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# Sustainable Scheduling of the Production in the Aluminum Furnace Hot Rolling Section with Uncertain Demand

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**Abstract:** In order to reduce the energy consumption of furnaces and save costs in the product delivery time, the focus of this paper is to discuss the uncertainty of demand in the rolling horizon and to globally optimize the sustainability of the production in the aluminum furnace hot rolling section in environmental and economic dimensions. First, the triples $\alpha/\beta/\gamma$ are used to describe the production scheduling in the aluminum furnace hot rolling section as the scheduling of flexible flow shop, satisfied to constraints of demand uncertainty, operation logic, operation time, capacity and demand, objectives of minimizing the residence time of the ingot in the furnace and minimizing the makespan. Second, on the basis of describing the uncertainty of demand in rolling horizon with the scenario tree, a multi-objective mixed integer linear programming (MILP) optimization model for sustainable production in the aluminum furnace hot rolling section is formulated. Finally, an aluminum alloy manufacturer is taken as an example to illustrate the proposed model. The computational results show that when the objective weight combination takes the value of $\alpha$= 0.7, $\beta$= 0.3, the sustainability indicators of the environmental and economic dimensions can be optimized to the maximum extent possible at the same time. Increasingly, managerial suggestions associated with the trade-off between environmental and economic dimensions are presented. Scheduling in the rolling horizon can optimize the production process of the aluminum furnace hot rolling section globally, indicating that it is more conducive to the sustainable development of the environment and economic dimensions than scheduling in a single decision time period.

**Keywords:** sustainability; uncertain demand; scenario tree; production scheduling; MILP

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**Citation:** Huang, Y.; Yang, Q.; Liu, J.; Li, X.; Zhang, J. Sustainable Scheduling of the Production in the Aluminum Furnace Hot Rolling Section with Uncertain Demand. *Sustainability* **2021**, *13*, 7708. https://doi.org/10.3390/su13147708

Academic Editors: Jose M. Framinan, Paz Perez-Gonzalez and Victor Fernandez-Viagas

Received: 23 April 2021
Accepted: 5 July 2021
Published: 9 July 2021



## 1. Introduction

As an important part of sustainable development, reasonable manufacturing operations scheduling is conducive to reducing energy consumption and saving production costs. It is a topic that has attracted more and more attention from practitioners to academia in recent years [1]. According to reports, and based on the data provided by the China National Bureau of Statistics [2], since 2000, the total energy consumption has been increasing steadily and reached 4,719,251,500 tons of standard coal in 2018, including the total industrial energy consumption of 311,151 million tons of standard coal, which accounts for 66% of the energy consumption in all industries. Meanwhile, the literature [3] proposed that 90% of energy consumption and 84% of $CO_2$ emission in industrial production are attributed to manufacturing activities. Therefore, optimizing industrial production scheduling plays an important role in sustainable development.

Hot rolling is a high energy-consuming process in industrial production (i.e., steel, aluminum and copper), which is closely related to product quality. More and more scholars are beginning to pay attention to the huge potential of the rolling process in energy saving and the reduction of greenhouse gas emissions [4]. The sustainable scheduling of the production in hot rolling has attracted significant research attention in both research and practice [5–7]. The amount of aluminum alloy used by industry is second only to steel, and its economic aspects in the production process due to high energy consumption and costs have also driven broad attention to this research field [8,9]. The heating procedure is followed by the hot rolling procedure in the aluminum hot rolling process. The energy consumption of furnaces accounts for about 60–70% of the total energy consumption of the entire hot rolling production process. As the main energy-consuming equipment, a furnace consumes a lot of energy. In addition, the aluminum furnace hot rolling section includes both heating and hot rolling processes and multiple kinds of equipment. Low utilization of equipment may lead to an increase in the total processing time, and enterprises cannot deliver products within the stipulated time. To sum up, in order to reduce the energy consumption of furnaces, save costs in the product delivery time and promote the sustainable development of the environmental and economic dimensions, the optimization of the production process in the aluminum furnace heat rolling section has become the current challenge and opportunity faced.

The sustainability of the production in the aluminum furnace hot rolling section is often hindered in two ways. On the one hand, according to the views of the literature [10], since there is no buffer zone between the furnace and the hot rolling mill, the ingots that have finished heating have to be kept in the furnace until the hot rolling mill is available, or the hot rolling mill has to be idle until the heating of the ingots is finished. Unreasonable scheduling of the production in the aluminum furnace hot rolling section is likely to cause such issues as waste of energy in the furnace and low utilization rate of equipment. On the other hand, due to the influence of seasonal changes and market fluctuations and other factors, the demand for products also faces uncertainties [11,12], which will cause dynamic changes in the production environment, resulting in the original optimal sustainable scheduling plan becoming non-optimal or even infeasible. In addition, from the perspective of long-term, multiple-decision time periods (rolling horizon), it is not possible to achieve global optimization of sustainable development by only considering the current demand, which is also a view agreed with in the literature [13,14]. Therefore, in order to alleviate the above-mentioned limitations in sustainable development, the need for the sustainable scheduling of the production in the aluminum furnace hot rolling section with uncertain demand is pressing.

In recent years, although the production scheduling in hot rolling has been a hot topic in academic and practical circles, the production scheduling in the aluminum furnace hot rolling section based uncertain environments is still in its infancy. The results come from the following aspects. Firstly, to tackle the scheduling problem in the furnace hot rolling section, most research concentrates on the iron and steel industry [10,15–27], and the aluminum industry is hardly considered in the literature [28]. Even if the aluminum and steel industries have similar production processes, which also have different characteristics, the two types of production cannot be treated as the same. Secondly, a few scholars such as Tang and Yang [19] and Parsunkin et al. [20] studied the production scheduling in the uncertain environments of the furnace hot rolling section, but they had only made discussions on the uncertain factors (excluding demand) in a single decision time period. Therefore, how to consider the uncertainty of demand in the rolling horizon and realize the global optimization regarding the production in the furnace hot rolling section, still remains to be an interesting and promising topic. Thirdly, researchers focus on such methods as robust optimization [29,30], machine learning [31–35], deep learning [36–38] and fuzzy planning [39] to describe uncertain factors in the production scheduling of hot rolling. However, due to the intrinsic characteristics of the methods themselves, they are not suitable for describing the uncertainty of demand in rolling horizon. The literature [40,41] delineated that a scenario tree is an effective way to describe the uncertain factors in the rolling horizon, and it has been widely used in other process system engineering [42,43].

Fourthly, the popular multi-objective optimization scheduling for the production in the hot rolling includes sustainable indicators such as environmental, economic and social dimensions [44–54], but only a few scholars have given thought to the sustainability of the production regarding the furnace hot rolling section in both the environmental and economic dimensions [10,18], which is not applicable to this paper, and further research is needed regarding how to determine the multi-objective [55–57] optimization scheduling model of environmental and economic dimensions according to the characteristics of specific problems.

In this context, this paper focuses on the uncertainty of demand in rolling horizon and is committed to the global optimization of the sustainability of the production in the aluminum furnace hot rolling section in the environmental and economic dimensions. First, the scenario tree is used to accurately describe the uncertainty of demand in the rolling horizon, and then a multi-objective MILP optimization model for the production in the aluminum furnace hot rolling section is established with demand in various scenarios as the known input parameter, taking into consideration of the sustainability indicators of the environmental and economic dimensions, to achieve the ultimate objectives of reducing the furnace energy consumption and save costs of product delivery time.

The contributions of this paper manifest the following aspects. Firstly, differing from the previous production scheduling of furnace hot rolling section, which focused on the iron and steel industry in a deterministic production environment with optimizing the furnace hot rolling section process in a single decision time period. This paper focuses on the uncertainty of demand in the rolling horizon in the aluminum industry, optimizes the production in the furnace hot rolling section globally and is devoted to solving a novel problem of sustainable scheduling in the aluminum furnace hot rolling section with uncertain demand. Secondly, the scenario tree method is employed to describe the uncertainty of demand in rolling horizon. To the best of our knowledge, this is the first time that the scenario tree method has been applied to describe the production in the furnace hot rolling section, which can appropriately and accurately predict the value and probability of demand under different production scenarios. Thirdly, a multi-objective MILP optimization model for sustainable production in the aluminum furnace hot rolling section is established by using the continuous time representation method, with the sustainability of the environmental dimension measured by minimizing the residence time of the ingot in the furnace and the sustainability of the economic dimension measured by minimizing the makespan.

The rest of this paper is organized as follows: The following section describes the related works on this topic. Section 3 uses a triple $\alpha/\beta/\gamma$ to present the problem description in detail. In Section 4, on the basis of describing the uncertain demand in rolling horizon with the scenario tree, a multi-objective MILP optimization model for sustainable scheduling of the production in the aluminum furnace hot rolling section is formulated. Section 5 uses a case study from an aluminum alloy manufacturer to illustrate the proposed model. Finally, the implication of the findings and future directions are concluded.

## 2. Literature Review

In recent years, the production optimization in the aluminum industry has attracted some researchers' attention, and some research has been done in the production scheduling of aluminum casting [58,59] and aluminum electrolytic cell [60,61]. However, the research regarding the production scheduling in aluminum hot rolling is still in its infancy, and bounded literature can be found [8,9,28,36,51,52]. The aluminum industry is an important component of the metallurgical industry. In contrast, the production scheduling of hot rolling in the metallurgical industry and that of aluminum hot rolling has a high correlation, which is favored by many scholars. Therefore, in this paper, based the literature on the production scheduling in hot rolling concerning the metallurgical industry in the past 10 years, the research progress is sorted into three aspects, including the production scheduling in the furnace hot rolling section, uncertainty description and multi-objective optimization scheduling. Then the implications for research are found.

