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Improved Intuitionistic Fuzzy Entropy and Its Application in the Evaluation of Regional Collaborative Innovation Capability

Xumei Yuan * and Cuicui Zheng

School of Economics and Management, Yanshan University, Qinhuangdao 066004, China; zhengcc0531@foxmail.com
* Correspondence: yxm@ysu.edu.cn

Abstract: Intuitionistic fuzzy entropy is an important concept to describe the uncertainty of intuitionistic fuzzy sets (IFSs). To fully measure the fuzziness of IFSs, this paper comprehensively considers the deviation between membership and non-membership and the influence of hesitation, constructs the general expression of intuitionistic fuzzy entropy based on special functions, and proves some of its major properties. Then, it is verified that some existing intuitionistic fuzzy entropies can be constructed by specific functions. Finally, based on a specific parametric intuitionistic fuzzy entropy, this paper applies it to evaluate the regional collaborative innovation capability, to verify the feasibility and practicability of the entropy. In addition, the effectiveness and practicability of this entropy in decision making are further illustrated by comparing it with other entropy measures.

Keywords: intuitionistic fuzzy sets; intuitionistic fuzzy entropy; regional collaborative innovation capability

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1. Introduction

Since Zadeh put forward the concept of a fuzzy set in 1965, the fuzzy set theory has been widely applied to neural networks, medical diagnosis, and electronic communication, and has made great progress. However, due to the complexity of objective things and the limitation of subjective cognition, there is often a lack of information. Because of this situation, the American scholar Atanassov [1] established an intuitionistic fuzzy set (IFS), which contains the following three aspects of information: membership degree, non-membership degree, and hesitation degree. The essence of intuitionistic fuzzy sets (IFSs) is to consider two aspects of fuzziness, so IFSs play an important role in multi-attribute decision making. The research on IFSs has attracted extensive attention from scholars. IFSs have been widely used in information measuring, supplier selection, decision making, pattern recognition, and many other fields [2–11]. De Luca and Termini combined the concept of entropy measures with the fuzzy set theory for the first time [12]. The main function of the entropy measure is to describe the extent of uncertainty. Recently, many researchers have combined entropy with different fuzzy sets [13–20]. The concept of intuitionistic fuzzy entropy was first proposed by Burillo and Bustince [15], which is mainly used to describe the uncertainty and fuzziness of IFSs. Intuitionistic fuzzy entropy provides a new way for us to measure the extent of uncertain information. In 2001, Szmidt et al. [16] redefined the distance of IFSs to emphasize the importance of hesitation in practical problems. In addition, on this basis, they also constructed the similarity measure for IFSs. Since then, Szmidt's research team [17–20] has described the concept of intuitionistic fuzzy sets more intuitively through geometric figures, and has defined various forms of entropy measures for IFSs. Meanwhile, some researchers [21–23] defined the entropy measure in a trigonometric function form, or in an exponential form, with a corresponding aggregation operator for IFSs, and verified the effectiveness and practicability of the above information measure through specific practical problems. However, the entropy measure proposed above often appears counterintuitive in particular cases. Therefore, it is necessary to further study intuitionistic fuzzy entropy to overcome the emergence of counterintuitive situations.

The multi-attribute decision-making method is a key problem in the research of the decision-making theory. The technique for order preference by similarity to an ideal solution (TOPSIS) decision-making method has been widely used in practical problems [24–29]. TOPSIS refers to using the ranking method to approach the ideal solution. If a scheme is closer to the ideal solution and further away from the negative ideal solution, it is the optimal scheme. Hu et al. [24] introduced the TOPSIS method into the research field of intuitionistic fuzzy multi-attribute decision making, and proposed an interval intuitionistic fuzzy TOPSIS multi-attribute decision-making method. Wu et al. [25] introduced the entropy weight method into the interval intuitionistic fuzzy environment, and obtained a new interval intuitionistic fuzzy TOPSIS decision-making method, combined with the entropy weight method. Zhou et al. [26] constructed a new TOPSIS decision-making method based on the characteristics of trapezoidal IFSs. Liu et al. [27] developed a new class of intuitionistic fuzzy entropy and proposed an improved multi-attribute decision-making method. Liu et al. [28] defined a new hesitation intuitionistic fuzzy distance measurement method and constructed the corresponding TOPSIS decision-making method. Yue et al. [29] studied the representation of knowledge measures for interval IFSs and extended the TOPSIS decision-making method. Since the TOPSIS decision-making method involves determining the ideal positive and negative solutions and distance measures, it is particularly important to set the evaluation criteria for multi-attribute index information when selecting alternatives.

In recent years, the Chinese government has attached great importance to innovation-driven development. It has formulated and promulgated a number of policy documents that promote innovation-driven development, including the Outline of the National Strategy on Innovation-Driven Development and the Outline of China's National Plan for Medium- and Long-term Education Reform and Development, and innovation has risen to the height of national strategy. Innovation can effectively replace the traditional human capital, material resources, and is an important factor for improving regional development. Regional collaborative innovation can realize resource sharing and risk sharing, improve the cooperation and exchange among innovation subjects, and has the synergistic effect of "1 + 1 > 2". Therefore, a scientific and comprehensive evaluation of regional collaborative innovation capability is conducive to clarifying the development direction of regional innovation, increasing the regional innovation performance output and promoting high-quality economic development.

Nevertheless, we found that the existing intuitionistic fuzzy entropy distance measurement methods have some defects, such as a counterintuitive phenomenon in some special cases, which are contrary to reality. Therefore, the above analysis provides a sufficient basis for further study on the information measure of intuitionistic fuzzy entropy. Based on the intuitionistic fuzzy entropy in the existing papers [16,21–23], this study comprehensively considers the deviation between membership and non-membership, and the influence of hesitation. Contrary to the existing research, this paper not only provides the calculation formula for entropy, but also offers a new approach for constructing entropy. Under the guidance of this idea, this paper constructs the general expression for an intuitionistic fuzzy entropy class, based on special functions, and proves some of its major properties. The new entropy is suitable for some special cases that cannot be distinguished. Finally, the newly constructed intuitionistic fuzzy entropy is applied to the evaluation of regional collaborative innovation capability, and the feasibility and effectiveness of the entropy are proved, which provides a new approach for decision makers to make decisions accurately.

