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Flood Classification Based on a Fuzzy Clustering Iteration Model with Combined Weight and an Immune Grey Wolf Optimizer Algorithm

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Received: 17 November 2018; Accepted: 28 December 2018; Published: 4 January 2019

Abstract: Flood classification is an important basis for flood forecasting, flood risk identification, flood real-time scheduling, and flood resource utilization. However, flood classification results may be not reasonable due to uncertainty, the fuzziness of evaluation indices, and the demerit of not comprehensively considering the index weight. In this paper, based on the fuzzy clustering iterative model, a sensitivity coefficient was applied to combine the subjective and objective weights into a combined weight, then the fuzzy clustering iterative model with combined weight (FCI-CW) was proposed for flood classification. Moreover, an immune grey wolf optimizer algorithm (IGWO) based on the standard grey wolf optimizer algorithm and an immune clone selection operator was proposed for the global search of the optimal fuzzy clustering center and the sensitivity coefficient of FCI-CW. Finally, simulation results at Nanjing station and Yichang station demonstrate that the proposed methodology, i.e., FCI-CW combined with IGWO, is reasonable and reliable, can effectively deal with flood classification problems with better fitness and a comprehensive consideration of the subjective and objective aspects, and has great application potential in sorting, evaluation, and decision-making problems without evaluation criteria.

Keywords: flood classification; FCI; combined weight; GWO; immune clone selection operator

1. Introduction

Floods are one of the most frequent and disastrous natural hazards around the world, and cause a serious loss of life and property every year [1–6]. Flood classification is an important basis for flood forecasting, flood risk identification, flood real-time scheduling, and flood resource utilization [7–10]. Usually, flood classification is performed according to the flood intensity, whose basic principle is to analyze the common points and differences of flood causes, spatial distribution, and the peak quantity relation according to the essential characteristics and objective laws of floods, and to explore the occurrence and possible consequences of different floods. In other words, a reasonable and effective method will be used to cluster the flood samples into different grades, which contributes to realizing the reasonable recognition, effective sorting, and comprehensive management of flood disasters [11–13]. Obviously, flood classification is a problem of multi-attribute and multi-stage fuzzy synthetic evaluation [14,15]. Therefore, it is very important to establish a unified flood classification index system and an advanced classification method to provide a scientific basis for decisions on flood risk management [16,17].

Nowadays, there are many methods for flood classification, such as projection pursuit [18] and its combinations with different optimization algorithms, including the particle swarm optimization algorithm [19], the artificial fish swarm algorithm [20], the shuffled frog leaping algorithm [21], the projection pursuit dynamic clustering model [22], the optimal curve projection dynamic model [23], the fuzzy clustering iterative model (FCI) [24–26], variable fuzzy sets theory (VFS) [8], the weighted fuzzy kernel-clustering algorithm (WFKCA) [27], and fuzzy c-means clustering [28]. However, the computational process of projection pursuit and its improved methods is complicated, and there is a lack of a complete theoretical basis for the subjective division based on projection values [26,27]; the establishment of relative membership in the VFS function depends on a physical analysis and an expert's instinct or experience according to different problems, leading to a hard decision during application [29]. WFKCA, which is based on the attribute weighted fuzzy clustering idea in the kernel feature space, has a good convergence property and its prototypes can be well-represented in the original space; however, its computational process is complex, and it is worth thinking about how to comprehensively obtain the index weight [30]. Fuzzy c-means clustering can classify and evaluate the flood samples; however, it ignores the weights of indicators [27].

Fortunately, the FCI, which was proposed by Chen [7], is a sample classification method for data analysis and pattern partition that not only considers the uncertainty and fuzziness of the flood classification index system, but also considers the weights of flood classification indices [26]. Moreover, it is difficult for the FCI to ensure the minimum general Euclidean weighted distance of the objective function, and its global search is generally combined with optimization algorithms to improve the possibility of a global optimal solution. Nevertheless, when the weights for the evaluation indices are coded as a population of individual variables in the optimization algorithm for the FCI, the obtained weights are calculated in terms of "mathematical weights", which are only derived from the attribute values of the sample, and cannot express the decision-maker's subjective consciousness and risk preference. In other words, this "mathematical weight" still requires repeated consistency tests and subjective adjustments in fuzzy decision-making [31–33]. Therefore, considering the fact that the spatial and temporal distributions of flood classification indices are uneven and serious, and even comprehensive weights for indices are relatively complicated to calculate, it is necessary to carry out a complete and integrated analysis on the index weight calculation before we implement flood classification based on the FCI.

Currently, in the study of flood classification, the commonly used methods to determine the weight of evaluation indexes are the Delphi method [8], the consistency theorem method [7,34], the entropy weight method [24] and the projection pursuit method [35,36], which can be divided into two types: subjective weighting methods, such as the Delphi method and the consistency theorem method; and objective weighting methods, such as the entropy weight method and the projection pursuit method. In other words, the subjective weighting method means that decision-makers combine their own subjective understanding, judgment, and experience to analyze the physical meaning of indices and then give the relative importance of indices; however, it has a larger subjective and arbitrary component. The objective weighting method calculates the weight by using the difference information of samples, has a strong mathematical theory, and may avoid subjective randomness; however, sometimes, the physical meaning of indices in the evaluation cannot be clarified and deviation may be caused by the amount of information in the data itself. Therefore, the subjective and objective weighting methods have their own advantages and disadvantages, and the integration of them can avoid the shortcomings and one-sidedness of a single method alone.

Therefore, based on the FCI and the thought of making full use of the subjective weight and objective weight information, a sensitivity coefficient is presented to integrate the subjective weights and objective weights into comprehensive weights, and a fuzzy clustering iteration model with combined weight (FCI-CW) is proposed in this paper. However, the computational process of FCI-CW with a fuzzy clustering iterative solution is time-consuming and intractable, and cannot even achieve global optimization with some initial conditions; so, an intelligent optimization technique is used to deal with the optimization problem of FCI-CW. In this way, an immune grey wolf optimizer algorithm (IGWO) is adopted for the fuzzy clustering to obtain better results.

The Grey Wolf Optimization algorithm (GWO) is a new metaheuristic optimization algorithm that was proposed by Mirjalili in 2014 [37]. It is a new swarm intelligence optimization algorithm that has superior performance in finding optimal solutions and less need to adjust the parameters, is easy to implement, and has a strong search ability. In terms of function optimization, GWO has an advantage over differential evolution algorithms in terms of the convergence accuracy and convergence speed. In order to solve the global optimization of a fuzzy clustering problem, GWO is employed in this paper. However, similar to other swarm intelligence optimization algorithms, GWO also has the disadvantages of being prone to local optimization and having low precision. As a result, after a detailed introduction, GWO is improved with the immune clonal method [38], and the immune grey wolf optimizer algorithm (IGWO) with better performance is proposed. On this basis, the clustering center and the sensitivity coefficient are combined with IGWO for the global optimal search to avoid the sensitive problem of initial clustering centers and obtain the optimal fuzzy clustering center and the optimal sensitivity coefficient.

Finally, the purpose of this paper is to establish a fuzzy clustering iterative model with combined weight (FCI-CW) by organically integrating the subjective and objective weights without evaluation criteria, and then executing the optimization solution of FCI-CW with IGWO. Additionally, we consider two case studies on flood classification with the combination of FCI-CW and IGWO at Yichang station and Nanjing station, respectively. The results demonstrate that the methodology proposed in this paper is more reasonable and reliable as it comprehensively considers the subjective and objective aspects, and thus provides a new effective approach for flood classification in a complex decision-making environment.

The main contributions of this article are as follows:

(1) The fuzzy clustering iterative model with combined weight (FCI-CW) is proposed by using the sensitivity coefficient to combine the subjective weight and objective weight.
(2) An immune grey wolf optimizer algorithm (IGWO) is proposed by employing an immune clone selection operator based on the grey wolf optimizer algorithm.
(3) IGWO was employed to obtain the optimal fuzzy class center matrix and optimal sensitivity coefficient of FCI-CW.
(4) Two case studies of flood classification at Nanjing station and Yichang station were carried out to demonstrate the reasonableness and effectiveness of the proposed methodology.

For the reader's convenience, the remainder of this paper is organized as follows. The FCI and the combined weighting method are first introduced, and then FCI-CW is presented in detail in Section 2. The basic principles of GWO are introduced, and then IGWO is carried out in detail in Section 3. Section 4 displays the procedure of the proposed methodology, i.e., flood classification using FCI-CW and IGWO. The proposed methodology is applied to the case studies in Section 5. Finally, conclusions are given in Section 6.

2. The Fuzzy Clustering Iteration Model with Combined Weight (FCI-CW)

2.1. Overview of the Fuzzy Clustering Iteration Model

As one of the most important concepts in fuzzy mathematics, the FCI model was proposed by Chen [7]. In this theory, a fuzzy membership function can be defined as a rule to explore a mapping between a given set of observations and their relevant factors. The implementation of the model consists of following steps [16,26,30].

