

Supporting Information

A modified constitutive model for isotropic hyperelastic polymeric materials and its parameter identification

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Table S1 The experimental data for natural unfilled vulcanized rubber from Treloar [24].

Uniaxial tension		Equal-biaxial tension		Pure shear	
λ	T/MPa	λ	T/MPa	λ	T/MPa
1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
1.1151	0.1135	1.0270	0.0926	1.0865	0.0992
1.2302	0.1987	1.0650	0.1566	1.1514	0.1839
1.3646	0.2839	1.1150	0.2371	1.2739	0.2744
1.5951	0.3785	1.1400	0.2570	1.4036	0.3649
1.8641	0.4731	1.2000	0.3258	1.8072	0.5431
2.1523	0.5583	1.3100	0.4339	2.3622	0.7214
2.4020	0.6434	1.4200	0.5079	2.9460	0.9056
2.9977	0.8232	1.6800	0.6470	3.4432	1.0926
3.5549	1.0124	1.9400	0.7624	3.9333	1.2621
3.9967	1.1922	2.4900	0.9590	4.3369	1.4462
4.7456	1.5612	3.0300	1.2382	4.6685	1.6185
5.3600	1.9113	3.4300	1.4387	4.9279	1.7966
5.7629	2.2709	3.7500	1.7052		
6.1657	2.6494	4.0700	1.9704		
6.4147	2.9995	4.2600	2.2001		
6.6252	3.3685	4.4500	2.4160		
6.8742	3.7281				
7.0654	4.0971				
7.1798	4.4472				
7.2942	4.8257				
7.4470	5.1947				
7.5229	5.5542				
7.6362	6.2828				

Note: The specific data is from the literature (Li, X.B. and Wei, Y.T.: Classic strain energy functions and constitutive tests of rubber-like materials. Rubber Chemistry & Technology 88(4), 604-627 (2015)). The stress T in the table is nominal stress. The equidistant data points with different deformation ranges from the dataset are shown in the file--Treloar.xlsx.

Table S2 The experimental data for isoprene vulcanized rubber from Kawabata[49]

Uniaxial tension		Equal-biaxial tension		Pure shear	
λ	σ /MPa	λ	σ /MPa	λ	σ /MPa
1.00	0	1.00	0	1.00	0
1.04	0.045	1.04	0.088	1.04	0.061
1.06	0.065	1.06	0.129	1.06	0.091
1.08	0.093	1.08	0.178	1.08	0.122
1.10	0.117	1.10	0.218	1.10	0.152
1.12	0.140	1.12	0.257	1.12	0.181
1.14	0.163	1.14	0.293	1.14	0.209
1.16	0.186	1.16	0.329	1.16	0.238
1.20	0.235	1.20	0.396	1.20	0.294
1.24	0.282	1.24	0.461	1.24	0.349
1.30	0.351	1.30	0.547	1.30	0.434
1.60	0.693	1.60	0.976	1.60	0.819
1.90	1.051	1.90	1.414	1.90	1.193
2.20	1.432	2.20	1.896	2.20	1.593
2.50	1.850	2.50	2.435	2.50	2.022
2.80	2.316	2.80	3.030	2.80	2.482
3.10	2.849	3.10	3.689	3.10	2.973
3.40	3.454			3.40	3.505
3.70	4.133			3.70	4.077

Note: The specific data is from the literature (Afshin Anssari-Benam, Andrea Bucchi. A generalised neo-Hookean strain energy function for application to the finite deformation of elastomers[J]. International Journal of Non-Linear Mechanics, 2021, 128:103626.). The stress σ in the table is true stress.

Table S3 The experimental data for unfilled silicone rubber from Meunier[17]

Uniaxial tension		Equal-biaxial tension		Pure shear	
λ	σ/MPa	λ	σ/MPa	λ	σ/MPa
0.49	-0.68	1	0	1	0
0.52	-0.62	1.05	0.10	1.02	0.05
0.55	-0.56	1.10	0.20	1.05	0.10
0.58	-0.51	1.18	0.32	1.10	0.17
0.61	-0.46	1.23	0.41	1.15	0.23
0.64	-0.41	1.34	0.54	1.22	0.30
0.67	-0.37	1.44	0.69	1.29	0.39
0.71	-0.33	1.54	0.83	1.38	0.48
0.74	-0.29	1.68	1.09	1.48	0.59
0.77	-0.26	1.77	1.31	1.57	0.71
0.80	-0.22	1.80	1.39	1.66	0.83
0.83	-0.18	1.84	1.52	1.75	0.96
0.86	-0.15	1.95	1.97	1.83	1.10
0.89	-0.11	2.06	2.72	1.90	1.24
0.93	-0.08			1.97	1.38
0.96	-0.04			2.02	1.51
1.00	0			2.07	1.66
1.05	0.05			2.12	1.80
1.11	0.11			2.15	1.94
1.18	0.17				
1.27	0.25				
1.36	0.33				
1.46	0.44				
1.56	0.54				
1.66	0.66				
1.76	0.78				
1.85	0.91				
1.93	1.06				
1.99	1.18				
2.04	1.32				
2.09	1.45				
2.14	1.60				
2.17	1.73				

Note: The specific data is from the literature (Afshin Anssari-Benam, Andrea Bucchi. A generalised neo-Hookean strain energy function for application to the finite deformation of elastomers[J]. International Journal of Non-Linear Mechanics, 2021, 128:103626.). The stress σ in the table is true stress.

Table S4 The experimental data for poly-acrylamide hydrogel from Yohsuke[50]

Uniaxial tension		Equal-biaxial tension		Pure shear	
λ	σ/KPa	λ	σ/KPa	λ	σ/KPa
1.00	0	1.00	0	1.00	0
1.06	1.08	1.06	2.06	1.06	1.67
1.13	2.20	1.13	4.08	1.13	2.93
1.25	4.17	1.25	7.19	1.25	5.10
1.38	6.28	1.38	10.51	1.38	7.69
1.54	9.16	1.54	14.60	1.54	10.73
1.72	12.24	1.72	18.92	1.72	14.47
1.91	16.08	1.91	24.57	1.91	18.56
2.10	20.04	2.10	30.74	2.10	22.76
2.29	24.63	2.29	37.37	2.29	27.39
2.47	29.10	2.47	44.68	2.47	32.54
2.72	36.23			2.72	39.25
3.03	46.581				
3.32	57.23				
3.60	69.59				
4.00	91.85				

Note: The specific data is from the literature (Afshin Anssari-Benam. Large isotropic elastic deformations: on a comprehensive model to correlate the theory and experiments for incompressible rubber-like materials[J]. Journal of Elasticity, 2023, 153:219-244.). The stress σ in the table is Cauchy stress.

Table S5 The experimental data for carbon-black-filled styrene butadiene rubber from Fujikawa[51]

Uniaxial tension		Equal-biaxial tension		Pure shear	
λ	σ/MPa	λ	σ/MPa	λ	σ/MPa
1	0	1	0	1	0
1.05	0.32	1.04	0.52	1.04	0.42
1.11	0.59	1.09	0.94	1.09	0.74
1.16	0.81	1.14	1.26	1.14	1.00

1.21	0.99	1.18	1.53	1.19	1.21
1.26	1.18	1.23	1.75	1.23	1.41
1.31	1.33	1.28	2.00	1.28	1.60
1.36	1.48	1.32	2.25	1.33	1.78
1.41	1.65	1.37	2.49	1.38	2.00
1.46	1.85	1.42	2.79	1.42	2.17
1.51	2.00	1.46	3.11	1.47	2.37
1.56	2.22	1.51	3.43	1.52	2.59
1.60	2.42	1.56	3.75	1.57	2.81
1.65	2.62	1.61	4.05	1.62	3.04
1.72	2.81	1.66	4.34	1.66	3.23
1.74	2.99	1.70	4.54	1.71	3.41
1.79	3.11			1.76	3.55

Note: The specific data is from the literature (Afshin Anssari-Benam. Comparative modelling results between a separable and a non-separable form of principal stretches-based strain energy functions for a variety of isotropic incompressible soft solids: Ogden model compared with a parent model[J]. Mechanics of Soft Materials, 2023, 5:2.). The stress σ in the table is true stress.

Table S6 The experimental data for human brain tissue from Budday[52]

Uniaxial tension		Simple shear	
λ	σ/KPa	γ	σ/KPa
0.90	-1.06	-0.20	-0.55
0.91	-0.82	-0.175	-0.38
0.92	-0.62	-0.15	-0.27
0.94	-0.46	-0.125	-0.19
0.95	-0.33	-0.10	-0.14
0.96	-0.225	-0.075	-0.1
0.975	-0.135	-0.05	-0.06
0.99	-0.06	-0.02	-0.04
1.00	0	0	0
1.01	0.04	0.02	0.03
1.025	0.08	0.05	0.06
1.04	0.12	0.07	0.1
1.05	0.15	0.10	0.14
1.06	0.19	0.12	0.19
1.075	0.25	0.15	0.27
1.09	0.34	0.17	0.37
1.10	0.46	0.20	0.54

Note: The specific data is from the literature (Afshin Anssari-Benam, Michel Destrade, Giuseppe Saccomandi. Modelling brain tissue elasticity with the Ogden model and an alternative family of constitutive models[J]. Philosophical Transactions A, 2022, 380:20210325). The stress σ in the table is true stress.

