Incentive Model Based on Cooperative Relationship in Sustainable Construction Projects

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Abstract: Considering the cooperative relationship between owners and contractors in sustainable construction projects, as well as the synergistic effects created by cooperative behaviors, a cooperative incentive model was developed using game theory. The model was formulated and analyzed under both non-moral hazard and moral hazard situations. Then, a numerical simulation and example were proposed to verify the conclusions derived from the model. The results showed that the synergistic effect increases the input intensity of one party’s resource transfer into the increase of marginal utility of the other party, thus the owner and contractor are willing to enhance their levels of effort. One party’s optimal benefit allocation coefficient is positively affected by its own output efficiency, and negatively affected by the other party’s output efficiency. The effort level and expected benefits of the owner and contractor can be improved by enhancing the cooperative relationship between the two parties, as well as enhancing the net benefits of a sustainable construction project. The synergistic effect cannot lower the negative effect of moral hazard behaviors during the implementation of sustainable construction projects. Conversely, the higher levels of the cooperative relationship, the wider the gaps amongst the optimal values under both non-moral hazard and moral hazard situations for the levels of effort, expected benefits and net project benefits. Since few studies to date have emphasized the effects of cooperative relationship on sustainable construction projects, this study constructed a game-based incentive model to bridge the gaps. This study contributes significant theoretical and practical insights into the management of cooperation amongst stakeholders, and into the enhancement of the overall benefits of sustainable construction projects.

Keywords: sustainable construction; cooperative relationship; incentive model; synergistic effect

1. Introduction

Sustainable construction is “construction that satisfies the demands of sustainable development”, and can be defined as a pursuit to ensure economic development and social health whilst reducing the negative impact of construction on the environment [1,2]. Sustainable development is one of the leading civilization ideas, which means such a development that satisfies the present needs without a limitation of the possibility of satisfying the needs in the future [3]. Based on ecological principles and resource efficiency, sustainable construction not only considers environmental issues, but also attempts to achieve a balance amongst the environmental, economic and social objectives [4–9]. Compared to traditional construction projects, sustainable construction projects require a higher level of cooperation amongst stakeholders. These projects are typically characterized by multidisciplinary collaboration, the early intervention of contractors, the liquidity of construction processes, and the objective’s
ambidexterity of stakeholders. Therefore, inter-organizational cooperation is essential to a balanced approach towards sustainability [10–12]. Sustainable construction is an adaptive complex system that requires the cooperation of numerous stakeholders during the different stages of a construction project [13]. A high level of cooperation in sustainable construction is essential because designs, technologies, and systems associated with sustainable construction are often found to be complicated and not straightforward to work with.

In sustainable construction projects, the key stakeholders include the owners, contractors, subcontractors, designers and supervisors (see Figure 1). The stakeholders are cooperative to pursue a high level of sustainable development with an ethical motive and a desire to achieve a high level of professionalism. The cooperation amongst these key stakeholders is critical for the success of sustainable construction. Over the past few years, a number of studies have been undertaken to examine the cooperation amongst stakeholders. However, there are still some concerns on the cooperation which is associated with organizational capabilities and behaviors [14–16]. In the process of project implementation, resource exchange and knowledge interaction are related to different stakeholders [17]. Since the endowment of resources is different amongst stakeholders, a single-function organization is not able to complete an overall project delivery independently [18]. These characteristics require a higher level of stakeholder engagement and collaboration to generate more intensive cooperative behavior than is found in traditional projects. While the stakeholders involved in sustainable construction projects are all independent legal entities or rational individuals. Whether the owner, the contractor or other stakeholders, they all have their own independent benefits [19]. Due to different core knowledge and capabilities, together with the information asymmetry in the course of project implementation, the owner and contractor may both exhibit moral hazard behaviors [20]. The behaviors of the owner and contractor heavily rely on the signed contractual terms, while moral hazard behaviors may occur where the actions of one party may change to the detriment of another after an undesirable behavior has taken place [21–23]. Therefore, cooperative strategies are being increasingly embraced by both the owner and contractor, in order to avoid such moral hazard behaviors [24]. Due to the lack of trusting relationships and incomplete contracts amongst various stakeholders involved in sustainable construction projects, many moral hazard behaviors, such as cutting corners and hiding defects, may remain hidden and unknown during the implementation of projects [25,26].

![Figure 1. Stakeholders and factors surrounding sustainable construction.](image-url)
Although it is well documented in the literature that cooperation can enhance project benefits [27–30], quantitative approaches have not been common for exploring the effects of cooperation on the benefits of sustainable construction projects. The research object of this study is the cooperative behavior between owners and contractors in sustainable construction projects. Due to the complementary knowledge of the two parties, the effects of cooperation on benefits are greater than the effects of competition [31]. Considering the inherent nature of sustainable construction projects, the cooperative relationship between the owner and contractor is dynamic, and can be described as either negative cooperation or positive cooperation. Therefore, a mathematical model using game theory was developed under different situations. Game theory has been used to examine the conflict and cooperation between rational decision makers [32,33]. Game theory provides a useful approach to study the conceptual aspects of construction projects [34–37], and provides useful insights into the way that the participants utilize the resources under different situations. The grounded model can be used to simulate and analyze the effects of cooperative behavior on the stakeholders’ net benefits, the overall project benefits, and the benefits’ allocation strategy. This study aims to: (i) investigate how cooperative relationships affect the two parties’ behaviors; (ii) explore how cooperative relationships affect the two parties’ expected benefits; (iii) explore how cooperative relationships affect the overall project benefits. Since few studies to date have explored the effects of cooperative relationships on project benefits in sustainable construction projects, this study provides theoretical and practical insights to achieve the success of sustainable construction projects in an inter-organizational context. This study can also provide a theoretical reference for properly handling the relationship amongst the inter-organizational cooperative behaviors of stakeholders and the realization of overall project benefits.