Assume that there are n samples to form the set $X = \{x_1, x_2, \ldots, x_n\}$, and any sample x_i has m indices, and the actual values of sample x_i can be denoted as

$$x_i = (x_{i1}, x_{i2}, \ldots, x_{im}). \tag{1}$$

Hence, the sample set is described as a $n \times m$ matrix $\mathbf{X}$

$$\mathbf{X} = (x_{ij})_{n\times m} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \tag{2}$$

where x_{ij} is the eigenvalue of index j for sample x_i, $i = 1, 2, \ldots, n; j = 1, 2, \ldots, m$; and n, m is the total number of assessment samples and assessment indices, respectively. In order to deal with values of different orders of magnitude, all of the eigenvalues of matrix $\mathbf{X}$ are transformed into normalized eigenvalues as:

$$r_{ij} = (x_{ij} - x_{\min}(j))/(x_{\max}(j) - x_{\min}(j)) \tag{3}$$

where r_{ij} is the normalized eigenvalue of index j for sample x_i, obviously $0 \leq r_{ij} \leq 1$; and $x_{\max}(j)$, $x_{\min}(j)$ denote the maximum and minimum eigenvalues of index j, respectively. After the normalization of all the eigenvalues, the normalized matrix $\mathbf{R}$ is calculated as:

$$\mathbf{R} = (r_{ij})_{n\times m} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{n1} \\ r_{21} & r_{22} & \cdots & r_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix}. \tag{4}$$

Assume that the m indices of n samples can be clustered with c classes, and the fuzzy clustering matrix is defined as follows:

$$\mathbf{U} = (u_{hi})_{c\times n} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ u_{21} & u_{22} & \cdots & u_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ u_{c1} & u_{c2} & \cdots & u_{cn} \end{bmatrix}, \sum_{h=1}^{c} u_{hi} = 1; 0 \leq u_{hi} \leq 1. \tag{5}$$

where u_{hi} denotes the relative membership degree of sample i belonging to class h. Assume that the eigenvalues of m indices for class h are denoted as the clustering center of the class h standard, then an index matrix can be obtained as the fuzzy class center matrix.

$$\mathbf{S} = \left(s_{jh}\right)_{m\times c} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1c} \\ s_{21} & s_{22} & \cdots & s_{2c} \\ \cdots & \cdots & \cdots & \cdots \\ s_{m1} & s_{m2} & \cdots & s_{mc} \end{bmatrix}, 0 \leq s_{jh} \leq 1 \tag{6}$$

where s_{jh} is the eigenvalue of index j of the class h standard. For depicting different indexes' effects, we induct weights into a cluster. The index weight vector is defined as follows:

$$\boldsymbol{\omega} = (\omega_1, \omega_2, \cdots, \omega_m), 0 \leq \omega_j \leq 1, \sum_{j=1}^{m} \omega_j = 1. \tag{7}$$

Here, a weighted general Euclidean distance $D(\mathbf{r}_i - \mathbf{s}_h)$ is used to represent the difference between sample i denoted as $\mathbf{r}_i$ and class h denoted as $\mathbf{s}_h$:

$$D(\mathbf{r}_i - \mathbf{s}_h) = u_{hi} \cdot \|\omega_j \cdot (\mathbf{r}_i - \mathbf{s}_h)\| = u_{hi} \cdot \left[\sum_{j=1}^{m} \left[\omega_j \cdot (r_{ij} - s_{jh}) \right]^2 \right]^{\frac{1}{2}}. \tag{8}$$

In order to gain the optimal fuzzy clustering matrix **U** and the optimal center matrix **S**, an objective function can be established to minimize the square sum of the general Euclidean weighted distance (GEWD) from the minimum class 1 to the maximum class c, and the optimal of the GEWD can be established as follows.

$$\begin{aligned} &\min\{F(\omega_j, u_{hi}, s_{jh}) = \sum_{i=1}^{n} \sum_{h=1}^{c} \left[u_{hi} \left\| \omega_j (r_{ij} - s_{jh}) \right\| \right]^2 \} \\ &s.t. \sum_{h=1}^{c} u_{hi} = 1; 0 \le u_{hi} \le 1; \sum_{j=1}^{m} \omega_j = 1; 0 \le \omega_j \le 1. \end{aligned} \tag{9}$$

Based on the Lagrange function approach, the index weight $\boldsymbol{\omega} = (\omega_1, \omega_2, \cdots, \omega_m)$, the fuzzy clustering matrix $\mathbf{U} = (u_{hi})_{c \times n}$, and the fuzzy class center matrix $\mathbf{S} = (s_{jh})_{m \times c}$ are described as follows:

$$\omega_j = \left[\sum_{j=1}^{m} \frac{\sum_{i=1}^{n} \sum_{h=1}^{c} u_{hi}^2 \left[(r_{ij} - s_{jh}) \right]^2}{\sum_{i=1}^{n} \sum_{h=1}^{c} u_{hi}^2 \left[(r_{ij} - s_{jh}) \right]^2} \right]^{-1} \tag{10}$$

$$u_{hi} = \left[\sum_{k=1}^{c} \frac{\sum_{j=1}^{m} \left[\omega_j (r_{ij} - s_{jh}) \right]^2}{\sum_{j=1}^{m} \left[\omega_j (r_{ij} - s_{jk}) \right]^2} \right]^{-1} \tag{11}$$

$$s_{jh} = \sum_{i=1}^{n} u_{hi}^2 \omega_j^2 r_{ij} / \sum_{i=1}^{n} u_{hi}^2 \omega_j^2 \tag{12}$$

Finally, the optimal weight vector $\boldsymbol{\omega} = (\omega_1, \omega_2, \cdots, \omega_m)^T$, the optimal fuzzy membership matrix $\mathbf{U} = (u_{hi})_{c \times n}$, and the optimal fuzzy clustering center matrix $\mathbf{S} = (s_{jh})_{m \times c}$ can be obtained through an iterative solution via Equations (11)–(13).

The details of the steps of the fuzzy clustering iterative solution are illustrated as follows [8,16,26]:

Step 1 Set the precision of ε_1, ε_2, and ε_3 for calculating ω_j, u_{hi}, and s_{jh}, and set the maximum iterative number T.

Step 2 Let the iterative number be $t = 0$, assume that the original weight matrix ω_j^t satisfies the constraint $0 \le \omega_j \le 1$, $\sum_{j=1}^{m} \omega_j = 1$ shown in Equation (7), and assume that the original fuzzy membership matrix u_{hi}^t satisfies the constraint $\sum_{h=1}^{c} u_{hi} = 1; 0 \le u_{hi} \le 1$ shown in Equation (5).

Step 3 Calculate the corresponding original clustering center s_{jh}^t by importing u_{hi}^t and ω_j^t into Equation (12).

Step 4 Seek an approximate clustering matrix ω_j^{t+1} by importing s_{jh}^t and u_{hi}^t into Equation (10).

Step 5 Seek an approximate clustering matrix u_{hi}^{t+1} by importing s_{jh}^t and ω_j^{t+1} into Equation (11).

Step 6 Seek an approximate clustering center matrix s_{jh}^{t+1} by importing u_{hi}^{t+1} and ω_j^{t+1} into Equation (12).

Step 7 Compare the corresponding values ω_j^t and ω_j^{t+1}, u_{hi}^t and u_{hi}^{t+1}, and s_{jh}^t and s_{jh}^{t+1}, respectively, and update the iteration counter by $t = t + 1$ until the termination conditions in Equation (13) are satisfied or the iteration counter reaches the maximum iterative number; otherwise go to **Step 3**.

$$\begin{cases} \max\left|\omega_j^t - \omega_j^{t+1}\right| \leq \varepsilon_1 \\ \max\left|u_{hi}^{t+1} - u_{hi}^t\right| \leq \varepsilon_2 \\ \max\left|s_{jh}^{t+1} - s_{jh}^t\right| \leq \varepsilon_3 \end{cases} \tag{13}$$

Step 8 Finally, ω_j^{t+1}, u_{hi}^{t+1}, and s_{jh}^{t+1} are obtained through the above computational steps. The objective value GEWD of the FCI is then calculated as the objective function fitness.

To be clear, the FCI obtains access to the index weight vector so as to minimize the GEWD by the above iterative calculation. However, this may lead to the objective function not being able to implement the global optimization with some initial conditions [31]. Intelligent optimization techniques are usually combined with the FCI to deal with the optimization problem, increase the possibility of searching for the global optimal solution, and avoid the problem of sensitivity to the initial clustering center. They improve the computational efficiency and optimization performance of the conventional iterative calculation approach [26,33].

Moreover, the optimal weight vector based on the above process is calculated only according to the sample data itself. Although its value is between 0 and 1 and its sum is 1, just the "mathematical weight" is given in the meaning of the data calculation, as it cannot reasonably reflect the decision-makers' subjective cognition and the objective difference information of samples [31,33].

Hence, to solve the problem of weight in the FCI, a comprehensive weight is used, and a fuzzy clustering iteration model with a combined weight is proposed.

2.2. Combined Weight for Flood Classification

In order to comprehensively reflect the knowledge of decision-makers' experience and judgment and the importance of evaluation indices, as well as reduce the subjective randomness of the evaluation process, the subjective weight and the objective weight are integrated to determine a more reasonable combined weight, so as to improve the utilization of information and the reliability of the results. In this way, a more practical evaluation result will be reached.

Generally speaking, the calculation methods for combined weight include the additive synthesis method [39], the multiplicative synthesis method [40], and the minimum relative entropy method [41].

Assume that the subjective weight and the objective weight are denoted as $\boldsymbol{\omega}_S = (\omega_{S1}, \omega_{S2}, \ldots, \omega_{Sm})$ and $\boldsymbol{\omega}_O = (\omega_{O1}, \omega_{O2}, \ldots, \omega_{Om})$, respectively. The main calculation procedures to obtain the combined weight $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_m)$ of the above three methods are as follows:

(1) the additive synthesis method [39]

$$\omega_j = \beta \cdot \omega_{Sj} + (1 - \beta) \cdot \omega_{Oj}, j = 1, 2, \ldots, m. \tag{14}$$

where β is the preference coefficient for the subjective and objective weight and $0 \leq \beta \leq 1$. Obviously, the combined weight changes with the selection of β. Usually, β is set as 0.5.