Table S7. The model parameters for large deformation in Table 1

Model	Parameters
Ogden-N3	$\alpha_1=4.720$; $\mu_1=0.005 \text{ MPa}$; $\alpha_2=1.334$; $\mu_2=0.357 \text{ MPa}$; $\alpha_3=-1.812$; $\mu_3=0.016 \text{ MPa}$;
Alexander	$C_1=0.139 \text{ MPa}$; $C_2=0.320 \text{ MPa}$; $C_3=0.002 \text{ MPa}$; $k=0.000338$; $\gamma=12.10744$;
Yeoh	$C_{10}=0.152 \text{ MPa}$; $C_{20}=0 \text{ MPa}$; $C_{30}=0.0000228 \text{ MPa}$;
Melly	$C_{10}=0.137 \text{ MPa}$; $C_{20}=0 \text{ MPa}$; $C_{30}=0.0000235 \text{ MPa}$; $D=0.094 \text{ MPa}$;
Modified Yeoh	$C_{10}=1.44\text{E}-7 \text{ MPa}$; $C_{20}=0.00148 \text{ MPa}$; $C_{30}=0.0000189 \text{ MPa}$; $\alpha=0.165 \text{ MPa}$; $\beta=0.0288$;
Generalized Yeoh	$K_1=1.2278 \text{ MPa}$; $m=1$; $K_2=-1.0309 \text{ MPa}$; $p=1.01847$; $K_3=0.000392 \text{ MPa}$; $q=2.3972$;
Anssari-Benam	$\mu=0.29086 \text{ MPa}$; $N=24.9907$; $n=2.0108$;
Modified Anssari-Benam	$\mu=1.1692 \text{ MPa}$; $N=0.05779$; $n=3.35977$; $\alpha=0.003166 \text{ MPa}$; $\beta=2.3647$;
My work	$C_{10}=0.112895 \text{ MPa}$; $C_{20}=0.00041 \text{ MPa}$; $C_{30}=0.000021669 \text{ MPa}$; $\alpha=0.792239 \text{ MPa}$; $\beta=0.3361846$;

Table S8. The model parameters for medium deformation in Table 1

Model	Parameters
Ogden-N3	$\alpha_1=0.39472$; $\mu_1=0.268358 \text{ MPa}$; $\alpha_2=3.010134$; $\mu_2=0.081632 \text{ MPa}$; $\alpha_3=-1.88910$; $\mu_3=0.00923 \text{ MPa}$;
Alexander	$C_1=0.132 \text{ MPa}$; $C_2=0.262585 \text{ MPa}$; $C_3=0.003018 \text{ MPa}$; $k=0.00045775$; $\gamma=6.62166$;
Yeoh	$C_{10}=0.1575635 \text{ MPa}$; $C_{20}=0 \text{ MPa}$; $C_{30}=0.000010285 \text{ MPa}$;
Melly	$C_{10}=0.13631957 \text{ MPa}$; $C_{20}=0 \text{ MPa}$; $C_{30}=0.0000203 \text{ MPa}$; $D=0.1120827 \text{ MPa}$;
Modified Yeoh	$C_{10}=0.148625 \text{ MPa}$; $C_{20}=0.0004428 \text{ MPa}$; $C_{30}=0.0000039786 \text{ MPa}$; $\alpha=0.032364 \text{ MPa}$; $\beta=0.47151486$;
Generalized Yeoh	$K_1=0.1917369 \text{ MPa}$; $m=1$; $K_2=-0.072778 \text{ MPa}$; $p=1.5876$; $K_3=0.0554836 \text{ MPa}$; $q=1.65405$;

Anssari-Benam	$\mu=0.313768 \text{ MPa}; N=12.41387; n=1.04495;$
Modified Anssari-Benam	$\mu=0.215047 \text{ MPa}; N=25.51408; n=85.0; \alpha=0.6880299 \text{ MPa}; \beta=0.37208;$
My work	$C_{10}=0.0938605 \text{ MPa}; C_{20}=0.00138968 \text{ MPa}; C_{30}=0 \text{ MPa}; \alpha=3.6362 \text{ MPa}; \beta=0.09307;$

Table S9. The model parameters for small deformation in Table 1

Model	Parameters
Ogden-N3	$\alpha_1=-0.42082; \mu_1=0.094627 \text{ MPa}; \alpha_2=2.822296; \mu_2=0.096908 \text{ MPa}; \alpha_3=0.56605; \mu_3=0.1682697 \text{ MPa};$
Alexander	$C_1=0.125446 \text{ MPa}; C_2=0.180328 \text{ MPa}; C_3=0.007073 \text{ MPa}; k=0.000964878; \gamma=3.7760124;$
Yeoh	$C_{10}=0.16289 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=0 \text{ MPa};$
Melly	$C_{10}=0.131834 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=0 \text{ MPa}; D=0.14487 \text{ MPa};$
Modified Yeoh	$C_{10}=0.009 \text{ MPa}; C_{20}=0.002555 \text{ MPa}; C_{30}=0.0000001147 \text{ MPa}; \alpha=0.16779856 \text{ MPa}; \beta=0.07253678;$
Generalized Yeoh	$K_1=0.46885 \text{ MPa}; m=1; K_2=-0.4695 \text{ MPa}; p=1.122683; K_3=0.186023 \text{ MPa}; q=1.22587;$
Anssari-Benam	$\mu=0.37157 \text{ MPa}; N=42.787; n=0.26312;$
Modified Anssari-Benam	$\mu=0.19653 \text{ MPa}; N=13.6234; n=3.10978; \alpha=1.236187 \text{ MPa}; \beta=0.2478425;$
My work	$C_{10}=0.09095 \text{ MPa}; C_{20}=0.00133 \text{ MPa}; C_{30}=0 \text{ MPa}; \alpha=8.7237 \text{ MPa}; \beta=0.04076;$

Table S10. The model parameters for isoprene vulcanized rubber in Table 2

Model	Parameters
Ogden-N3	$\alpha_1=-1.5722; \mu_1=0.01719 \text{ MPa}; \alpha_2=10.5078; \mu_2=0.000001 \text{ MPa}; \alpha_3=1.6233; \mu_3=0.37751 \text{ MPa};$
Alexander	$C_1=0.1444 \text{ MPa}; C_2=0.16637 \text{ MPa}; C_3=0.00337 \text{ MPa}; k=0; \gamma=3.535169;$
Yeoh	$C_{10}=0.1575 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=0.00000259 \text{ MPa};$
Melly	$C_{10}=0.147067 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=0 \text{ MPa}; D=0.095079 \text{ MPa};$
Modified Yeoh	$C_{10}=0.0056 \text{ MPa}; C_{20}=0.000337 \text{ MPa}; C_{30}=0.00011 \text{ MPa}; \alpha=0.1860795 \text{ MPa}; \beta=0.05467;$
Generalized Yeoh	$K_1=1.16528 \text{ MPa}; m=1; K_2=-0.97185 \text{ MPa}; p=1.01405; K_3=0.0000001 \text{ MPa}; q=5.08984;$
Anssari-Benam	$\mu=0.22303 \text{ MPa}; N=0.601714; n=0.751607;$
Modified Anssari-Benam	$\mu=0.24728 \text{ MPa}; N=0.6888547; n=0.897568; \alpha=0.191818 \text{ MPa}; \beta=0.76695;$
My work	$C_{10}=2.494 \text{ MPa}; C_{20}=0.0034 \text{ MPa}; C_{30}=-0.00004348 \text{ MPa}; \alpha=4.475518 \text{ MPa}; \beta=-2.053077;$

Table S11. The model parameters for unfilled silicone rubber in Table 3

Model	Parameters
Ogden-N3	$\alpha_1=0.3587; \mu_1=0.28225 \text{ MPa}; \alpha_2=-6.45; \mu_2=0.000245 \text{ MPa}; \alpha_3=5.6138; \mu_3=0.036298 \text{ MPa};$
Alexander	$C_1=0.16297 \text{ MPa}; C_2=0.0261936 \text{ MPa}; C_3=0 \text{ MPa}; k=0.021613; \gamma=1.672377859;$
Yeoh	$C_{10}=0.10936 \text{ MPa}; C_{20}=0.047917 \text{ MPa}; C_{30}=0 \text{ MPa};$
Melly	$C_{10}=0.16308897 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=0.00159675 \text{ MPa}; D=0.0141 \text{ MPa};$
Modified Yeoh	$C_{10}=0.155636 \text{ MPa}; C_{20}=0.00338289 \text{ MPa}; C_{30}=0.0013389 \text{ MPa}; \alpha=0.03732 \text{ MPa}; \beta=2.509305;$
Generalized Yeoh	$K_1=0.90577 \text{ MPa}; m=1; K_2=-0.737696 \text{ MPa}; p=1.021726; K_3=0.006919 \text{ MPa}; q=2.421487;$
Anssari-Benam	$\mu=0.287468 \text{ MPa}; N=3.716448; n=1.62188;$
Modified Anssari-Benam	$\mu=0.272177 \text{ MPa}; N=3.658636; n=1.58244; \alpha=0.54795 \text{ MPa}; \beta=0.084;$
My work	$C_{10}=0.17927 \text{ MPa}; C_{20}=-0.00731 \text{ MPa}; C_{30}=0.00322 \text{ MPa}; \alpha=-0.00068986 \text{ MPa}; \beta=4.70799;$