Firstly, a significant issue of this research is the production scheduling in the furnace hot rolling section. Heating and hot rolling are important processes in the metallurgical industry, and extensive research attention has been devoted to the production scheduling in the furnace hot rolling section. Tang and Wang [15] discussed a multistage production scheduling problem from the integrated hot rolling production in the iron and steel industry and formulated this problem as a mixed integer linear programming model. Xu et al. [16] presented a generic model formulation for multi-stage batch schedule coordination problems in order to reduce the storage time of the slabs. Chakravarty et al. [17] and Jiang et al. [18] devoted to reducing the fuel consumption in the production of furnace hot rolling process in iron and steel industry. Tang and Yang [19] studied the uncertainty of steel slab temperature control of the reheating furnace process and employed the support vector machine to establish a nonlinear predictive model based on the real production data. Parsunkin et al. [20] forecasted the time of charging billets into a furnace to tackle the problem of energy-saving optimal control of continuous cast hot rolling process in the iron and steel industry. Xia et al. [21] addressed the production scheduling problem of furnace hot rolling section in the iron and steel industry with minimizing temperature deviation of slab, energy consumption and oxidation loss. Li and Tian [22] and Li et al. [23] formulated the production scheduling problem of the furnace hot rolling section in the iron and steel industry as a mixed integer linear programming model. Tan et al. [24] established nonlinear mathematical models for a continuous casting furnace hot rolling production scheduling problem in the iron and steel industry with the objectives of minimizing energy waste and energy requirement. Peng et al. [25] considered deducing the heating energy and the rolling energy during modeling in the iron and steel industry. Wang et al. [26] formulated an integrated scheduling problem for steelmaking continuous casting hot rolling processes as an integrated two-stage mathematical programming model. Ding et al. [27] presented a multi-objective optimization method for furnace temperature setting so as to obtain a reasonable heating process temperature of a slab in the furnace. De Ladurantaye et al. [28] aimed at minimizing the idle time on the mill and penalties for soft constraint violations related to production quality and studied the production scheduling of furnace hot rolling section in the aluminum industry.

Secondly, uncertainty description is another important stream in this paper. It is found that uncertain factors can affect the feasibility and optimization of decision-making schemes, and how to describe the uncertain factors has become a hot topic in the metallurgical industry [62]. At present, researchers mainly use methods such as robust optimization, machine learning (ML), deep learning and fuzzy planning to describe the uncertain factors such as processing time and rolling force. For the robust optimization method, Kong et al. [29] and Zhang et al. [30] established a scheduling model for steel continuous casting hot rolling production and a scheduling model for steel hot rolling production, respectively, considering the uncertain processing time of the product in the hot rolling process. For machine learning methods, Lohmar et al. [31] and Zhang et al. [32] used nonlinear regression methods to determine the parameters of the model and obtained analytical expressions for steel hot rolling force prediction. Liu et al. [33] and Chen et al. [34] performed numerical simulations with the mathematical models created to predict the separation force of the rolls and grain size respectively, which provided valuable guidance for the optimization of steel rolling process. Cao et al. [35] formulated a model for work roll wear prediction, a hot roll profile model, and a three-dimensional finite element model of the roll system and strip steel with MATLAB and ABAQUS software. For deep learning methods, Hu et al. [36] and Bagheripoor and Bisadi [37] created rolling force prediction models with adaptive neural networks and artificial neural networks, respectively, based on the classification systems in the aluminum and steel industries. Wang et al. [38] formulated a model for the prediction of the bending force of hot-rolled strip steel using an artificial neural network optimized by the genetic algorithm. For the fuzzy planning method, Wang et al. [39] made a description of fuzzy logic considering the uncertainty of the arrival time of the workpiece, the heating value of the gas and the processing time and the delivery date in steel hot rolling.

Thirdly, this paper reviews the literature on multi-objective optimization scheduling. It can be found that the managers of manufacturing enterprises have begun to pay attention to indicators in environmental, economic and social dimensions [63], and multi-objective optimization scheduling was a very popular stream in the metallurgical industry. Pan et al. [44] and Chen et al. [45] decomposed steel hot rolling scheduling into two sub-problems and formulated a mixed-integer linear programming model and a mixed-integer nonlinear programming model, respectively, for the two sub-problems. Jia et al. [46,47] described the steel hot rolling batch scheduling as multi-objective vehicle routing and adopted a solution model using a hierarchical optimization algorithm based on decomposition and a multi-objective optimization algorithm based on Pareto advantages. Liu et al. [48] formulated a multi-objective traveling salesman problem model for steel hot rolling production scheduling with the objective of the minimum rolling unit plan, process specification and minimum power consumption per ton of steel. Qi et al. [49] formulated a mathematical model for the optimization of steel finishing rolling procedure with relatively equal rolling power and better slab flatness as the objective function. Li et al. [50] proposed a multi-objective optimization model for draft scheduling of steel hot strip mill aiming at minimizing rolling power, rolling force ratio distribution and good strip shape. Jing et al. [51] and Che et al. [52] showed a rolling schedule model based on the slippage factor to equal power margin and prevent the slip phenomenon. Tan et al. [53] studied the scheduling of steel hot rolling based on time-of-use electricity pricing, in which the objective is to minimize electricity costs while considering penalties caused by jumps between adjacent slabs. Zhang et al. [54] formulated a mixed-integer nonlinear programming model aiming at minimizing the number of rolling turns and the average thickness change of adjacent slats and proposed a hybrid variable neighborhood search algorithm for a solution.

In summary, Table 1 summarizes the related literature to the production scheduling in the furnace hot rolling section from various perspectives. The column of "Process" represents the technical processes, including (1) furnace; (2) hot rolling; (3) furnace hot rolling; (4) multi-stage process including the furnace hot rolling section. The column of "Period" indicates the decision time periods considered in the model, including (1) a single decision time period; (2) the rolling horizon. The column of "Obj." indicates a single or multiple decision objective(s) that the model contains. The column of "Main obj." represents the objective function of the model, which can be (1) environment (such as energy consumption, etc.); (2) economy (such as cost, etc.); (3) society (such as delivery time, etc.); (4) Miscellaneous (such as roll wear, etc.). The column of "Model" indicates the type of mathematical model, including (1) non-linear; (2) linear; (3) integer; (4) mixed-integer; (5) mixed-integer non-linear.

The following conclusions can be summarized:

(1) To the best of our knowledge, the existing studies focus on the production scheduling of furnace hot rolling section in the iron and steel industry [10,15–27]. A few scholars such as De Ladurantaye et al. [28] discussed the production scheduling of furnace hot rolling section in the aluminum industry, but this research field has received rather limited attention in the scientific literature. Similar to the iron and steel industry, the production process of the furnace hot rolling section in the aluminum industry also includes heating with the furnace, hot rolling and hot continuous rolling production links, however, there are differences between the two processes in the processing path, the rules of the slabs entering and exiting the furnace and the type of furnace used, and the above types of production cannot be treated as the same. Therefore, it is necessary to make a separate study according to the production process and characteristics in the aluminum furnace hot rolling section.

**Table 1.** Literature Review of the Production Scheduling in the Furnace Hot Rolling Section.

| Reference | Year | Problem | | | Uncertainty Method | | Mathematical Model | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Industry | Process | Period | Uncertain Factors | Method | Obj. | Main Obj. | Model |
| Tang and Wang [10] | 2010 | Steel | 3 | 1 | - | - | Multi | 1, 2 | 3 |
| Tang and Wang [15] | 2011 | Steel | 4 | 1 | - | - | Single | 2 | 4 |
| Xu et al. [16] | 2012 | Steel | 4 | 1 | - | - | Single | 4 | 4 |
| Chakravarty et al. [17] | 2013 | Steel | 3 | 1 | - | - | Single | 1 | 2 |
| Jiang et al. [18] | 2013 | Steel | 3 | 1 | - | - | Multi | 1, 2 | 5 |
| Tang and Yang [19] | 2014 | Steel | 1 | 1 | Temperature | ML | Single | 1 | 1 |
| Parsunkin et al. [20] | 2015 | Steel | 3 | 1 | Time | ML | Single | 1 | 1 |
| Xia et al. [21] | 2016 | Steel | 3 | 2 | - | - | Multi | 1, 4 | 1 |
| Li and Tian [22] | 2018 | Steel | 3 | 1 | - | - | Multi | 1, 4 | 4 |
| Li et al. [23] | 2019 | Steel | 3 | 1 | - | - | Single | 2 | 4 |
| Tan et al. [24] | 2019 | Steel | 4 | 1 | - | - | Multi | 1, 1 | 1, 4 |
| Peng et al. [25] | 2020 | Steel | 3 | 1 | - | - | Multi | 1, 1 | 1 |
| Wang et al. [26] | 2020 | Steel | 4 | 1 | - | - | Single | 1 | 2, 3, 4 |
| Ding et al. [27] | 2021 | Steel | 1 | 1 | - | - | Single | 1 | 1 |
| De Ladurantaye et al. [28] | 2007 | Alu | 3 | 1 | - | - | Multi | 2, 4 | 5 |
| This paper | - | Alu | 3 | 2 | Demand | Scenario tree | Multi | 1, 2 | 4 |

(2) The production scheduling in the furnace hot rolling section based uncertain environments is still in its infancy. Tang and Yang [19] and Parsunkin et al. [20] used machine learning to predict the uncertainty of the slab temperature and processing time, focused on optimizing the furnace hot rolling process in a single decision time period. In contrast, how to consider the uncertainty of demand in rolling horizon and realize the global optimization in the furnace hot rolling section is the focus of this paper.

(3) At present, researchers mainly use methods such as robust optimization [29,30], machine learning [31–35], deep learning [36–38] and fuzzy planning [39] to describe the uncertain factors in hot rolling production. Unfortunately, the existing methods are not suitable for describing the uncertainty of demand in the production of the aluminum furnace hot rolling section. Robust optimization is suitable for situations where demand is unpredictable, but in practice, historical demand data may be used for prediction. Machine learning and deep learning can predict a value of demand and are suitable for finding a sustainable production scheduling plan in a single decision time period, but it is difficult to achieve the objectives in the rolling horizon. Fuzzy planning selects the fuzzy interval of demand, and the description may be inaccurate. In recent years, the scenario tree [40,41] received more attention from process system engineering researchers [42,43] can accurately predict the uncertainty in rolling horizon and obtain the values and probabilities in different production scenarios, which is suitable to describe demand in the production of the aluminum furnace hot rolling section.