The content of this research paper is as follows: Section 2 introduces the literature review of entropy, IFSs, and the application of these methods in decision making and evaluation. Section 3 reviews the basic concepts and existing studies of IFSs. Section 4 constructs a new intuitionistic fuzzy entropy and compares its advantages with other entropy measures. Section 5 evaluates the regional collaborative innovation capability, based on the improved intuitionistic fuzzy entropy, and verifies the effectiveness and practicability of the improved intuitionistic fuzzy entropy decision method. Finally, Section 6 provides the conclusions of this study.

2. Literature Review

The IFS theory can flexibly describe the evaluation of information through membership degree, non-membership degree, and hesitation degree. Entropy can be used as a measure of uncertainty. Therefore, Burillo and Bustince first used entropy to measure the fuzziness of IFSs, and proposed the concept of intuitionistic fuzzy entropy [15]. After that, some scholars introduced intuitionistic fuzzy entropy into many application fields. For example, Verma et al. [23] proposed an exponential intuitionistic fuzzy entropy measurement method based on the IFS theory. Wang et al. [30] applied the concepts of entropy and IFSs, and proposed an intuitionistic fuzzy entropy measurement method for supplier selection and ranking. Yin et al. [31] proposed a dynamic multi-attribute decision-making method based on an improved weight function and scoring function, by using an interval intuitionistic fuzzy geometric-weighted Heronian means operator, and applied it to partner selection for collaborative innovation. Joshi et al. [3] introduced a new metric, (δ, γ)-Norm, for intuitionistic fuzzy entropy, to solve the multi-attribute decision-making problem of supplier selection. Hashemi et al. [32] proposed a decision model that integrates the IFS theory, ELECTRE, VIKOR, and GRA, and applied it to the selection of builders. Wang et al. [33] applied intuitionistic fuzzy entropy to an intuitionistic linguistic set, and constructed a new intuitionistic fuzzy entropy to evaluate the performance of different types of vehicles. Jiang et al. [34] constructed a new intuitionistic fuzzy entropy under the condition of the same fuzziness, and analyzed its properties. Yin et al. [35] used an interval intuitionistic fuzzy hybrid geometric operator and a new score function to calculate the composite index value of each decision option, based on an improved entropy formula, to rank a company's decision options. Chen et al. [36] established an intuitionistic fuzzy linear regression model, considering the explanatory and response variables in an observational data set and the model parameters, such as intuitionistic fuzzy numbers. Fu et al. [37] proposed a group decision-making method based on intuitionistic fuzzy entropy and a VIKOR framework, and applied it to the selection and decision making of power companies. Chutia et al. [38] proposed a new IFN ranking method based on the concept of value and fuzziness at different (α, β) decision levels. Rahimi et al. [39] proposed a new intuitionistic fuzzy entropy measure method and applied it to supplier selection. Furthermore, Hashemi et al. [40] considered the risk attitude and entropy of experts in a triangular intuitionistic fuzzy environment, to deal with the inherent uncertainty and fuzziness in the process of supplier selection. Li et al. [41] established a selection model for decision makers, based on an improved interval-valued IFS method, and used gray correlation and TOPSIS methods to select collaborative innovation partners in a military–civilian scientific and technological collaborative innovation. Thao et al. [42] combined the exponential membership function with the negative non-membership function, proposed some new IFS similarity measures, and constructed a new entropy measure to evaluate the quality of software projects. Although the existing studies have examined the importance of entropy and IFSs for evaluation, there are still deficiencies in the literature on these issues. This article attempts to provide a comprehensive review of these deficiencies, based on the existing studies.

3. IFS

3.1. Preliminaries

IFSs are not only the generalization of fuzzy sets, but they also play a key role in the study of fuzzy decision problems. This section mainly introduces the related concepts of IFSs.

Definition 1. *An IFS A defined on X is as follows* [1]*:*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\},$$

when X is a nonempty set, called the universe of discourse, $\mu_A : X \to [0,1]$ *and* $\nu_A : X \to [0,1]$, $0 \le \mu_A(x) + \nu_A(x) \le 1, \forall x \in X$, μ_A *and* ν_A *are the membership and non-membership degrees of the element x in X to A, respectively.*

In addition, the hesitation degree and intuitionistic index of x in X, belonging to A, are defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ *and, obviously,* $0 \le \pi_A(x) \le 1$, respectively. *The totality of IFSs in universe X is denoted as* $IFS(X)$.

Definition 2. *Let the family of all IFSs in a universe of discourse X be* $IFS(X)$. *Let* $A, B \in IFS(X)$ *be* $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$, $B = \{(x, \mu_B(x), \nu_B(x)) | x \in X\}$, *and the operations defined on* $IFS(X)$ *are given for every* $x \in X$, *as follows* [43]*:*

(1) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle | x \in X\}$;

(2) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle | x \in X\}$;

(3) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\}$.

Definition 3. *(Extended operation of IFSs). Let A and B be two IFSs on the theoretical domain X, as follows* [15]*:*

(1) $A + B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x), v_A(x) \cdot v_B(x)\rangle | x \in X\}$;

(2) $A \cdot B = \{\langle x, \mu_A(x) \cdot \mu_B(x), v_A(x) + v_B(x) - v_A(x) \cdot v_B(x)\rangle | x \in X\}$;

(3) $k \cdot A = \left\{\left\langle x, 1 - \left[1 - \mu_A(x)\right]^k, [v_A(x)]^k \right\rangle \middle| x \in X\right\}$, *where* $k \in R$.