Table S12. The model parameters for poly-arcylamide hydrogel in Table 4

Model	Parameters
Ogden-N3	$\alpha_1=1.99617; \mu_1=0.004565 \text{ MPa}; \alpha_2=-1.26655; \mu_2=0.0010889 \text{ MPa}; \alpha_3=7.226208; \mu_3=0.000002788 \text{ MPa};$
Alexander	$C_1=0.0023 \text{ MPa}; C_2=0.004895 \text{ MPa}; C_3=0.00008 \text{ MPa}; k=0.001156; \gamma=12.0408;$
Yeoh	$C_{10}=0.0027 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=3.874E-7 \text{ MPa};$
Melly	$C_{10}=0.0022 \text{ MPa}; C_{20}=0.000006824 \text{ MPa}; C_{30}=8.49E-7 \text{ MPa}; D=0.0022786 \text{ MPa};$
Modified Yeoh	$C_{10}=0.00251 \text{ MPa}; C_{20}=0.0000152 \text{ MPa}; C_{30}=0 \text{ MPa}; \alpha=0.000321 \text{ MPa}; \beta=0.35653;$

Generalized Yeoh	$K_1=0.050987 \text{ MPa}; m=1.0755; K_2=-0.0491 \text{ MPa}; p=1.08738; K_3=0.001006 \text{ MPa}; q=1.338512;$
Anssari-Benam	$\mu=0.096 \text{ MPa}; N=0.00618; n=16.68516;$
Modified Anssari-Benam	$\mu=0.004677 \text{ MPa}; N=7.81967; n=1.09826; \alpha=0.0012 \text{ MPa}; \beta=1.369;$
My work	$C_{10}=0.00187 \text{ MPa}; C_{20}=0.0000316 \text{ MPa}; C_{30}=0 \text{ MPa}; \alpha=0.00628 \text{ MPa}; \beta=0.59589;$

Table S13. The model parameters for carbon-black-filled styrene butadiene rubber in Table 5

Model	Parameters
Ogden-N3	$\alpha_1=0.909289; \mu_1=1.25589 \text{ MPa}; \alpha_2=-4.18085; \mu_2=0.0101 \text{ MPa}; \alpha_3=0.8449; \mu_3=0.25837 \text{ MPa};$
Alexander	$C_1=0.49284 \text{ MPa}; C_2=0.07262 \text{ MPa}; C_3=0.11140 \text{ MPa}; k=1.49E-8; \gamma=0.165188;$
Yeoh	$C_{10}=0.655183 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=0.00693 \text{ MPa};$
Melly	$C_{10}=0.5132 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=0 \text{ MPa}; D=0.67044 \text{ MPa};$
Modified Yeoh	$C_{10}=0.4509 \text{ MPa}; C_{20}=0.07017 \text{ MPa}; C_{30}=0 \text{ MPa}; \alpha=0.49347 \text{ MPa}; \beta=2.17857;$
Generalized Yeoh	$K_1=3.16754 \text{ MPa}; m=1; K_2=-2.4819 \text{ MPa}; p=1.05753; K_3=0.046104 \text{ MPa}; q=2.39379;$
Anssari-Benam	$\mu=1.27325 \text{ MPa}; N=2.20960; n=1.04265;$
Modified Anssari-Benam	$\mu=1.52426 \text{ MPa}; N=7.80347; n=0.38067; \alpha=0.379 \text{ MPa}; \beta=1.8624;$
My work	$C_{10}=0.164 \text{ MPa}; C_{20}=-0.0333 \text{ MPa}; C_{30}=0.0089348 \text{ MPa}; \alpha=-4.16605 \text{ MPa}; \beta=-0.6066;$

Table S14. The model parameters for human brain cortex tissue in Table 6

Model	Parameters
Ogden-N3	$\alpha_1=61.2136; \mu_1=0.01101 \text{ KPa}; \alpha_2=-21.351585; \mu_2=1.2009 \text{ KPa}; \alpha_3=2.49218; \mu_3=0.00122 \text{ KPa};$
Generalized Anssari-Benam	$\mu=0.02 \text{ KPa}; N=7.52; n=19.99; \alpha=-15.93;$
Modified Anssari-Benam	$\mu=0.01145 \text{ KPa}; N=1.01874; n=1.46566; \alpha=0.09726 \text{ KPa}; \beta=21.49197;$
My work	$C_{10}=10.3115 \text{ KPa}; C_{20}=6.07355 \text{ KPa}; C_{30}=87.4228 \text{ KPa}; \alpha=9.83259 \text{ KPa}; \beta=-3.93031;$

Table S15. The predictive results of different models calibrated by the data from UT of natural vulcanized rubber [24].

Model	Large deformation				Medium deformation				Small deformation			
	Error	UT	ET	PS	Error	UT	ET	PS	Error	UT	ET	PS
Ogden-N3	--	0.982	--	--	0.032	0.993	0.928	0.985	--	0.993	--	--
Alexander												
Yeoh	0.131	0.953	0.775	0.880	0.119	0.982	0.751	0.909	0.145	0.965	0.721	0.878
Melly	0.241	0.959	0.421	0.895	0.188	0.993	0.542	0.900	0.082	0.990	0.820	0.946
Biderman	--	0.960	--	0.553	--	0.993	--	0.663	--	0.990	--	0.831
Polynomial-N2	--	0.916	--	--	--	0.993	--	0.568	--	0.992	--	0.208
GeneralizedYeoh	0.095	0.972	0.783	0.960	0.117	0.991	0.751	0.909	0.124	0.994	0.727	0.908
Arruda-Boyce	0.126	0.966	0.773	0.884	0.124	0.977	0.751	0.899	0.145	0.965	0.721	0.878
Exp-Ln	0.127	0.957	0.773	0.889	0.114	0.989	0.751	0.916	0.125	0.986	0.727	0.913
Modified-Yeoh	0.112	0.965	0.779	0.920	0.115	0.993	0.752	0.912	0.128	0.990	0.726	0.901
This work	0.041	0.960	0.966	0.952	0.022	0.989	0.959	0.987	0.026	0.988	0.949	0.987

Notes: Error indicates the total error in predicting the three modes of deformation; UT, ET and PS represent the goodness of fit of different models for uniaxial tension, equi-biaxial tension and pure shear; "--" indicates that the goodness of fit is negative and the prediction is invalid.

Table S16. The model parameters for large deformation in Table 7

Model	Parameters
Ogden-N3	$\alpha_1=2.38743; \mu_1=0.17449 \text{ MPa}; \alpha_2=-0.69033; \mu_2=0.16442 \text{ MPa}; \alpha_3=7.931477; \mu_3=0.00001 \text{ MPa};$
Alexander	$C_1=0.14023 \text{ MPa}; C_2=0.023797 \text{ MPa}; C_3=0.0118826 \text{ MPa}; k=0.0003355; \gamma=0.651445;$
Yeoh	$C_{10}=0.14971 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=0.0000225 \text{ MPa};$
Melly	$C_{10}=0.12486 \text{ MPa}; C_{20}=3.979E-8 \text{ MPa}; C_{30}=0.000026179 \text{ MPa}; D=0.17694 \text{ MPa};$
Modified Yeoh	$C_{10}=2.099E-7 \text{ MPa}; C_{20}=0.0001419 \text{ MPa}; C_{30}=0.00002889 \text{ MPa}; \alpha=0.1643997 \text{ MPa}; \beta=0.01107;$
Generalized Yeoh	$K_1=0.1606 \text{ MPa}; m=1; K_2=-1.78E-7 \text{ MPa}; p=1.00064; K_3=0.0000001 \text{ MPa}; q=4.28993;$
Anssari-Benam	$\mu=0.288323 \text{ MPa}; N=24.36469; n=1.848835;$

Modified Anssari-Benam	$\mu=6.033266 \text{ MPa}$; $N=0.02078$; $n=16.496647$; $\alpha=0.00002889 \text{ MPa}$; $\beta=0.0000419$;
My work	$C_{10}=0.121556 \text{ MPa}$; $C_{20}=0.0001045 \text{ MPa}$; $C_{30}=0.0000244 \text{ MPa}$; $\alpha=0.2246026 \text{ MPa}$; $\beta=0.91653$;

Table S17. The model parameters for medium deformation in Table 7

Model	Parameters
Ogden-N3	$\alpha_1=3.1880758$; $\mu_1=0.05834 \text{ MPa}$; $\alpha_2=1.495376$; $\mu_2=0.10143955 \text{ MPa}$; $\alpha_3=-0.3198387$; $\mu_3=0.198828 \text{ MPa}$;
Alexander	$C_1=0.13477 \text{ MPa}$; $C_2=0.04137 \text{ MPa}$; $C_3=0.01422 \text{ MPa}$; $k=0.000425$; $\gamma=1.17507$;
Yeoh	$C_{10}=0.154369 \text{ MPa}$; $C_{20}=0 \text{ MPa}$; $C_{30}=0.00001068 \text{ MPa}$;
Melly	$C_{10}=0.126125 \text{ MPa}$; $C_{20}=0.0003406 \text{ MPa}$; $C_{30}=0.00001655 \text{ MPa}$; $D=0.1538359 \text{ MPa}$;
Modified Yeoh	$C_{10}=0.15151 \text{ MPa}$; $C_{20}=0.000195 \text{ MPa}$; $C_{30}=0.00000748 \text{ MPa}$; $\alpha=0.0291045 \text{ MPa}$; $\beta=0.5246785$;
Generalized Yeoh	$K_1=0.849356 \text{ MPa}$; $m=1$; $K_2=-0.6802765 \text{ MPa}$; $p=1.00653$; $K_3=0.0000001 \text{ MPa}$; $q=4.434723$;
Anssari-Benam	$\mu=0.3092459 \text{ MPa}$; $N=11.41556785$; $n=1.0312297$;
Modified Anssari-Benam	$\mu=0.217254657 \text{ MPa}$; $N=25.766939$; $n=85$; $\alpha=0.379018689 \text{ MPa}$; $\beta=0.6344856$;
My work	$C_{10}=0.09435 \text{ MPa}$; $C_{20}=0.00141 \text{ MPa}$; $C_{30}=0 \text{ MPa}$; $\alpha=0.9361 \text{ MPa}$; $\beta=0.35528$;