(2) the multiplicative synthesis method (MS) [40]

$$\omega_j = (\omega_{Sj} \times \omega_{Oj}) / \sum_{j=1}^{m} (\omega_{Sj} \times \omega_{Oj}). \tag{15}$$

(3) the minimum relative entropy method (MRE) [41]

According to the principle of minimum relative information entropy, the following objective function is constructed to make ω_j as close as possible to ω_{Sj} and ω_{Oj}. Based on the Lagrange function approach, ω_j is calculated as follows:

$$\omega_j = (\omega_{Sj} \cdot \omega_{Oj})^{0.5} / \left[\sum_{j=1}^{m} (\omega_{Sj} \cdot \omega_{Oj})^{0.5} \right], j = 1, 2, \ldots, m. \tag{16}$$

2.3. The Fuzzy Clustering Iteration Model with Combined Weight

In this section, the additive synthesis method mentioned above is adopted after the subjective weight and the objective weight have been obtained by the corresponding weighting methods. For the subjective weight, there is the Delphi method [8], the consistency theorem method [34], and so on; for the objective weight, there is the entropy weight method [24], the projection pursuit method [35,36], etc. In other words, the fusion of the subjective and objective weights is carried out to calculate the combined weight with the sensitivity coefficient β. Therefore, the objective function of the GEWD in Equation (9) is correspondingly transformed to Equation (17) as follows:

$$\begin{aligned} &\min F = \min\{F(\beta, \omega_{Sj}, \omega_{Oj}, u_{hi}, s_{jh}) = \sum_{i=1}^{n} \sum_{h=1}^{c} \left[u_{hi} \left\| (\beta \cdot \omega_{Sj} + (1-\beta) \cdot \omega_{Oj}) \cdot (r_{ij} - s_{jh}) \right\| \right]^2 \} \\ &s.t. \sum_{h=1}^{c} u_{hi} = 1; 0 \le u_{hi} \le 1 \end{aligned} \tag{17}$$

According to the Lagrange function approach, the Lagrange function is constructed as follows:

$$L(\mathbf{U}, \mathbf{S}, \lambda, \beta) = \sum_{i=1}^{n} \sum_{h=1}^{c} \left[u_{hi} \left\| (\beta \cdot \omega_{Sj} + (1-\beta) \cdot \omega_{Oj}) \cdot (r_{ij} - s_{jh}) \right\| \right]^2 - \lambda \left(\sum_{h=1}^{c} u_{hi} - 1 \right). \tag{18}$$

Set $\frac{\partial L}{\partial u_{hi}} = 0$, $\frac{\partial L}{\partial s_{jh}} = 0$, $\frac{\partial L}{\partial \beta} = 0$, $\frac{\partial L}{\partial \lambda} = 0$. Based on a theoretical derivation, a new iteration model of FCI-CW can be obtained:

$$u_{hi} = \left[\sum_{k=1}^{c} \frac{\sum_{j=1}^{m} \left[(\beta \cdot \omega_{Sj} + (1-\beta) \cdot \omega_{Oj}) \cdot (r_{ij} - s_{jh}) \right]^2}{\sum_{j=1}^{m} \left[(\beta \cdot \omega_{Sj} + (1-\beta) \cdot \omega_{Oj}) \cdot (r_{ij} - s_{jk}) \right]^2} \right]^{-1} \tag{19}$$

$$s_{jh} = \sum_{i=1}^{n} u_{hi}^2 \cdot (\beta \cdot \omega_{Sj} + (1-\beta) \cdot \omega_{Oj})^2 \cdot r_{ij} / \sum_{i=1}^{n} u_{hi}^2 (\beta \cdot \omega_{Sj} + (1-\beta) \cdot \omega_{Oj})^2 \tag{20}$$

$$\beta = \frac{\sum_{i=1}^{n} \sum_{h=1}^{c} \sum_{j=1}^{m} u_{hi}^2 \cdot (r_{ij} - s_{jh})^2 \cdot \omega_{Oj} (\omega_{Oj} - \omega_{Sj})}{\sum_{i=1}^{n} \sum_{h=1}^{c} \sum_{j=1}^{m} u_{hi}^2 \cdot (r_{ij} - s_{jh})^2 \cdot (\omega_{Oj} - \omega_{Sj})^2}. \tag{21}$$

2.4. The Procedure of FCI-CW

We aim to minimize the objective function in Equation (17) of FCI-CW. The subjective and objective weights are denoted $\boldsymbol{\omega}_S = (\omega_{S1}, \omega_{S2}, \ldots, \omega_{Sm})$ and $\boldsymbol{\omega}_O = (\omega_{O1}, \omega_{O2}, \ldots, \omega_{Om})$, respectively; the sensitivity coefficient is denoted β; the fuzzy clustering matrix is denoted $\mathbf{U} = (u_{hi})_{c \times n}$; and the fuzzy class center matrix is denoted $\mathbf{S} = (s_{jh})_{m \times c}$. We adopt the sensitivity coefficient and the clustering center matrix to calculate the optimal fuzzy clustering matrix. Its steps are illustrated as follows:

Step 1 Set the iteration counter to $g = 0$;

Step 2 Obtain the subjective weight ω_{Sj} and the objective weight ω_{Oj} according to the subjective weighting method and the objective weighting method separately; initialize s_{jh}^0 and β^0, set the objective function value of Equation (17) to $F^0 = \xi$, where ξ is a large constant; and set the precision ε;

Step 3 Calculate the combined weight ω_j^g based on Equation (14);

Step 4 Calculate the membership matrix u_{hi}^{g} based on Equation (19);
Step 5 Calculate the fuzzy class center matrix s_{ih}^{g} based on Equation (20);
Step 6 Calculate the sensitivity coefficient β^{g} based on Equation (21);
Step 7 Calculate the objective function value $F^{g} = F(\beta^{g}, \omega_{Sj}, \omega_{Oj}, u_{hi}{}^{g}, s_{jh}{}^{g})$;
Step 8 Compare F^{g} with F^{g-1}. If $\left|F^{g} - F^{g-1}\right| < \varepsilon$, stop the calculation; otherwise $g = g + 1$ and return to **Step 3**.

From the above procedure, we can see that the computational process of the FCI is time-consuming and intractable when the number of data in the dataset is large. So, GWO is adopted for the fuzzy clustering to obtain better results. Taking into consideration the disadvantages of GWO, IGWO is presented after the introduction of GWO.

3. The Immune Grey Wolf Optimizer Algorithm

3.1. Overview of the Grey Wolf Optimizer Algorithm

GWO is a biologically inspired optimization algorithm that simulates the social hierarchy and hunting mechanism of grey wolves in nature, and was proposed by Mirjalili [37,42]. The grey wolves are gregarious animals, and there are usually a dozen wolves in each pack, which build a strict grey wolf pyramid hierarchy. In GWO, the crowd is split into four different groups (alpha, beta, delta, and omega), which are employed for simulating the leadership hierarchy, i.e., $\alpha \succ \beta \succ \delta \succ \omega$, as shown in Figure 1.

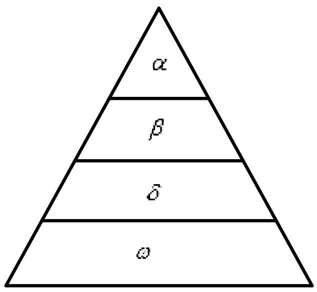

Figure 1. The grey wolf social hierarchy.

Specifically, firstly, the alpha wolf (α) is the leader of the wolves, and it is mostly responsible for making decisions about hunting, the sleeping place, the time to wake, and so on. The alpha's decisions are dictated to the pack. Secondly, the beta wolf (β) is a subordinate wolf that helps the alpha wolf in decision-making or other pack activities. The beta wolf should respect and advise the alpha, but commands the other lower-level wolves as well. Thirdly, the delta wolf (δ) has to command to the alpha wolf and beta wolf, but can also dominate the omega wolves. Finally, the lowest ranking grey wolf is an omega wolf. The omega wolves (ω) always have to submit to all the other dominant wolves, such as the alpha wolf, the beta wolf, and the delta wolf.

For mathematical modeling of the GWO algorithm, which simulates the hunting behavior of grey wolves, we need to first generate a group of wolves randomly in the search space, then use α, β, and δ wolves to estimate the position of the prey. As for other wolves, they are ordered to calculate the distance between themselves and the α, β, and δ wolves, then get close to the prey and encircle it, and finally capture the prey successfully. The modeling steps of GWO are detailed as follows:

Assume that there are N wolves in a pack, denoted as $\mathbf{Y} = \{\mathbf{Y}_1, \mathbf{Y}_2, \cdots, \mathbf{Y}_N\}$, and the searching space has D dimensions. The position of grey wolf i at the generation g can be expressed as $\mathbf{Y}_i^{g} = (y_{i,1}^{g}, y_{i,2}^{g}, \ldots, y_{i,D}^{g})$, and the position of α, β, and δ wolves can be respectively denoted as $\mathbf{Y}_{\alpha}$, $\mathbf{Y}_{\beta}$, and $\mathbf{Y}_{\delta}$, who are in, respectively, the best, the second best, and the third best current position of grey wolves.

Hence, the behavior of grey wolves encircling the prey can be mathematically expressed using the following equations:

$$\mathbf{D}_i^g = \left| \mathbf{C} \cdot \mathbf{Y}_p^g - \mathbf{Y}_i^g \right| \tag{22}$$

$$\mathbf{Y}_i^{g+1} = \mathbf{Y}_i^g - \mathbf{A} \cdot \mathbf{D}_i^g \tag{23}$$

where $\mathbf{Y}_p^g$ is the position of the prey at the generation g. The vectors $\mathbf{C}$ and $\mathbf{A}$ can respectively be obtained by Equations (24) and (25) as follows.

$$\mathbf{C} = 2 \cdot \mathbf{r}_1 \tag{24}$$

$$\mathbf{A} = 2a \cdot \mathbf{r}_2 - a \tag{25}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are random vectors ranging from 0 to 1. As the time of iteration increases, a decreases from 2 to 0.

