 + 1.8820702 \cdot \tau_3^7$$

Pseudo-Weibull:

$$\tau_4 = 0.1132189 - 0.1242052 \cdot \tau_3 + 1.1329458 \cdot \tau_3^2 - 0.4716246 \cdot \tau_3^3 + 0.3449906 \cdot \tau_3^4$$

Wakeby:

$$\tau_4 = -0.07347 + 0.14443 \cdot \tau_3 + 1.03879 \cdot \tau_3^2 - 0.14602 \cdot \tau_3^3 + 0.03357 \cdot \tau_3^4$$

Pareto:

$$\tau_4 = -0.0003668 + 0.2070484 \cdot \tau_3 + 0.9264 \cdot \tau_3^2 - 0.133564 \cdot \tau_3^3$$

GEV:

$$\tau_4 = 0.1072214 + 0.1143838 \cdot \tau_3 + 0.8341466 \cdot \tau_3^2 - 0.0632425 \cdot \tau_3^3 + 0.0074607 \cdot \tau_3^4$$

Frechet:

$$\tau_4 = 0.1069938 + 0.1155235 \cdot \tau_3 + 0.8294258 \cdot \tau_3^2 - 0.0528083 \cdot \tau_3^3$$

Weibull:

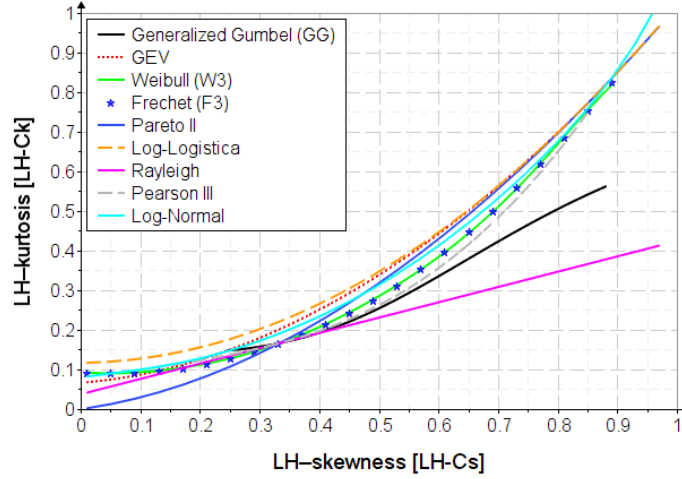
$$\tau_4 = 0.1057425 - 0.0753465 \cdot \tau_3 + 0.6176919 \cdot \tau_3^2 + 0.5065127 \cdot \tau_3^3 - 0.1788008 \cdot \tau_3^4$$

Log-normal:

$$\tau_4 = 0.1238145 - 0.032954 \cdot \tau_3 + 0.9783895 \cdot \tau_3^2 - 0.3929245 \cdot \tau_3^3 + 0.3174611 \cdot \tau_3^4$$

Log-logistic: 
$$\tau_4 = \frac{1 + 5 \cdot \tau_3^2}{6} \approx 0.16667 + 0.83333 \cdot \tau_3^2$$

In Figure S2 are presented the variation of LH-skewness and LH-kurtosis.



**Figure S2.** The variation diagram for LH-skewness and LH-kurtosis.

Pearson III:

$$\tau_{H4} = 0.1012322 - 0.2437927 \cdot \tau_{H3} + 3.5538482 \cdot \tau_{H3}^2 - 13.8996553 \cdot \tau_{H3}^3 + 31.3422272 \cdot \tau_{H3}^4 - 35.5781144 \cdot \tau_{H3}^5 + 20.677076 \cdot \tau_{H3}^6 - 4.8315272 \cdot \tau_{H3}^7$$

Log-normal:

$$\tau_{H4} = 0.0801496 + 0.2283728 \cdot \tau_{H3} - 0.9987353 \cdot \tau_{H3}^2 + 8.8369201 \cdot \tau_{H3}^3 - 24.2488462 \cdot \tau_{H3}^4 + 37.7635295 \cdot \tau_{H3}^5 - 30.6596349 \cdot \tau_{H3}^6 + 10.1236239 \cdot \tau_{H3}^7$$

GEV:

$$\tau_{H4} = 0.066585 + 0.1207821 \cdot \tau_{H3} + 0.8711194 \cdot \tau_{H3}^2 - 0.0456666 \cdot \tau_{H3}^3 - 0.0033066 \cdot \tau_{H3}^4 + 0.0170713 \cdot \tau_{H3}^5 - 0.0094267 \cdot \tau_{H3}^6 + 0.0009899 \cdot \tau_{H3}^7$$

Weibull:

$$\tau_{H4} = 0.0940624 - 0.167248 \cdot \tau_{H3} + 2.2686976 \cdot \tau_{H3}^2 - 9.8557774 \cdot \tau_{H3}^3 + 35.7330334 \cdot \tau_{H3}^4 - 66.7529615 \cdot \tau_{H3}^5 + 61.9426076 \cdot \tau_{H3}^6 - 22.3935866 \cdot \tau_{H3}^7$$

Rayleigh:

$$\tau_{H4} = \frac{7 \cdot (\sqrt{10} + 4 - 5 \cdot \sqrt{2}) \cdot (3 \cdot \tau_{H3} - 1)}{30 \cdot \sqrt{6} - 10 \cdot \sqrt{2} - 60} + \frac{99 \cdot \tau_{H3} - 28}{30}$$

Log-logistic:

$$\tau_{H4} = \frac{170 \cdot \tau_{H3}^2 + 36 \cdot \tau_{H3} + 224}{1920}$$

Fréchet:

$$\tau_{H4} = 0.0903692 + 0.0382575 \cdot \tau_{H3} - 0.9531845 \cdot \tau_{H3}^2 + 11.6781608 \cdot \tau_{H3}^3 - 35.9472026 \cdot \tau_{H3}^4 + 57.6701516 \cdot \tau_{H3}^5 - 45.5569267 \cdot \tau_{H3}^6 + 14.0532691 \cdot \tau_{H3}^7$$

Kappa (generalized Gumbel, Jeong 2009):

$$\tau_{H4} = 0.2623693 - 1.8726094 \cdot \tau_{H3} + 12.2859097 \cdot \tau_{H3}^2 - 45.0169093 \cdot \tau_{H3}^3 + 100.4606818 \cdot \tau_{H3}^4 - 122.9124059 \cdot \tau_{H3}^5 + 76.5421309 \cdot \tau_{H3}^6 - 19.1260088 \cdot \tau_{H3}^7$$