Table S18. The model parameters for small deformation in Table 7

Model	Parameters
Ogden-N3	$\alpha_1=0.222160$; $\mu_1=0.058955 \text{ MPa}$; $\alpha_2=2.632$; $\mu_2=0.1235447 \text{ MPa}$; $\alpha_3=0.0444$; $\mu_3=0.176199 \text{ MPa}$;
Alexander	$C_1=0.129286 \text{ MPa}$; $C_2=0.20462658 \text{ MPa}$; $C_3=0.00149566 \text{ MPa}$; $k=0.0005238$; $\gamma=4.23884$;
Yeoh	$C_{10}=0.15199 \text{ MPa}$; $C_{20}=0 \text{ MPa}$; $C_{30}=2.755E-10 \text{ MPa}$;
Melly	$C_{10}=0.128929 \text{ MPa}$; $C_{20}=0 \text{ MPa}$; $C_{30}=0.00001755 \text{ MPa}$; $D=0.1580353 \text{ MPa}$;
Modified Yeoh	$C_{10}=0.00063265 \text{ MPa}$; $C_{20}=0.001563569 \text{ MPa}$; $C_{30}=0.000005727 \text{ MPa}$; $\alpha=0.17600515 \text{ MPa}$; $\beta=0.057149$;
Generalized Yeoh	$K_1=1.07821848 \text{ MPa}$; $m=1$; $K_2=-0.9109487 \text{ MPa}$; $p=1.011924$; $K_3=0.0079585 \text{ MPa}$; $q=1.012909$;
Anssari-Benam	$\mu=0.3785835 \text{ MPa}$; $N=63.874$; $n=0.168621$;
Modified Anssari-Benam	$\mu=0.19769 \text{ MPa}$; $N=12.6129$; $n=1.82063$; $\alpha=2.03253 \text{ MPa}$; $\beta=0.150266$;
My work	$C_{10}=0.0971 \text{ MPa}$; $C_{20}=0.00114 \text{ MPa}$; $C_{30}=0 \text{ MPa}$; $\alpha=1.90129 \text{ MPa}$; $\beta=0.1717$;

Table S19. The predictive results of different models calibrated by the data from UT and PS of isoprene vulcanized rubber [49].

Models	The number of coefficient	Error	UT	ET	PS
Ogden-N3	6	0.061	0.985	0.847	0.986
Alexander	5	0.068	0.990	0.829	0.977
Yeoh	3	0.095	0.971	0.798	0.947
Melly	4	0.045	0.977	0.935	0.954
ModifiedYeoh	5	0.078	0.981	0.800	0.984
GeneralizedYeoh	6	0.081	0.986	0.791	0.980
Anssari-Benam	3	0.082	0.985	0.787	0.983
Modified Anssari-Benam	5	0.047	0.982	0.903	0.973
This work	5	0.041	0.974	0.911	0.993

Notes: Error indicates the total error of different models; UT, ET and PS represent the goodness of fit of different models for uniaxial tension, equi-biaxial tension and pure shear. “--” indicates that the goodness of fit is negative and the prediction is invalid. The model parameters are shown in Table S23 in supporting information.

Table S20. The predictive results of different models calibrated by the data from UT and PS of unfilled silicone rubber [17].

Models	The number of coefficient	Error	UT	ET	PS
Ogden-N3	6	0.197	0.979	0.460	0.971
Alexander	5	0.323	0.986	0.072	0.974
Yeoh	3	0.523	0.708	--	0.820
Melly	4	0.128	0.980	0.665	0.973
ModifiedYeoh	5	0.117	0.933	0.734	0.981

GeneralizedYeoh	6	0.089	0.931	0.821	0.982
Anssari-Benam	3	0.566	0.937	--	0.973
Modified Anssari-Benam	5	0.666	0.979	--	0.973
This work	5	0.134	0.982	0.642	0.974

Notes: Error indicates the total error of different models; UT, ET and PS represent the goodness of fit of different models for uniaxial tension, equi-biaxial tension and pure shear. "--" indicates that the goodness of fit is negative and the prediction is invalid. The model parameters are shown in Table S24 in supporting information.

Table S21. The predictive results of different models calibrated by the data from UT and PS of poly-arcylamide hydrogel [50].

Models	The number of coefficient	Error	UT	ET	PS
Ogden-N3	6	0.051	0.992	0.864	0.990
Alexander	5	0.057	0.988	0.849	0.991
Yeoh	3	0.091	0.971	0.771	0.985
Melly	4	0.043	0.989	0.891	0.990
ModifiedYeoh	5	0.090	0.973	0.768	0.990
GeneralizedYeoh	6	0.091	0.974	0.764	0.990
Anssari-Benam	3	0.097	0.954	0.770	0.986
Modified Anssari-Benam	5	0.043	0.990	0.890	0.991
This work	5	0.035	0.992	0.914	0.990

Notes: Error indicates the total error of different models; UT, ET and PS represent the goodness of fit of different models for uniaxial tension, equi-biaxial tension and pure shear. "--" indicates that the goodness of fit is negative and the prediction is invalid. The model parameters are shown in Table S25 in supporting information.

Table S22. The predictive results of different models calibrated by the data from UT and PS of carbon-black-filled styrene butadiene rubber [51].

Models	The number of coefficient	Error	UT	ET	PS
Ogden-N3	6	0.041	0.968	0.952	0.957
Alexander	5	0.037	0.981	0.927	0.982
Yeoh	3	0.126	0.925	0.790	0.907
Melly	4	0.066	0.963	0.901	0.937
ModifiedYeoh	5	0.113	0.967	0.767	0.927
GeneralizedYeoh	6	0.109	0.968	0.776	0.927
Anssari-Benam	3	0.157	0.942	0.660	0.925
Modified Anssari-Benam	5	0.046	0.966	0.940	0.957
This work	5	0.049	0.968	0.938	0.949

Notes: Error indicates the total error of different models; UT, ET and PS represent the goodness of fit of different models for uniaxial tension, equi-biaxial tension and pure shear. "--" indicates that the goodness of fit is negative and the prediction is invalid. The model parameters are shown in Table S26 in supporting information.

Table S23. The parameters of different model calibrated by the data from UT and PS of isoprene vulcanized rubber

Model	Parameters
Ogden-N3	$\alpha_1=9.72856$; $\mu_1=0.0000012$ MPa; $\alpha_2=4.88164$; $\mu_2=0.00112$ MPa; $\alpha_3=1.58782$; $\mu_3=0.38452$ MPa;
Alexander	$C_1=0.14810$ MPa; $C_2=0.08285$ MPa; $C_3=0$ MPa; $k=0$; $\gamma=1.2627859$;
Yeoh	$C_{10}=0.155407$ MPa; $C_{20}=1E-8$ MPa; $C_{30}=0$ MPa;
Melly	$C_{10}=0.1491316$ MPa; $C_{20}=0$ MPa; $C_{30}=0$ MPa; $D=0.06775$ MPa;
Modified Yeoh	$C_{10}=0.12979$ MPa; $C_{20}=0.000289$ MPa; $C_{30}=0.00001556$ MPa; $\alpha=0.059919$ MPa; $\beta=0.161331$;
Generalized Yeoh	$K_1=0.2295233$ MPa; $m=1$; $K_2=-0.065447$ MPa; $p=1.397285937$; $K_3=0.026765$ MPa; $q=1.5599$;
Anssari-Benam	$\mu=0.081174$ MPa; $N=0.357115$; $n=0.28277$;
Modified Anssari-Benam	$\mu=0.719328$ MPa; $N=0.0000026$; $n=2.6356154$; $\alpha=6.55401847$ MPa; $\beta=0.0289229$;
My work	$C_{10}=1.376388$ MPa; $C_{20}=0.00104$ MPa; $C_{30}=-0.0000041$ MPa; $\alpha=2.29319$ MPa; $\beta=-2.064195$;

Table S24. The parameters of different model calibrated by the data from UT and PS of unfilled silicone rubber

Model	Parameters
Ogden-N3	$\alpha_1=1.11700$; $\mu_1=0.308509$ MPa; $\alpha_2=8.022885$; $\mu_2=0.00587$ MPa; $\alpha_3=-4.23368$; $\mu_3=0.013435$ MPa;
Alexander	$C_1=0.126247$ MPa; $C_2=0.09678$ MPa; $C_3=0.00278596$ MPa; $k=0.05734279$; $\gamma=2.2575024$;
Yeoh	$C_{10}=0.1206157$ MPa; $C_{20}=0$ MPa; $C_{30}=0.0158067$ MPa;
Melly	$C_{10}=0.127741686$ MPa; $C_{20}=0$ MPa; $C_{30}=0.002757$ MPa; $D=0.12116189$ MPa;
Modified Yeoh	$C_{10}=0.1298157$ MPa; $C_{20}=0.000001266$ MPa; $C_{30}=0.00327815$ MPa; $\alpha=0.057514488$ MPa; $\beta=0.681829$;