(4) The multi-objective optimization scheduling in hot rolling as a hot subject is being debated at present [44–54], but only Tang and Wang [10] and Jiang et al. [18] have considered the sustainability of production regarding the furnace hot rolling section in both the environmental and economic dimensions. This paper is unique in a way that it creates a multi-objective [55–57] MILP optimization scheduling model for the sustainable production of the aluminum furnace hot rolling section, with the environmental dimension measured by minimizing the residence time of the ingot in the furnace, and the economic dimension measured by minimizing the makespan.

## 3. Problem Description

This paper uses a triple $\alpha/\beta/\gamma$ to describe the sustainable scheduling of the production in the aluminum furnace hot rolling section with uncertain demand. The $\alpha$ field describes the machine environment, explaining the production process and characteristics of the aluminum furnace hot rolling section. The $\beta$ field describes the processing characteristics and constraints, including the characteristics of demand uncertainty, operation logic, operation time, capacity and demand constraints. The $\gamma$ field describes the scheduling objectives of minimizing the residence time of the ingot in the furnace and minimizing the makespan.

Machine environment. The production in the aluminum furnace hot rolling section is closely related to product quality. Heating and hot rolling are key procedures in the aluminum production process. They are two serial stations; each station has one or more furnaces and hot rolling units. It can be attributed to the scheduling of the flexible flow shop. The production process of the aluminum furnace hot rolling section is shown in Figure 1. It takes a semi-continuous process to cast ingots as raw materials. Firstly, the ingots to be processed are batched and then distributed to pusher furnaces for heating. After the soaking process is completed, the ingots are heated and the ingots in the furnace are cooled to the initial rolling temperature for heat preservation. Finally, the ingots are distributed to the hot rolling unit for processing. Compared with the iron and steel industry, the heating and hot rolling processes of the aluminum industry show the following different characteristics: firstly, for the production process path, semi-continuous casting-heating-hot rolling is used in the aluminum industry, while in the iron and steel industry, there are four processes between continuous casting and hot rolling, among which three processes are heated and one is not. Then, in terms of the rules of the slabs entering and exiting the furnace, the billet distribution to a furnace in the aluminum industry does not need to be in the same rolling order, while in the iron and steel industry, billet distribution to the furnace in the rolling order is required, and the discharging sequence of furnace is first in first out. Finally, for the type of furnace, the aluminum industry uses the pusher furnace, while the iron and steel industries employ a walking beam furnace.

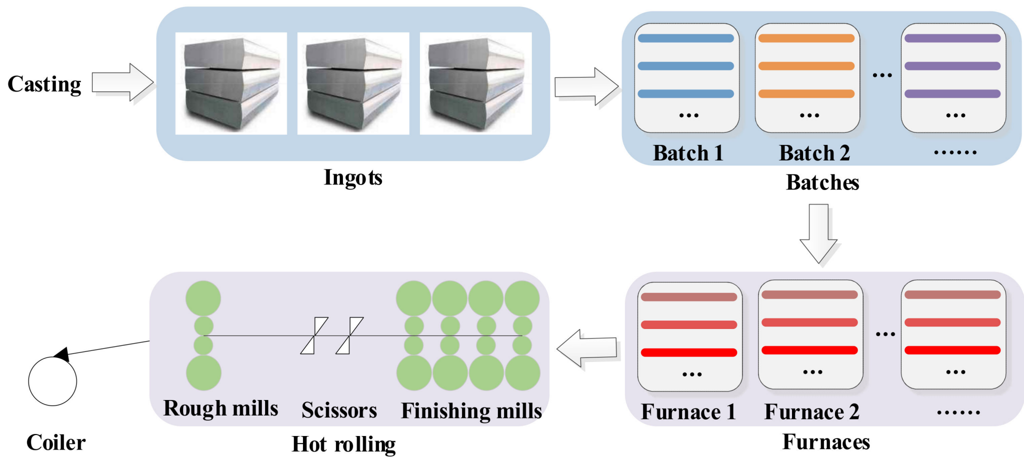


**Figure 1.** Production process.

Processing features and constraints. Due to the influence of seasonal changes and market fluctuations, the changing demand for aluminum products in various decision time periods, presents the following uncertain characteristics: the manager receives new orders or order modifications at least a few days before the expiration date. Therefore, the demand for the product is certain from current decision time period to the first scheduling cycle (that is decision time period 1). With the increase of the decision time period, the uncertainty of demand becomes larger and larger. In addition, the production in the aluminum furnace hot rolling section also contains the following constraints: operation logic (for example, the ingots processed in the furnace must be sent to the hot rolling mill for further processing); operating time (such as the production completion time of the ingots in the furnace is greater than or equal to the start time of production plus the processing time); capacity constraints (such as the total weight of the ingots processed in the furnace is less than or equal to the maximum capacity of the furnace); demand constraints (such as the total number of ingots processed in the furnace must be greater than or equal to the demand).

Scheduling objectives. This paper considers the sustainability indicators of the environmental and economic dimensions regarding the production scheduling in the aluminum furnace hot rolling section. On the one hand, the sustainability of the environmental dimension is measured by minimizing the residence time of the ingot in the furnace. The ideas and insights from Tang and Wang [10], Xu et al. [16], Tan et al. [24] are leveraged and extended to define the formulation of environmental sustainability. More specifically, the energy consumption of furnaces accounts for about 60–70% of the total energy consumption of the entire hot rolling production process, hence, reducing the energy consumption of the furnace can reduce environmental pollution. The residence time of the ingot in the furnace is closely related to the energy consumption of the furnace, which is equal to the time when the ingot leaves the furnace minus the time when the ingot enters the furnace. Under the premise that the ingot reaches the rolling temperature, the longer the residence time is, the more energy is consumed. On the other hand, the sustainability of the economic dimension is measured by minimizing the makespan. Pinedo and Hadavi [64] portrayed that a minimum makespan usually implies a good utilization of the machine(s), and efficient production can shorten the total processing time. According to the insights of Akbar and Irohara [63], product delivery time defines the sustainability of the economic dimension, which represents the service level of the company [65]. It is worth noting that the above two objectives are time-dependent, and the same dimension makes it easy to calculate.

In summary, the production scheduling in the aluminum furnace hot rolling section is a complex operation considering multi-period, multi-process, multi-workpieces, multi-constraints and multi-objectives and that aims at minimizing the residence time of the ingot in the furnace and minimizing the makespan. On the premise of satisfying the constraints, the following decisions need to be made. Firstly, taking the demand in rolling horizon as the guide, consider different production scenarios and decide which ingots should be arranged in which decision time period. Secondly, consider when and which ingots are arranged to form a batch and are assigned to which furnace. Thirdly, consider when and which ingots are assigned to which hot rolling mill. In this paper, a multi-objective MILP optimization model for the sustainable scheduling of the production in the aluminum furnace hot rolling section is established by combining the machine environment, processing characteristics and constraints and scheduling objectives to obtain a globally optimized sustainable production scheduling plan.

## 4. A Multi-Objective Optimization Model Formulation

Aiming at the sustainable scheduling of the production in the aluminum furnace hot rolling section with uncertain demand, and on the basis of the uncertainty of demand in the rolling horizon described with a scenario tree, a multi-objective MILP optimization model for sustainable scheduling of the production in the aluminum furnace hot rolling section is formulated in this paper.

### *4.1. Demand in Rolling Horizon Described by the Scenario Tree*

For the production process in the aluminum furnace hot rolling section, the demand in the rolling horizon of type $c$ ingots is uncertain. The scenario trees describing the demand of type $c$ ingots are shown in Figure 2. Based on Figure 2, this section illustrates how to use scenario trees to describe demands in rolling horizon in the following three parts: what is the scenario tree, how to describe demands in the rolling horizon and how to generate the scenario tree.