2. Stakeholders and Factors Surrounding Sustainable Construction

Sustainable construction can be treated as a result of sustainable development in the comprehensive project cycle [38], rather than a specific stage or a site activity of the project lifecycle. According to existing studies [39,40] and the current practices of sustainable construction, Figure 1 was developed to show the key stakeholders and factors surrounding sustainable construction in China. Figure 1 shows the key stakeholders of sustainable construction (i.e., governments, financiers, developers, consultants, suppliers, designers, owners, supervisors, contractors, subcontractors and end users) and the critical factors affecting the success of sustainable construction (i.e., awareness, knowledge, demand, commitment, implementation and communication). Awareness and knowledge are two important factors promoting sustainable development in the construction industry [39]. The extent of the implementation of a construction towards sustainability mainly depends on improving the stakeholders’ awareness and knowledge of sustainability [41]. The raising of awareness about the potential benefits of sustainable construction in the long term could help the construction industry turn more and more towards sustainability [42]. The stakeholders’ contributions to achieving sustainability is dependent on their possession of core knowledge and project experience related to the concept of sustainable construction [43]. Knowledge exchange could help improve the motivations and performances of the stakeholders [44], thereby encouraging the construction process towards sustainability. The demand of the stakeholders is another important factor affecting the success of sustainable construction projects. Each stakeholder often has their own interest, whilst their demands do not stay exactly the same during the whole project lifecycle [45], thereby increasing the complexity of sustainable construction projects. It is important to take the demands and expectations of all stakeholders into consideration. These stakeholders work by their own interest to fulfill the overarching project objectives, and seek better opportunities via the attitudinal and behavioral changes [46]. Commitment reflects stakeholders’ need throughout the project implementation which changes along with internal or external environment of projects, construction duration, and output [47]. It is necessary to improve the commitment and knowledge of all the interested stakeholders involved in the construction industry in order to achieve sustainable construction [48]. According to Jamil and Fathi (2016), commitment and knowledge are the elements crucial to the successful implementation
of sustainable construction projects [44]. Communication is also an essential and critical factor affecting the stakeholders’ perceptions of sustainability. Knowledge of sustainable construction is often fragmented amongst different stakeholders [49], but frequent communication amongst them could help integrate the information and knowledge, align objectives, and reduce conflicts. In fact, the quality of communication in the context of the construction industry plays an important role in the process of construction. Poor communication is considered to be one of the most common risks in construction projects [50]. The complexity of the sustainability aspect of construction projects increases the need for communication amongst the stakeholders in order to integrate knowledge and share ideas.

Cooperation refers to a reciprocal process in which two or more individuals or organizations work together [51]. Cooperative behaviors generally refer to behaviors that help to advance the goals of a cooperative network of individuals and/or organizations [52]. Xue et al. [19] proposed that human behavior plays a crucial role in facilitating cooperation in construction projects. Stewart and Waroonkun [53] found that trust, understanding and communication are key enablers to value creation in construction projects. Erdogan et al. [54] presented an organizational changes model to improve cooperative environments. Game theory has been applied in a number of studies to explore the conceptual aspects of construction projects. Zhang et al. [55] proposed that the contractor’s fairness perception had a positive effect on the contractor’s cooperative behavior. Asgari et al. [25] developed a cooperative game theoretic framework for joint resource management in construction projects. Son et al. [51] introduced an agent-based simulation method using game theory and social network to simulate the evolution of cooperation in temporary project teams. Khanzadi et al. [16] constructed a mathematical model using game theory to investigate the behaviors and strategies of the owner and contractor in construction projects with time overruns. Dzeng and Wang [56] developed a web-based negotiation game to analyze the decision-making process in construction procurement. Ahmed et al. [57] presented a game theory approach to analyze bidding in construction projects. Liu et al. [58] presented an evolutionary game approach to determine the opportunistic behaviors in Public-Private Partnership (PPP) projects. Ho [59] developed a game-theory based model for financial renegotiation in PPP projects. Chiou et al. [60] proposed two bi-level programming models based on behavioral conjectures in game theory to facilitate the planning intercity passenger transport systems.

Due to the increasing challenges of sustainability, stakeholders involved in sustainable construction must seek new solutions to obtain competitive advantages. Cooperation has been identified as a significant approach to achieving sustainability, which, if used effectively, could provide a higher outcome. Cooperation can lead to an increasing use of collaborative arrangements in order to improve the construction development. Sustainable construction project delivery should be supported by cooperation amongst stakeholders with a clear understanding of the challenges of sustainability [61]. Cooperation is considered to be essential to stakeholders if they are to consider the whole lifecycle of the construction process. However, a high level of cooperation does not result from the implementation of new information technologies alone, but also requires any organizational structure and behavioral issues to be solved [62]. In sustainable construction projects, cooperative behaviors are essential elements in the success of projects. Even though different stakeholders look for different benefits, as long as a common objective exists in their relationships, cooperative behaviors inevitably occur [63].

3. Model Description and Solution

3.1. Model Description

In this study, an assumption is made that in a specific sustainable construction project, there exists two equal cooperative parties—the owner and the contractor. Both their efforts concentrate on the overall project benefits with a consideration of the sustainable construction project’s inherent nature. On this basis and without considering the time value of investments, this study comprehensively weighs up the influence of the cooperative relationship on the project benefits. Three hypotheses are proposed as follows:
Hypothesis 1: The levels of the cooperative efforts of the owner and contractor are \( e = (e_1, e_2) \), while \( e > 0 \). The cooperative behaviors between the owner and contractor can generate a synergistic effect, thus adding benefits to the sustainable construction. The added benefits can be shared by the owner and contractor. Assuming that the cooperative behavior began at time \( t \), the project outcome function at \( s \) \((t > s)\) is [64]:

\[
\pi = A_1e_1 + A_2e_2 + ke_1e_2 + \theta
\]