In order to mathematically simulate the hunting behavior, assume that α, β, and δ wolves are closest to the prey and have better knowledge about the potential location of the prey, and that we can rely on the position of these three kinds of wolves to estimate the prey's position. The way to update the position of grey wolves at the generation g is as shown in Equation (26):

$$\mathbf{Y}_i^{g+1} = (\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3)/3 \tag{26}$$

$$\mathbf{Y}_1 = \mathbf{Y}_\alpha - \mathbf{A}_1 \cdot \mathbf{D}_1 \quad \mathbf{Y}_2 = \mathbf{Y}_\beta - \mathbf{A}_2 \cdot \mathbf{D}_2 \quad \mathbf{Y}_3 = \mathbf{Y}_\delta - \mathbf{A}_3 \cdot \mathbf{D}_3 \tag{27}$$

$$\mathbf{D}_1 = \left| \mathbf{C}_1 \cdot \mathbf{Y}_\alpha - \mathbf{Y}_i^g \right| \quad \mathbf{D}_2 = \left| \mathbf{C}_2 \cdot \mathbf{Y}_\beta - \mathbf{Y}_i^g \right| \quad \mathbf{D}_3 = \left| \mathbf{C}_3 \cdot \mathbf{Y}_\delta - \mathbf{Y}_i^g \right| \tag{28}$$

$$\mathbf{A}_1 = 2a \cdot \mathbf{r}_3 - a \quad \mathbf{A}_2 = 2a \cdot \mathbf{r}_4 - a \quad \mathbf{A}_3 = 2a \cdot \mathbf{r}_5 - a \tag{29}$$

$$\mathbf{C}_1 = 2 \cdot \mathbf{r}_6 \quad \mathbf{C}_2 = 2 \cdot \mathbf{r}_7 \quad \mathbf{C}_3 = 2 \cdot \mathbf{r}_8 \tag{30}$$

where $\mathbf{r}_3$, $\mathbf{r}_4$, $\mathbf{r}_5$, $\mathbf{r}_6$, $\mathbf{r}_7$, and $\mathbf{r}_8$ are random vectors ranging from 0 to 1, similar to the above-mentioned $\mathbf{r}_1$ and $\mathbf{r}_2$; obviously, the calculation of $\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{A}_3$ is similar to the calculation of $\mathbf{A}$, and the calculation of $\mathbf{C}_1$, $\mathbf{C}_2$, and $\mathbf{C}_3$ is similar to the calculation of $\mathbf{C}$.

With these equations, the position would be in a random place within a circle that is defined by the positions of α, β, and δ in the search space. In other words, α, β, and δ estimate the position of the prey, and the other wolves update their positions randomly around the prey. Finally, when the generation g reaches the maximum generation G, $\mathbf{Y}_\alpha$ is put out as the optimal positon of GWO.

GWO carries out the process of encircling, hunting, and attacking prey based on α, β, and δ with a simple structure, a lower number of control parameters, and easy programming. However, it also easily falls into the local optimal. The reason is that, with the deepening of the optimization iteration and the rapid decline of the population diversity, the differences among the individuals become smaller and smaller, and cannot be found in the solution space, thus resulting in a premature convergence in the global search. Therefore, based on the immune clonal theory, this paper proposes an immune grey wolf optimizer algorithm (IGWO), which chooses the elite individuals for immune clone selection to improve the global convergence accuracy and the overall optimization ability.

3.2. Immune Clone Selection Operator

The immune clone selection operator is a deep search for elite individuals. Its essence is to clone the elite individuals according to their fitness, thus producing a certain number of mutant individuals to expand the range of the search and improve the diversity of the population [43]. The details of the calculation procedure are as follows [38].

Step 1 The current population is ranked according to fitness, and the best nn individuals are selected to form the elite population.

Step 2 All the individuals in the elite population are respectively cloned to form a temporary population S. The clone size is directly proportional to the affinity, and the population number N_c of S is calculated as:

$$N_c = \sum_{i=1}^{nn} round(\frac{\beta \times nn}{i} + b) \tag{31}$$

where the $round()$ function results in round numbers; the number of β is between 0 and 1; b is an integer constant; and $b \geq 1$, which ensures that each individual from the elite population has a certain number of clones.

Step 3 All the individuals in S are successively implemented based on a mutation operator mm times to obtain better candidate solutions nearby to themselves. The mutation operator is shown in Equations (32)–(34) as follows:

$$new\ yx_{i,j}^{g} = y_{i,j}^{g} + p \times \eta \times y_{i,j}^{g} \times r_1 - p \times \eta \times y_{i,j}^{g} \times r_2 \tag{32}$$

$$p = \begin{cases} 1, & r_3 \leq 0.5 \\ 0, & else \end{cases} \tag{33}$$

$$\eta = 1 - \exp(1 - G/(g+1)) \tag{34}$$

where $y_{i,j}^{g}$ is the j-th dimension variable of the i-th individual at generation g, $new\ yx_{i,j}^{g}$ is a new variable generated from $y_{i,j}^{g}$ based on the mutation operator; r_1, r_2, and r_3 are random numbers between 0 and 1; and η is clonal variation parameter. From Equation (34), we can see that as the generation number increases, the clonal variation parameter decreases, which indicates that η is close to 1 at the beginning, and the variation range is large. A global search is implemented to maintain the diversity of the population at this time, and, when the generation number equals the maximum evolution generation, η is close to 0. Then, a local search is carried out in a small range to improve the local fine tuning ability and to ensure the search's accuracy.

Step 4 The best nn individuals are selected from S to replace the elite population in the next generation.

Hence, IGWO is developed to optimize the fuzzy clustering objective function of FCI-CW. For your convenience, the immune clone selection operator is performed every 20 generations, and we set $nn = N/4$, $mm = 200$.

3.3. The Pseudo Code of IGWO

The Pseudo code of IGWO can be expressed as in Table 1.

Table 1. The pseudo code of the immune grey wolf optimizer algorithm (IGWO).

Algorithm Immune Grey Wolf Optimizer Algorithm (IGWO)
Set generation $g = 0$
Initialize the grey wolf population $\mathbf{Y}_i^0$, $i = 1, 2, \ldots, N$.
Initialize a, $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$, $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{C}_3$, nn, mm.
Calculate the fitness of each individual
$\mathbf{Y}_\alpha$ = the best search individual
$\mathbf{Y}_\beta$ = the second best search individual
$\mathbf{Y}_\delta$ = the third best search individual
While (g < Max number of generations)
For each search individual
Update position of each current search individual by Equation (26)
End for
Update a, $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$, $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{C}_3$
Calculate the fitness of all search individuals
Choose the best nn individuals to form the elite population
Clone the elite population to form the temporary population S by Equation (31)
execute the mutation operator by Equation (32)
replace the elite population and the GWO population
Update $\mathbf{Y}_\alpha$, $\mathbf{Y}_\beta$, and $\mathbf{Y}_\delta$,
$g = g + 1$
End while
Output $\mathbf{Y}_\alpha$

3.4. Simulation of IGWO for Solving Benchmark Optimization Problems

In this section, six well-known benchmark optimization problems (shown in Table 2) are employed to evaluate the search ability of the IGWO algorithm by a comparison with the simple GWO and differential evolution (DE) algorithms [44,45].

Table 2. The benchmark functions.

Function	Expression	Dim	Shift Position	Optimal Value
Sphere	$f_1(x) = \sum_{i=1}^{D} x_i^2$	30	[−100,100]	0
Rastrigin	$f_2(x) = \sum_{i=1}^{D} [x_i^2 - 10\cos(2\pi x_i) + 10]$	30	[−5.12,5.12]	0
Ackley	$f_3(x) = -20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}) - \exp[\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)] + 20 + e$	30	[−30,30]	0
Schwefel 2.21	$f_4(x) = \max_{1 \le i \le D}\{\lvert x_i \rvert\}$	30	[−100,100]	0
Schwefel 2.22	$f_5(x) = \sum_{i=1}^{D} \lvert x_i \rvert + \prod_{i=1}^{D} \lvert x_i \rvert$	30	[−10,10]	0
Schaffer's f7	$f_6(x) = \frac{1}{4000}\sum_{i=1}^{D-1} (x_i^2 + x_{i+1}^2)^{0.25}[\sin^2(50(x_i^2 + x_{i+1}^2)^{0.1}) + 1]$	30	[−100,100]	0

For the sake of comparison, the algorithm parameters are set as below: the number of individuals is 30; and the maximum generation number is 1000. For DE, its mutation parameter and crossover parameter are set as 0.5 and 0.4, respectively. The simulation results are shown in Table 3, and each function is independently processed 50 times. The convergence curves for each function are respectively shown in Figures 2–7. The Y-axis is the value of the corresponding function.

Table 3. The test function results for the differential evolution (DE), grey wolf optimization (GWO), and IGWO algorithms.

Function	Algorithm	Best	Worst	Mean	Standard Deviation
Sphere	DE	1.15×10^{-19}	1.49×10^{-18}	5.60×10^{-19}	6.33×10^{-19}
	GWO	1.35×10^{-60}	6.54×10^{-59}	2.77×10^{-59}	3.11×10^{-59}
	IGWO	1.05×10^{-223}	8.78×10^{-220}	4.51×10^{-220}	0
Rastrigin	DE	84.5	109	99.5	11.9
	GWO	0	3.22	8.04×10^{-1}	1.61
	IGWO	0	0	0	0
Ackley	DE	1.09×10^{-10}	1.41×10^{-10}	1.29×10^{-10}	1.53×10^{-11}
	GWO	1.51×10^{-14}	2.93×10^{-14}	1.87×10^{-14}	7.11×10^{-15}
	IGWO	4.44×10^{-15}	7.99×10^{-15}	6.22×10^{-15}	2.05×10^{-15}
Schwefel 2.21	DE	1.00	5.69	3.20	2.05
	GWO	8.27×10^{-15}	4.68×10^{-14}	2.83×10^{-14}	1.58×10^{-14}
	IGWO	4.52×10^{-42}	4.21×10^{-36}	1.05×10^{-36}	2.10×10^{-36}
Schwefel 2.22	DE	2.96×10^{-12}	9.44×10^{-12}	5.94×10^{-12}	3.14×10^{-12}
	GWO	4.06×10^{-35}	7.89×10^{-35}	6.29×10^{-35}	1.69×10^{-35}
	IGWO	1.24×10^{-135}	1.80×10^{-132}	9.01×10^{-133}	1.02×10^{-132}
Schaffer's f_7	DE	2.36×10^{-02}	9.19×10^{-02}	4.74×10^{-02}	3.15×10^{-02}
	GWO	1.09×10^{-15}	4.99×10^{-15}	3.11×10^{-15}	1.74×10^{-15}
	IGWO	2.00×10^{-67}	7.36×10^{-67}	3.72×10^{-67}	2.46×10^{-67}

Table 3 lists the best, worst, mean, and standard deviation results from the different algorithms over 50 independent runs. As inferred from Table 3, compared with DE and GWO, IGWO can obtain the best values, the smallest averages, and the smallest variances in terms of optimal results. What is more, Figures 2–7 show the convergence curves of DE, GWO, and IGWO, respectively. It can be observed that the proposed method is able to effectively avoid premature convergence and obtain better convergence precision. On the whole, the optimization performance of IGWO is obviously better than that of DE and GWO, and it can be illustrated that the targeted improvement strategy for IGWO is effective. Hence, the paper proposes a new and feasible method with good search capability to solve optimization problems.