Generalized Yeoh	$K_1=0.20068558 \text{ MPa}; m=1; K_2=-0.1 \text{ MPa}; p=1.973845; K_3=0.075507 \text{ MPa}; q=2.17729;$
Anssari-Benam	$\mu=0.318809 \text{ MPa}; N=2.1757257; n=1.0477485;$
Modified Anssari-Benam	$\mu=0.1794 \text{ MPa}; N=2.4882355; n=1.4125138; \alpha=0.822476 \text{ MPa}; \beta=0.270858;$
My work	$C_{10}=0.101608 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=0.0030489 \text{ MPa}; \alpha=1.484448 \text{ MPa}; \beta=0.1724878;$

Table S25. The parameters of different model calibrated by the data from UT and PS of poly-acrylamide hydrogel

Model	Parameters
Ogden-N3	$\alpha_1=1.80246; \mu_1=0.00231 \text{ MPa}; \alpha_2=0.025249; \mu_2=0.0026228 \text{ MPa}; \alpha_3=3.34351; \mu_3=0.000819 \text{ MPa};$
Alexander	$C_1=0.002433 \text{ MPa}; C_2=0.0027534 \text{ MPa}; C_3=0.0000001639 \text{ MPa}; k=0.0009197; \gamma=7.36328;$
Yeoh	$C_{10}=0.00268 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=3.725\text{E-}7 \text{ MPa};$
Melly	$C_{10}=0.00244 \text{ MPa}; C_{20}=1.34\text{E-}7 \text{ MPa}; C_{30}=7.7486\text{E-}7 \text{ MPa}; D=0.00122 \text{ MPa};$
Modified Yeoh	$C_{10}=0.00221165 \text{ MPa}; C_{20}=0.000023585 \text{ MPa}; C_{30}=0 \text{ MPa}; \alpha=0.000607 \text{ MPa}; \beta=0.1799296;$
Generalized Yeoh	$K_1=0.92755 \text{ MPa}; m=1; K_2=-0.924766 \text{ MPa}; p=1.07449; K_3=0.0000001 \text{ MPa}; q=3.6293;$
Anssari-Benam	$\mu=0.066565 \text{ MPa}; N=0.0036; n=11.987967;$
Modified Anssari-Benam	$\mu=0.0039 \text{ MPa}; N=18.9523; n=32.1626; \alpha=0.024685 \text{ MPa}; \beta=0.12702178;$
My work	$C_{10}=0.0018258 \text{ MPa}; C_{20}=0.0000306 \text{ MPa}; C_{30}=0 \text{ MPa}; \alpha=1.7097385 \text{ MPa}; \beta=0.002398;$

Table S26. The parameters of different model calibrated by the data from UT and PS of carbon-black-filled styrene butadiene rubber

Model	Parameters
Ogden-N3	$\alpha_1=0.75172; \mu_1=0.8629886 \text{ MPa}; \alpha_2=0.7517155; \mu_2=0.59426 \text{ MPa}; \alpha_3=0.751706; \mu_3=0.09592 \text{ MPa};$
Alexander	$C_1=0.488956 \text{ MPa}; C_2=0.04274595 \text{ MPa}; C_3=0.1425747 \text{ MPa}; k=5.2357\text{E-}8; \gamma=0.0801276;$
Yeoh	$C_{10}=0.64822856 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=0 \text{ MPa};$
Melly	$C_{10}=0.465135 \text{ MPa}; C_{20}=0 \text{ MPa}; C_{30}=0 \text{ MPa}; D=0.9250067 \text{ MPa};$
Modified Yeoh	$C_{10}=0.62163597 \text{ MPa}; C_{20}=2.23\text{E-}8 \text{ MPa}; C_{30}=0 \text{ MPa}; \alpha=0.344689 \text{ MPa}; \beta=4.89252;$
Generalized Yeoh	$K_1=1.78476899 \text{ MPa}; m=1; K_2=-1.32311778 \text{ MPa}; p=1.15276; K_3=0.23359 \text{ MPa}; q=1.555475;$
Anssari-Benam	$\mu=2.0111 \text{ MPa}; N=51.836617; n=0.067696;$
Modified Anssari-Benam	$\mu=0.87155 \text{ MPa}; N=13.5647; n=0.274455; \alpha=16.056696 \text{ MPa}; \beta=0.10759;$
My work	$C_{10}=0.26596 \text{ MPa}; C_{20}=3.159\text{E-}6 \text{ MPa}; C_{30}=0 \text{ MPa}; \alpha=16.17489 \text{ MPa}; \beta=0.12210187;$

Table S27 The parameters of the constitutive model corresponding to the Table 9.

Methods	Arruda-Boyce model	Ogden-3N model	Yeoh model
ABAQUS	$\mu=0.3053 \text{ MPa}, \lambda_m=5.1322 \text{ MPa}$	$\alpha_1=1.3913 \text{ MPa}; \mu_1=0.3430 \text{ MPa};$ $\alpha_2=4.7869 \text{ MPa}; \mu_2=0.0044 \text{ MPa};$ $\alpha_3=-1.6503 \text{ MPa}; \mu_3=0.0240 \text{ MPa};$	$C_{10}=0.1712 \text{ MPa}$ $C_{20}=-0.0008 \text{ MPa}$ $C_{30}=0.00003 \text{ MPa}$
This work	$\mu=0.2797 \text{ MPa}, \lambda_m=4.7589 \text{ MPa}$	$\alpha_1=4.7203 \text{ MPa}; \mu_1=0.0053 \text{ MPa};$ $\alpha_2=1.3336 \text{ MPa}; \mu_2=0.3572 \text{ MPa};$ $\alpha_3=-1.8124 \text{ MPa}; \mu_3=0.0163 \text{ MPa};$	$C_{10}=0.1631 \text{ MPa}$ $C_{20}=-0.0006 \text{ MPa}$ $C_{30}=0.00003 \text{ MPa}$

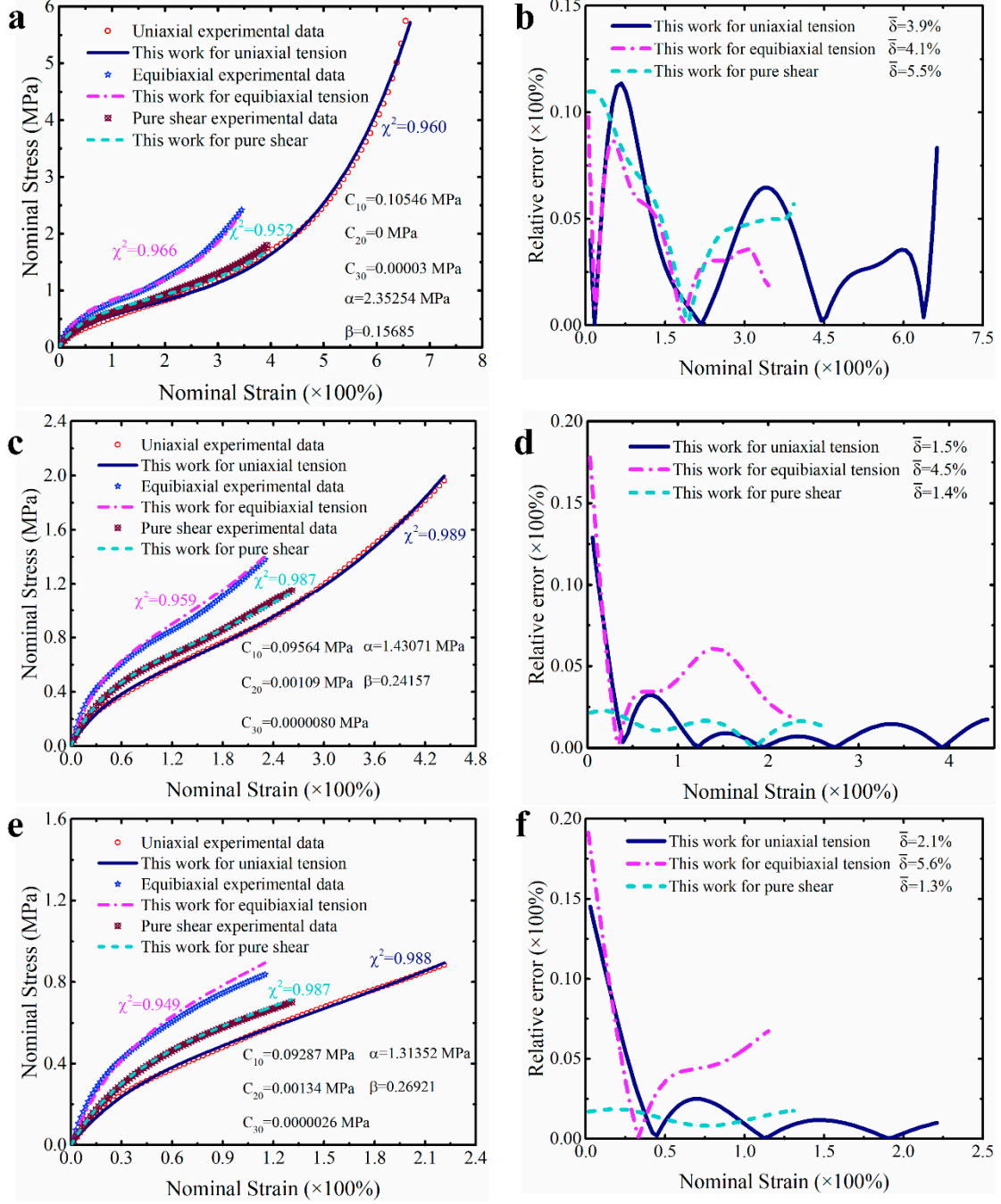


Figure S1. The prediction effects of the modified model for different deformation modes with different deformation ranges. (a-b) in large deformation, (c-d) in medium deformation, (e-f) in small deformation. Here, the parameters of the modified model are calibrated only by the Treloar's uniaxial experimental data with different deformation ranges.