where \( A_1 \) and \( A_2 \) are the output coefficients of the levels of the efforts of both parties, i.e., the integrated technical level and comprehensive management ability of the owner and contractor. The output coefficient manifests the stakeholders’ capabilities to transform the input resources into project outcomes, which is associated with the levels of operational management, qualifications and competencies. The effort levels of \( e_1 \) and \( e_2 \) are difficult to observe by the other parties, while can be verified by the input resources and the degree of the achievement of the project’s objectives. \( \theta \) denotes the external random factors affecting outcome function, and is mutually independent of \( e_1 \) and \( e_2 \). To ensure the Hyers–Ulam stability of the outcome function, \( \theta \) is set to follow a normal distribution \((0, \delta^2)\). \( k \) presents the cooperative relationship between the owner and contractor. \( K = 0 \) \((0 \leq k \leq 1)\) means that the owner and contractor do not adopt cooperative behaviors, and so, one party may chase their own benefit maximization, thereby damaging the other party’s benefits. \( K = 1 \) means that the owner and contractor adopt seamless cooperative behavior, and so both parties are willing to select their behavior in accordance with the overall project benefits. This study aims to investigate the effects of a cooperative relationship \((0 < k < 1)\) on overall project benefits, namely the effects of cooperation on project outcomes. When \( 0 < k \leq 1 \), the cooperative behavior between the owner and contractor results in synergistic effects. Under this condition, when one party increases its input resources, the other party’s marginal benefit progressively increases with a given level of effort. \( ke_1e_2 \) refers to the added benefits created by synergistic effects, and \( e_1e_2 \) represents the cooperative willingness between the owner and contractor. When \( e_i = 0 \) \((i = 1, 2)\), the owner and contractor cannot generate synergistic effects, because neither of them is willing to adopt cooperative behavior. Therefore, \( k \) can also be named as the coefficient of synergistic effects, and can be explained as the ability to create added benefits under a given condition of input resources.

Hypothesis 2: The effort cost function \( c(e_i) \) of the owner and contractor is a strictly monotonic increasing function of \( e_i \) \((i = 1, 2)\) [37,65]:

\[
c(e_i) = \frac{1}{2}\eta_i e_i^2 (i = 1, 2)
\]

where \( \eta \) \((\eta > 0)\) is the effort cost coefficient. To ensure the existence of an arithmetic solution, the relationship between \( \eta_1, \eta_2 \) and \( k^2 \) should meet the requirement of \( \eta_1, \eta_2 > k^2 \). Since the synergistic effect makes the project outcome increase in terms of scale, an arithmetic solution only exists when the cost function presents a more strictly convex space than does the outcome function. Otherwise, the input resources of the owner and contractor tend towards infinitude, which is inconsistent with sustainable construction practice. Meanwhile, this study sets the output coefficients of effort levels as \( A_1 = A_2 = 1 \) in the outcome function; thus, the effort cost coefficient manifests as the output efficiency \((\frac{A_i}{\eta_i})\) of the owner and contractor [66]. In other words, the greater the effort cost coefficient, the lower is the output efficiency.

Hypothesis 3: The owner and contractor are risk neutral and aim to maximize the overall project benefits. The owner’s payment to the contractor follows a linear function. The benefit allocation coefficients for the owner and contractor are \( \beta_1 \) and \( \beta_2 \) \((0 \leq \beta_1, \beta_2 \leq 1)\), respectively, and meet the requirement of \( \beta_1 + \beta_2 = 1 \). The constant payments to the owner and contractor are \( \omega_1 \) and \( \omega_2 \), respectively, and meet the requirement of \( \omega_1 + \omega_2 = 0 \).

Based on the above hypotheses, the expected benefit functions of the owner and contractor, as well as the project net benefit functions, can be calculated as follows:

\[
\begin{align*}
U_i &= \beta_1 (e_1 + e_2 + ke_1e_2) + \omega_1 - \frac{1}{2}\eta_1 e_1^2 (i = 1, 2) \\
E\pi(e_1, e_2) &= e_1 + e_2 + ke_1e_2 - \frac{1}{2}\eta_1 e_1^2 - \frac{1}{2}\eta_2 e_2^2
\end{align*}
\]
3.2. Model Solution

3.2.1. Solution under Non-Moral Hazard

Under a non-moral hazard situation, the owner and contractor jointly maximize the overall project benefits. This is equivalent to solving the following linear optimization problem:

$$\max_{e_1, e_2} E\pi(e_1, e_2) = e_1 + e_2 + ke_1e_2 - \frac{1}{2}\eta_1e_1^2 - \frac{1}{2}\eta_2e_2^2 \quad (4)$$

To solve Equation (2), the first partial derivatives of $e_1$ and $e_2$ were calculated respectively, then set the values to zero. Thus, the researchers obtain:

$$\begin{cases} \frac{\partial E\pi}{\partial e_1} = 1 + ke_2 - \eta_1e_1 = 0 \\ \frac{\partial E\pi}{\partial e_2} = 1 + ke_1 - \eta_2e_2 = 0 \end{cases} \quad (5)$$

To solve the above simultaneous equation about $e_1$ and $e_2$, the researchers obtain:

$$\begin{cases} e_1 = \frac{\eta_2 + k}{\eta_1\eta_2 - k^2} \\ e_2 = \frac{\eta_1 + k}{\eta_1\eta_2 - k^2} \end{cases} \quad (6)$$

The researchers calculate the second partial derivatives of $e_1$ and $e_2$ to obtain:

$$\frac{\partial^2 E\pi}{\partial e_1\partial e_1} = -\eta_1, \quad \frac{\partial^2 E\pi}{\partial e_1\partial e_2} = k, \quad \frac{\partial^2 E\pi}{\partial e_2\partial e_2} = -\eta_2. \quad (7)$$

According to the hypothesis $\eta_1\eta_2 > k^2$, the researchers can conclude that the optimal effort levels under a non-moral hazard situation for the owner and contractor are:

$$\begin{cases} e_{1*} = \frac{\eta_2 + k}{\eta_1\eta_2 - k^2} \\ e_{2*} = \frac{\eta_1 + k}{\eta_1\eta_2 - k^2} \end{cases} \quad (8)$$

Substituting Equation (3) into Equation (2), the optimal net project benefits under a non-moral hazard situation can be obtained:

$$E\pi(e_{1*}, e_{2*}) = \frac{\eta_1 + \eta_2 + 2k}{2(\eta_1\eta_2 - k^2)} \quad (9)$$