Definition 4. *For any* $A, B \in IFS(X)$, *a mapping* $E : IFS(X) \to [0,1]$ *is an intuitionistic fuzzy entropy on* $IFS(X)$, *if E satisfies the following axioms* [43]*:*

(1) $E(A) = 0$ *if, and only if, A is a crisp set, i.e.,* $\mu_A(x_i) = 1, \nu_A(x_i) = 0$ *or* $\mu_A(x_i) = 0, \nu_A(x_i) = 1$;

(2) $E(A) = 1$ *if, and only if,* $\mu_A(x_i) = \nu_A(x_i) = 0$;

(3) $E(A) = E(A^c)$;

(4) $E(A) \le E(B)$ *if* $\mu_A(x_i) - \nu_A(x_i) \ge \mu_B(x_i) - \nu_B(x_i)$ *and* $\mu_A(x_i) + \nu_A(x_i) \ge \mu_B(x_i) + \nu_B(x_i)$.

To define the distance between the scheme, ideal solution, and negative ideal solution, this paper will use the standard Hamming distance formula between two IFSs given by Szmidt et al. [16], namely, the following:

Let X be a finite theoretical domain, $\forall A, B \in IFS(X)$, and define the metric according to the Hamming distance, as follows:

$$D(A,B) = \frac{1}{2n} \sum_{i=1}^{n} [|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|], \quad (1)$$

This distance considers not only the membership degree and non-membership degree, but also the hesitation degree, which makes the measurement result more consistent with the actual situation. The lack of information can be expressed more effectively and completely in the decision-making process.

3.2. Intuitionistic Fuzzy Entropy Is Commonly Used

Intuitionistic fuzzy entropy is an important tool for describing the uncertainty of information, and for dealing with fuzzy information, and plays an important role in the decision theory [44]. Many researchers have conducted in-depth research on it and constructed different forms of intuitionistic fuzzy entropy.

In 2010, Ye [21] established the following formula of specific entropy by considering the membership degree and non-membership degree:

$$\begin{aligned} E_1(A) = \frac{1}{n} \sum_{i=1}^{n} \Big[& (\sin \tfrac{\mu_A(x_i) + 1 - \nu_A(x_i)}{4} \pi \\ & + \sin \tfrac{\nu_A(x_i) + 1 - \mu_A(x_i)}{4} \pi - 1) \times \tfrac{1}{\sqrt{2} - 1} \Big], \end{aligned} \quad (2)$$

$$E_2(A) = \frac{1}{n}\sum_{i=1}^{n}\left[\left(\cos\frac{\mu_A(x_i)+1-\nu_A(x_i)}{4}\pi + \cos\frac{\nu_A(x_i)+1-\mu_A(x_i)}{4}\pi - 1\right) \times \frac{1}{\sqrt{2}-1}\right], \tag{3}$$

Subsequently, Zhang [22] proposed a new intuitionistic fuzzy entropy based on the previous formula, the specific form of which is as follows:

$$E_3(A) = \frac{1}{n}\sum_{i=1}^{n}\left[\left(\cos\frac{\mu_A(x_i)-\nu_A(x_i)}{4}\pi - 1\right) \times \frac{1}{\sqrt{2}-1}\right], \tag{4}$$

In 2013, Verma et al. [23] improved the above intuitionistic fuzzy entropy and defined the following new intuitionistic fuzzy entropy:

$$E_4(A) = \frac{1}{n(\sqrt{e}-1)}\sum_{i=1}^{n}\left[\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}e^{1-\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}} + \frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}e^{1-\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}}\right) - 1\right], \tag{5}$$

In 2001, Szmidt et al. [16] concluded that the fuzziness of information in IFSs is mainly affected by two factors, according to the geometric significance of IFSs. One is the influence of hesitation on the IFS itself, and the other is the difference between the membership degree and non-membership degree. Based on these two factors, the following new intuitive fuzzy entropy was constructed:

$$E_5(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{1-|\mu_A(x_i)-\nu_A(x_i)|+\pi_A(x_i)}{1+|\mu_A(x_i)-\nu_A(x_i)|+\pi_A(x_i)}, \tag{6}$$

In 2007, Wang et al. [45] defined intuitive fuzzy entropy as follows:

$$E_6(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{\min(\mu_A(x_i),\nu_A(x_i))+\pi_A(x_i)}{max(\mu_A(x_i),\nu_A(x_i))+\pi_A(x_i)}, \tag{7}$$

where $A \in IFS(X)$, n represents the number of elements in X.

In Equations (2)–(5), the membership degree and non-membership degree are taken into account when carrying out entropy measures. However, in many complicated practical problems, people are uncertain, due to the limitations of their cognition and the complexity of objective things. Therefore, the role of hesitation in intuitionistic fuzzy entropy should not be underestimated. In addition, some counterintuitive situations still exist for the entropy measure mentioned above, that is, when the difference between the membership degree and non-membership degree is equal, it cannot effectively distinguish the fuzziness of two IFSs. Some examples are as follows:

Example 1. *Let the theoretical domain X be a single point set, and $A = \{\langle x, 0.2, 0.5\rangle\}$ and $B = \{\langle x, 0.3, 0.6\rangle\}$ be two IFSs on X. Equations (2)–(5) will be used to calculate the intuitionistic fuzzy entropy of A and B.*

$$E_1(A) = E_1(B) = E_2(A) = E_2(B) = 0.9060;$$
$$E_3(A) = E_3(B) = 0.9058;$$
$$E_4(A) = E_4(B) = 0.9138.$$

Obviously, IFS $A \neq B$, but the calculated result shows that the entropy of two IFSs is equal, which is contrary to the actual situation. Therefore, it is further explained that Equations (2)–(5) may be counterintuitive in some cases.

