

## 1. Organic Carbon

The present model has three state variables for organic carbon: refractory particulate, labile particulate, and dissolved.

### 1.1. RPOC and LPOC

Labile and refractory distinctions are based on the time scale of decomposition. Labile particulate organic carbon with a decomposition time scale of days to weeks decomposes rapidly in the water column or in the sediments. Refractory particulate organic carbon with a longer than- weeks decomposition time scale decomposes slowly, primarily in the sediments, and may contribute to sediment oxygen demand years after decomposition.

For labile and refractory particulate organic carbon, sources and sinks included in the model are:

- Algal predation
- Dissolution to dissolved organic carbon
- Settling
- External loads.

The governing equations for refractory and labile particulate organic carbons are:

$$\frac{\partial RPOC}{\partial t} = \sum_{x=c,d,g,m} FCRP_x PR_x B_x - K_{RPOC} RPOC + \frac{\partial}{\partial Z} (WS_{RP} RPOC) + \frac{WRPOC}{V}$$

$$\frac{\partial LPOC}{\partial t} = \sum_{x=c,d,g,m} FCLP_x PR_x B_x - K_{LPOC} LPOC + \frac{\partial}{\partial Z} (WS_{LP} LPOC) + \frac{WLPOC}{V}$$

Where

RPOC = concentration of refractory particulate organic carbon (g C m<sup>-3</sup>)

LPOC = concentration of labile particulate organic carbon (g C m<sup>-3</sup>)

FCRP = fraction of predated carbon produced as refractory particulate organic carbon

FCLP = fraction of predated carbon produced as labile particulate organic carbon

K<sub>RPOC</sub> = dissolution rate of refractory particulate organic carbon (day<sup>-1</sup>)

K<sub>LPOC</sub> = dissolution rate of labile particulate organic carbon (day<sup>-1</sup>)

$WS_{RP}$  = settling velocity of refractory particulate organic matter ( $m\ day^{-1}$ )

$WS_{LP}$  = settling velocity of labile particulate organic matter ( $m\ day^{-1}$ )

$WRPOC$  = external loads of refractory particulate organic carbon ( $g\ C\ day^{-1}$ )

$WLPOC$  = external loads of labile particulate organic carbon ( $g\ C\ day^{-1}$ ).

## 1.2 DOC

Sources and sinks for dissolved organic carbon included in the model are:

- Algal excretion (exudation) and predation
- Dissolution from refractory and labile particulate organic carbon
- Heterotrophic respiration of dissolved organic carbon (decomposition)
- Denitrification
- External loads

The kinetic equation describing these processes is:

$$\begin{aligned} \frac{\partial DOC}{\partial t} = & \sum_{x=c,d,g,m} \left( \left[ FCD_x + (1 - FCD_x) \left( \frac{KHR_x}{KHR_x + DO} \right) \right] BM_x + FCDP_x PR_x \right) B_x \\ & + K_{RPOC} RPOC + K_{LPOC} LPOC - K_{HR} DOC - Denit\ DOC + \frac{WDOC}{V} \end{aligned}$$

Where

$DOC$  = concentration of dissolved organic carbon ( $g\ C\ m^{-3}$ )

$FCD_x$  = fraction of basal metabolism exuded as dissolved organic carbon at infinite dissolved oxygen concentration for algal group  $x$

$KHR_x$  = half-saturation constant of dissolved oxygen for algal dissolved organic carbon excretion for group  $x$  ( $g\ O_2\ m^{-3}$ )

$DO$  = dissolved oxygen concentration ( $g\ O_2\ m^{-3}$ )

$FCDP$  = fraction of predated carbon produced as dissolved organic carbon

$K_{HR}$  = heterotrophic respiration rate of dissolved organic carbon ( $day^{-1}$ )

$Denit$  = denitrification rate ( $day^{-1}$ )

BFDOD = benthic flux of dissolved organic carbon in bottom layer only (g C m<sup>-2</sup> day<sup>-1</sup>)

WDOC = external loads of dissolved organic carbon (g C day<sup>-1</sup>).

## 2. Nitrogen

The present model has five state variables for nitrogen: three organic forms (refractory particulate, labile particulate, and dissolved) and two inorganic forms (ammonium and nitrate).

The nitrate state variable in the model represents the sum of nitrate and nitrite.

### 2.1. RPON and LPON

For refractory and labile particulate organic nitrogen, sources and sinks included in the model are:

- Algal basal metabolism and predation
- Dissolution to dissolved organic nitrogen
- Settling
- External loads

The kinetic equations for refractory and labile particulate organic nitrogen are:

$$\frac{\partial RPON}{\partial t} = \sum_{x=c,d,g,m} (FNR_x BM_x + FNR P_x PR_x) ANC_x B_x - K_{RPON} RPON +$$

$$\frac{\partial}{\partial z} (WS_{RP} RPON) + \frac{WRPON}{V}$$

$$\frac{\partial LPON}{\partial t} = \sum_{x=c,d,g,m} (FNL_x BM_x + FNL P_x PR_x) ANC_x B_x - K_{LPON} LPON +$$

$$\frac{\partial}{\partial z} (WS_{LP} LPON) + \frac{WLPON}{V}$$

Where

RPON = concentration of refractory particulate organic nitrogen (g N m<sup>-3</sup>)