3.2.2. Solution under Moral Hazard

Contracts in construction projects are inevitably incomplete mainly because arrangements are complex within dynamic project environments and high levels of uncertainty [67–69]. Contractual relationships are mainly based on the assumption of confrontational situations, and the level of trust is reflected in the contract documents [70]. The binding force of a contract cannot guarantee the achievement of the Pareto optimality of project benefits under the situation of a moral hazard. In the process of cooperation, both the owner and contractor are rational agents; therefore, they will tend to maximize and prioritize their own benefits. Therefore, this study explores the sub-optimal arrangements of construction contracts in sustainable construction projects. When either party has achieved their own maximum benefits, the owner and contractor are likely to pursue the overall project benefits. This is equivalent to solving the following linear optimization problem under certain constraint conditions:

$$\max_{e_1, e_2, k} E\pi(e_1, e_2) = e_1 + e_2 + ke_1e_2 - \frac{1}{2}\eta_1e_1^2 - \frac{1}{2}\eta_2e_2^2$$

s.t. $e_i \in \arg\max U_i \quad (10)$
The first-order condition of Equation (5) is:

\[
\begin{align*}
\frac{\partial U_1}{\partial x} &= \beta_1 (1 + ke_2) - \eta_1 e_1 = 0 \\
\frac{\partial U_2}{\partial e_2} &= \beta_2 (1 + ke_1) - \eta_2 e_2 = 0
\end{align*}
\]  

(11)

To solve the above simultaneous equation about \(e_1\) and \(e_2\), the researchers obtain:

\[
\begin{align*}
e_1 &= \frac{\eta_2 \beta_2 + k \beta_2^2 \beta_1}{\eta_1 \beta_2 - k \beta_2^2 \beta_1} \\
e_2 &= \frac{\eta_1 \beta_1 + k \beta_1^2 \beta_2}{\eta_1 \beta_2 - k \beta_2^2 \beta_1}
\end{align*}
\]

(12)

According to Hypothesis 3, the researchers can substitute \(\beta_1 + \beta_2 = 1\) into Equation (6) and obtain:

\[
\frac{\eta_1 e_1}{1 + ke_2} + \frac{\eta_2 e_2}{1 + ke_1} = 1
\]

(13)

Moving the denominator to the right side of Equation (8), the researchers obtain:

\[
(1 + ke_1)(1 + ke_2) = \eta_1 e_1 (1 + ke_1) + \eta_2 e_2 (1 + ke_2)
\]

(14)

To solve Equation (5), a Lagrange function was constructed under the constraint condition of Equation (8):

\[
F(e_1, e_2, \xi) = E \pi(e_1, e_2) - \xi \left( \frac{\eta_1 e_1}{1 + ke_2} + \frac{\eta_2 e_2}{1 + ke_1} - 1 \right)
\]

(15)

To solve the above equation about \(e_1\) and \(e_2\), the researchers order \(F' e_1 = 0\) and \(F' e_2 = 0\), then make a simplification:

\[
\frac{\eta_1 (1 + ke_1)^2 (1 + ke_2) - \eta_2 ke_2 (1 + ke_2)^2}{\eta_2 (1 + ke_1) (1 + ke_2)^2 - \eta_1 ke_1 (1 + ke_1)^2} = \frac{1 + ke_2 - \eta_1 e_1}{1 + ke_1 - \eta_2 e_2}
\]

(16)

Substituting Equation (9) into the left side of Equation (10), the researchers obtain:

\[
\frac{\eta_1 (1 + ke_1)}{\eta_2 (1 + ke_2)} = \frac{1 + ke_2 - \eta_1 e_1}{1 + ke_1 - \eta_2 e_2}
\]

(17)

Substituting Equation (7) and \(\beta_2 = 1 - \beta_1\) into Equation (11) transforms the latter into a simple cubic equation about \(\beta_1\) as follows:

\[
(\eta_1 + \eta_2)^2 \beta_1^3 - 3\eta_1 k^2 \beta_1^2 + (\eta_1 \eta_2^2 + \eta_1^2 \eta_2 + 4\eta_1 \eta_2 k + 3\eta_1 k^2) \beta_1 - \eta_1 (\eta_2 + k)^2 = 0
\]

(18)

To solve Equation (12), the different values of \(k\) are discussed.

(I) When \(k = 0\), Equation (12) can be simplified to:

\[
(\eta_1 + \eta_2) \beta_1 - \eta_2 = 0
\]

(19)

Thus, the researchers obtain:

\[
\begin{align*}
\beta_1 &= \frac{\eta_2}{\eta_1 + \eta_2} \\
\beta_2 &= \frac{\eta_1}{\eta_1 + \eta_2}
\end{align*}
\]

(20)

(II) When \(k > 0\), the Cardano formula [71] is applied to solve Equation (12). First, a transformation was made:

\[
\beta_1 = x - \frac{3\eta_1 k^2}{3(\eta_1 + \eta_2) k^2} = x + \frac{\eta_1}{\eta_1 + \eta_2}
\]

(21)
Thus, Equation (12) can be converted to a simplified form:

$$x^3 + px + q = 0$$  \hspace{1cm} (22)

where

$$p = \frac{\eta_1 \eta_2 (\eta_1 + \eta_2 + k)(\eta_1 + \eta_2 + 3k)}{(\eta_1 + \eta_2)^2 k^2} > 0$$  \hspace{1cm} (23)

and

$$q = \frac{\eta_1 \eta_2 (\eta_1 - \eta_2)(\eta_1 + \eta_2 + k)^2}{(\eta_1 + \eta_2)^2 k^2}$$  \hspace{1cm} (24)

The discriminant of Equation (14), is obviously greater than zero. This means that Equation (14) has one real root and two complex roots. According to the Cardano formula, the real root of Equation (14) can be written as follows:

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$  \hspace{1cm} (25)

$$x(\eta_1, \eta_2, k) = \frac{3}{2} \sqrt[3]{\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \frac{3}{2} \sqrt[3]{\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$  \hspace{1cm} (26)