Further analysis of Equations (6) and (7) shows that $E_5(A) = E_6(A)$. We all know that $\forall a, b \in R$, $\max\{a,b\} = \frac{1}{2}(a+b+|a-b|)$ and $\min\{a,b\} = \frac{1}{2}(a+b-|a-b|)$. Therefore, after simplification of Equation (7), we obtain $E_5(A) = E_6(A)$. In some special cases, Equations (6) and (7) may also be counterintuitive, as illustrated below:

Example 2. *Let the theoretical domain X be a single point set, and the two IFSs on X are* $A = \{\langle x, 0.4, 0.5\rangle\}$ *and* $B = \{\langle x, 0.1, 0.25\rangle\}$. *From Equations (6) and (7), we can calculate their entropies as* $E_5(A) = E_5(B) = E_6(A) = E_6(B) = 0.8333$. *Then,* E_5 *and* E_6 *are counterintuitive.*

4. Improvement of Intuitionistic Fuzzy Entropy

In this section, a new intuitionistic fuzzy entropy class is constructed to address the deficiency of the existing intuitionistic fuzzy entropy. First, we let the universe be $X = \{x_1, x_2, \cdots, x_n\}$, any IFS on X is $A = \{\langle x, \mu_A(x_i), \nu_A(x_i)\rangle | x \in X\}$, and the difference between membership degree and non-membership degree is $g_A(x_i) = |\mu_A(x_i) - \nu_A(x_i)|$; $g_A(x_i)$ indicates the difference in information content. Next, a new intuitionistic fuzzy entropy class is constructed by combining $g_A(x_i)$ and hesitation $\pi_A(x_i)$, in order to more effectively describe the uncertainty of information.

Theorem 1. *Let A be the IFS in the universe, and mapping* $E : IFS(X) \to [0,1]$ *is the intuitionistic fuzzy entropy, which is defined as follows:*

$$E(A) = \frac{1}{n}\sum_{i=1}^{n} f(g_A(x_i), \pi_A(x_i)), \tag{8}$$

where function $f : [0,1] \times [0,1] \to [0,1]$ *is a bivariate continuous function and satisfies the following conditions:*

(1) $f(x,y)$ *is monotonically decreasing with respect to x and monotonically increasing with respect to y;*

(2) $f(0,1) = 1$;

(3) $f(1,0) = 0$.

Proof for Theorem 1. To prove that the measure given by Equation (8) is an intuitionistic fuzzy entropy, we only need to prove that it satisfies all the axioms given in Definition 4.

(1) Let A be a crisp set, i.e., for $\forall x_i \in X$, we have $\mu_A(x_i) = 1, \nu_A(x_i) = 0$ or $\mu_A(x_i) = 0, \nu_A(x_i) = 1$. It is obvious that $E(A) = 0$.

If $E(A) = 0$, i.e., $E(A) = \frac{1}{n}\sum_{i=1}^{n} f(g_A(x_i), \pi_A(x_i)) = 0$, then $\forall x_i \in X$, we have $f(g_A(x_i), \pi_A(x_i) = 0$, thus $g_A(x_i) = |\mu_A(x_i) - \nu_A(x_i)| = 1, \pi_A(x_i) = 0$, then we have $\mu_A(x_i) = 1, \nu_A(x_i) = 0$ or $\mu_A(x_i) = 0, \nu_A(x_i) = 1$. Therefore, A is a crisp set.

(2) Let $\mu_A(x_i) = \nu_A(x_i) = 0$, $\forall x_i \in X$, we have $g_A(x_i) = |\mu_A(x_i) - \nu_A(x_i)| = 0$, $\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i) = 1$, i.e., $f(g_A(x_i), \pi_A(x_i) = 1$, from Equation (8), we have $E(A) = 1$.

Now we assume that $E(A) = 1$, then for all $\forall x_i \in X$, we have $f(g_A(x_i), \pi_A(x_i) = 1$, then $g_A(x_i) = |\mu_A(x_i) - \nu_A(x_i)| = 0, \pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i) = 1$; we can obtain the conclusion $\mu_A(x_i) = \nu_A(x_i) = 0$ for all $\forall x_i \in X$.

(3) $E(A) = E(A^c)$.

We know that $A^c = \{\langle x_i, \nu_A(x_i), \mu_A(x_i)\rangle | x_i \in X\}$ for all $\forall x_i \in X$, that is, $\mu_A(x_i) = \mu_{A^c}(x_i), \nu_A(x_i) = \nu_{A^c}(x_i)$; therefore, $g_A(x_i) = g_{A^c}(x_i)$ and $\pi_A(x_i) = \pi_{A^c}(x_i)$.

Thus, from Equation (8), we have the following:

$$E(A) = \frac{1}{n}\sum_{i=1}^{n} f(g_A(x_i), \pi_A(x_i)) = \frac{1}{n}\sum_{i=1}^{n} f(g_{A^c}(x_i), \pi_{A^c}(x_i)) = E(A^c),$$

that is, $E(A) = E(A^c)$.

(4) If $\mu_A(x_i) + \nu_A(x_i) \geq \mu_B(x_i) + \nu_B(x_i)$, then $\pi_A(x_i) \leq \pi_B(x_i)$; if $|\mu_A(x_i) - \nu_A(x_i)| \geq |\mu_B(x_i) - \nu_B(x_i)|$, then $g_A(x_i) \geq g_B(x_i)$.

Therefore, we have $f(g_A(x_i), \pi_A(x_i)) \leq f(g_B(x_i), \pi_B(x_i))$; it is proved that $E(A) \leq E(B)$.

This completes the proof. □

Particular Cases:

(1) If $f(x,y) = \frac{1-x+y}{1+x+y}$, then Equation (8) becomes Equation (6), which was studied by Szmidt et al. [16].

(2) If $f(x,y) = 1 - x$, then Equation (8) becomes the following:

$$E_7(A) = 1 - \frac{1}{n}\sum_{i=1}^{n} |\mu_A(x_i) - \nu_A(x_i)|. \tag{9}$$

This was studied by Zeng et al. [14].

