Supplementary Materials: Dynamic Mapping of Evapotranspiration Using an Energy Balance-Based Model over an Andean Páramo Catchment of Southern Ecuador

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1. METRIC\textsubscript{L} and METRIC\textsubscript{M} Implementation Flowchart

![Flowchart Image]

* NDVI with bands 3, 4 (for Landsat) & 1, 2 (for MODIS)
** SAVI is no needed for MODIS
*** A LAINDVI function is needed to obtain LAI for MODIS

**Figure S1.** METRIC\textsubscript{L} and METRIC\textsubscript{M} algorithms flowchart (adapted from equations of Allen et al. [1] and Trezza et al. [2]).
2. Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Name</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>ET</td>
<td>Evapotranspiration</td>
<td>mm</td>
</tr>
<tr>
<td>LE</td>
<td>Latent heat flux</td>
<td>W·m⁻²</td>
</tr>
<tr>
<td>RN</td>
<td>Net radiation</td>
<td>W·m⁻²</td>
</tr>
<tr>
<td>G</td>
<td>Soil heat flux</td>
<td>W·m⁻²</td>
</tr>
<tr>
<td>H</td>
<td>Sensible heat flux</td>
<td>W·m⁻²</td>
</tr>
<tr>
<td>RS, in</td>
<td>Incoming shortwave radiation</td>
<td>W·m⁻²</td>
</tr>
<tr>
<td>RL, in</td>
<td>Incoming longwave thermal radiation from the atmosphere</td>
<td>W·m⁻²</td>
</tr>
<tr>
<td>RL, out</td>
<td>Outgoing longwave thermal radiation</td>
<td>W·m⁻²</td>
</tr>
<tr>
<td>ε₀</td>
<td>Broad-band surface emissivity</td>
<td>dimensionless</td>
</tr>
<tr>
<td>Ts</td>
<td>Temperature of the surface</td>
<td>K</td>
</tr>
<tr>
<td>LAI</td>
<td>Leaf Area Index</td>
<td>m²·m⁻²</td>
</tr>
<tr>
<td>SAVI</td>
<td>Soil Adjusted Vegetation Index</td>
<td>dimensionless</td>
</tr>
<tr>
<td>NDVI</td>
<td>Normalized Difference Vegetation Index</td>
<td>dimensionless</td>
</tr>
<tr>
<td>τ₁ₕ</td>
<td>Aerodynamic resistance to heat transfer</td>
<td>s·m⁻¹</td>
</tr>
<tr>
<td>dT</td>
<td>Near surface temperature difference</td>
<td>K</td>
</tr>
<tr>
<td>AWSs</td>
<td>Automatic weather stations</td>
<td>-</td>
</tr>
<tr>
<td>u'</td>
<td>Friction velocity</td>
<td>m·s⁻¹</td>
</tr>
<tr>
<td>u₂₀₀</td>
<td>Wind speed at a blending height (200 m)</td>
<td>m·s⁻¹</td>
</tr>
<tr>
<td>zom</td>
<td>Momentum roughness length</td>
<td>m</td>
</tr>
<tr>
<td>zom(mtn)</td>
<td>Mountain adjusted roughness length</td>
<td>m</td>
</tr>
<tr>
<td>ε</td>
<td>Wind speed weighting coefficient</td>
<td>dimensionless</td>
</tr>
<tr>
<td>TₛDEM</td>
<td>Lapse rate corrected surface temperature</td>
<td>K</td>
</tr>
<tr>
<td>L</td>
<td>Monin-Obukhov Length</td>
<td>m</td>
</tr>
</tbody>
</table>

| ETₜᵣ | ASCE-ERWI Standardized Reference Evapotranspiration for alfalfa | mm·h⁻¹ |
| ETₜ₁ │ Instantaneous actual ET                               | mm·h⁻¹ |
| λ    | Latent heat of vaporization of water                  | J·kg⁻¹   |
| ETₜᵣ | Reference Evapotranspiration fraction (crop coefficient at reference alfalfa basis) | dimensionless |
| ETₜₐ₅ | ET at a daily basis (24 h)                             | mm·day⁻¹|
| Kc   | Grass-based crop coefficient                          | dimensionless |
| K_ratio | Conversion value for ETₜᵣ to Kc | dimensionless |