Thus, the real root of Equation (12) is $$\beta_1^*(\eta_1, \eta_2, k) = x(\eta_1, \eta_2, k) + \frac{\eta_1}{\eta_1 + \eta_2}$$. Taking $$\beta_1 + \beta_2 = 1$$ into consideration, the researchers obtain:

$$\left\{ \begin{array}{l}
\beta_1^*(\eta_1, \eta_2, k) = x(\eta_1, \eta_2, k) + \frac{\eta_1}{\eta_1 + \eta_2} \\
\beta_2^*(\eta_1, \eta_2, k) = -x(\eta_1, \eta_2, k) + \frac{\eta_2}{\eta_1 + \eta_2} 
\end{array} \right.$$  \hspace{1cm} (27)

Substituting Equation (16) into Equation (7), the optimal levels of effort under moral hazard for the owner and contractor were obtained:

$$\left\{ \begin{array}{l}
e_1^* = \eta_2 \beta_1^* + k \beta_1^* \beta_2^* \\
e_2^* = \eta_1 \beta_2^* + k \beta_1^* \beta_2^* 
\end{array} \right.$$  \hspace{1cm} (28)

4. Model Analysis and Simulations

4.1. Model Analysis

**Proposition 1:** Under a non-moral hazard situation, the cooperative relationship between the owner and contractor is positively associated with the level of effort, reflecting the fact that higher levels of cooperation lead to a stronger willingness to commit resources to sustainable construction. During the implementation of sustainable construction, enhancing a cooperative relationship could effectively affect the intensity of the input resources, presenting high levels of effort on the parts of the owner and contractor.

**Proof.** According to Equation (3), the researchers obtain:

$$\left\{ \begin{array}{l}
\frac{\partial e_1^*}{\partial k} = \frac{1}{\eta_1 \eta_2 - k^2} + \frac{2k(\eta_2 - k)}{(\eta_1 \eta_2 - k^2)^2} > 0 \\
\frac{\partial e_2^*}{\partial k} = \frac{1}{\eta_1 \eta_2 - k^2} + \frac{2k(\eta_1 - k)}{(\eta_1 \eta_2 - k^2)^2} > 0 
\end{array} \right.$$  \hspace{1cm} (29)

It is obvious that the functions of $$e_1^{**}$$ and $$e_2^{**}$$ are monotonic increasing functions about $$k$$. The greater the value of $$k$$, the greater are the values of $$e_1^{**}$$ and $$e_2^{**}$$. 


Proposition 2: When both parties do not maintain a cooperative relationship \((k = 0)\) in sustainable construction projects, their benefit allocation coefficients are positively correlated with their own output efficiency and negatively with the other party’s output efficiency. The costs of the higher levels of effort lead to lower levels of benefit allocation. In this circumstance, an optimal approach to increasing the benefit allocation is to enhance the output efficiency, such as improving the core capability and ameliorating the management ability.

Proof. According to Equation (13), the researchers obtain:

\[
\begin{align*}
\frac{\partial \beta_1}{\partial \eta_1} &= -\frac{\eta_2}{(\eta_1 + \eta_2)^2} < 0 \\
\frac{\partial \beta_1}{\partial \eta_2} &= \frac{1}{(\eta_1 + \eta_2)^2} > 0 \\
\frac{\partial \beta_2}{\partial \eta_1} &= -\frac{\eta_2}{(\eta_1 + \eta_2)^2} < 0 \\
\frac{\partial \beta_2}{\partial \eta_2} &= \frac{1}{(\eta_1 + \eta_2)^2}
\end{align*}
\]  

It is obvious that the function of \(\beta_1\) is a monotonic increasing function about \(\eta_2\), and a monotonic decreasing function about \(\eta_1\). The function of \(\beta_2\) is a monotonic increasing function about \(\eta_1\), and a monotonic decreasing function about \(\eta_2\).

Corollary 1: According to Equation (15), when \(k \to 0\), then \(x(\eta_1, \eta_2, k) \to \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}\).

Thus, the researchers obtain \(\frac{q}{p} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}\), which reflects Equation (13) as a limit form of Equation (16).

Proof. According to Equation (15), evaluated its limit value when \(k \to 0\), and the researchers obtain:

\[
\begin{align*}
\lim_{k \to 0} x(\eta_1, \eta_2, k) &= \lim_{k \to 0} \sqrt{\frac{q}{p}} \left[ -\frac{1}{2} + \frac{\sqrt{\left(\frac{q}{p}\right)^2 + \left(\frac{q}{p}\right)^3}}{2} + \frac{1}{2} - \sqrt{\frac{q}{p}} \right] \\
&= \lim_{k \to 0} \frac{\sqrt{\frac{q}{p}}}{\sqrt{2}} \left[ -\frac{1}{2} + \frac{\sqrt{\left(\frac{q}{p}\right)^2 + \left(\frac{q}{p}\right)^3}}{2} + \frac{1}{2} - \sqrt{\frac{q}{p}} \right] \\
&= \frac{\sqrt{\frac{q}{p}}}{\sqrt{2}} \lim_{k \to 0} \sqrt{1 - \frac{\sqrt{\frac{q}{p}}}{\sqrt{2}} + \frac{1}{\sqrt{\frac{q}{p}}}} + \sqrt{1 + \frac{\sqrt{\frac{q}{p}}}{\sqrt{2}} - \frac{1}{\sqrt{\frac{q}{p}}}} \\
&= -\frac{q}{p}
\end{align*}
\]  

In the roots of the last equation in the Taylor series of the former equation, the high order terms above order three are abandoned. Substituting the equation of \(p\) and \(q\), Corollary 1 can be proven.

Corollary 2: Whether or not there is a cooperative relationship between the owner and contractor, if they possess the same cost effort coefficients (\(\eta_1 = \eta_2\)), then they will obtain the same benefit allocation coefficients (\(\beta_1^* = \beta_2^* = 0.5\)). If the effort cost of the owner approaches positive infinity (\(\eta_1 \to +\infty\)), the benefit allocation coefficient approaches zero (\(\beta_1^* \to 0\)). If the effort cost of the contractor approaches positive infinity (\(\eta_2 \to +\infty\)), the owner is likely to have all the project benefits (\(\beta_1^* \to 1\)). In other words, if the owner and contractor possess the same output efficiency, they will obtain the share ratio. If one party has no efficiency, the other party will obtain all the benefits.