(3) If $f(x,y) = \frac{(1-x+y)}{2}$, then Equation (8) becomes the following:

$$E_8(A) = \frac{1}{n}\sum_{i=1}^{n} \frac{1 - |\mu_A(x_i) - \nu_A(x_i)| + \pi_A(x_i)}{2}. \tag{10}$$

This was defined by Wu et al. [46].

(4) If $f(x,y) = \frac{(1-x)(1+y)}{2}$, then Equation (8) becomes the following:

$$E_9(A) = \frac{1}{n}\sum_{i=1}^{n} (1 - |\mu_A(x_i) - \nu_A(x_i)|)\frac{1+\pi_A(x_i)}{2}. \tag{11}$$

This was constructed by Guo et al. [47].

In this paper, when the selected binary function is $f(x,y) = \alpha(1-x) + (1-\alpha)log_2{}^{1+y}$, a new intuitionistic fuzzy entropy can be written as follows:

$$E_{10}(A) = \frac{1}{n}\sum_{i=1}^{n}\left[\alpha(1 - |\mu_A(x_i) - v_A(x_i)|) + (1-\alpha)\log_2{}^{(\pi_A(x_i)+1)}\right], \tag{12}$$

where the parameter $\alpha \in [0,1]$ is the attitude coefficient, that is, α represents the subjective attitude of decision makers. When $\alpha \in [0, 0.5]$, it represents the positive attitude of decision makers; when $\alpha \in (0.5, 1]$, it indicates the pessimistic attitude of decision makers [48]. With different values of α, different intuitionistic fuzzy entropies can be obtained, which will not be listed here.

Following Example 1 above, the entropy measures for the IFSs A and B are calculated by Equation (12), as follows:

$$E_{10}(A) = 0.7\alpha + (1-\alpha)\log_2^{1.3} \neq E_{10}(B) = 0.7\alpha + (1-\alpha)\log_2^{1.1}.$$

Obviously, E_{10} overcomes the counterintuitive situation of E_1 and E_2.

Similarly, following Example 2, $E_{10}(A) = 0.9\alpha + (1-\alpha)\log_2^{1.1} \neq E_{10}(B) = 0.85\alpha + (1-\alpha)\log_2^{1.65}$, the counterintuitive situation of E_5 and E_6 is overcome.

Therefore, through Examples 1 and 2, it can be observed that the entropy measure proposed in this paper not only considers the deviation between the membership degree and non-membership degree, but also fully considers the hesitation of decision makers. It more comprehensively and objectively reflects the fuzzy degree of the fuzzy set, in terms of uncertainty and unknown aspects. In addition, without considering the effect of the hesitation degree on intuitionistic fuzzy entropy, Equation (12) can be used to effectively distinguish the case where the deviation of the membership degree and non-membership degree is equal. At the same time, the use of Equation (12) can effectively avoid the emergence of counterintuitive phenomena, which further illustrates the effectiveness of the new entropy measure.

5. Evaluation of Regional Collaborative Innovation Capability Based on Improved Intuitionistic Fuzzy Entropy

5.1. TOPSIS Decision-Making Method Based on Intuitionistic Fuzzy Entropy

The essence of multi-attribute decision making is to comprehensively evaluate each scheme for a given group of schemes under the constraints of multiple attributes, and, finally, sort this group of schemes, or select a relatively satisfactory scheme. At present, it is widely used in the real world, such as in venture capital, medical diagnosis, site selection, and so on. The concrete steps of intuitionistic fuzzy entropy in decision theory are given below:

Suppose there are n evaluation schemes $S = \{A_1, A_2, \cdots, A_n\}$, and T attributes $C = \{C_1, C_2, \cdots, C_T\}$. The evaluation value of scheme A_i with the attribute C_j is an intuitionistic fuzzy number $s_{ij} = (\mu_{ij}, v_{ij})$, where μ_{ij} and v_{ij} represent the membership and non-membership degrees, respectively, and $0 \leq \mu_{ij} \leq 1$, $0 \leq v_{ij} \leq 1$, $0 \leq \mu_{ij} + v_{ij} \leq 1$, $i = 1, 2, \cdots, n$; $j = 1, 2, \cdots, T$. Let the weight of the corresponding attribute be $W = \{w_1, w_2, \cdots, w_T\}$, which satisfies $\sum_{j=1}^{T} w_j = 1$ and $0 \leq w_j \leq 1$.

Next, this paper will use the decision method of TOPSIS to solve the multi-attribute decision-making problem and calculate the attribute weights with the given entropy measure. The specific decision-making steps are shown in Figure 1.

Step 1: Under the condition of comprehensively considering T attributes, the decision maker evaluates the scheme and obtains the following intuitionistic fuzzy decision matrix:

$$D = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1T} \\ a_{21} & a_{22} & \cdots & a_{2T} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nT} \end{pmatrix}$$

All elements in the row i of the intuitionistic fuzzy decision matrix D can be aggregated into $A_i = (a_{i1}, a_{i2}, \cdots, a_{iT})$, where $i = 1, 2, \cdots, n$.

Step 2: Use the entropy measure E_9 to calculate the weight of attributes, as follows:

$$w_j = \frac{1 - e_j}{\sum_{j=1}^{T} (1 - e_j)}, \tag{13}$$

where $e_j = \frac{1}{n} \sum_{i=1}^{n} E_9(a_{ij})$ and $j = 1, 2, \cdots, T$.