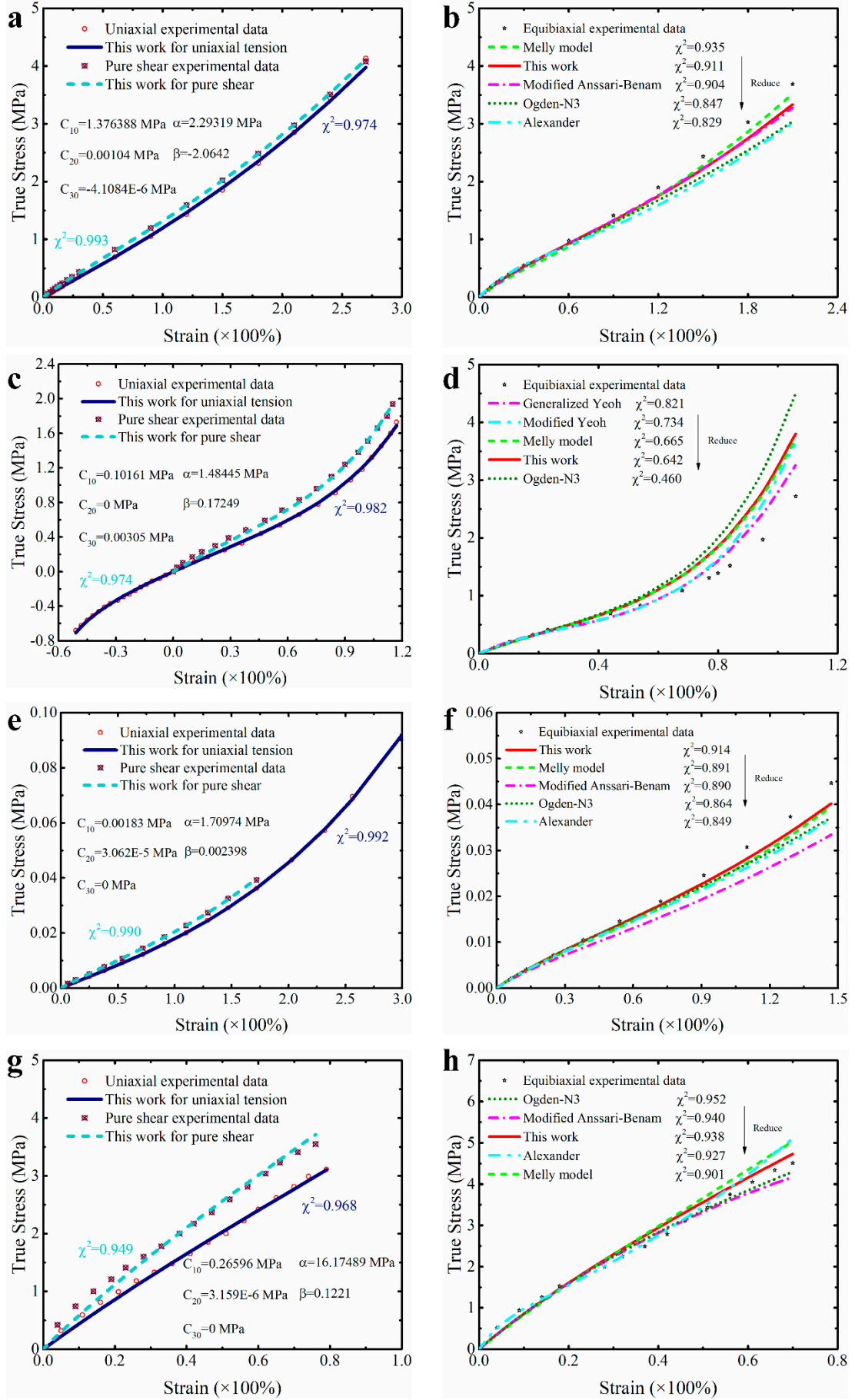


Figure S2. The prediction effects of the modified model for different deformation modes of different materials in the case where the parameters of model are calibrated by the data only from UT and PS. (a-b) Isoprene vulcanized rubber. (c-d) Unfilled silicone rubber. (e-f)

Poly-acrylamide hydrogel. (g-h) Carbon-black-filled styrene butadiene rubber. Only the top five constitutive models are listed on the right side of the figure.

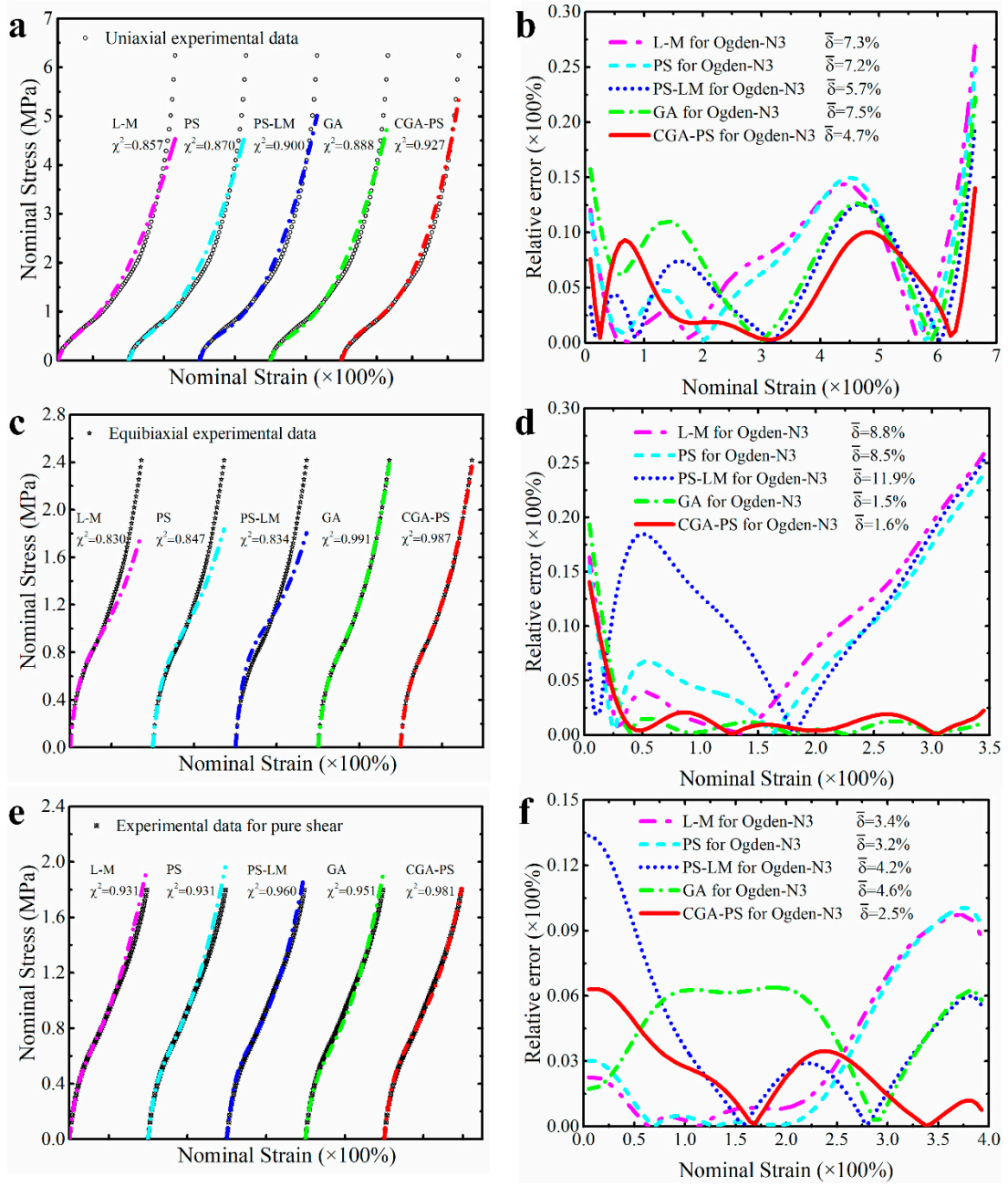


Figure S3. The comparison of prediction performance by the third-order Ogden model based on different methods of parameter identification. (a-b) the prediction performance of uniaxial tension by the modified model based on different methods of parameter identification. (c-d) the prediction performance of equi-biaxial tension by the modified model based on different methods of parameter identification. (e-f) the prediction performance of pure shear by the modified model based on different methods of parameter identification. $\bar{\delta}$ means the average relative error.