3. Description of the METRIC Landsat-Based Implementation

According to the methodology of Allen et al. [1] METRIC delivers the instantaneous latent heat flux (LE) on a pixel-by-pixel basis as a residual of the energy balance Equation (S1):

\[
LE = RN - G - H \quad (W \cdot m^{-2}) \tag{S1}
\]

Here \(LE\) can be expressed as actual evapotranspiration \(ET\) by dividing it by the latent heat of vaporization. We calculated \(RN\) as actual evapotranspiration \(ET\) by dividing it by the latent heat of vaporization. We calculated \(RN\) as:

\[
RN = RS,in - \alpha \times RS,in + RL,in - RL,out - (1 - \varepsilon_0) \times RL,in \quad (W \cdot m^{-2}) \tag{S2}
\]

\(RS,in\) was calculated using Equation (S3).

\[
RS,in = \frac{G_{sc} \times \cos \theta_{rel} \times \tau_{sw}}{d^2} \quad (W \cdot m^{-2}) \tag{S3}
\]
where $G_{sc}$ is the solar constant (1367 W·m$^{-2}$), $\cos \theta$ considers the slope and aspect terrain adjustments according [1,3]. $\tau_{sw}$ is the atmospheric transmissivity obtained with the ASCE-ERWI method [4], and $d^2$ is the square of the relative earth-sun distance at the day of the image. $\alpha$ was calculated by integrating satellite spectral reflectance values from bands 1 to 5 and 7 of Landsat image according [5]. $R_{L,\text{out}}$ was calculated using the Stephan-Boltzmann equation (Equation (S4)).

$$R_{L,\text{out}} = \sigma \times \epsilon_0 \times T_s^4 \quad (\text{W} \cdot \text{m}^{-2})$$  \hspace{1cm} (S4)

where $\sigma$ is the Stephan-Boltzmann constant ($5.67 \times 10^{-8}$ W·m$^{-2}$·K$^{-4}$). $\epsilon_0$ was computed using an empirical equation developed by Tasumi [6] (Equation (S5)).

$$\epsilon_0 = 0.95 + 0.01 \times \text{LAI} \quad \text{for LAI} \leq 3$$  \hspace{1cm} (S5)

The equation is limited to $\epsilon_0 = 0.98$ when $\text{LAI} > 3$. LAI was computed using the equation from Bastiaanssen [7] (Equation (S6)).

$$\text{LAI} = \frac{-\ln \left( \frac{0.69 - \text{SAVI}}{0.59} \right)}{0.91}$$  \hspace{1cm} (S6)

$\text{SAVI}$ considers the top-of-atmosphere reflectance bands number 3 and 4 of the images (Equation (S7)).

$$\text{SAVI} = \frac{(1 - L_{\text{const}}) \times (\rho_{\lambda,4} - \rho_{\lambda,3})}{(\rho_{\lambda,4} + \rho_{\lambda,3})}$$  \hspace{1cm} (S7)

$L_{\text{const}}$ constant value represents the soil-brightness dependent factor. We calibrated $L_{\text{const}}$ for our study area and a value of 0.05 was found after processing the images according to the method recommended by Huete [8]. $T_s$ was estimated for Landsat imagery using the methodology proposed by Markham and Barker [9] which accounts for emissivity and atmospheric specific corrections (Equation (S8)).

$$T_s = \frac{K_2}{\ln \left( \frac{\epsilon_{NB} \times K_1}{L_{CA,6}} + 1 \right)} \quad (\text{K})$$  \hspace{1cm} (S8)

Here, $K_1$ and $K_2$ constants were 666.09 and 1282.71 W·m$^{-2}$·sr$^{-1}$ respectively for Landsat, and the corrected thermal radiance $L_{CA,6}$ was calculated using the approach from Wukelic et al. [10]. In this calculation we applied the default values recommended by Allen et al. [1] for the path radiance ($R_p = 0$), downward thermal radiation from clear sky ($R_{sky} = 0$), and narrow band transmissivity or air ($\tau_{NB} = 1$) due to the lack of radio sounding data for the study area.