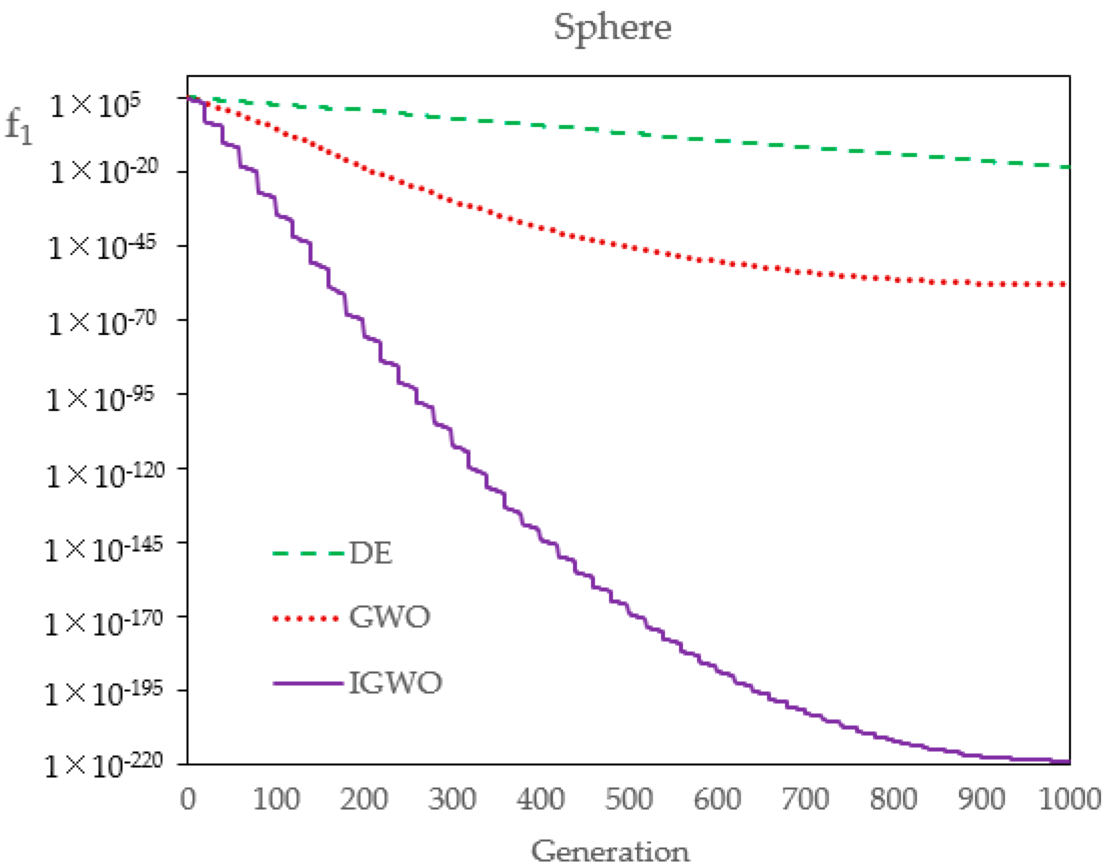

Figure 2. The convergence curve of the function Sphere.

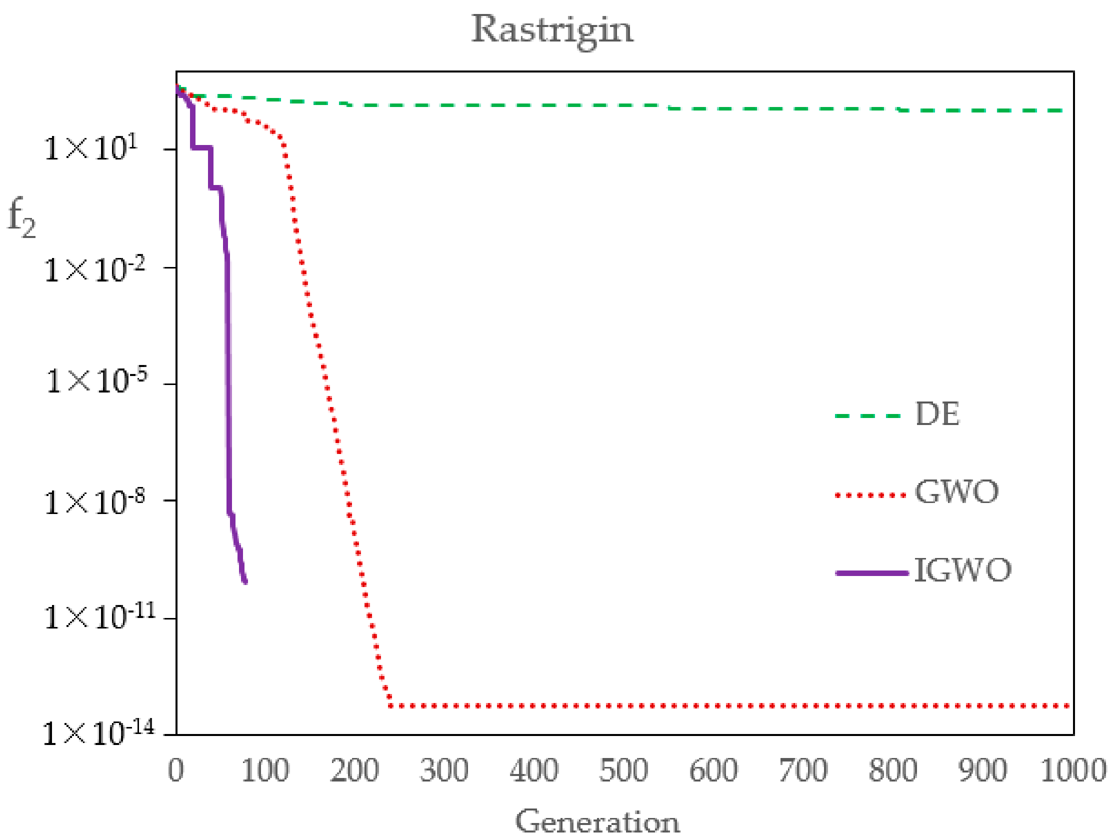

Figure 3. The convergence curve of the function Rastrigin.

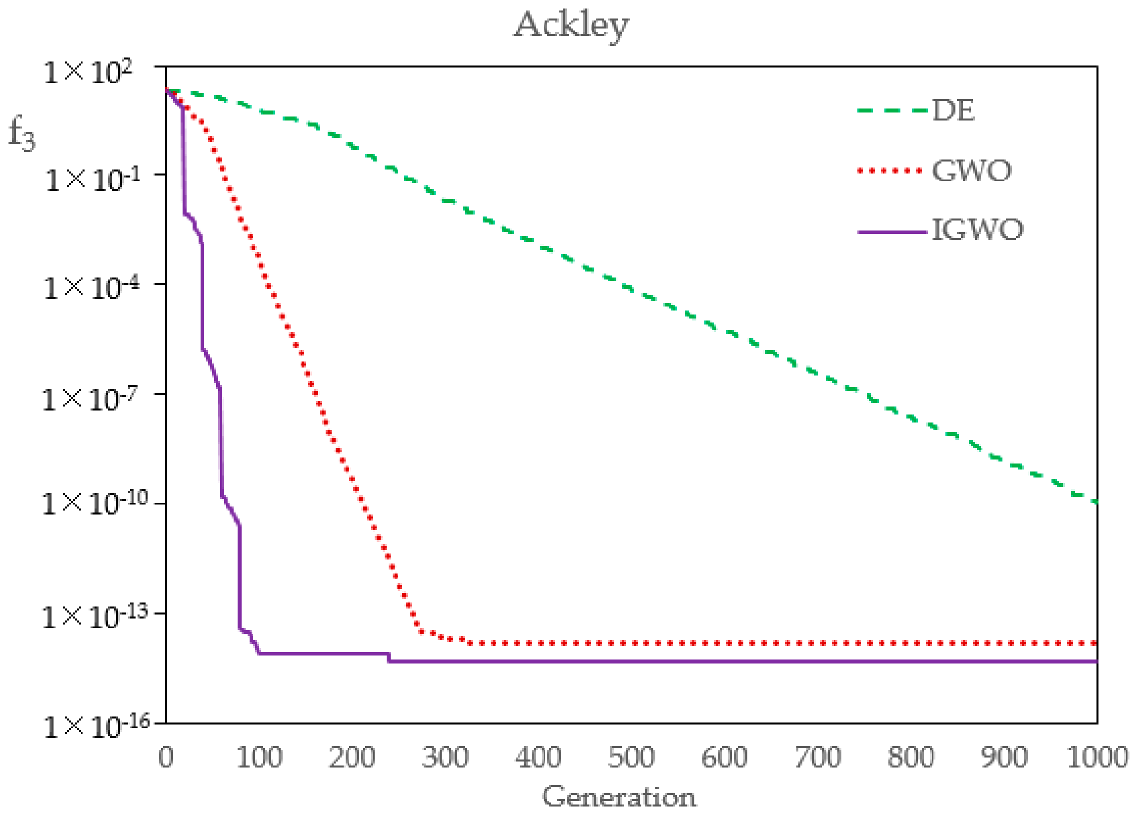

Figure 4. The convergence curve of the function Ackley.