Step 3: The intuitionistic fuzzy decision matrix is transformed into an intuitionistic fuzzy benefit matrix. First, if a_{ij} is a benefit attribute, then $\bar{a}_{ij} = a_{ij}$; if a_{ij} is a cost attribute, then $\bar{a}_{ij} = a_{ij}^{c}$. Therefore, the intuitionistic fuzzy benefit matrix is as follows:

$$\overline{D} = \begin{pmatrix} \bar{a}_{11} & \bar{a}_{12} & \cdots & \bar{a}_{1n} \\ \bar{a}_{21} & \bar{a}_{22} & \cdots & \bar{a}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \bar{a}_{n1} & \bar{a}_{n2} & \cdots & \bar{a}_{nn} \end{pmatrix}$$

Step 4: Construct a weighted intuitionistic fuzzy benefit matrix, as follows:

$$\begin{pmatrix} \widetilde{a}_{11} & \widetilde{a}_{12} & \cdots & \widetilde{a}_{1n} \\ \widetilde{a}_{21} & \widetilde{a}_{22} & \cdots & \widetilde{a}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \widetilde{a}_{n1} & \widetilde{a}_{n2} & \cdots & \widetilde{a}_{nn} \end{pmatrix} = \begin{pmatrix} \bar{a}_{11} & \bar{a}_{12} & \cdots & \bar{a}_{1n} \\ \bar{a}_{21} & \bar{a}_{22} & \cdots & \bar{a}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \bar{a}_{n1} & \bar{a}_{n2} & \cdots & \bar{a}_{nn} \end{pmatrix} \cdot \begin{pmatrix} w_1 & 0 & \cdots & 0 \\ 0 & w_2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & w_T \end{pmatrix}$$

Step 5: From step 4, we can obtain the ideal positive solution $S^{+} = \{s_1^{+}, s_2^{+}, \cdots, s_T^{+}\}$, and the ideal negative solution $S^{-} = \{s_1^{-}, s_2^{-}, \cdots, s_T^{-}\}$, where

$$s_j^{+} = (\overset{\overset{i}{n}}{m}ax(\overset{\overset{m}{l_{ij}}}{m}axu_{ij}(x)), \overset{\overset{i}{n}}{m}in(\overset{\overset{m}{l_{ij}}}{m}inv_{ij}(x))), s_j^{-} = (\overset{\overset{i}{n}}{m}in(\overset{\overset{m}{l_{ij}}}{m}inu_{ij}(x)), \overset{\overset{i}{n}}{m}ax(\overset{\overset{m}{l_{ij}}}{m}axv_{ij}(x))).$$

Step 6: Calculate the distance from each scheme to the ideal positive solution and the ideal negative solution, according to the following formula:

$$J_i^+ = D(S^+, A_i) = \sum_{j=1}^{T} D\left(\bar{a}_{ij}, s_j^+\right),\ J_i^- = D(S^-, A_i) = \sum_{j=1}^{T} D\left(\bar{a}_{ij}, s_j^-\right).$$

Step 7: Calculate the gain/loss benefit rate of the scheme. The specific form is as follows:

$$J_i = \left| \frac{J_i^-}{J_i^+ + J_i^-} \right|$$

The schemes are comprehensively sorted according to the size of the J_i value. The larger the J_i value, the better the corresponding scheme.

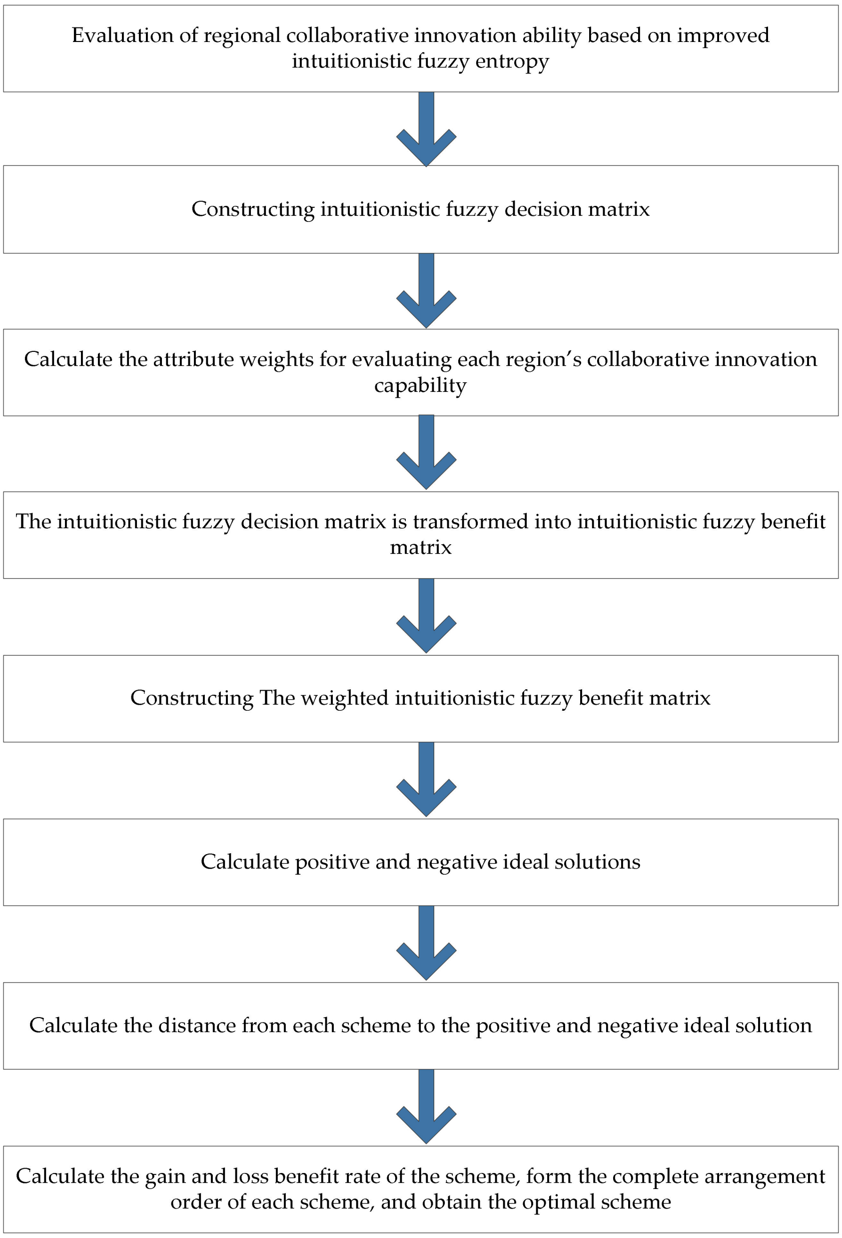

Figure 1. The diagram of specific decision-making steps.