The $T_s$ was internally adjusted during the creation of the $dT$ function according Allen et al [1]. The narrow-band surface emissivity ($\epsilon_{NB}$) was calculated using the approach from Tasumi [6] similarly to $\epsilon_0$. The $R_{L,\text{in}}$ was calculated by Equation (S9).

$$R_{L,\text{in}} = \sigma \times \epsilon_a \times T_a^4 \quad (\text{W} \cdot \text{m}^{-2})$$  \hspace{1cm} (S9)

where $\epsilon_a$ is the effective atmospheric emissivity and $T_a$ is the air temperature. The $\epsilon_a$ was obtained according Allen et al. [1] and Bastiaanssen [11]. METRIC applications generally use $T_s$ as a surrogate of $T_a$ in the calculation of $R_{L,\text{out}}$ and $R_{L,\text{in}}$. This assumption was appropriate considering the high altitude and predominant wet conditions of the Andean páramo, where differences between the surface and near-surface air temperatures tends to be small [1]. $G$ in W·m$^{-2}$ was calculated using the approach of Bastiaanssen et al. [12] (Equation (S10)).

$$G = (T_s - 273.15) \times (0.0038 + 0.0074 \times \alpha) \times (1 - 0.98 \times \text{NDVI}^4) \times R_N$$  \hspace{1cm} (S10)

This approach was considered because $\text{NDVI}$ input is more appropriate for heterogeneous terrain instead of $\text{LAI}$ when estimating $G$ [13]. The $\text{NDVI}$ was calculated according to Equation (S11).

$$\text{NDVI} = \frac{(\rho_{\lambda,4} - \rho_{\lambda,3})}{(\rho_{\lambda,4} + \rho_{\lambda,3})}$$  \hspace{1cm} (S11)
where $\rho_{\lambda,4}$ and $\rho_{\lambda,4}$ are the Landsat reflectances from near-infrared and red bands respectively. We estimated $H$ employing an aerodynamic function (Equation (S12)):

$$H = \rho_{\text{air}} \times C_p \times \frac{dT}{r_{ah}} \text{ (W} \cdot \text{m}^{-2})$$

(S12)

where $\rho_{\text{air}}$ is the air density (kg·m$^{-3}$), $C_p$ is the specific heat of air at constant pressure (J·kg$^{-1}$·K$^{-1}$), $r_{ah}$ in s·m$^{-1}$ was retrieved between two near surface heights $z_1 = 0.1$ m and $z_1 = 2$ m, and $dT$ in Kelvin was retrieved similarly between $z_1$ and $z_2$. In Equation (S12) both $r_{ah}$ and $dT$ were unknown and required an iterative process to solve it.

The $r_{ah}$ was calculated by extrapolating wind speed to some blending heights above the surface (usually 100 to 200 m) and corrected with an iterative stability scheme based on the Monin-Obukhov functions (Equation (S13)):

$$r_{ah} = \frac{\ln\left(\frac{z_1}{z_2}\right)}{k \times u^*}$$

(S13)

$k$ is Von Karman’s constant (0.41). $u^*$ in m·s$^{-1}$ was applied for neutral atmospheric conditions and was computed during the first iteration using Equation (S14).

$$u^* = \frac{k \times u_{200}}{\ln\left(\frac{200}{z_{om}}\right)}$$

(S14)

The term $u_{200}$ was calculated using Equation (S15).

$$u_{200} = \frac{u_{AWS} \times \ln\left(\frac{200}{z_{omAWS}}\right)}{\ln\left(\frac{z_{AWS}}{z_{omAWS}}\right)}$$

(S15)

In the equation, $u_{AWS}$ is wind speed measured at $z_{AWS}$ height (m) above the surface, and $z_{omAWS}$ is the estimated roughness length (m) at the same place. To represent $z_{om}$ we employ a land use and land cover map (LULC) where we assigned $z_{om}$ values according to land use type and vegetation cover. The assigned $z_{om}$ values were: Water (0.0005 m), páramo grasslands (0.020 m), páramo flooded grasslands (0.020 m), evergreen shrublands with grasslands (0.125 m), high montane evergreen forest (0.20 m) and inter-Andean montane evergreen forest (0.25 m). These values were selected according to several studies [14–17].

To account for mountainous effects we implemented the empirical methodology recommended by Allen et al. [18] where an adjustment of $z_{om}$ was applied using Equation (S16).

$$z_{om(mtn)} = z_{om} \times \left(1 + \frac{s_d - 5}{20}\right) \text{; for } s_d \geq 5$$

(S16)

where $s_d$ is the slope of each pixel in degrees. In the same way, the term $u_{200}$ for mountainous image pixels was multiplied by a wind speed weighting coefficient ($\omega$), before computations of $u^*$ and $r_{ah}$. The function is on Equation (S17).

$$\omega = 1 + 0.1 \times \left(\frac{E_{DEM} - E_{AWS}}{1000}\right)$$

(S17)

In this equation, $E_{DEM}$ and $E_{AWS}$ are the elevation of each pixel and elevation of the AWSs respectively. $T_s$ was corrected using the difference in elevation in relation to an arbitrary point in the image where $T_{sDEM} = T_s$ according Allen et al. [18] (Equation (S18)).

$$T_{sDEM} = T_s - CT_{IR} \times \Delta z$$

(S18)

Specific calibration of the temperature lapse rate $CT_{IR}$ was performed using the methodology proposed by Allen et al. [18] and was applied for each image date ($CT_{IR}$ mean = 0.68 °C each 100 m).