LPON = concentration of labile particulate organic nitrogen (g N m<sup>-3</sup>)

FNR<sub>x</sub> = fraction metabolized nitrogen by algal group x as refractory particulate

organic nitrogen

$FNL_x$  = fraction of metabolized nitrogen by algal group  $x$  produced as labile particulate organic nitrogen

$FNRP$  = fraction of predated nitrogen produced as refractory particulate organic nitrogen

$FNLP$  = fraction of predated nitrogen produced as labile particulate organic nitrogen

$ANC_x$  = nitrogen-to-carbon ratio in algal group  $x$  (g N per g C)

$K_{RPON}$  = hydrolysis rate of refractory particulate organic nitrogen ( $\text{day}^{-1}$ )

$K_{LPON}$  = hydrolysis rate of labile particulate organic nitrogen ( $\text{day}^{-1}$ )

$WRPON$  = external loads of refractory particulate organic nitrogen ( $\text{g N day}^{-1}$ )

$WLPON$  = external loads of labile particulate organic nitrogen ( $\text{g N day}^{-1}$ )

## 2.2. DON

Sources and sinks for dissolved organic nitrogen included in the model are:

- Algal basal metabolism and predation
- Dissolution from refractory and labile particulate organic nitrogen
- Mineralization to ammonium
- External loads.

The kinetic equation describing these processes is:

$$\frac{\partial DON}{\partial t} = \sum_{x=c,d,g,m} (FND_x BM_x + FNDP_x PR_x) ANC_x B_x + K_{RPON} RPON + K_{LPON} LPON - K_{DON} DON + \frac{BFDON}{\Delta Z} + \frac{WDON}{V}$$

Where

$DON$  = concentration of dissolved organic nitrogen ( $\text{g N m}^{-3}$ )

$FND_x$  = fraction of metabolized nitrogen by algal group  $x$  produced as dissolved organic nitrogen

$FNDP$  = fraction of predated nitrogen produced as dissolved organic nitrogen

$K_{DON}$  = mineralization rate of dissolved organic nitrogen ( $\text{day}^{-1}$ )

$BFDON$  = benthic flux of dissolved organic nitrogen in bottom layer only ( $\text{g C m}^{-2} \text{ day}^{-1}$ ).

$WDON$  = external loads of dissolved organic nitrogen ( $\text{g N day}^{-1}$ ).

### 2.3. $\text{NH}_4$

Sources and sinks for ammonia nitrogen included in the model are:

- Algal basal metabolism, predation, and uptake
- Mineralization from dissolved organic nitrogen
- Nitrification to nitrate
- Sediment-water exchange for the bottom layer only
- External loads

$$\frac{\partial \text{NH}_4}{\partial t} = \sum_{x=c,d,g,m} (FNI_x BM_x + FNIP_x PR_x - PN_x P_x) ANC_x B_x + K_{DON} DON -$$
$$KNit \text{NH}_4 + \frac{BFNH_4}{\Delta Z} + \frac{W\text{NH}_4}{V}$$

Where

$FNI_x$  = fraction of metabolized nitrogen by algal group  $x$  produced as inorganic nitrogen

$FNIP$  = fraction of predated nitrogen produced as inorganic nitrogen

$PN_x$  = preference for ammonium uptake by algal group  $x$  ( $0 \leq PN_x \leq 1$ )

$KNit$  = nitrification rate ( $\text{day}^{-1}$ )

$BFNH_4$  = sediment-water exchange flux of ammonium ( $\text{g N m}^{-2} \text{ day}^{-1}$ ), applied to the bottom layer only

$W\text{NH}_4$  = external loads of ammonium ( $\text{g N day}^{-1}$ )

### 2.4. $\text{NO}_3$

Sources and sinks for nitrate nitrogen included in the model are (Figure 2.1):

- Algal uptake

- Nitrification from ammonium
- Denitrification to nitrogen gas
- Sediment-water exchange for the bottom layer only
- External loads

The kinetic equation describing these processes is:

$$\frac{\partial NO_3}{\partial t} = \sum_{x=c,d,g,m} (PN_x - 1) P_x ANC_x B_x + KNit NH_4 - ANDC Denit DOC + \frac{BFNO_3}{\Delta Z} + \frac{WNO_3}{V}$$

Where

ANDC = mass of nitrate nitrogen reduced per mass of dissolved organic carbon oxidized (0.933 g N per g C)

BFNO<sub>3</sub> = sediment-water exchange flux of nitrate (g N m<sup>-2</sup> day<sup>-1</sup>), applied to the bottom layer only

WNO<sub>3</sub> = external loads of nitrate (g N day<sup>-1</sup>)

### 3. Phosphorus

The present model has four state variables for phosphorus: three organic forms (refractory particulate, labile particulate, and dissolved) and one inorganic form representing the sum of dissolved and particulate phosphate in the water phase, but exclude phosphate in algae cells.