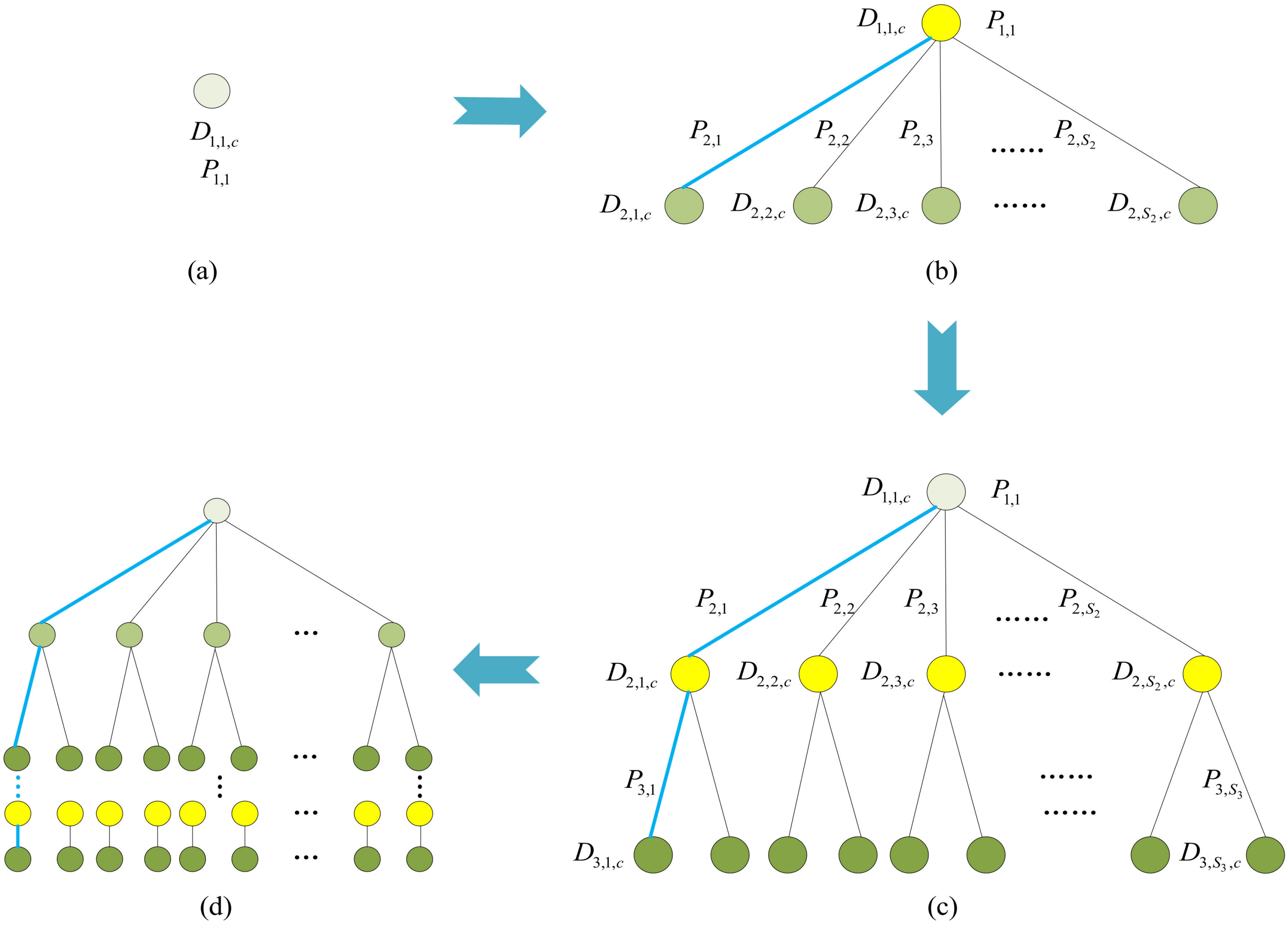


**Figure 2.** Schematic diagram of the demand for the type $c$ described by the scenario tree. (**a**) shows the unique parent node, (**b**) shows a two-stage scenario tree, (**c**) shows a three-stage scenario tree, (**d**) shows a multi-stage scenario tree.

The scenario tree. The structure of the scenario tree is based on a unique parent node, branching to form multiple child nodes. The child node is then set to the parent node and branch to form multiple successor child nodes, until the end of the decision time period. How we obtained the scenario tree presented in Figure 2a–d is described as follows. Each scenario tree contains several future scenarios, and each path formed by the parent node-child node connection represents each scenario (blue line in Figure 2). Each scenario has specific values and probabilities, which indicate the value of the demand and the probability of its occurrence. For a certain decision time period, the demand has several values and corresponding probability, which can be regarded as the discrete probability distribution of the demand. By traversing every scenario in the scenario tree, the uncertain demand can be transformed into the value and probability of several deterministic scenarios. Then, the uncertainty of demands can be described.

The scenario trees describing demands in rolling horizon. The decision time period 1 is from the current time to the first scheduling cycle, and $t \in T = \{1\}$, $s \in S = \{1\}$, $S_t = \{1(1)\}$, the demand for the type $c$ ingots is a certain value $D_{1,1,c}$ and the probability of the occurrence of the scenario is $P_{1,1} = 1$, as presented in Figure 2a. The decision time period 2 is from the current time to the second scheduling cycle, and $t \in T = \{1,2\}$, $s \in S = \{1,2,\ldots,s_2\}$, $S_t = \{1(1), 2(1,\ldots,s_2)\}$, the demand for the type $c$ ingots is $D_{2,s,c}$ and the probability of the occurrence of the scenario is $P_{2,s}$, as shown in Figure 2b. Figure 2b represents a two-stage scenario tree, which considers the values and probabilities of the demands in two decision time periods. The decision time period 3 is from the current time to the third scheduling cycle, and $t \in T = \{1,2,3\}$, $s \in S = \{1,2,\ldots,s_3\}$,

$S_t = \{1(1), 2(1, \ldots, s_2), 3(1, \ldots, s_3)\}$, the demand for the type $c$ ingot is $D_{3,s,c}$ and the probability of the occurrence of the scenario is $P_{3,s}$, as shown in Figure 2c. Figure 2c represents a three-stage scenario tree, which considers the values and probabilities of the demands in three decision time periods. The results can be deduced in the same manner until the end of the decision time period.

Scenario tree generation method. To require the probability of the occurrence of each scenario and the demand for the type $c$ ingots, in this paper, the distribution matching method proposed in the literature [41], is used to generate the scenario tree. For the distribution matching method, first the statistical properties of the historical demand data are calculated, such as the mean, variance, skewness, kurtosis and cumulative distribution function. Then, matching the statistical properties of the historical data with those of the scenario trees, the two-stage scenario tree is generated by minimizing the error between them to predict the possible scenarios in a certain decision time period whose values and probabilities are obtained to generate the scenario tree. However, due to space limitation, more details are only provided in the literature [41]. The steps of which are as follows.

Step 1: for $t \in T = \{1\}$, the demand for type $c$ ingots is a certain value $D_{1,1,c}$, as shown in Figure 2a.

Step 2: for $t \in T = \{1, 2\}$, set the node of $t = 1$ as the parent node and incorporate it into the historical data set (yellow node in Figure 2b), and the distribution matching method is used to generate a two-stage scenario tree to obtain the demand $D_{2,s,c}$ and probability $P_{2,s}$ of each scenario at $t = 2$, as shown in Figure 2b.

Step 3: for $t \in T = \{1, 2, 3\}$, go through each node of $t = 2$, set it as the parent node and incorporate it in the historical data set (yellow node in Figure 2c). Generate a two-stage scenario tree with the distribution matching method, so that the demand $D_{3,s,c}$ and probability $P_{3,s}$ of each scenario at $t = 3$ is obtained, as shown in Figure 2c.

Step 4: Repeat Step 3 until the decision time period is over, as shown in Figure 2d.

### *4.2. MILP Model for the Production in the Aluminum Furnace Hot Rolling Section*

The objective function and constraints of the model for the scheduling of the production in the aluminum furnace hot rolling section are linear equations, including continuous variables such as the start time of the ingot production in the equipment, the production completion time and the discrete variables including whether the ingot is assigned to the equipment for processing and whether the equipment is idle. A MILP model is formulated.

Model assumptions. When the residence time of the ingot in the furnace reaches the rated processing time, it can be considered as reaching the rolling temperature. The ingot can only be kept in the furnace before rolling. The ingot will leave the hot rolling mill immediately after processing without delay. The furnace is the main energy-consuming equipment in the production process, so more consideration is given to the energy consumption of the furnace.

Mathematical model. A multi-objective MILP optimization model for sustainable production in the aluminum furnace hot rolling section is formulated.

The objective function of environmental sustainability is the sum regarding the residence time of the ingot in the furnace in each scenario multiplied by the probability of that scenario. The objective function of economic sustainability is the sum concerning the makespan in each scenario multiplied by the probability of that scenario, which denotes the minimum expectation of the sustainable indicators, as shown in Formulas (1) and (2):

$$Min \sum_{t \in T, s \in S_t} P_{t,s} \left[ \sum_{i \in I, j1 \in J1, n1 \in N1} (TF1_{t,s,i,j1,n1} - TS1_{t,s,i,j1,n1}) \right] \tag{1}$$

$$Min \sum_{t \in T, s \in S_t} P_{t,s} MS_{t,s} \tag{2}$$

Each ingot must be assigned to a furnace (hot rolling mill) for processing and can only be assigned to one event point of each furnace (hot rolling mill), as shown in Formulas (3) and (4):

$$\sum_{t\in T, j1\in J1, n1\in N1} W1_{t,s,i,j1,n1} = 1 \forall s \in S, i \in I \tag{3}$$

$$\sum_{t\in T, j2\in J2, n2\in N2} W2_{t,s,i,j2,n2} = 1 \forall s \in S, i \in I \tag{4}$$

After each ingot is processed in the furnace, it must be processed in the hot rolling mill. The formula is shown in (5):

$$\sum_{j1\in J1, n1\in N1} W1_{t,s,i,j1,n1} = \sum_{j2\in J2, n2\in N2} W2_{t,s,i,j2,n2} \forall t \in T, s \in S_t, i \in I \tag{5}$$

Multiple ingots can be assigned to each event point of each furnace for processing, and the formula is shown in (6). The maximum of ingot can be assigned to each event point of each hot rolling mill is one, and the formula is shown in (7):

$$\sum_{i\in I} W1_{t,s,i,j1,n1} + V1_{t,s,j1,n1} \geq 1 \forall t \in T, s \in S_t, j1 \in J1, n1 \in N1 \tag{6}$$

$$\sum_{i\in I} W2_{t,s,i,j2,n2} + V2_{t,s,j2,n2} = 1 \forall t \in T, s \in S_t, j2 \in J2, n2 \in N2 \tag{7}$$

The event points with no assignment of ingot are arranged at the end of the furnace (hot rolling mill), as shown in Formulas (8) and (9):

$$V1_{t,s,j1,n1-1} \leq V1_{t,s,j1,n1} \forall t \in T, s \in S_t, j1 \in J1, n1 \in N1, n1 > 1 \tag{8}$$

$$V2_{t,s,j2,n2-1} \leq V2_{t,s,j2,n2} \forall t \in T, s \in S_t, j2 \in J2, n2 \in N2, n2 > 1 \tag{9}$$

The ingots of the same batch have the same starting time in the same furnace, and the formulas are shown in (10) and (11):

$$\begin{gathered} TS1_{t,s,i,j1,n1} \geq TS1_{t,s,i\prime,j1,n1} - SC(2 - W1_{t,s,i,j1,n1} - W1_{t,s,i\prime,j1,n1}) \\ \forall t \in T, s \in S_t, i, i\prime \in I, i \neq i\prime, j1 \in J1, n1 \in N1 \end{gathered} \tag{10}$$

$$\begin{gathered} TS1_{t,s,i,j1,n1} \leq TS1_{t,s,i\prime,j1,n1} + SC(2 - W1_{t,s,i,j1,n1} - W1_{t,s,i\prime,j1,n1}) \\ \forall t \in T, s \in S_t, i, i\prime \in I, i \neq i\prime, j1 \in J1, n1 \in N1 \end{gathered} \tag{11}$$