Equation S1. Components of the Hessian matrix \mathbf{H}

$$\begin{aligned}
H_{11} &= C_{10} \cdot (2 + 6\lambda_1^{-4} \lambda_2^{-2}) + 2C_{20} \cdot (2\lambda_1 - 2\lambda_1^{-3} \cdot \lambda_2^{-2}) \cdot (2\lambda_1 - 2\lambda_1^{-3} \cdot \lambda_2^{-2}) \\
&\quad + 2C_{20} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3) \cdot (2 + 6\lambda_1^{-4} \cdot \lambda_2^{-2}) \\
&\quad + 6C_{30} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3) \cdot (2\lambda_1 - 2\lambda_1^{-3} \cdot \lambda_2^{-2}) \cdot (2\lambda_1 - 2\lambda_1^{-3} \cdot \lambda_2^{-2}) \\
&\quad + 3C_{30} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3)^2 \cdot (2 + 6\lambda_1^{-4} \cdot \lambda_2^{-2}) \\
&\quad + \alpha \cdot [(\beta - 1)\lambda_1^{\beta-2} \cdot \lambda_2^\beta + (\beta + 1)\lambda_1^{-\beta-2}] \\
H_{22} &= C_{10} \cdot (2 + 6\lambda_1^{-2} \lambda_2^{-4}) + 2C_{20} \cdot (2\lambda_2 - 2\lambda_1^{-2} \cdot \lambda_2^{-3}) \cdot (2\lambda_2 - 2\lambda_1^{-2} \cdot \lambda_2^{-3}) \\
&\quad + 2C_{20} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3) \cdot (2 + 6\lambda_1^{-2} \cdot \lambda_2^{-4}) \\
&\quad + 6C_{30} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3) \cdot (2\lambda_2 - 2\lambda_1^{-2} \cdot \lambda_2^{-3}) \cdot (2\lambda_2 - 2\lambda_1^{-2} \cdot \lambda_2^{-3}) \\
&\quad + 3C_{30} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3)^2 \cdot (2 + 6\lambda_1^{-2} \cdot \lambda_2^{-4}) \\
&\quad + \alpha \cdot [(\beta - 1)\lambda_1^\beta \cdot \lambda_2^{\beta-2} + (\beta + 1)\lambda_2^{-\beta-2}] \\
H_{12} = H_{21} &= C_{10} \cdot (4\lambda_1^{-3} \lambda_2^{-3}) + 2C_{20} \cdot (2\lambda_2 - 2\lambda_1^{-2} \cdot \lambda_2^{-3}) \cdot (2\lambda_1 - 2\lambda_1^{-3} \cdot \lambda_2^{-2}) \\
&\quad + 2C_{20} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3) \cdot (4\lambda_1^{-3} \cdot \lambda_2^{-3}) \\
&\quad + 6C_{30} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3) \cdot (2\lambda_2 - 2\lambda_1^{-2} \cdot \lambda_2^{-3}) \cdot (2\lambda_1 - 2\lambda_1^{-3} \cdot \lambda_2^{-2}) \\
&\quad + 3C_{30} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3)^2 \cdot (4\lambda_1^{-3} \cdot \lambda_2^{-3}) \\
&\quad + \alpha \cdot (\beta \lambda_1^{\beta-1} \cdot \lambda_2^{\beta-1})
\end{aligned}$$

Equation S2. Components of the matrix \mathbf{D}

$$\begin{aligned}
D_{11} &= [C_{10} \cdot (4\lambda_1 + 4\lambda_1^{-3} \lambda_2^{-2}) + 2C_{20} \cdot (2\lambda_1 - 2\lambda_1^{-3} \cdot \lambda_2^{-2}) \cdot (2\lambda_1^2 - 2\lambda_1^{-2} \cdot \lambda_2^{-2}) \\
&\quad + 2C_{20} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3) \cdot (4\lambda_1 + 4\lambda_1^{-3} \cdot \lambda_2^{-2}) \\
&\quad + 3C_{30} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3)^2 \cdot (4\lambda_1 + 4\lambda_1^{-3} \cdot \lambda_2^{-2}) \\
&\quad + 6C_{30} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3) \cdot (2\lambda_1 - 2\lambda_1^{-3} \cdot \lambda_2^{-2}) \cdot (2\lambda_1^2 - 2\lambda_1^{-2} \cdot \lambda_2^{-2}) \\
&\quad + \alpha \cdot (\beta \lambda_1^{\beta-1} \cdot \lambda_2^\beta + \beta \lambda_1^{-\beta-1})] \cdot \lambda_1 \\
D_{22} &= [C_{10} \cdot (4\lambda_2 + 4\lambda_1^{-2} \lambda_2^{-3}) + 2C_{20} \cdot (2\lambda_2 - 2\lambda_1^{-2} \cdot \lambda_2^{-3}) \cdot (2\lambda_2^2 - 2\lambda_1^{-2} \cdot \lambda_2^{-2}) \\
&\quad + 2C_{20} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3) \cdot (4\lambda_2 + 4\lambda_1^{-2} \cdot \lambda_2^{-3}) \\
&\quad + 3C_{30} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3)^2 \cdot (4\lambda_2 + 4\lambda_1^{-2} \cdot \lambda_2^{-3}) \\
&\quad + 6C_{30} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3) \cdot (2\lambda_2 - 2\lambda_1^{-2} \cdot \lambda_2^{-3}) \cdot (2\lambda_2^2 - 2\lambda_1^{-2} \cdot \lambda_2^{-2}) \\
&\quad + \alpha \cdot (\beta \lambda_1^\beta \cdot \lambda_2^{\beta-1} + \beta \lambda_2^{-\beta-1})] \cdot \lambda_2
\end{aligned}$$

$$\begin{aligned}
D_{12} = D_{21} = & C_{10} \cdot 4\lambda_1^{-2} \lambda_2^{-2} + 2C_{20} \cdot (2\lambda_2^2 - 2\lambda_1^{-2} \cdot \lambda_2^{-2}) \cdot (2\lambda_1^2 - 2\lambda_1^{-2} \cdot \lambda_2^{-2}) \\
& + 2C_{20} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3) \cdot (4\lambda_1^{-2} \cdot \lambda_2^{-2}) \\
& + 3C_{30} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3)^2 \cdot (4\lambda_1^{-2} \cdot \lambda_2^{-2}) \\
& + 6C_{30} \cdot (\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \cdot \lambda_2^{-2} - 3) \cdot (2\lambda_2^2 - 2\lambda_1^{-2} \cdot \lambda_2^{-2}) \cdot (2\lambda_1^2 - 2\lambda_1^{-2} \cdot \lambda_2^{-2}) \\
& + \alpha \cdot (\beta \lambda_1^\beta \cdot \lambda_2^\beta)
\end{aligned}$$

Equation S3. Some constitutive models for comparison

1) Third-order Ogden model

$$\begin{aligned}
W &= \sum_{i=1}^3 \frac{2\mu_i}{\alpha_i^2} (\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3) \\
T_{UT} &= \sum_{i=1}^n \frac{2\mu_i}{\alpha_i} \left(\lambda^{\alpha_i-1} - \lambda^{-\frac{\alpha_i-1}{2}} \right) \\
T_{ET} &= \sum_{i=1}^n \frac{2\mu_i}{\alpha_i} (\lambda^{\alpha_i-1} - \lambda^{-2\alpha_i-1}) \\
T_{PS} &= \sum_{i=1}^n \frac{2\mu_i}{\alpha_i} (\lambda^{\alpha_i-1} - \lambda^{-\alpha_i-1})
\end{aligned}$$

2) Alexander model

$$\begin{aligned}
W &= C_1 \int \exp \left[k(I_1 - 3)^2 \right] dI_1 + C_2 \ln \left[\frac{(I_2 - 3) + \gamma}{\gamma} \right] + C_3 (I_2 - 3) \\
T_{UT} &= 2 \bullet C_1 \bullet e^{\left[k(I_1 - 3)^2 \right]} \bullet \left(\lambda - \frac{1}{\lambda^2} \right) + 2 \bullet \left[\frac{C_2}{(I_2 - 3) + \gamma} + C_3 \right] \bullet \left(1 - \frac{1}{\lambda^3} \right) \\
T_{ET} &= 2 \bullet C_1 \bullet e^{\left[k(I_1 - 3)^2 \right]} \bullet \left(\lambda - \frac{1}{\lambda^5} \right) + 2 \bullet \left[\frac{C_2}{(I_2 - 3) + \gamma} + C_3 \right] \bullet \left(\lambda^3 - \frac{1}{\lambda^3} \right) \\
T_{PS} &= 2 \bullet C_1 \bullet e^{\left[k(I_1 - 3)^2 \right]} \bullet \left(\lambda - \frac{1}{\lambda^3} \right) + 2 \bullet \left[\frac{C_2}{(I_2 - 3) + \gamma} + C_3 \right] \bullet \left(\lambda - \frac{1}{\lambda^3} \right)
\end{aligned}$$