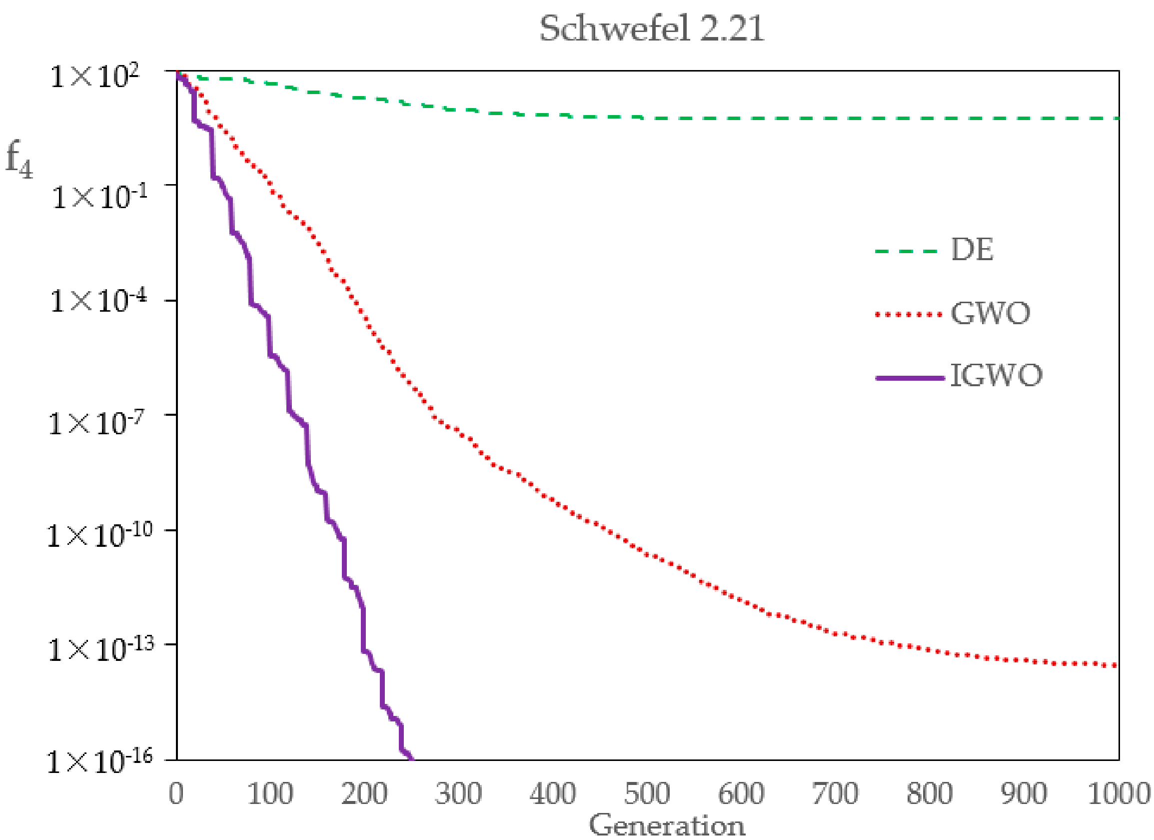

Figure 5. The convergence curve of the function Schwefel 2.21.

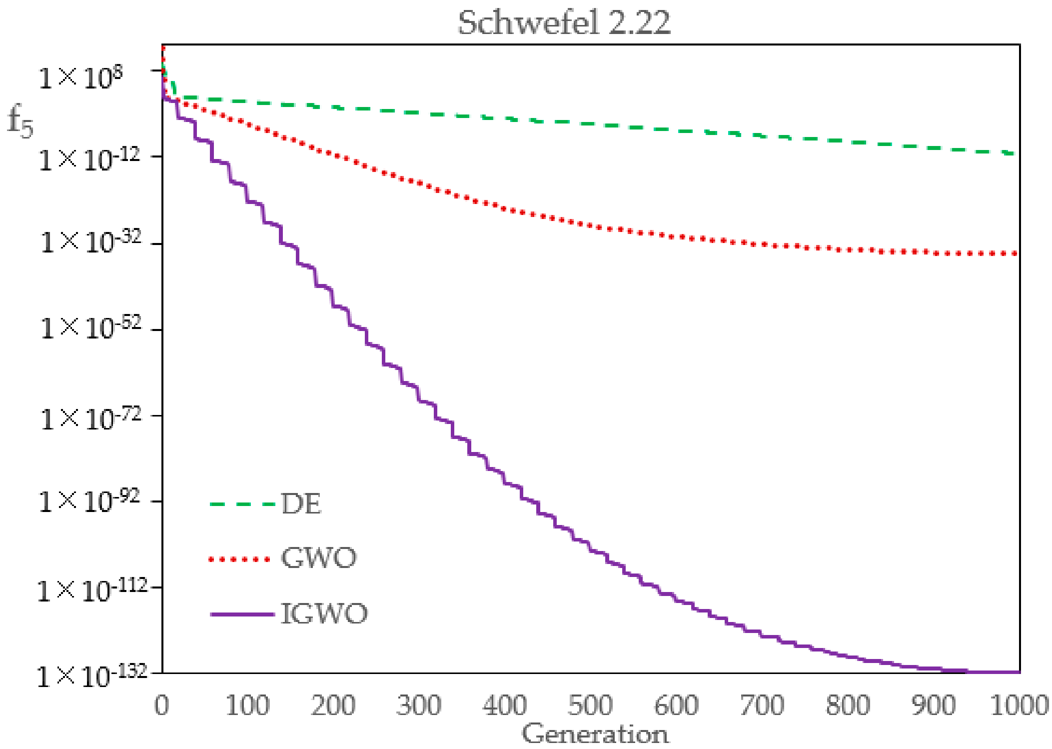

Figure 6. The convergence curve of the function Schwefel 2.22.

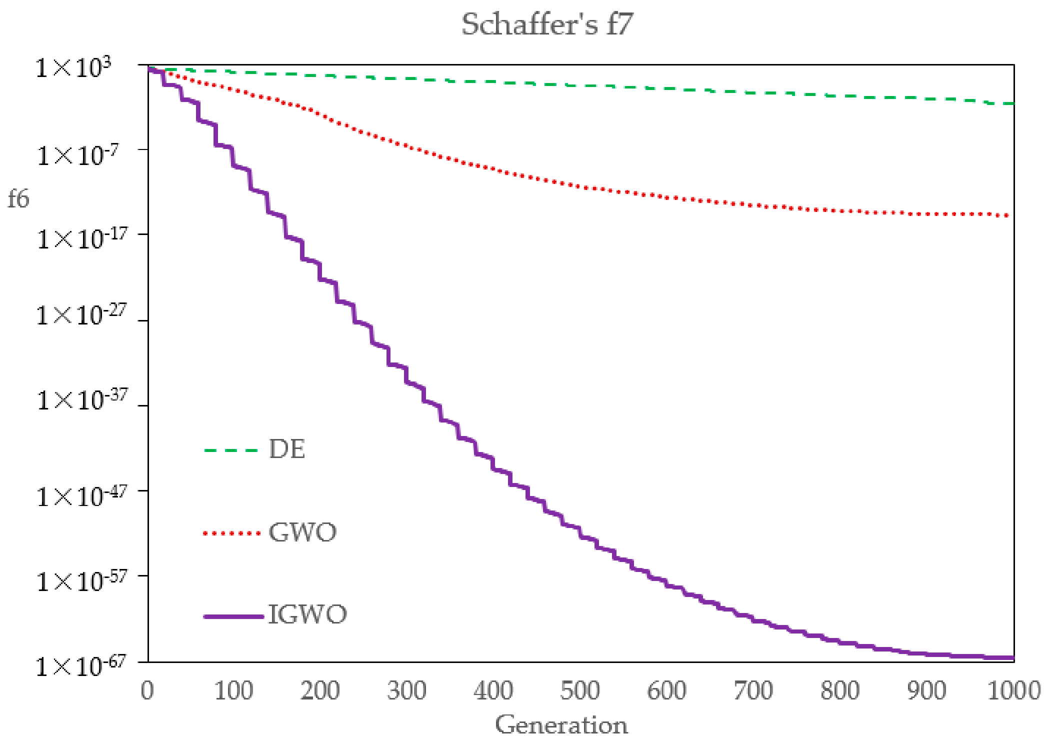

Figure 7. The convergence curve of the function Schaffer's f7.

4. The Procedure for Flood Classification Using FCI-CW and IGWO

4.1. Search-Variable Representation and Fitness Function

Since IGWO is a real-parameter optimization algorithm, when IGWO is adopted to optimize the objective function of FCI-CW, the search-variable representation should be considered first. Here, the fuzzy class center matrix $\mathbf{S} = \left(s_{jh}\right)_{m \times c}$ as well as the sensitivity coefficient β are both chosen to be optimization variables and encoded as a position of individuals. This means that a grey wolf in GWO is a string of real numbers of the $mc + 1$ dimension vector, which can be described as:

$$\mathbf{Y}_i = \{s_{11}, s_{12} \ldots, s_{1c}, s_{21}, s_{11}, \ldots, s_{2c}, \ldots, s_{m1}, s_{m2}, \ldots, s_{mc}, \beta\} \tag{35}$$

where the first c elements $s_{11}, s_{12} \ldots, s_{1c}$ represent the first cluster, the next c elements represent the second cluster center, and so on; the last element represents the sensitivity coefficient. In this way, $\mathbf{S}$ and β can be obtained by IGWO.

When using IGWO to solve the objective function of the GEWD shown in Equation (17), it will be considered as the fitness function, and a smaller fitness leads to a better effect of fuzzy clustering. Therefore, the purpose of IGWO is to obtain the minimum optimal solution by continuous iterative optimization.

4.2. The Procedure of FCI-CW and IGWO

Finally, the procedure for flood classification using FCI-CW and IGWO can be described as in Figure 8.