During ten iterations corrected values for $u^*$ were computed using Equation (S19). We choose this number of iterations to ensure that the change in $r_{ah}$ is less than 0.1% at the end of the process.
Herein, $\Psi_{m(200)}$ is the stability correction for momentum transport at 200 m. Then $r_{ah}$ was recalculated using Equation (S20):

$$
r_{ah} = \frac{\ln \left( \frac{z_1}{z_2} \right) - \Psi_{H(z_2)} + \Psi_{H(z_1)}}{k \times u^*}
$$

where $\Psi_{H(z_2)}$ and $\Psi_{H(z_1)}$ are stability corrections for the heat transport at heights $z_2$ & $z_1$. The terms $\Psi_{m(200)}$, $\Psi_{H(z_2)}$ and $\Psi_{H(z_1)}$ were updated in the iterative process following the approaches outlined by Paulson [19] and Webb [20], which employ the Monin-Obukhov length $L$ defined by Equation (S21).

$$
L = -\frac{\rho_{air} \times C_p \times u^* \times T_s}{k \times g \times H}
$$

This function contemplates the variables mentioned above, and also the gravitational acceleration $g$ (9.807 m s$^{-2}$) and the air density $\rho_{air}$ at the pixel elevation (kPa). A comprehensive explanation of this process is available in Allen et al. [1].

The $dT$ function assumes a linear relationship with $T_s$ where coefficients $a$ and $b$ were used to correct the function in an iterative computation.

$$
dT = a + b \times T_s
$$

In Equation (S22) the $a$ and $b$ coefficients have an empirical determination based on two extreme conditions in the image. The named hot and cold pixels were selected by two specific criteria: the hot pixel was a relative dry, non-vegetated, and with higher temperature pixel, and the cold pixel was a wet and fully vegetated pixel, both located in the proximity of the three AWSs. The selection of these pixels for each image was based on a rigorous process of visual and software assisted selection of pixels in $T_s$, $a$, $NDVI$, $LAI$, and slope images. Most of the hot pixels were located on non- or sparsely-vegetated areas in the mid and high areas of the grasslands, and the wet pixels were mostly selected from the Polylepis sp. tree patches near wetlands. Temperature differences between the cold and hot pixels ranged in 5 to 7 °C for our study. Hence, the estimations for $dT$ were assessed using Equations (S23) and (S24):

$$
dT_{hot} = \frac{(R_N - G) \times r_{ah,hot}}{\rho_{air,hot} \times C_p}
$$

$$
dT_{cold} = \frac{(R_N - G - 1.05 \times ET_r) \times r_{ah,cold}}{\rho_{air,cold} \times C_p}
$$

In the Equation (S24), an adjustment with the instantaneous $ET_r$ obtained according the methodology of ASCE-ERWI [4] accounts for an increased condition of ET in the cold pixel (5% greater) [1]. The Equation (S25) calculates $H$ for the hot and cold pixels.

$$
H_{hot} = (R_N - G)_{hot} - LE_{hot} \quad ; \quad H_{cold} = (R_N - G)_{cold} - LE_{cold}
$$

At the end of the iterative process, $a$ and $b$ coefficients were determined using Equations (S26) and (S27).

$$
a = \frac{dT_{hot} - dT_{cold}}{T_{sDEM,hot} - T_{sDEM,cold}}
$$

$$
b = \frac{dT_{hot} - a}{T_{sDEM,hot}}
$$

where $T_{sDEM,hot}$ and $T_{sDEM,cold}$ are the lapse rate corrected surface temperatures for the hot and cold pixels. These two conditions tie the calculations for all pixels in the image.
Once \( R_N, H, \) and \( G \) were calculated, we applied the Equation (S1) to obtain \( LE \) as the residual of the balance. Hereafter, we divided \( LE \) by the \( \lambda \) to obtain the instantaneous actual ET (\( E_{T_{\text{inst}}} \)) at the satellite image time. Equation (S28) describes this:

\[
E_{T_{\text{inst}}} = \frac{3600 \times LE}{\lambda \times \rho_w} \quad \text{(mm h}^{-1})
\]  