The production completion time of the ingot in the furnace is greater than or equal to the start time of production plus the processing time and is less than or equal to the start time of production plus the maximum residence time. The formulas are shown in (12) and (13). The production completion time of the ingot in the hot rolling mill is equal to the start time of production plus the processing time, and the formula is shown in (14):

$$TF1_{t,s,i,j1,n1} \geq TS1_{t,s,i,j1,n1} + TP1_c W1_{t,s,i,j1,n1} \forall t \in T, s \in S_t, i \in I_c, j1 \in J1, n1 \in N1, c \in C \tag{12}$$

$$TF1_{t,s,i,j1,n1} \leq TS1_{t,s,i,j1,n1} + TMW1_{t,s,i,j1,n1} \forall t \in T, s \in S_t, i \in I, j1 \in J1, n1 \in N1 \tag{13}$$

$$\begin{gathered} TF2_{t,s,i,j2,n2} = TS2_{t,s,i,j2,n2} + TP2_c W2_{t,s,i,j2,n2} \\ \forall t \in T, s \in S_t, i \in I_c, j2 \in J2, n2 \in N2, c \in C \end{gathered} \tag{14}$$

The start time of production for the ingot in the hot rolling mill is equal to the production completion time of the ingot in the furnace plus the procedure conversion time. The formula is shown in (15) and (16):

$$\begin{gathered} TS2_{t,s,i,j2,n2} \geq TF1_{t,s,i,j1,n1} + TC - SC(2 - W1_{t,s,i,j1,n1} - W2_{t,s,i,j2,n2}) \\ \forall t \in T, s \in S_t, i \in I, j1 \in J1, j2 \in J2, n1 \in N1, n2 \in N2 \end{gathered} \tag{15}$$

$$TS2_{t,s,i,j2,n2} \leq TF1_{t,s,i,j1,n1} + TC + SC(2 - W1_{t,s,i,j1,n1} - W2_{t,s,i,j2,n2}) \quad \forall t \in T, s \in S_t, i \in I, j1 \in J1, j2 \in J2, n1 \in N1, n2 \in N2 \tag{16}$$

In the furnace (hot rolling mill), the start time of production for the ingot assigned to the next event point is greater than or equal to the production completion time of the ingot assigned to the current event point, as shown in Formulas (17) and (18):

$$TS1_{t,s,i\prime,j1,n1} \geq TF1_{t,s,i,j1,n1-1} - SC(2 - W1_{t,s,i\prime,j1,n1} - W1_{t,s,i,j1,n1-1}) \quad \forall t \in T, s \in S_t, i, i\prime \in I, i \neq i\prime, j1 \in J1, n1 \in N1, n1 > 1 \tag{17}$$

$$TS2_{t,s,i\prime,j2,n2} \leq TF2_{t,s,i,j2,n2-1} + SC(2 - W2_{t,s,i\prime,j2,n2} - W2_{t,s,i,j2,n2-1}) \quad \forall t \in T, s \in S_t, i, i\prime \in I, i \neq i\prime, j2 \in J2, n2 \in N2, n2 > 1 \tag{18}$$

The start time of production and production completion time for the ingot in the furnace (hot rolling mill) are less than or equal to the makespan, and the formulas are shown in (19)–(22):

$$TS1_{t,s,i,j1,n1} \leq MS_{t,s} \forall t \in T, s \in S_t, i \in I, j1 \in J1, n1 \in N1 \tag{19}$$

$$TF1_{t,s,i,j1,n1} \leq MS_{t,s} \forall t \in T, s \in S_t, i \in I, j1 \in J1, n1 \in N1 \tag{20}$$

$$TS2_{t,s,i,j2,n2} \leq MS_{t,s} \forall t \in T, s \in S_t, i \in I, j2 \in J2, n2 \in N2 \tag{21}$$

$$TF2_{t,s,i,j2,n2} \leq MS_{t,s} \forall t \in T, s \in S_t, i \in I, j2 \in J2, n2 \in N2 \tag{22}$$

The total weight of the ingots processed by the furnace (hot rolling mill) is less than or equal to the maximum capacity of the furnace (hot rolling mill), and the formula is shown in (23) and (24). The total number of ingots assigned to the furnace for processing is less than or equal to the maximum number of ingots per batch that can be heated by one furnace. The formula is shown in (25):

$$\sum_{i \in I_c} W1_{t,s,i,j1,n1} H_c \leq M1 \forall t \in T, s \in S_t, j1 \in J1, n1 \in N1, c \in C \tag{23}$$

$$\sum_{i \in I_c} W2_{t,s,i,j2,n2} H_c \leq M2 \forall t \in T, s \in S_t, j2 \in J2, n2 \in N2, c \in C \tag{24}$$

$$\sum_{i \in I} W1_{t,s,i,j1,n1} \leq B(1 - V1_{t,s,j1,n1}) \forall t \in T, s \in S_t, j1 \in J1, n1 \in N1 \tag{25}$$

The total number of ingots processed in the furnace (hot rolling mill) must be greater than or equal to the demand, and the formulas are shown in (26) and (27):

$$\sum_{t\prime \leq t, i \in I_c, j1 \in J1, n1 \in N1} W1_{t\prime,s,i,j1,n1} \geq \sum_{t\prime \leq t} D_{t\prime,s,c} \forall t \in T, s \in S_t, c \in C \tag{26}$$

$$\sum_{t\prime \leq t, i \in I_c, j2 \in J2, n2 \in N2} W2_{t\prime,s,i,j2,n2} \geq \sum_{t\prime \leq t} D_{t\prime,s,c} \forall t \in T, s \in S_t, c \in C \tag{27}$$

## 5. Computational Studies

In order to verify the feasibility and effectiveness of the multi-objective optimization model, based on the description of the case data, this paper expounds the experimental results from the following three aspects: multi-objective optimization model solution, comparison of computational results in rolling horizon or a single decision time period, and analysis of the solution efficiency. The computer is configured with INTEL Core i5-5200U CPU @ 2.72GHz, 4G memory and GAMS software (XPRESS solver) version 23.7.3 under Windows 10 operating system.

### *5.1. Description of the Case Data*

This paper takes an aluminum alloy manufacturer as an example. There are two push-type furnaces, the heating time of the ingot in the furnace, which is related to the alloy type and ingot specification is between 420 and 900 min, and the maximum residence time is 2880 min. The maximum loading capacity of the furnace is 450 tons, with no more than 30 ingots inside. There is a "1 + 4" hot rolling production line, and the maximum weight of ingot that can be borne is 30 tons. The average rolling speed of a hot rough rolling mill is 96,000 mm/min, and 27 passes are needed. The average rolling speed of four hot finishing mills is 194,000 mm/min, and one pass is required. The production capacity is 40/10,000 t·a-1. The average adjustment time of the ingot from the furnace to the hot rolling mill is 6 min. The ingot specification is selected mainly based on the product size, specifications in the user's order, the capacity of the equipment and the technological level of the manufacturer. Six types of ingots are listed in Table 2 as examples, and the items shown in the table include the weight of the ingot, the processing time of the ingot in the furnace and the processing time of the ingot in the hot rolling mill. Compared with other ingots, ingot type 1 has a greater change in product demand. The scenario tree is used to describe the uncertainty of demand in rolling horizon, the demand $D_{3,s,c}$ and probability $P_{3,s}$ at $t = 3$ are obtained, as shown in Figure 3. In addition to the above parameters, other parameters may be arbitrarily valued. Table 3 shows the parameter settings of 10 groups of different data scales for subsequent computational research.

**Table 2.** Examples of Various Types of Ingots.

| Ingot Type | Alloy Type | Ingot Specification (mm) | Density (kg/m$^3$) | $H_c$(t) | $TP1_c$ (min) | $TP2_c$ (min) |
|---|---|---|---|---|---|---|
| c1 | 1100 | 480 × 1260 × 5000 | 2710 | 8 | 438 | 2 |
| c2 | 1145 | 600 × 2040 × 5000 | 2700 | 16 | 490 | 2 |
| c3 | 3003 | 600 × 2060 × 5000 | 2730 | 17 | 495 | 2 |
| c4 | 3104 | 465 × 1320 × 5000 | 2720 | 8 | 432 | 2 |
| c5 | 3104 | 610 × 1600 × 4150 | 2720 | 11 | 494 | 2 |
| c6 | 5083 | 600 × 1800 × 5000 | 2660 | 14 | 503 | 2 |