#### 3.1. RPOP and LPOP

For refractory and labile particulate organic phosphorus, sources and sinks included in the model are:

- Algal basal metabolism and predation
- Dissolution to dissolved organic phosphorus
- Settling
- External loads.

The kinetic equations for refractory and labile particulate organic phosphorus are:

$$\frac{\partial RPOP}{\partial t} = \sum_{x=c,d,g,m} (FPR_x BM_x + FPRP_x PR_x) APC_x B_x - K_{RPOP} RPOP +$$

$$\frac{\partial}{\partial Z} (WS_{RP} RPOP) + \frac{WRPOP}{V}$$

$$\frac{\partial LPOP}{\partial t} = \sum_{x=c,d,g,m} (FPL_x BM_x + FPLP_x PR_x) APC_x B_x - K_{LPOP} LPOP +$$

$$\frac{\partial}{\partial Z} (WS_{RP} LPOP) + \frac{WLPOP}{V}$$

Where

RPOP = concentration of refractory particulate organic phosphorus (g P m<sup>-3</sup>)

LPOP = concentration of labile particulate organic phosphorus (g P m<sup>-3</sup>)

FPR<sub>x</sub> = fraction of metabolized phosphorus by algal group x produced as refractory particulate organic phosphorus

FPL<sub>x</sub> = fraction of metabolized phosphorus by algal group x produced as labile particulate organic phosphorus

FPRP = fraction of predated phosphorus produced as refractory particulate organic phosphorus

FPLP = fraction of predated phosphorus produced as labile particulate organic phosphorus

APC = mean algal phosphorus-to-carbon ratio for all algal groups (g P per g C)

K<sub>RPOP</sub> = hydrolysis rate of refractory particulate organic phosphorus (day<sup>-1</sup>)

K<sub>LPOP</sub> = hydrolysis rate of labile particulate organic phosphorus (day<sup>-1</sup>)

WRPOP = external loads of refractory particulate organic phosphorus (g P day<sup>-1</sup>)

WLPOP = external loads of labile particulate organic phosphorus (g P day<sup>-1</sup>).

### 3.2. DOP

Sources and sinks for dissolved organic phosphorus included in the model are:

- Algal basal metabolism and predation
- Dissolution from refractory and labile particulate organic phosphorus
- Mineralization to phosphate phosphorus
- External loads.

The kinetic equation describing these processes is:

$$\frac{\partial DOP}{\partial t} = \sum_{x=c,d,g,m} (FPD_x BM_x + FPDP_x PR_x) APC_x B_x + K_{RPOP} RPOP + K_{LPOP} LPOP - K_{DOP} DOP + \frac{WDOP}{V}$$

Where

DOP = concentration of dissolved organic phosphorus (g P m<sup>-3</sup>)

FPD<sub>x</sub> = fraction of metabolized phosphorus by algal group x produced as dissolved organic phosphorus

FPDP<sub>x</sub> = fraction of predated phosphorus produced as dissolved organic phosphorus

K<sub>DOP</sub> = mineralization rate of dissolved organic phosphorus (day<sup>-1</sup>)

WDOP = external loads of dissolved organic phosphorus (g P day<sup>-1</sup>)

### 3.3. PO<sub>4</sub>

For total phosphate that includes both dissolved and sorbed phosphate in the water phase, sources and sinks included in the model are:

- Algal basal metabolism, predation, and uptake
- Mineralization from dissolved organic phosphorus
- Settling of sorbed phosphate
- Sediment-water exchange of dissolved phosphate for the bottom layer only
- External loads

The kinetic equation describing these processes is:

$$\begin{aligned} \frac{\partial}{\partial t} (PO4p + PO4d) \\ = \sum_{x=c,d,g,m} (FPI_x BM_x + FPIP_x PR_x - P_x) APC_x B_x + K_{DOP} DOP \\ + \frac{\partial}{\partial Z} (WS_{TSS} PO4p) + \frac{BFPO4d}{\Delta Z} + \frac{WPO4p}{V} + \frac{WPO4d}{V} \end{aligned}$$

Where

PO<sub>4t</sub> = total phosphate (g P m<sup>-3</sup>) = PO<sub>4d</sub> + PO<sub>4p</sub>



$\text{PO4d}$  = dissolved phosphate ( $\text{g P m}^{-3}$ )

$\text{PO4p}$  = particulate (sorbed) phosphate ( $\text{g P m}^{-3}$ )

$\text{FPI}_x$  = fraction of metabolized phosphorus by algal group  $x$  produced as inorganic phosphorus

$\text{FPIP}$  = fraction of predated phosphorus produced as inorganic phosphorus

$\text{WS}_{\text{TSS}}$  = settling velocity of suspended solid ( $\text{m day}^{-1}$ ), provided by the hydrodynamic model

$\text{BFPO4d}$  = sediment-water exchange flux of phosphate ( $\text{g P m}^{-2} \text{ day}^{-1}$ ), applied to the bottom layer only

$\text{WPO4t}$  = external loads of total phosphate ( $\text{g P day}^{-1}$ ).