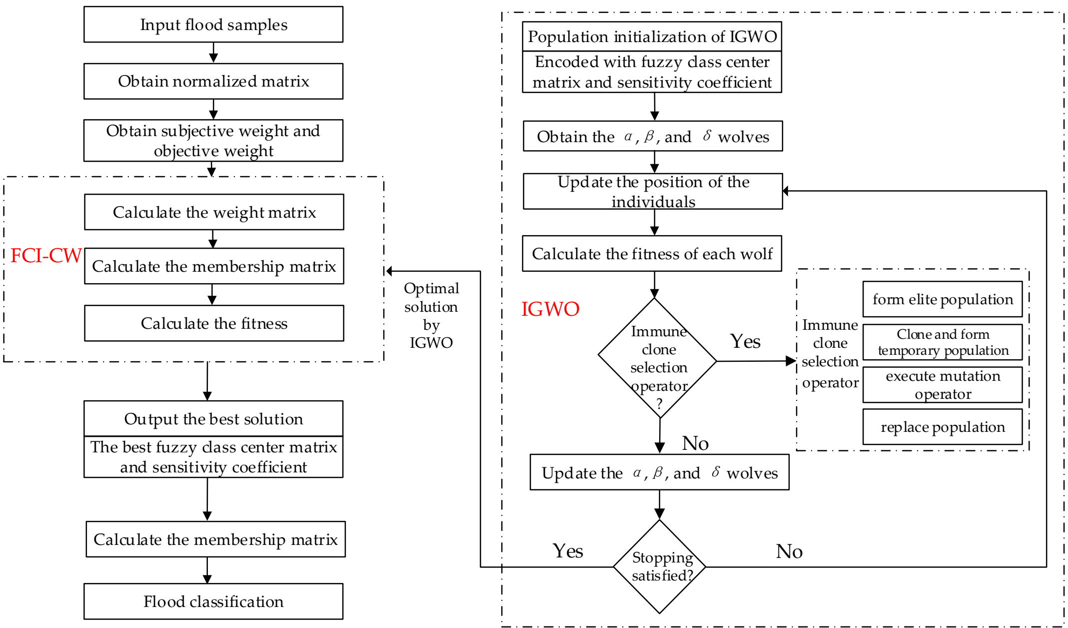

Figure 8. The procedure of flood classification using the fuzzy clustering iterative model with combined weight (FCI-CW) and IGWO.

5. Case Study

5.1. The First Case at Nanjing Station

According to the flood records of Nanjing station in the lower reaches of the Yangtze River from 1951 to 2005, the flood samples are as shown in Table 4 [8,26]. There are 10 floods, with data on the flood peak level, the number of days that the flood level was over 9 m, the flood discharge in DaTong station, the flood volume from May to September, and the synthetic index of discharge and time, where the final index is the composite value of the flood discharge and its duration according to References [8,26].

Table 4. The flood classification index values of 10 flood samples from Nanjing station.

Number of Floods	Year	Flood Peak Level (m)	The Days of Flood Level over 9 m (day)	Flood Peak Discharge in DaTong Station ($m^3 \cdot s^{-1}$)	Flood Volume from May to September (10^8 m^3)	The Synthetic Index of Discharge and Time
(1)	1954	10.22	87	92,600	8891	7800
(2)	1969	9.20	8	67,700	5447	1710
(3)	1973	9.19	7	70,000	6623	3280
(4)	1980	9.20	10	64,000	6340	2730
(5)	1983	9.99	27	72,600	6641	3560
(6)	1991	9.70	17	63,800	5576	1930
(7)	1992	9.06	13	67,700	5295	1575
(8)	1995	9.66	23	75,500	6162	2390
(9)	1996	9.89	34	75,100	6206	2702
(10)	1998	10.14	81	82,100	7773	5283

On the basis of the Chinese National Standard for Hydrological Forecasting, which was implemented on January 1, 2009, and according to the flood recurrence periods of less than 5 years, 5–20 years, 20–50 years, and more than 50 years, the floods are divided into four grades, i.e., small floods, common floods, large floods, and catastrophic floods. Considering the fact that the flood samples in this paper are at least common floods, the aim of the first study is to divide the flood records into three classes; in other words, the floods are also clustered into catastrophic floods, large floods, and common floods, also denoted as I, II, and III.

Hence, in this case study, FCI-CW and IGWO are applied to flood classification at Nanjing Station.

The parameters employed for FCI-CW are set as follows: the number of flood samples, indices, and clusters is $n = 10$, $m = 5$, and $c = 3$, respectively. According to an analysis of the historical flood characteristics of Nanjing station and the influence of various indices on the flood intensity [8], the subjective weight by the Delphi method [35,36] is $\boldsymbol{\omega}_S = (0.16, 0.25, 0.19, 0.15, 0.25)$. Meanwhile, according to the projection pursuit method [35,36], the objective weight is $\boldsymbol{\omega}_O = (0.17, 0.23, 0.20, 0.19, 0.21)$.

The parameters employed for IGWO are set as follows: population size $N = 50$; maximum evolution generation $G = 600$.

Firstly, in order to compare the optimization performance of DE, GWO, and IGWO, Table 5 shows their statistical results over 30 runs. The results show that the three kinds of evolutionary algorithms have different precisions, to a certain extent. The standard deviation and average value of IGWO are both the smallest, and can basically achieve the same optimal solution every time, which indicates that, compared with DE and GWO, IGWO has better robustness and higher convergence precision. Of course, the number of flood samples in this case study is very small, which leads to a classification effect at an order of magnitude. If the number of samples and the index dimension increases, the optimization effect of IGWO will be more obvious and effective.

Table 5. The comparison results of objective fitness by the different algorithms for the first case study.

Algorithms	Minimum	Maximum	Average	Standard Deviation
DE	2.032220×10^{-2}	2.324090×10^{-2}	2.099263×10^{-2}	1.266623×10^{-3}
GWO	2.030084×10^{-2}	2.030152×10^{-2}	2.030128×10^{-2}	1.364308×10^{-8}
IGWO	2.023083×10^{-2}	2.023083×10^{-2}	2.023083×10^{-2}	3.878990×10^{-18}

In this case study, the minimum objective function value is 2.023083×10^{-2}, and the optimal search result of IGWO was output to obtain the optimal fuzzy class center matrix $\mathbf{S}^*$ and the optimal sensitivity coefficient β^* shown in Equations (36) and (37), respectively. Afterwards, the index weight matrix was calculated as $\boldsymbol{\omega}^* = (0.1644, 0.2412, 0.1944, 0.1676, 0.2324)$, which was combined with the subjective weight and the objective weight by using β^* to obtain the best optimal value of GEWD.

Finally, the optimal fuzzy clustering matrix $\mathbf{U}^*$ was achieved as shown in Equation (39).

$$\mathbf{S}^* = \begin{bmatrix} 0.9683\ 0.9652\ 0.8356\ 0.8597\ 0.8178 \\ 0.6653\ 0.2561\ 0.3429\ 0.2788\ 0.2040 \\ 0.1336\ 0.0432\ 0.1080\ 0.1526\ 0.1034 \end{bmatrix} \tag{36}$$

$$\beta^* = 5.595729 \times 10^{-1} \tag{37}$$

$$\boldsymbol{\omega}^* = (0.1644, 0.2412, 0.1944, 0.1676, 0.2324) \tag{38}$$

$$\mathbf{U}^* = \begin{bmatrix} 0.9477\ 0.0052\ 0.0240\ 0.0093\ 0.0227\ 0.0273\ 0.0111\ 0.0131\ 0.0088\ 0.8543 \\ 0.0327\ 0.0456\ 0.2021\ 0.0781\ 0.9078\ 0.4074\ 0.0820\ 0.8664\ 0.9593\ 0.0992 \\ 0.0196\ 0.9492\ 0.7740\ 0.9125\ 0.0696\ 0.5653\ 0.9069\ 0.1204\ 0.0319\ 0.0466 \end{bmatrix} \tag{39}$$

According to the clustering results in Equation (39), we can conclude that there was a huge flood disaster in 1954 and 1998; there was a medium flood disaster in 1983, 1995, and 1996; and there was a small flood disaster in 1969, 1973, 1980, 1991, and 1992, as shown in Table 6. This sorting result is identical to that obtained with the optimal curve projection dynamic cluster method (OC-PDC) [23], variable fuzzy set theory (VFS) [8], the fuzzy clustering iteration model with a chaotic differential evolution algorithm (FCI-CDE) [26], and the weighted fuzzy kernel-clustering algorithm with an adaptive differential evolution algorithm (WFKCA-ADE) [27], which demonstrates that the proposed methodology for flood classification is reasonable and reliable.

Table 6. The results from the comparison of the proposed method with other methods at Nanjing station.

Number of Floods	Year	OC-PDC	VFS	FCI-CDE	WFKCA-ADE	The Proposed Method
(1)	1954	I	I	I	I	I
(2)	1969	III	III	III	III	III
(3)	1973	III	III	III	III	III
(4)	1980	III	III	III	III	III
(5)	1983	II	II	II	II	II
(6)	1991	III	III	III	III	III
(7)	1992	III	III	III	III	III
(8)	1995	II	II	II	II	II
(9)	1996	II	II	II	II	II
(10)	1998	I	I	I	I	I

OC-PDC, optimal curve projection dynamic cluster; VFS, variable fuzzy set theory; FCI-CDE, fuzzy clustering iteration model with a chaotic differential evolution algorithm; WFKCA-ADE, weighted fuzzy kernel-clustering algorithm with an adaptive differential evolution algorithm.

Moreover, VFS only considers the subjective weight; however, there is unavoidable factual evidence, which means that an objective weight is also needed to describe the distribution characteristics of flood samples. Moreover, the index weight is coded as a search-variable representation in FCI-CDE [26], and its optimal fitness, i.e., the GEWD, is 1.277551×10^{-2}, which is less than the optimal value 2.020046×10^{-2} by the proposed method. However, its optimal weight vector, denoted as $\boldsymbol{\omega}' = (0.0908, 0.6267, 0.1141, 0.0801, 0.0883)$, is just the "mathematical weight" in the sense of sample data calculation, which indicates that the importance of the second index is much larger than the sum of the other four indices. This is contrary to the decision-maker's subjective cognition and the actual situation, so it is necessary to reasonably modify the weight results.

Furthermore, the effect of the sensitivity coefficient on the classification results using FCI-CW and IGWO was evaluated, as shown in Table 7. Here, the sensitivity coefficients were chosen to be different values, such as 0, 0.2, 0.4, 0.45, 0.5, 0.55, 0.6, 0.8, and 1. In other words, when $\beta = 0$ in Equation (17), it means that only the objective weight was employed, and the FCI-CW was degraded into a fuzzy clustering iteration model with objective weight (FCI-OW); when $\beta = 1$ in Equation (17), it means that only the subjective weight was employed, and the FCI-CW was degraded into a fuzzy clustering iteration model with subjective weight (FCI-SW); and when $\beta = 0.5$ in Equation (17), it is the traditional parameter selection using the additive synthesis method. According to Table 7, when β was calculated as 5.595729×10^{-1}, the GEWD was the smallest of all, which indicates that the proposed methodology to calculate the fuzzy class center matrix and the sensitivity coefficient for flood classification is reasonable. This renders it superior to the conventional methods, since only considering subjectivity or objectivity, or just setting β = 0.5, lacks a powerful mathematical basis.