3) Yeoh model

$$\begin{aligned}
W &= C_{10} (I_1 - 3) + C_{20} (I_1 - 3)^2 + C_{30} (I_1 - 3)^3 \\
T_{UT} &= 2 \bullet \left[C_{10} + 2C_{20} \left(\lambda^2 + \frac{2}{\lambda} - 3 \right) + 3C_{30} \left(\lambda^2 + \frac{2}{\lambda} - 3 \right)^2 \right] \bullet \left(\lambda - \frac{1}{\lambda^2} \right) \\
T_{ET} &= 2 \bullet \left[C_{10} + 2C_{20} \left(2\lambda^2 + \frac{1}{\lambda^4} - 3 \right) + 3C_{30} \left(2\lambda^2 + \frac{1}{\lambda^4} - 3 \right)^2 \right] \bullet \left(\lambda - \frac{1}{\lambda^5} \right) \\
T_{PS} &= 2 \bullet \left[C_{10} + 2C_{20} \left(\lambda^2 + \frac{1}{\lambda^2} - 2 \right) + 3C_{30} \left(\lambda^2 + \frac{1}{\lambda^2} - 2 \right)^2 \right] \bullet \left(\lambda - \frac{1}{\lambda^3} \right)
\end{aligned}$$

4) Melly model

$$\begin{aligned}
 W &= C_{10} (I_1 - 3) + C_{20} (I_1 - 3)^2 + C_{30} (I_1 - 3)^3 + D(\sqrt{I_2} - \sqrt{3}) \\
 T_{UT} &= 2 \bullet \left[C_{10} + 2C_{20} \left(\lambda^2 + \frac{2}{\lambda} - 3 \right) + 3C_{30} \left(\lambda^2 + \frac{2}{\lambda} - 3 \right)^2 \right] \left(\lambda - \frac{1}{\lambda^2} \right) + D \bullet \left(\frac{1}{\lambda^2} + 2\lambda \right)^{-\frac{1}{2}} \bullet \left(1 - \frac{1}{\lambda^3} \right) \\
 T_{ET} &= 2 \bullet \left[C_{10} + 2C_{20} \left(2\lambda^2 + \frac{1}{\lambda^4} - 3 \right) + 3C_{30} \left(2\lambda^2 + \frac{1}{\lambda^4} - 3 \right)^2 \right] \left(\lambda - \frac{1}{\lambda^5} \right) + D \bullet \left(\frac{2}{\lambda^2} + \lambda^4 \right)^{-\frac{1}{2}} \bullet \left(\lambda^3 - \frac{1}{\lambda^3} \right) \\
 T_{PS} &= 2 \bullet \left[C_{10} + 2C_{20} \left(\lambda^2 + \frac{1}{\lambda^2} - 2 \right) + 3C_{30} \left(\lambda^2 + \frac{1}{\lambda^2} - 2 \right)^2 \right] \left(\lambda - \frac{1}{\lambda^3} \right) + D \bullet \left(\frac{1}{\lambda^2} + \lambda^2 + 1 \right)^{-\frac{1}{2}} \bullet \left(\lambda - \frac{1}{\lambda^3} \right)
 \end{aligned}$$

5) Modified Yeoh model

$$\begin{aligned}
 W &= C_{10} (I_1 - 3) + C_{20} (I_1 - 3)^2 + C_{30} (I_1 - 3)^3 + \frac{\alpha}{\beta} (1 - e^{-\beta(I_1 - 3)}) \\
 T_{UT} &= 2 \bullet \left[C_{10} + 2C_{20} \left(\lambda^2 + \frac{2}{\lambda} - 3 \right) + 3C_{30} \left(\lambda^2 + \frac{2}{\lambda} - 3 \right)^2 + \alpha \bullet e^{-\beta \left(\lambda^2 + \frac{2}{\lambda} - 3 \right)} \right] \left(\lambda - \frac{1}{\lambda^2} \right) \\
 T_{ET} &= 2 \bullet \left[C_{10} + 2C_{20} \left(2\lambda^2 + \frac{1}{\lambda^4} - 3 \right) + 3C_{30} \left(2\lambda^2 + \frac{1}{\lambda^4} - 3 \right)^2 + \alpha \bullet e^{-\beta \left(2\lambda^2 + \frac{1}{\lambda^4} - 3 \right)} \right] \left(\lambda - \frac{1}{\lambda^5} \right) \\
 T_{PS} &= 2 \bullet \left[C_{10} + 2C_{20} \left(\lambda^2 + \frac{1}{\lambda^2} - 2 \right) + 3C_{30} \left(\lambda^2 + \frac{1}{\lambda^2} - 2 \right)^2 + \alpha \bullet e^{-\beta \left(\lambda^2 + \frac{1}{\lambda^2} - 2 \right)} \right] \left(\lambda - \frac{1}{\lambda^3} \right)
 \end{aligned}$$

6) Generalized Yeoh model

$$\begin{aligned}
 W &= K_1 (I_1 - 3)^m + K_2 (I_1 - 3)^p + K_3 (I_1 - 3)^q \\
 T_{UT} &= 2 \bullet \left(\lambda - \frac{1}{\lambda^2} \right) \bullet \left[K_1 \bullet m \bullet \left(\lambda^2 + \frac{2}{\lambda} - 3 \right)^{m-1} + K_2 \bullet p \bullet \left(\lambda^2 + \frac{2}{\lambda} - 3 \right)^{p-1} + K_3 \bullet q \bullet \left(\lambda^2 + \frac{2}{\lambda} - 3 \right)^{q-1} \right] \\
 T_{ET} &= 2 \bullet \left(\lambda - \frac{1}{\lambda^5} \right) \bullet \left[K_1 \bullet m \bullet \left(2\lambda^2 + \frac{1}{\lambda^4} - 3 \right)^{m-1} + K_2 \bullet p \bullet \left(2\lambda^2 + \frac{1}{\lambda^4} - 3 \right)^{p-1} + K_3 \bullet q \bullet \left(2\lambda^2 + \frac{1}{\lambda^4} - 3 \right)^{q-1} \right] \\
 T_{PS} &= 2 \bullet \left(\lambda - \frac{1}{\lambda^3} \right) \bullet \left[K_1 \bullet m \bullet \left(\lambda^2 + \frac{1}{\lambda^2} - 2 \right)^{m-1} + K_2 \bullet p \bullet \left(\lambda^2 + \frac{1}{\lambda^2} - 2 \right)^{p-1} + K_3 \bullet q \bullet \left(\lambda^2 + \frac{1}{\lambda^2} - 2 \right)^{q-1} \right]
 \end{aligned}$$

7) Anssari-Benam model

$$\begin{aligned}
 W &= \frac{3(n-1)}{2n} \mu N \left[\frac{1}{3N(n-1)} (I_1 - 3) - \ln \left(\frac{I_1 - 3N}{3 - 3N} \right) \right] \\
 T_{UT} &= \frac{1}{\lambda} \bullet \left\{ \frac{\mu}{n} \left(\frac{\lambda^2 + 2\lambda^{-1} - 3nN}{\lambda^2 + 2\lambda^{-1} - 3N} \right) \left(\lambda^2 - \frac{1}{\lambda} \right) \right\}
 \end{aligned}$$

$$T_{ET} = \frac{1}{\lambda} \bullet \left\{ \frac{\mu}{n} \left(\frac{2\lambda^2 + \lambda^{-4} - 3nN}{2\lambda^2 + \lambda^{-4} - 3N} \right) \left(\lambda^2 - \frac{1}{\lambda^4} \right) \right\}$$

$$T_{PS} = \frac{1}{\lambda} \bullet \left\{ \frac{\mu}{n} \left(\frac{\lambda^2 + \lambda^{-2} + 1 - 3nN}{\lambda^2 + \lambda^{-2} + 1 - 3N} \right) \left(\lambda^2 - \frac{1}{\lambda^2} \right) \right\}$$

8) Modified Anssari-Benam model

$$W = \frac{3(n-1)}{2n} \mu N \left[\frac{1}{3N(n-1)} (I_1 - 3) - \ln \left(\frac{I_1 - 3N}{3 - 3N} \right) \right] + \frac{\alpha}{\beta} \left[(\lambda_1 \lambda_2)^\beta + (\lambda_2 \lambda_3)^\beta + (\lambda_1 \lambda_3)^\beta - 3 \right]$$

$$T_{UT} = \frac{1}{\lambda} \bullet \left\{ \frac{\mu}{n} \left(\frac{\lambda^2 + 2\lambda^{-1} - 3nN}{\lambda^2 + 2\lambda^{-1} - 3N} \right) \left(\lambda^2 - \frac{1}{\lambda} \right) + \alpha \bullet \left(\lambda^{\frac{\beta}{2}} - \lambda^{-\beta} \right) \right\}$$

$$T_{ET} = \frac{1}{\lambda} \bullet \left\{ \frac{\mu}{n} \left(\frac{2\lambda^2 + \lambda^{-4} - 3nN}{2\lambda^2 + \lambda^{-4} - 3N} \right) \left(\lambda^2 - \frac{1}{\lambda^4} \right) + \alpha \bullet \left(\lambda^{2\beta} - \lambda^{-\beta} \right) \right\}$$

$$T_{PS} = \frac{1}{\lambda} \bullet \left\{ \frac{\mu}{n} \left(\frac{\lambda^2 + \lambda^{-2} + 1 - 3nN}{\lambda^2 + \lambda^{-2} + 1 - 3N} \right) \left(\lambda^2 - \frac{1}{\lambda^2} \right) + \alpha \bullet \left(\lambda^\beta - \lambda^{-\beta} \right) \right\}$$