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# Supplementary Materials:

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**Modified Mann-Kendall test**

The null hypothesis for the Mann-Kendall test (Mann, 1945; Kendall, 1948) is that there is no trend in the data assuming the data are independent and randomly ordered. The Mann-Kendall test statistic S can be calculated as follows:

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} sgn(x_j - x_i) \tag{1}$$

where, n = the number of data points in the observation series, $x_i$ and $x_j$ = values of the data at time $i$ and $j$ respectively ($j > i$), and $sgn(x_j - x_i)$ is the sign function given by:

$$sgn(x_j - x_i) = \begin{cases} -1 & for\ (x_j - x_i) < 0 \\ 0 & for\ (x_j - x_i) = 0 \\ +1 & for\ (x_j - x_i) > 0 \end{cases} \tag{2}$$

For large values of n, the statistic S tends to normality with mean and variance are given by:

$$E(S) = 0 \tag{3}$$

$$Var(S) = \frac{n(n-1)(2n+5) - \sum_{p=1}^{q} t_p(t_p - 1)(2t_p + 5)}{18} \tag{4}$$

where, $t_p$ is the number of ties for the $p^{th}$ values and $q$ is the number of tied values.

The standardized test statistic Z is calculated as:

$$Z = \begin{cases} \frac{S-1}{\sqrt{V(S)}}, & if\ S > 0 \\ 0, & if\ S = 0 \\ \frac{S+1}{\sqrt{V(S)}}, & if\ S < 0 \end{cases} \quad \text{when n > 10} \tag{5}$$

The standardized test statistic Z is compared with the standard normal variate at the desired significance level to determine the significance of the trends. Also, the positive values of Z represent an increasing trend, while the negative values represent a decreasing trend.