5.2. Application of Improved Intuitionistic Fuzzy Entropy in the Evaluation of Regional Collaborative Innovation Capability

Regional collaborative innovation refers to a collaborative innovation system in which enterprises, universities, academic research institutions, financial institutions, intermediaries, and other innovation subjects, under the guidance of government policies and market demand, realize resource sharing and information exchange [49]. The scientific evaluation and positioning for regional collaborative innovation capability are conducive to clarifying the development strategy of regional collaborative innovation, improving innovation performance, and promoting the high-quality development of the regional economy.

Therefore, this part considers applying intuitionistic fuzzy entropy to the evaluation of regional collaborative innovation capability, and evaluates five regions, A_1, A_2, A_3, A_4, and A_5, including four primary indicators, C_1, C_2, C_3, and C_4, that is, collaborative innovation input, collaborative innovation cooperation, collaborative innovation output, and collaborative innovation auxiliary conditions [50], as shown in Figure 2. The following is the evaluation of five regions by experts, in the form of IFS, to obtain the decision matrix, as shown in Table 1. In this paper, $\alpha = 0.3$.

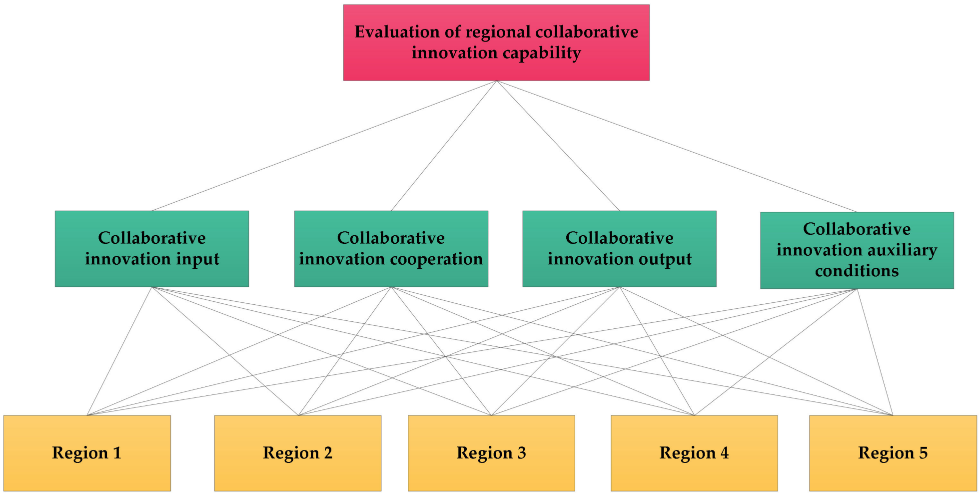

Figure 2. The diagram of evaluation criteria for regional collaborative innovation capability.

Table 1. Decision matrix D.

	C_1	C_2	C_3	C_4
A_1	<0.8, 0.1>	<0.5, 0.3>	<0.6, 0.2>	<0.8, 0.1>
A_2	<0.5, 0.2>	<0.6, 0.3>	<0.7, 0.1>	<0.6, 0.2>
A_3	<0.6, 0.1>	<0.5, 0.4>	<0.4, 0.5>	<0.6, 0.1>
A_4	<0.4, 0.2>	<0.7, 0.1>	<0.4, 0.3>	<0.7, 0.2>
A_5	<0.7, 0.1>	<0.5, 0.2>	<0.7, 0.2>	<0.5, 0.4>

According to Equations (12) and (13), the weights of each attribute can be calculated as follows:

$$e_1 = \frac{1}{5} \times (0.1863 + 0.4750 + 0.4150 + 0.5798 + 0.3041) = 0.3920;$$
$$e_2 = \frac{1}{5} \times (0.4241 + 0.3063 + 0.3663 + 0.3041 + 0.4750) = 0.3751;$$
$$e_3 = \frac{1}{5} \times (0.3641 + 0.3041 + 0.3663 + 0.5350 + 0.2463) = 0.3631;$$
$$e_4 = \frac{1}{5} \times (0.1863 + 0.3641 + 0.4150 + 0.2463 + 0.3663) = 0.3156.$$

That is, the weights of attributes are $\omega_1 = 0.2711, w_2 = 0.2595, w_3 = 0.2512$, and $w_4 = 0.2183$, respectively. Then, according to step 4, multiply the intuitionistic fuzzy benefit matrix by the weight to obtain the weighted benefit decision matrix, as shown in Table 2, where the benefit matrix $\bar{D} = D$.

Table 2. Weighted benefit decision matrix $\overline{D}$.