Table 7. The comparison results of the general Euclidean weighted distance (GEWD) with different sensitivity coefficients for the first case study.

β	0	0.2	0.4	0.45	0.5
GEWD	2.037210×10^{-2}	2.028914×10^{-2}	2.024230×10^{-2}	2.0236234×10^{-2}	2.02324×10^{-2}
β	0.55	5.595729×10^{-1}	0.6	0.8	1
GEWD	2.023087×10^{-2}	2.023083×10^{-2}	2.023157×10^{-2}	2.025693×10^{-2}	2.031831×10^{-2}

Finally, we also calculate the GEWD for the combined weights that were obtained by the multiplicative synthesis method shown in Equation (15) and the minimum relative entropy method shown in Equation (16). Their GEWDs are 1.019555 and 1.020171, respectively, which are larger than the adopted combined weight method, i.e., the additive synthesis method with a sensitivity coefficient.

5.2. The Second Case of Yichang Station

According to the flood records of Yichang station, which is the representative hydrological station for the Three Gorges Reservoir in the middle reaches of the Yangtze River, the flood samples are as shown in Table 8 [8]. There are 12 floods, with data on the flood peak level, the flood peak discharge, three-day floods, seven-day floods, and fifteen-day floods. The aim of the study is to divide the flood records into three classes, also denoted as I, II, and III, similarly to the first case.

Table 8. The flood classification index values of 12 flood samples from Yichang station.

Number of Floods	Year	Flood Peak Level (m)	Flood peak Discharge ($m^3 \cdot s^{-1}$)	Three-Day Flood (10^8 m^3)	Seven-Day Flood (10^8 m^3)	Fifteen-Day Flood (10^8 m^3)
(1)	1931	55.0	64,600	163.2	350.4	621.3
(2)	1935	54.6	56,900	137.1	283.3	509.5
(3)	1954	55.7	66,800	170.1	385.3	785.1
(4)	1958	53.5	59,500	148.8	305.1	550.2
(5)	1966	54.0	59,600	151.7	334.2	592.4
(6)	1969	51.5	41,900	105.1	217.4	412.2
(7)	1974	54.8	61,000	151.8	301.6	545.7
(8)	1980	54.0	54,600	139.7	300.8	545.6
(9)	1981	55.4	70,800	172.5	334.8	558.3
(10)	1982	54.6	59,000	146.9	303.8	583.8
(11)	1983	53.3	52,600	129.9	268.1	491.3
(12)	1998	54.5	63,600	151.3	347.8	728.2

In this study, FCI-CW and IGWO are applied to flood classification at Yichang station. The parameters employed for FCI-CW are set as follows: the number of flood samples, indices, and clusters is $n = 12$, $m = 5$, and $c = 3$, respectively. According to an analysis of the historical flood characteristics of Yichang station and the influence of various indices on the flood intensity, the subjective weight by the Delphi method [36] is $\boldsymbol{\omega}_S = (0.3388, 0.2042, 0.2004, 0.1283, 0.1283)$. Meanwhile, according to the projection pursuit method [24], the objective weight is $\boldsymbol{\omega}_O = (0.188, 186, 0.200, 0.197, 0.229)$. The parameters employed for IGWO are set as follows: population size $N = 50$; maximum evolution generation $G = 600$.

Firstly, in order to compare the optimization performance of DE, GWO, and IGWO, Table 9 shows their statistical results over 30 runs. The results show that the three kinds of evolutionary algorithm have different precisions, to a certain extent. The standard deviation and the average value of IGWO are both the smallest, and can basically achieve the same optimal solution every time, which indicates that, compared with DE and GWO, IGWO has better robustness and higher convergence precision.

Table 9. The comparison results of objective fitness by the different algorithms for the second case study.

Algorithms	Minimum	Maximum	Average	Standard Deviation
DE	2.750000×10^{-2}	3.150000×10^{-2}	2.888000×10^{-2}	1.213095×10^{-3}
GWO	2.648600×10^{-2}	2.990000×10^{-2}	2.732100×10^{-2}	1.142702×10^{-6}
IGWO	2.644227×10^{-2}	2.644230×10^{-2}	2.644227×10^{-2}	6.633250×10^{-16}

In this case study, the minimum objective function value is 2.644227×10^{-2}, and the optimal search result of IGWO was output to obtain the optimal fuzzy class center matrix $\mathbf{S}^*$ and the optimal sensitivity coefficient β^* shown in Equations (40) and (41), respectively. Afterwards, the index weight matrix was calculated as $\boldsymbol{\omega}^* = (0.2789, 0.1970, 0.2002, 0.1556, 0.1683)$, which was combined with the subjective weight and the objective weight. Finally, the optimal fuzzy clustering matrix $\mathbf{U}^*$ was achieved as shown in Equation (43).

$$\mathbf{S}^* = \begin{bmatrix} 0.024 & 0.021 & 0.021 & 0.018 & 0.013 \\ 0.626 & 0.554 & 0.584 & 0.506 & 0.377 \\ 0.870 & 0.843 & 0.884 & 0.809 & 0.668 \end{bmatrix} \quad (40)$$

$$\beta^* = 6.027070 \times 10^{-1} \quad (41)$$

$$\boldsymbol{\omega}^* = (0.2789, 0.1970, 0.2002, 0.1556, 0.1683) \quad (42)$$

$$\mathbf{U}^* = \begin{bmatrix} 0.004 & 0.030 & 0.025 & 0.034 & 0.021 & 0.998 & 0.022 & 0.015 & 0.022 & 0.012 & 0.225 & 0.030 \\ 0.049 & 0.884 & 0.127 & 0.877 & 0.834 & 0.001 & 0.755 & 0.951 & 0.153 & 0.895 & 0.658 & 0.306 \\ 0.947 & 0.087 & 0.848 & 0.089 & 0.144 & 0.001 & 0.223 & 0.033 & 0.824 & 0.093 & 0.116 & 0.665 \end{bmatrix} \quad (43)$$

Moreover, according to the clustering results in Equation (43), we can conclude that there was a huge flood disaster in 1931, 1954, 1981, and 1998; there was a medium flood disaster in 1935, 1958, 1966, 1974, 1980, 1982, and 1996; and there was a small flood disaster in 1969, as shown in Table 10. This sorting result is identical to that obtained with VFS [8], which demonstrates that the proposed methodology for flood classification is reasonable and reliable.

Table 10. The comparison results of the proposed method with VFS at Yichang station.

Number of Floods	Year	VFS	The Proposed Method
(1)	1931	I	I
(2)	1935	II	II
(3)	1954	I	I
(4)	1958	II	II
(5)	1966	II	II
(6)	1969	III	III
(7)	1974	II	II
(8)	1980	II	II
(9)	1981	I	I
(10)	1982	II	II
(11)	1983	II	II
(12)	1998	I	I

Finally, the effect of the sensitivity coefficient on the classification results using FCI-CW and IGWO was evaluated, as shown in Table 11. Here, the sensitivity coefficients were chosen to be different values, such as 0, 0.2, 0.4, 0.45, 0.5, 0.55, 0.6, 0.8, and 1. According to Table 11, When β was calculated as 6.027070×10^{-1}, the GEWD was the smallest of all, which indicates that the proposed methodology is reasonable. This renders it superior to the conventional methods, since only considering subjectivity or objectivity, or just setting β = 0.5, lacks a powerful mathematical basis.

Table 11. The comparison results of the GEWD with different sensitivity coefficients for the second case study.

β	0	0.2	0.4	0.45	0.5
GEWD	2.740795×10^{-2}	2.687555×10^{-2}	2.655286×10^{-2}	2.650519×10^{-2}	2.647082×10^{-2}
β	0.55	0.6	6.027070×10^{-1}	0.8	1
GEWD	2.644981×10^{-2}	2.644229×10^{-2}	2.644227×10^{-2}	2.655053×10^{-2}	2.689421×10^{-2}

Therefore, the simulation and analysis results of the second case are identical with those of the first case, and illustrate that it is necessary to adopt the sensitivity coefficient to effectively and comprehensively consider subjectivity and objectivity in classification problems.

6. Conclusions

In view of the problem that flood classification has no evaluation standard and the comprehensive weight is not easy to calculate, a fuzzy clustering iterative model based on a combined weight was proposed by organically integrating the subjective and objective weights into a combined weight with the sensitivity coefficient. At the same time, a better-performance IGWO was put forward based on GWO and the immune clonal theory. On this basis, the optimal fuzzy clustering center matrix and the sensitivity coefficient of FCI-CW were obtained by IGWO. The simulation results show that the proposed methodology, i.e., FCI-CW with IGWO, is simple and feasible, and the classification results are reasonable, reliable, and robust. The proposed methodology can not only effectively deal with the uncertainties and fuzziness of flood classification, but also consider the subjective and objective weights of the evaluation indices, and has a good and wide application in sorting, evaluation, and decision-making problems without an evaluation standard.

Last but not the least, a balance between efficiency and accuracy is needed to be reached in the future, and our future work is to establish a comprehensive evaluation index system, propose subjective weight description methods and objective weight calculation methods in a complex environment, and combine other intelligent evolution techniques for flood classification, especially for the risk assessment of urban floods.

Author Contributions: The research presented here was carried out in collaboration between all authors. Q.Z. and L.L. designed and conducted the experiments. Q.Z. wrote the initial draft of the paper. L.L. reviewed and edited the paper. Y.D. analyzed and discussed the results. H.Q. prepared the figures for this paper. All authors read and approved the final paper.

Funding: This research was funded by the Open Research Program of Changjiang River Scientific Research Institute (Grant No. CKWV2017505/KY), the National Key R&D Program of China (Item No. 2016YFC0402202), and the National Natural Science Foundation of China (Grant No. 91647114).

Conflicts of Interest: The authors declare no conflict of interest.

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