Proof. When \(\eta_1 = \eta_2\) , if \(k = 0\), according to Equation (13) there has \(\beta_1^* = \beta_2^* = 0.5\). If \(k > 0, q = 0\) can be obtained. Substituting \(\eta_1 = \eta_2\) and \(q = 0\) into Equation (15), \(x = 0\) can be obtained. Thus, there still has \(\beta_1^* = \beta_2^* = 0.5\). When \(\eta_1 \to +\infty\), the following equation can be obtained:

\[
\lim_{\eta_1 \to +\infty} \frac{q}{p} = \lim_{\eta_1 \to +\infty} \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = 1
\]  

Similar to the proof of Corollary 1, \(\beta_1^* \to 0\) can be obtained according to Equation (16). The proof of \(\eta_2 \to +\infty\) can be aligned with the former method.
Proposition 3: According to Equation (17), the constant payments to the owner and contractor \( (\omega^*) \) do not produce incentive effects. This is consistent with the reference [52,66], meaning that there are no distinctions amongst the traditional construction projects. In sustainable construction, the constant payments are jointly negotiated by the owner and contractor, and documented in a specific contract. Therefore, in the majority of construction contracts, emphasis should be placed on the incentive terms, and constant payment should be arranged in accordance with a bargaining agreement.

Proof. According to Equation (5), the first-order conditions of Proposition 4

\begin{align*}
\partial \pi(e_1, k_1) \partial e_1 &= \frac{\partial U_1(e_1, k_1)}{\partial e_1} + \frac{\partial U_2(e_1, k_1)}{\partial e_1} > 0 \\
\partial \pi(e_2, k_2) \partial e_2 &= \frac{\partial U_1(e_1, k_1)}{\partial e_2} + \frac{\partial U_2(e_1, k_1)}{\partial e_2} > 0 \quad (33)
\end{align*}

Thus, the following is obtained:

\begin{align*}
\frac{\partial \pi(e_1, e_2, k) \partial k}{\partial k} &= \frac{\partial U_1(e_1, k_1)}{\partial e_1} + \frac{\partial U_2(e_1, k_1)}{\partial e_2} > 0 \quad (34)
\end{align*}

Proposition 4: Under a moral hazard situation, high levels of cooperative relationship between the owner and contractor lead to high levels of effort and project benefits. The synergistic effects generated by cooperation will stimulate the owner and contractor to increase their resources input. As long as the effort levels do not achieve the optimal levels in a non-moral hazard situation, the project benefit created by the input of the resources of the owner and contractor can be greater than the corresponding marginal cost. Under this circumstance, enhancing the cooperative relationship between the owner and contractor can improve the net project benefits.

Proof. According to Equation (3), let \( e_1^* - e_1^* \), then the following is obtained:

\begin{align*}
(\eta_2 + k) - (\eta_2 \beta_1 + k \beta_1 \beta_2) &= \eta_2(1 - \beta_1^*) + k(1 - \beta_1^* \beta_2^*) > 0 \\
(\eta_1 \eta_2 - k^2) - (\eta_1 \eta_2 - k^2 \beta_1^* \beta_2^*) &= -k^2(1 - \beta_1^* \beta_2^*) < 0 \quad (35)
\end{align*}

Obviously, the researchers can conclude that \( e_1^* > e_1^* \) and \( e_2^* > e_2^* \), which can be proven in a similar way.

Corollary 3: Integrating Propositions 4 and 5, the expected benefits of the owner and contractor, as well as the net project benefits under a non-moral hazard situation, are higher than the counterparts under a moral hazard situation. Therefore, the owner and contractor should reduce their moral hazard behaviors in order to enhance a cooperative relationship, which is beneficial to improving the project benefits.
4.2. Model Simulations

4.2.1. The Effects of Cooperative Relationship (k) on Benefit Allocation Coefficient (β₁)

According to Equations (12) and (16), the researchers set \( \eta_2 = 2 \) and \( \omega_1 = \omega_2 = 0 \), then the researchers simulate the relationships between the benefits allocation coefficient (\( \beta_1 \)) of the owner and his/her effort cost coefficient (\( \eta_1 \)) under different \( k \) values. The results are shown in Figure 2. The researchers set \( \eta_1 = 1 \) and \( \omega_1 = \omega_2 = 0 \), then simulate the relationships between the benefits allocation coefficient (\( \beta_1 \)) of the owner and the contractor’s effort cost coefficient (\( \eta_2 \)) under different \( k \) values. The results are shown in Figure 3. The researchers set \( \eta_1 = 1 \), \( \eta_2 = 2 \) and \( \omega_1 = \omega_2 = 0 \), then simulate the relationships between the benefits allocation coefficient (\( \beta_1 \)) of the owner and the cooperative relationship (\( k \)). The result is shown in Figure 4.

![Figure 2](image1.png)

**Figure 2.** The effects of \( \eta_1 \) on \( \beta_1 \) under different values of \( k \).

![Figure 3](image2.png)