(S28)

where 3600 is the conversion from seconds to hours and \( \rho_w \) is the density of water (1000 kg m\(^{-3}\)). \( \lambda \) was calculated using the Equation (S29).

\[
\lambda = [2.501 - 0.00236 \times (T_s - 273.15)] \times 10^6 \quad \text{(J kg}^{-1})
\]  

(S29)

A requirement to convert hourly \( E_{T_{\text{inst}}} \) into a daily basis is to obtain \( E_{T_r} \). This variable was calculated dividing \( E_{T_{\text{inst}}} \) by the \( E_{T_r} \) (Equation (S30)). \( E_{T_r} \) is also the equivalent of the crop coefficient \( (K_c) \) when used with an alfalfa reference basis [1,4]. The advantage of this calibration via \( E_{T_r} \) is that each pixel of the \( E_{T_r} \) image retains a unique value for \( E_{T_{\text{inst}}} \) which can be interpolated in the time using representative dataset from the weather stations.

\[
E_{T_r} = \frac{E_{T_{\text{inst}}}}{E_{T_r}}
\]  

(S30)

Prior to the calculation of daily evaporation the minor areas of the \( E_{T_r} \) images had to be masked to avoid contamination due to clouds [1,21,22]. This was done in two steps: (1) identification and masking of clouds and cloud-shadows (considering a buffer of 200 m surrounding the cloud/shadow) with the application of the FMASK algorithm proposed by Zhu and Woodcock [23] and (2) implementing the methodology recommended by Kjaersgaard et al. [24] which fills in the masked areas using a time weighted interpolation of the \( E_{T_r} \) values from the precedent and following satellite images containing valid \( E_{T_r} \) estimates. This method considers an adjustment for vegetation (using \( NDVI \)) to account for reflectance changes over the time.

Equation (S31) shows calculation of \( E_{T_{\text{day}}} \). It was obtained by multiplying \( E_{T_r} \) times hourly aggregated \( E_{T_r} \) in 24-h periods times a correction term \( C_{\text{Solar24}} \) to account for the variation of solar radiation over sloping terrain in 24 h.

\[
E_{T_{\text{day}}} = E_{T_r} \times \sum_{i=1}^{24} E_{T_r,(i)} \times C_{\text{Solar24}} \quad \text{(mm day}^{-1})
\]  

(S31)

\( C_{\text{Solar24}} \) was calculated using daily extraterrestrial and clear-sky solar energy using the methodology suggested by Allen et al. [25].

Monthly \( ET \) was calculated assuming that \( ET \) changes in proportion to the change in \( E_{T_r} \) at the AWSs. Last, according to Kjaersgaard et al. [26], a cubic spline interpolation of the daily \( E_{T_r} \) maps over the study period was implemented, as well as an spatial inverse weighted interpolation (IDW) to recreate maps of \( E_{T_r} \) using the information of the three AWSs in the same period.

4. The \( K_{\text{ratio}} \) Conversion Factor Methodology

According to the methodology of Allen et al. [26] a crop coefficient based on the alfalfa reference (in our case \( E_{T_r} \)) can be converted to a crop coefficient with a grass reference \( (K_c) \) by multiplying it by a conversion value \( (K_{\text{ratio}}) \) which usually ranges between 1.0 to 1.3. The Equation (S32) uses averaged climate data for the period of study and a constant value of the alfalfa crop coefficient for the mid-season stage \( (K_{c_{\text{mid}}} \)):

\[
K_{\text{ratio}} = K_{c_{\text{mid}}} + [0.04 \times (u_{AWS_{\text{mean}}} - 2) - 0.004 \times (RH_{\text{min}} - 45)] \times \left(\frac{h}{3}\right)^{0.3}
\]  

(S32)

Herein \( K_{c_{\text{mid}}} = 1.20 \) was obtained from tables in [27], \( u_{AWS_{\text{mean}}} \) was the daily mean value for wind speed during the study period, \( RH_{\text{min}} \) was the mean value for daily minimum relative humidity (%) during the study period, and \( h \) is the height for alfalfa reference crop (\( h = 0.5 \)).
References


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