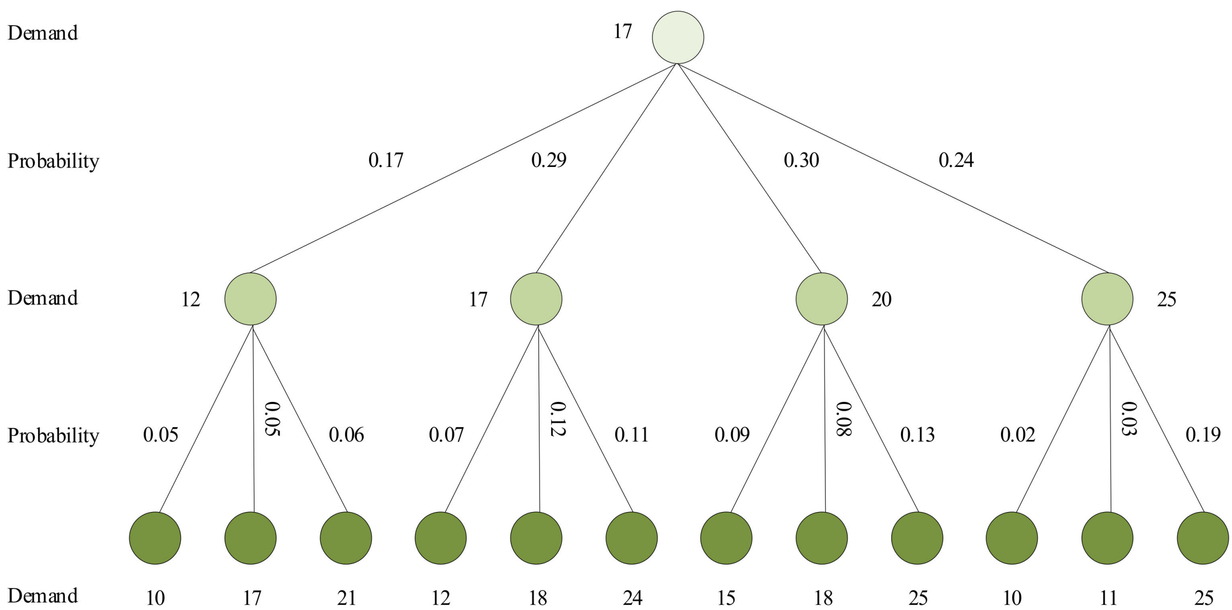


**Figure 3.** Demand for ingot type 1.

**Table 3.** Parameter Settings Under Different Data Scales.

| NO. | $\bar{t}$ | $\bar{s}$ | $\bar{i}$ | $\overline{n1}$ | $\overline{n2}$ | $S_t$ | $I_c$ | $D_{t,s,c}$ | SC (min) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 30 | 1 | 30 | 1 (1) | c1 (17); c2 (10); c3 (3) | c2 (10); c3 (3); c4 (0); c5 (0); c6 (0) | 14,400 |
| 2 | 1 | 1 | 36 | 1 | 36 | 1 (1) | c1 (17); c2 (10); c3 (9) | c2 (10); c3 (9); c4 (0); c5 (0); c6 (0) | 14,400 |
| 3 | 2 | 4 | 42 | 1 | 42 | 1 (1); 2 (1–4) | c1 (42) | c2 (0); c3 (0); c4 (0); c5 (0); c6 (0) | 14,400 |
| 4 | 2 | 4 | 48 | 1 | 48 | 1 (1); 2 (1–4) | c1 (42); c2 (6) | c2 (3); c3 (0); c4 (0); c5 (0); c6 (0) | 14,400 |
| 5 | 2 | 4 | 54 | 1 | 54 | 1 (1); 2 (1–4) | c1 (42); c2 (6); c3 (6) | c2 (3); c3 (3); c4 (0); c5 (0); c6 (0) | 14,400 |
| 6 | 2 | 4 | 60 | 1 | 60 | 1 (1); 2 (1–4) | c1 (42); c2 (6); c3 (6); c4 (6) | c2 (3); c3 (3); c4 (3); c5 (0); c6 (0) | 14,400 |
| 7 | 2 | 4 | 66 | 1 | 66 | 1 (1); 2 (1–4) | c1 (42); c2 (6); c3 (6); c4 (6); c5 (6) | c2 (3); c3 (3); c4 (3); c5 (3); c6 (0) | 14,400 |
| 8 | 2 | 4 | 72 | 1 | 72 | 1 (1); 2 (1–4) | c1 (42); c2 (6); c3 (6); c4 (6); c5 (6); c6 (6) | c2 (3); c3 (3); c4 (3); c5 (3); c6 (3) | 14,400 |
| 9 | 2 | 4 | 78 | 1 | 78 | 1 (1); 2 (1–4) | c1 (42); c2 (6); c3 (6); c4 (6); c5 (12); c6 (6) | c2 (3); c3 (3); c4 (3); c5 (6); c6 (3) | 28,800 |
| 10 | 2 | 4 | 84 | 1 | 84 | 1 (1); 2 (1–4) | c1 (42); c2 (6); c3 (6); c4 (6); c5 (12); c6 (12) | c2 (3); c3 (3); c4 (3); c5 (6); c6 (6) | 28,800 |

### 5.2. Multi-Objective Optimization Model Solution

As mentioned above, the production scheduling in the aluminum furnace hot rolling section is expressed as a multi-objective optimization model. In general, handling strategies designed to cope with multiple objectives are very critical, and the common methods include the weighted sum, epsilon-constraints and other relevant methods [66,67]. In addition, Pareto-optimal scheduling [64] is another important stream in multi-objective optimization.

#### 5.2.1. Analysis of the Weights Assigned to Different Objectives

This paper sets the objective weight $\alpha$ of the objective function $f1$ in Formula (1) and the objective weight $\beta$ of the objective function $f2$ in Formula (2). A linear weighting method is used to integrate the objective functions in Formulas (1) and (2) into one objective function, that is, $f = \alpha f1 + \beta f2$, $\alpha + \beta = 1$. In order to find proper objective weights, the objective function values of 10 groups of experiments were calculated with various objective weights, as shown in Table 4.

**Table 4.** Objective Function Values with Various Weights.

| $\alpha$ | Obj. (min) | NO. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Ave. (min) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | *f1* | 15,549 | 18,860 | 20,118 | 24,624 | 28,860 | 33,000 | 36,828 | 40,104 | 44,966 | 48,888 | 31,180 |
| | *f2* | 549 | 562 | 528 | 540 | 560 | 572 | 606 | 630 | 639 | 655 | 584 |
| 0.1 | *f1* | 15,310 | 18,528 | 19,854 | 23,770 | 27,642 | 30,660 | 34,918 | 38,456 | 43,036 | 47,874 | 30,005 |
| | *f2* | 554 | 562 | 528 | 566 | 599 | 592 | 624 | 647 | 661 | 677 | 601 |
| 0.2 | *f1* | 15,181 | 18,588 | 19,950 | 23,406 | 27,380 | 30,000 | 35,151 | 39,540 | 42,816 | 47,697 | 29,971 |
| | *f2* | 554 | 568 | 528 | 548 | 576 | 594 | 623 | 647 | 656 | 669 | 596 |
| 0.3 | *f1* | 15,349 | 18,566 | 20,022 | 23,458 | 27,122 | 31,710 | 35,314 | 39,102 | 42,598 | 47,713 | 30,095 |
| | *f2* | 552 | 569 | 528 | 562 | 592 | 621 | 631 | 645 | 652 | 667 | 602 |
| 0.4 | *f1* | 15,270 | 18,598 | 20,028 | 23,526 | 27,516 | 29,682 | 34,880 | 39,810 | 42,989 | 47,874 | 30,017 |
| | *f2* | 552 | 566 | 528 | 586 | 608 | 610 | 630 | 647 | 649 | 677 | 605 |
| 0.5 | *f1* | 15,270 | 18,566 | 19,910 | 24,144 | 27,651 | 32,910 | 35,980 | 38,926 | 42,598 | 46,219 | 30,217 |
| | *f2* | 552 | 565 | 528 | 580 | 607 | 616 | 633 | 633 | 652 | 659 | 603 |
| 0.6 | *f1* | 15,327 | 18,496 | 20,060 | 23,626 | 26,916 | 30,108 | 35,112 | 40,004 | 43,476 | 47,211 | 30,034 |
| | *f2* | 552 | 587 | 528 | 560 | 580 | 576 | 624 | 653 | 665 | 675 | 600 |
| 0.7 | *f1* | 15,327 | 18,505 | 19,998 | 23,222 | 26,872 | 30,540 | 34,174 | 38,669 | 42,798 | 46,704 | 29,681 |
| | *f2* | 552 | 566 | 528 | 542 | 564 | 594 | 608 | 643 | 654 | 666 | 592 |
| 0.8 | *f1* | 15,327 | 18,511 | 19,986 | 23,598 | 27,407 | 30,738 | 35922 | 39,325 | 43,966 | 46,843 | 30,162 |
| | *f2* | 552 | 567 | 528 | 552 | 587 | 593 | 630 | 641 | 656 | 675 | 598 |
| 0.9 | *f1* | 15,308 | 18,660 | 19,988 | 23,682 | 27,906 | 30,520 | 34,584 | 38,958 | 41,932 | 47,697 | 29,924 |
| | *f2* | 552 | 568 | 528 | 566 | 588 | 584 | 619 | 645 | 652 | 669 | 597 |
| 1 | *f1* | 14,847 | 18,487 | 19,654 | 23,106 | 26,672 | 29,600 | 32,388 | 33,880 | 35,844 | 46,019 | 28,050 |
| | *f2* | 561 | 590 | 972 | 1088 | 1110 | 1122 | 1134 | 1162 | 1174 | 1186 | 1010 |

The following conclusions can be drawn from Table 4: First, with any combination of objective weights, when the decision time period $t$ is the same, the value of the objective function $f1$ (or $f2$) continues to rise with the increase of the data scale. This is because the increase in the total number of ingots makes the total residence time of the ingot in the furnace (or the total makespan) longer. Second, the data scale of experiment 3 is larger than that of experiments 1 and 2. Why is the value of the objective function $f2$ smaller? The reason is that experiment 3 only processed ingot type c1, whose processing time in the furnace was 438 min, while experiments 1 and 2 processed ingot types c1, c2 and c3, whose processing times in the furnace were 438 min, 490 min and 495 min, respectively. The processing time of experiment 1 and 2 in the furnace is longer than that of experiment 3, so the makespan of experiment 1 and 2 is longer than that of experiment 3. Third, with any combination of objective weights, for $\alpha = 0, \beta = 1$, the value of the objective function $f1$ is the largest and the value of the objective function $f2$ is the smallest. For $\alpha = 1, \beta = 0$, the result is just the opposite, indicating that in the solution process, one objective is optimized to the greatest extent, while the other objective is not optimized at all.

In order to show the trend of the objective function value changing with the weight more clearly, in Figure 4, the horizontal coordinate represents the weight of the objective function $f1$, that is, the value of the objective weights $\alpha$, correspondingly, $\beta = 1 - \alpha$. The left vertical coordinate represents the residence time of the ingot in the furnace (environmental dimension index), the right vertical coordinate represents the makespan (economic dimension index), the blue solid line represents the average value of the objective function $f1$ of the 10 experiments, and the orange solid line represents the mean value of the objective function $f2$ of 10 experiments.