	C_1	C_2	C_3	C_4
A_1	<0.8, 0.1>	<0.5, 0.3>	<0.6, 0.2>	<0.8, 0.1>
A_2	<0.5, 0.2>	<0.6, 0.3>	<0.7, 0.1>	<0.6, 0.2>
A_3	<0.6, 0.1>	<0.5, 0.4>	<0.4, 0.5>	<0.6, 0.1>
A_4	<0.4, 0.2>	<0.7, 0.1>	<0.4, 0.3>	<0.7, 0.2>
A_5	<0.7, 0.1>	<0.5, 0.2>	<0.7, 0.2>	<0.5, 0.4>

According to the weighted benefit decision matrix and step 5, the ideal positive solution $S^+ = \left(s_1^+, s_2^+, s_3^+, s_4^+\right)$ and the ideal negative solution $S^- = \left(s_1^-, s_2^-, s_3^-, s_4^-\right)$ can be calculated as follows:

$$\begin{cases} s_1^+ =< 0.3536, 0.5356 > \\ s_2^+ =< 0.2683, 0.5502 > \\ s_3^+ =< 0.2609, 0.5608 > \\ s_4^+ =< 0.2962, 0.6050 > \end{cases} \begin{cases} s_1^- =< 0.1293, 0.6464 > \\ s_2^- =< 0.1646, 0.7884 > \\ s_3^- =< 0.1204, 0.8402 > \\ s_4^- =< 0.1404, 0.8187 > \end{cases}.$$

The distances $D_i^+(\bar{A}_i, S^+)$ and $D_i^-(\bar{A}_i, S^-)$ can be obtained from Equation (1), and the value of J_i can be calculated according to step 7. The specific values are shown in Table 3.

Table 3. Distance and gain/loss benefit rate.

	$D_i^+\left(\bar{A}_i, S^+\right)$	$D_i^-\left(\bar{A}_i, S^-\right)$	J_i
A_1	$D_1^+\left(\bar{A}_1, S^+\right) = 1.5624$	$D_1^-\left(\bar{A}_1, S^-\right) = 2.5180$	0.6171
A_2	$D_2^+\left(\bar{A}_2, S^+\right) = 1.5332$	$D_2^-\left(\bar{A}_2, S^-\right) = 2.2938$	0.5994
A_3	$D_3^+\left(\bar{A}_3, S^+\right) = 1.3114$	$D_3^-\left(\bar{A}_3, S^-\right) = 1.9938$	0.6032
A_4	$D_4^+\left(\bar{A}_4, S^+\right) = 1.4517$	$D_4^-\left(\bar{A}_4, S^-\right) = 2.2938$	0.6124
A_5	$D_5^+\left(\bar{A}_5, S^+\right) = 1.4299$	$D_5^-\left(\bar{A}_5, S^-\right) = 2.2180$	0.6080

Obviously, according to the size of the gain and the loss efficiency rate, $J_1 > J_4 > J_5 > J_3 > J_2$, and the corresponding five regions are sorted as $A_1 \succ A_4 \succ A_5 \succ A_3 \succ A_2$, so the optimal region is A_1 and the poorest region is A_2.

To verify the effectiveness of the decision-making method based on improved intuitionistic fuzzy entropy in this paper, using the example in Section 5.2 and the entropy decision-making method in [21–23], the results are outlined in the following sections.

From Table 4, it is not difficult to find that the results obtained by using the method proposed in [21–23] are the same as the ranking of the decision-making method based on the improved intuitionistic fuzzy entropy, which also proves the practicability of the entropy proposed in this paper. In addition, the intuitionistic fuzzy entropy defined in this paper is constructed based on a special function, and the newly given entropy measure considers the decision maker's subjective attitude in the decision-making process, through the selection of parameters. A change in the decision maker's subjective attitude will directly affect the selection and ranking of the final scheme. Therefore, the introduction of the attitude coefficient is more in line with the actual situation.

Table 4. Sorting results.

Entropy Measure of IFSs	Sorting Results
E_1 [21]	$A_1 \succ A_4 \succ A_5 \succ A_3 \succ A_2$
E_2 [21]	$A_1 \succ A_4 \succ A_5 \succ A_3 \succ A_2$
E_3 [22]	$A_1 \succ A_4 \succ A_5 \succ A_3 \succ A_2$
E_4 [23]	$A_1 \succ A_4 \succ A_5 \succ A_3 \succ A_2$

6. Conclusions

According to the characteristics of regional collaborative innovation development, and based on the analysis of intuitionistic fuzzy entropy in the existing papers, this paper redefines a new kind of intuitionistic fuzzy entropy based on special functions, which considers the influence of the deviation between the membership degree and non-membership degree, as well as hesitation, on entropy, and also includes other forms of entropy, which solves the counterintuitive phenomena that appear in some cases in the existing papers [16,21–23,45]. Finally, the multi-attribute decision-making method of TOPSIS, based on the intuitionistic fuzzy entropy measure, was used to rank the alternatives, as well as select the optimal solution, and it is applied to the evaluation of regional collaborative innovation capability. The following conclusions were obtained:

(1) Using the improved intuitionistic fuzzy entropy, we can comprehensively and effectively describe the fuzzy information for the evaluation of regional collaborative innovation capability from both uncertain and unknown aspects, which improves the accuracy and objectivity of the evaluation results, to a certain extent, and provides a way to solve the intuitionistic fuzzy multi-attribute problem.

(2) Taking the evaluation of regional collaborative innovation capability as an example, this paper illustrates the feasibility of the entropy measure, compares it with other decision-making methods of entropy measure, and obtains consistent results with this paper, which further emphasizes the effectiveness and reliability of the method proposed in this paper. Meanwhile, the entropy measure proposed in this paper can be applied to image processing, pattern recognition, and medical diagnosis.

(3) Through the selection of parameters, the entropy measure given in this paper considers the subjective attitude of the decision maker in the decision-making process. A change in the decision maker's subjective attitude will directly affect the selection and ranking of the final scheme. Therefore, the introduction of an attitude coefficient is more consistent with the actual situation.

However, the discussion on the evaluation of regional collaborative innovation capability in this paper is not extensive enough, and we will continue to study the intuitionistic fuzzy information measurement and the corresponding decision-making methods, and apply them to practical problems. In addition, the research model does not consider interval-valued and trapezoidal intuitionistic fuzzy numbers, so the measurement of these two intuitionistic fuzzy entropies is also the direction of future research [6–9].

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Abbreviations

The following abbreviations are used in this manuscript:

TOPSIS	Technique for order preference by similarity to an ideal solution;
IFS	Intuitionistic fuzzy set.

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