**Figure 3.** The effects of \( \eta_2 \) on \( \beta_1 \) under different values of \( k \).
When the condition $\eta_1 \eta_2 > k^2$ is satisfied, the researchers can conclude from Figures 2 and 3 that $\beta_1$ is a decreasing function of $\eta_1$, as well as an increasing function of $\eta_2$. It shows that the lower the effort cost coefficient of the owner, the higher is the output efficiency, and the greater is the benefit allocation coefficient ($\beta_1$). The greater the effort cost coefficient of the contractor, the lower is the output efficiency, and the greater is the benefit allocation coefficient ($\beta_1$). Therefore, Proposition 2 was verified, which means that the owner and contractor could control their share ratio of project benefits through adjusting their effort cost and output efficiency. The researchers investigated the effects of $\eta_1$ and $\eta_2$ under different $k$ values, and found that the variation tendency of the function ($\beta_1$) curve of $k = 0$ is consistent with $k \neq 0$. Thus, Corollaries 1 and 2 were verified, which means that the ideal situation for the cooperative relationship is a non-moral hazard situation between the owner and contractor. Meanwhile, the function curves under different $k$ values intersect at one point, as shown in Figures 2 and 3. This means that the benefit allocation coefficient ($\beta_1$) is also affected by the ratio of the effort cost coefficients of the owner and contractor ($\eta_1/\eta_2$). When the ratio of the effort cost coefficients is greater than a specific value, the increasing or decreasing trends of the benefit allocation coefficient will slow down. The researchers explored the effects of $\eta_1/\eta_2$ on the benefit allocation coefficient ($\beta_1$) under different $\eta_1$ and $\eta_2$ values, and find that $\eta_1/\eta_2 = 0.5$ is the threshold value. Then, the researchers simulated the relationship between the cooperative relationship ($k$) and the benefit allocation coefficient ($\beta_1$) when $\eta_1/\eta_2 = 0.5$. The result shows that the benefit allocation coefficient is an increasing function of the cooperative relationship. The higher the level of cooperation, the greater is the benefit allocation coefficient. This means that the owner is willing to promote cooperation, because greater benefits can be gained from the synergistic effects of cooperation.

4.2.2. The Effects of Cooperative Relationship (k) on Effort Levels (e)

According to Equations (3) and (17), the researchers set $\eta_1 = 1, \eta_2 = 2$ and $\omega_1 = \omega_2 = 0$, then simulate the relationships between the cooperative relationship ($k$) and effort levels ($e$) of the owner and contractor. The results are shown in Figures 5 and 6. The non-moral and moral hazard situations are distinguished in the figures.
From Figures 5 and 6, the researchers can conclude that the effort levels under the non-moral hazard situation are higher than the counterparts under moral hazard. As for the owner and contractor, the variation trends are consistent under different situations. Whether there exists a moral hazard or not, enhancing the cooperative relationship can improve the effort levels of the owner and contractor. A higher level of cooperative relationship between the owner and contractor can lead to higher levels of synergistic effects, which can add more project benefits to sustainable construction. The synergistic effects stimulate the owner and contractor to increase their input resources. It is worth noting that the moral hazard behaviors still exist with the increase in cooperation. Though enhancing the cooperative relationship can improve the effort levels, the velocity of the increase is slower than that of the counterparts under a non-moral hazard situation. In other words, enhancing the cooperative relationship and the synergistic effects cannot eliminate moral hazard risks to an extent, but increases the probability of the occurrence of such a risk. This could explain why the soft factors are increasingly important in sustainable construction projects. In construction practice, except for the iron indicators, such as...
environmental, social and economic objectives, the soft indicators, such as cooperation, communication and trust, are also critical to achieving sustainability.

4.2.3. The Effects of Cooperative Relationship (k) on Project Benefit (E\(\pi\))

The researchers set \(\eta_1 = 1\), \(\eta_2 = 2\) and \(\omega_1 = \omega_2 = 0\), then substitute Equations (3), (16) and (17) into the formulae of \(U_i\) and \(E\pi\) accordingly. Thus, the researchers can simulate the relationship between the owner’s expected benefit (\(U_1\)) and the cooperative relationship (\(k\)), the relationship between the contractor’s expected benefit (\(U_2\)) and the cooperative relationship (\(k\)), and the relationship between the net project benefit (\(E\pi\)) and the cooperative relationship (\(k\)), respectively. The results are shown in Figures 7–9. The different situations of non-moral hazard and moral hazard are distinguished in the figures.

![Figure 7](image1.png)  
**Figure 7.** The effects of \(k\) on \(U_1\).

![Figure 8](image2.png)  
**Figure 8.** The effects of \(k\) on \(U_2\).
4.2.3. The Effects of Cooperative Relationship (k) on Project Benefit (Eπ)

The researchers set \( \omega_1 = 1 \), \( \omega_2 = 2 \) and \( \omega_1 = \omega_2 = 0 \), then substitute Equations (3), (16) and (17) into the formulae of \( U_i \) and \( E\pi \) accordingly. Thus, the researchers can simulate the relationship between the owner's expected benefit \( (U_1) \) and the cooperative relationship \( (k) \), the relationship between the contractor's expected benefit \( (U_2) \) and the cooperative relationship \( (k) \), and the relationship between the net project benefit \( (E\pi) \) and the cooperative relationship \( (k) \), respectively. The results are shown in Figures 7–9. The different situations of non-moral hazard and moral hazard are distinguished in the figures.

From Figures 7–9, the researchers can conclude that the expected benefits of the owner and contractor under non-moral hazard are higher than the counterparts under moral hazard. Accordingly, the net project benefit of sustainable construction is higher than the counterpart under moral hazard. Whether there exists a moral hazard or not, enhancing the cooperative relationship can improve the expected benefits of the owner and contractor, thus improving the net project benefits. This means that the synergistic effect created by cooperative behaviors can give additional benefits to the sustainable construction. Since the owner and contractor can share the added benefits, their expected benefits are accordingly improved. It is worth noting that due to the existence of a moral hazard, the benefit's gap between non-moral hazard and moral hazard will become bigger with the reinforcement of the cooperative relationship. This means that even though cooperative behaviors can improve the expected benefits of the owner and contractor, it cannot eliminate their moral hazard behaviors. With the increase in the synergistic effect, the owner and contractor are more willing to increase input resources with the aim of improving their own benefits and the overall project benefits. With the increase in the overall project benefits, the owner and contractor are more likely to adopt moral hazard behaviors. Under this circumstance, selecting moral hazard behaviors can bring more benefits. This, to some extent, provokes the owner and contractor to adopt moral hazard behaviors. When there is a moral hazard, enhancing the cooperative relationship between the owner and contractor is not absolutely positive in sustainable construction. In practice, the owner and contractor are more likely to select moral hazard behaviors at the completion stage, because it is their last chance. Therefore, the project contracts should include these related terms to prevent moral hazard behaviors.

4.2.4. The Numerical Example of the Incentive Model

According to the incentive model and the related values of parameters, the researchers calculated the effort levels and expected benefits of the owner and contractor, the benefit allocation coefficients, and the net project benefits with different \( k \) values under both non-moral and moral hazard situations. The results are shown in Table 1.
Table 1. The incentive model results under different $k$ values.