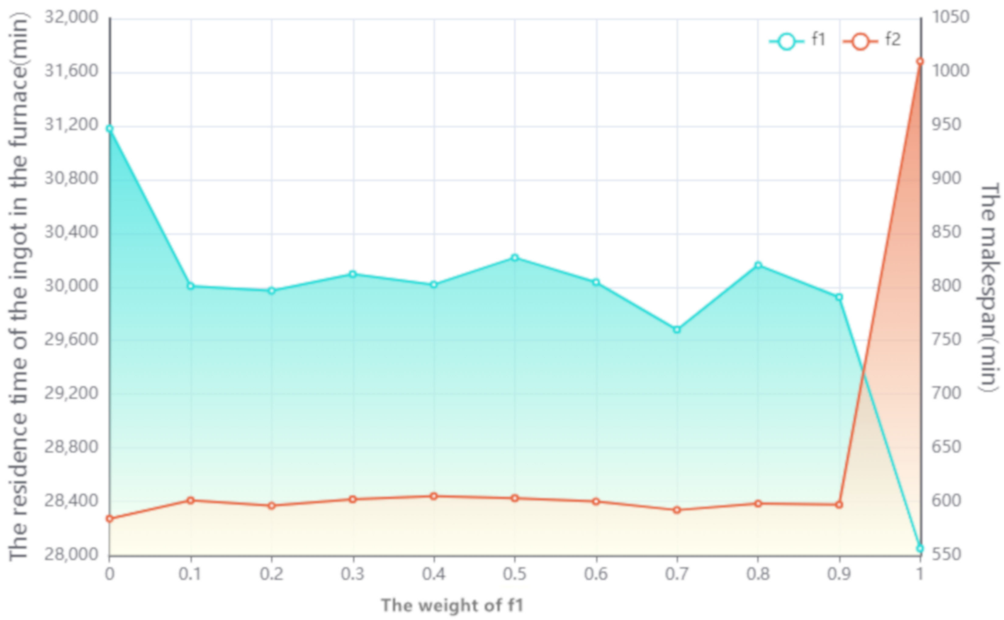


**Figure 4.** The trend of the objective function value changing with the weight.

The following conclusions can be drawn from Figure 4. Firstly, optimizing one indicator in the environmental or economic dimension will make the other indicator rise rapidly, and it is more conducive to sustainable comprehensive development when considering environmental and economic dimensions at the same time. Secondly, the change of the economic dimension indicators with the weights shows a trend of straight line. The change in the average value of the 10 experiments is very small. In contrast, the environmental dimension indicators are more sensitive and have a wider range of fluctuations. Hence, we can pay more attention to the environmental dimension indicator. Thirdly, for $\alpha = 0.7, \beta = 0.3$, the values of the environmental and economic dimension indicators are 29,681 min and 592 min, respectively, which are both the minimum within

$\alpha \in [0.1, 0.9]$. It is able to optimize both the environmental and economic dimension indicators to the greatest extent, indicating that the objective weight combination of $\alpha = 0.7$, $\beta = 0.3$ is optimal.

5.2.2. Analysis of the Pareto-Optimal Scheduling

The linear weighting method used in this paper only finds a "corner" point, to determine a line of a set of efficient solutions (in the Pareto-optimal sense) between the two objectives, a medium-scale instance is presented in this paper. Set $\bar{t} = 2$, $\bar{s} = 4$, $\bar{i} = 50$, $\overline{n1} = 1$, $\overline{n2} = 50$, $S_t = 1(1); 2(1-4)$, $I_c = c1(42); c2(6); c3(2)$, $D_{t,s,c2} = 3$, $D_{t,s,c3} = 1$, $SC = 14,400\,\text{min}$, the Pareto frontier regarding the residence time of the ingot in the furnace and the makespan are obtained, as shown in Figure 5.

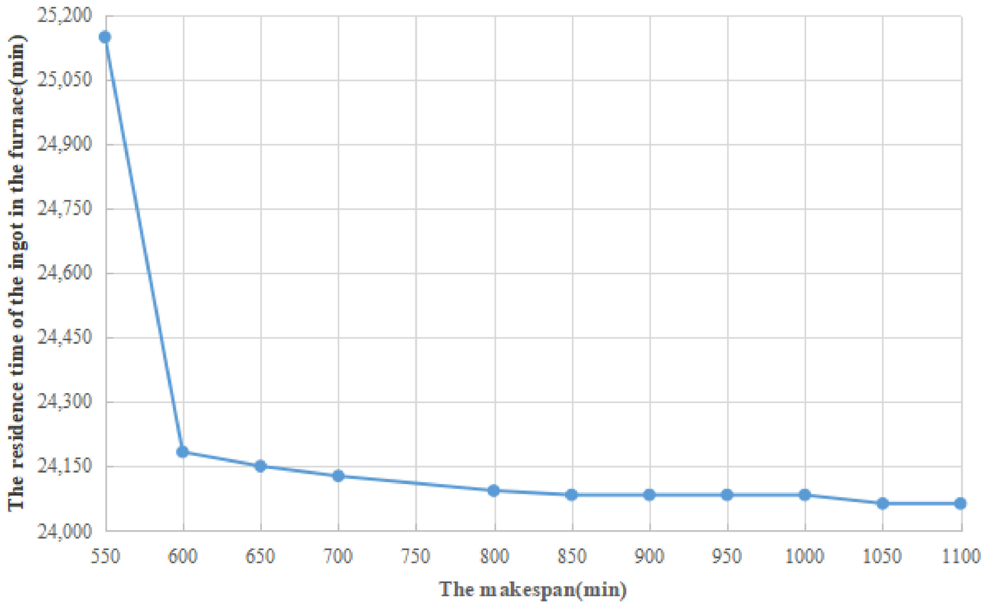


**Figure 5.** The Pareto frontier regarding the sustainability of environmental and economic dimensions.

The following can be observed in Figure 5. Firstly, as the makespan increases, the residence time of the ingot in the furnace first shows a downward trend and then remains stable. Secondly, when the $makespan \in [550,\ 600]$ min and the $makespan \in [600,\ 700]$ min the makespan increased by 50 min and 100 min, respectively, whereas the residence time of the ingot in the furnace decreased by 966 min and 56 min. Overall, the appropriate relaxation of the makespan can impose a significant decrease on the residence time of the ingot in the furnace, so considering the trade-off between environmental and economic dimensions in the light of practical demands is necessary. Thirdly, when the $makespan \in [700,\ 850]$ min, $makespan \in [850,\ 1000]$ min,$makespan \in [1000,\ 1050]$ min and $makespan \in [1050,\ 1100]$ min, the makespan increased by 150 min, 150 min, 50 min and 50 min, respectively, whereas the residence time of the ingot in the furnace decreased by 44 min, 0 min, 20 min and 0 min. Therefore, even though the requirements for the makespan are relaxed, the residence time of the ingot in the furnace only has a slight reduction. At this point, the optimization of the makespan should be highlighted. Fourthly, the residence time of the ingot in the furnace remains stable when the $makespan \in [850,\ 1000]$ min. However, why did the residence time of the ingot in the furnace decrease by 20 min when the $makespan \in [1000,\ 1050]$ min? When the makespan is increased to about 1050 min, the production scheduling is changed from a single decision time period to a two-decision time period, which results in a slight decrease of the residence time of the ingot in the furnace. Similar examples can be found in Figure 6.

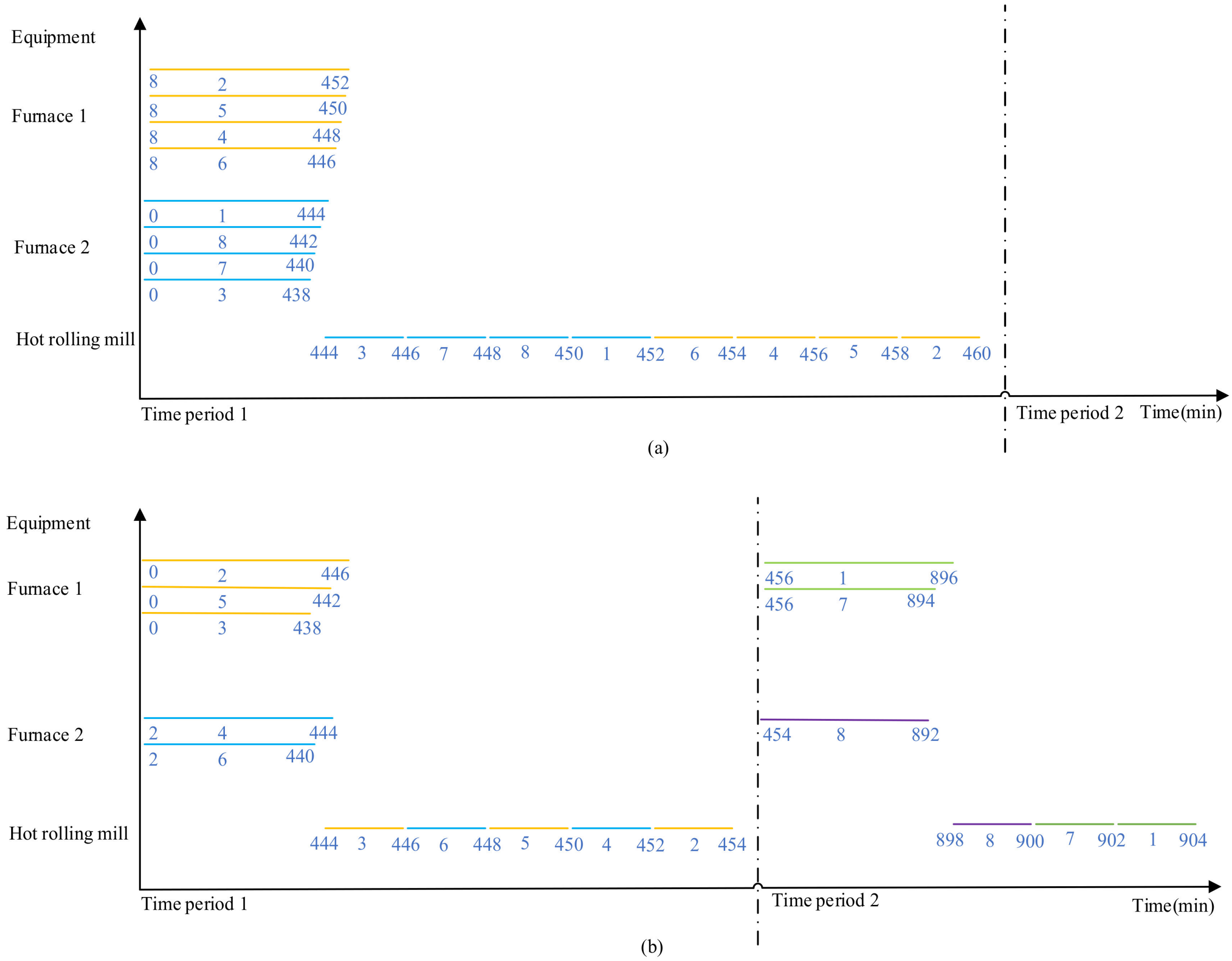


**Figure 6.** Scheduling schemes in rolling horizon or a single decision time period. (**a**) shows the scheduling schemes in rolling horizon, (**b**) shows the scheduling schemes in a single decision time period.