<table>
<thead>
<tr>
<th></th>
<th>$K = 0$</th>
<th>$K = 0.1$</th>
<th>$K = 0.2$</th>
<th>$K = 0.3$</th>
<th>$K = 0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(e_1^<em>, e_2^</em>)$</td>
<td>(0.333, 0.333)</td>
<td>(0.189, 0.659)</td>
<td>(0.210, 0.656)</td>
<td>(0.231, 0.657)</td>
<td>(0.252, 0.663)</td>
</tr>
<tr>
<td>$(e_1^{<strong>}, e_2^{</strong>})$</td>
<td>(0.500, 1.000)</td>
<td>(0.553, 1.055)</td>
<td>(0.612, 1.122)</td>
<td>(0.681, 1.204)</td>
<td>(0.761, 1.304)</td>
</tr>
<tr>
<td>$(\beta_1^<em>, \beta_2^</em>)$</td>
<td>(0.333, 0.667)</td>
<td>(0.354, 0.646)</td>
<td>(0.371, 0.629)</td>
<td>(0.385, 0.615)</td>
<td>(0.398, 0.602)</td>
</tr>
<tr>
<td>$(\beta_1^{<strong>}, \beta_2^{</strong>})$</td>
<td>(0.167, 0.833)</td>
<td>(0.183, 0.817)</td>
<td>(0.201, 0.799)</td>
<td>(0.217, 0.783)</td>
<td>(0.235, 0.765)</td>
</tr>
<tr>
<td>$(U_1^<em>, U_2^</em>)$</td>
<td>(0.333, 0.167)</td>
<td>(0.268, 0.339)</td>
<td>(0.287, 0.347)</td>
<td>(0.306, 0.358)</td>
<td>(0.327, 0.371)</td>
</tr>
<tr>
<td>$(U_1^{<strong>}, U_2^{</strong>})$</td>
<td>(0.125, 0.625)</td>
<td>(0.147, 0.657)</td>
<td>(0.174, 0.693)</td>
<td>(0.204, 0.738)</td>
<td>(0.243, 0.790)</td>
</tr>
<tr>
<td>$(E\pi^*, E\pi^{**})$</td>
<td>(0.500, 0.750)</td>
<td>(0.607, 0.804)</td>
<td>(0.634, 0.867)</td>
<td>(0.664, 0.942)</td>
<td>(0.698, 1.033)</td>
</tr>
</tbody>
</table>

5. Conclusions and Implications

Considering the dynamic cooperative relationship between the owner and contractor, this study proposed that a cooperative relationship could create synergistic effects on sustainable construction projects. Based on these theoretical hypotheses, an incentive model using game theory in sustainable construction was formulated and analyzed under non-moral hazard and moral hazard situations. Then, a simulation and numerical example were proposed to verify the model. The results show that high levels of cooperation could lead to an increase in the input resources of one party, and to increases in the marginal utility of the other party at the same time. Hence, the cooperative relationship is positively related to the levels of the efforts of the owner and contractor. When there are synergistic effects, the optimal benefit allocation coefficient is positively affected by its own output efficiency, and negatively affected by the other party’s output efficiency. Enhancing the cooperative relationship can improve the expected benefits of the owner and contractor, and thus, accordingly improve the overall project benefits. Remarkably, a higher level of cooperation cannot eliminate the negative effects of a moral hazard. Conversely, with the increase in synergistic effects, the levels of the efforts of the owner and contractor under a moral hazard situation increasingly deviate from the counterparts under a non-moral hazard situation. Accordingly, the expected benefits of the owner and contractor, as well as the net project benefits, are consistent with this condition. Under this circumstance, the owner and contractor are more likely to adopt moral hazard behaviors to maximize their own benefits, even though this may damage the other party’s and overall project benefits.

In sustainable construction, the core abilities of owners and contractors are mutually complementary. Both the owner and contractor are willing to cooperate and achieve the objective of sustainability. Both parties should tap into their respective advantages to better achieve synergistic effects. This study provides a game-based incentive model to explore how a cooperative relationship affects sustainable construction, and has validated the model by data simulation and a numerical example. The conclusions of this study highlight the implications for management as follows: (I) To better improve the overall project benefits, the owner should take the output efficiency of the contractor into consideration. The gap between the owner’s output efficiency and the contractor’s should not be too large; (II) During the implementation of a sustainable construction project, the owner and contractor should strengthen their resource integration capability and construct an effective communication mechanism, thereby reinforcing the synergistic effect created by cooperation; (III) The owner should strive to construct a fair and reasonable benefit allocation mechanism that can optimize the levels of the input resources of both parties and reduce the negative effects of a moral hazard; (IV) The owner and contractor should construct a mutual trust mechanism to reduce information asymmetry, thereby increasing their cooperation; and (V) The owner should strive to build a favorable environment for cooperation, and be adept at using bonus-penalty measures, thereby reducing the negative effects of moral hazard behaviors.

As few studies to date have emphasized the effects of a cooperative relationship on sustainable construction, this study constructed a game-based incentive model to bridge this gap. This study contributes significant theoretical and practical insights into managing cooperation amongst stakeholders in sustainable construction and enhancing overall project benefits. However, the proposed
model and conclusions must be considered in light of the study’s limitations. First, a game-based incentive model was constructed based on a series of theoretical hypotheses whilst some other factors that may affect synergistic effects were ignored. Second, the game-based incentive model only considers the cooperative relationship between the owner and contractor, but in practice, the relationship amongst the stakeholders is more complex. Third, this study does not take into account how cooperative relationships achieve synergistic effects. These points are also directions for future studies. In any case, this study contributes to the existing knowledge by proposing and validating a game-based incentive model that can be used by project managers when selecting partners for sustainable construction development. When selecting partners, project managers should consider the core abilities, knowledge structure, risk preferences and cooperative capabilities of the potential partners. Practical implications can be drawn based on this model, which provides a clear understanding of the effects of cooperative relationships on the benefits in sustainable construction projects.

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References