### 5.3. Comparison of Computational Results in Rolling Horizon or a Single Decision Time Period

A single decision time period divides the time evenly and only considers the constraints and objectives in each decision time period. The rolling horizon puts the decision time periods together and considers the constraints and objectives in all decision time periods globally. In order to explore the superiority of the scheduling scheme in the rolling horizon, 10 experiments with the objective weights of $\alpha = 0.7, \beta = 0.3$ were conducted in this paper, and the computational results in the rolling horizon or a single decision time period obtained are shown in Table 5.

**Table 5.** Computational results in Rolling Horizon or a Single Decision Time Period.

| Periods | Obj. (min) | NO. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Ave. (min) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rolling horizon | *f1* | 15,327 | 18,505 | 19,998 | 23,222 | 26,872 | 30,540 | 34,174 | 38,669 | 42,798 | 46,704 | 29,681 |
| | *f2* | 552 | 566 | 528 | 542 | 564 | 594 | 608 | 643 | 654 | 666 | 592 |
| | *f* | 10,895 | 13,123 | 14,157 | 16,418 | 18,980 | 21,556 | 24,104 | 27,261 | 30,155 | 32,893 | 20,954 |
| Single | *f1* | 15,327 | 18,505 | 19,828 | 23,204 | 26,852 | 30,523 | 34,136 | 38,667 | 42,783 | 46,688 | 29,651 |
| | *f2* | 552 | 566 | 965 | 1066 | 1071 | 1098 | 1124 | 1159 | 1167 | 1182 | 995 |
| | *f* | 10,895 | 13,123 | 14,169 | 16,563 | 19,118 | 21,696 | 24,232 | 27,415 | 30,298 | 33,036 | 21,054 |

In addition, a simple example can straightforwardly illustrate the superiority of the scheduling scheme in the rolling horizon. Let $\bar{t} = 2$, $\bar{s} = 4$, $\bar{i} = 8$, $\overline{n1} = 1$, $\overline{n2} = 8$, $S_t = 1(1); 2(1-4)$, $I_c = c1(8)$, $D_{1,s,c1} = 5$, $D_{2,s,c1} = 3$, $SC = 14,400\,\text{min}$, $B = 4$ and $\alpha = 0.7, \beta = 0.3$, and the scheduling plan in rolling horizon or a single decision time period can be obtained. In Figure 6, the horizontal coordinate represents time, the vertical coordinate represents the equipment, the numbers just below the line segment represents the ingot labels, the lower left of the line segment represents the start time of production and the lower right of the line segment represents the completion time of production.

The following conclusions can be seen. First, in experiments 3–10 in Table 5, the comparison between the rolling horizon and a single decision time period show that the objective function value of the former ($f1$) is slightly larger than that of the latter, while the objective function value of the latter ($f2$) is much larger than that of the former. The value of the objective function of the former ($f$) obtained is better than that of the latter, and the mean value of the objective function ($f$) is reduced by 100 min, which verified that the scheduling in rolling horizon can globally optimize the production in the aluminum furnace hot rolling section while satisfying the constraint conditions and is more conducive to the sustainable development of the environmental and economic dimensions. Second, Figure 6a,b show the scheduling schemes in rolling horizon or a single decision time period, respectively. This example can illustrate in detail that scheduling in the rolling horizon is better than scheduling in a single decision time period. Compared with Figure 6b, it can be seen from Figure 6a that, assuming the capacity constraint conditions of the furnace are satisfied, when the ingots 1, 7 and 8 that should have been processed in the decision time period 2 are processed in the decision time period 1, the total makespan is reduced from 904 min to 460 min, and the total residence time of the ingot in the furnace is increased from 3523 min to 3528 min. In general, the overall indicators of environmental and economic dimensions can be effectively optimized.

### 5.4. Analysis of the Solution Efficiency

It is well known that the model scale and the solution time are closely linked. To better analyze the solution efficiency of the production scheduling model of the aluminum furnace hot rolling section, 10 experiments with the objective weights of $\alpha = 0.7, \beta = 0.3$ were conducted in this paper. The total number of discrete variables, continuous variables, equations and the solution time for each experiment are shown in Table 6.

**Table 6.** Experimental Results of Different Data Scales.

| NO. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Ave. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Discrete variables | 992 | 1406 | 15,004 | 19,450 | 24,472 | 30,070 | 36,777 | 43,575 | 50,949 | 58,899 | 28,159 |
| Continuous variables | 2916 | 4146 | 33,492 | 43,458 | 54,720 | 67,278 | 82,325 | 97,583 | 114,137 | 131,987 | 63,204 |
| Equations | 35,508 | 58,806 | 474,260 | 688,282 | 958,308 | 1,290,818 | 1,780,899 | 2,274,514 | 2,851,078 | 3,517,426 | 1,392,990 |
| Solution time(s) | 8 | 34 | 45 | 960 | 987 | 1080 | 1116 | 1178 | 1271 | 1515 | 819 |

The following conclusions can be inferred based on the results in Table 6. Firstly, with regard to the average number of the experiments solved by GAMS (Xpress solver), there are 28,159 for discrete variables, 63,204 for continuous variables and 1,392,990 for equations on average. The large-scale model indicates the complexity of the scheduling model. Secondly, with the increase of data scale, the total number of discrete variables, continuous variables, equations and the solution time increases. Thirdly, when the values of decision time period and the scenario are 1, the average solution time is 21 s, where there is a high responding time. When the values of the decision time period and the scenario are two and four, respectively, the average solution time is 1019 s. Compared with manual scheduling, the response time solved in this paper is acceptable. When the values of decision time period and the scenario are 3 and 12, respectively, the solution time is more than 24 h. When the value of decision time period is greater than 3 and the value of the scenario is more than 12, the solution time may be longer. It can be seen that with the increasing number of decision time periods and scenarios, the solution becomes more and more difficult. Fourthly, it is sufficient and reasonable to consider 3 decision time periods and 12 scenarios in the practical application and to not recommend further increase the data scale for solving. The reasons are shown as follows. The demand is predicted based on historical data, when the decision time period is less, the predicted value of the demand is more accurate. As the decision time period continuously increases, the predicted value accuracy will be lower and lower, which accords with the law of data analysis. Therefore, if the number of decision time periods and scenarios is too large, even though the scheduling scheme is solved, it is not of great reference to managers.

## 6. Conclusions

This paper mainly studies the sustainable scheduling of the production in the aluminum furnace hot rolling section with uncertain demand. The scenario tree is used to describe the uncertainty of demand in the rolling horizon. A sustainable scheduling model for the production in the aluminum furnace hot rolling section in environmental and economic dimensions is formulated. Therefore, the energy consumption of furnaces and the time cost of product delivery can be reduced.

On the one hand, in view of the uncertainty of the demand in the production process of the aluminum furnace hot rolling section, the scenario tree method is used for description. To the best of our knowledge, this is the first time that the scenario tree method is applied to the production scheduling in the furnace hot rolling section. In actual application, the method used can not only accurately predict the demand in rolling horizon, but also globally optimize the production process in the aluminum furnace hot rolling section, indicating that it is more conducive to the sustainable development in environment and economic dimensions than scheduling in a single decision time period.

On the other hand, according to the production process and characteristics of the aluminum furnace hot rolling section, to minimize the residence time of the ingot in the furnace and minimize the makespan, a multi-objective MILP optimization model for the sustainable scheduling of the production in the aluminum furnace hot rolling section is formulated. The model not only expands the multi-objective optimization model library in this field theoretically, but can also provide managers with a sustainable scheduling plan that is optimized in both environmental and economic dimensions in practical applications. Increasingly, managerial suggestions associated with the trade-off between environmental and economic dimensions are presented.

Future research is necessary to focus on the following: When making decisions on scheduling plans, managers must not only judge the rationality of the plan, but also consider the response time of the solution model in order to choose a high-quality and high-efficiency scheduling plan. A high-quality scheduling scheme can be obtained in this paper currently, however, with the increase in the data scale and in the total number of variables and equations, the model solving time will be longer or even unable to be solved. Therefore, more attention will be focused on improving the optimization algorithm and improving the solving efficiency of the production scheduling model in the aluminum furnace hot rolling section in future research. Fortunately, the intelligent optimization algorithm [15,21,22,26,27] proposed by similar research works have provided excellent references.

**Author Contributions:** All of the authors have contributed to the idea of this paper. Y.H. developed the mathematical model, wrote the whole manuscript and designed the experiments. Q.Y. has significantly contributed to the design of the system. J.L., X.L. and J.Z. have contributed equally to the supervision of all the activities of this work. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research is supported by the projects including the Research Project of Science and Technology of Chengdu (Soft Science Research) (No. 2020-RK00-00052-ZF), the Planning Project of Philosophy and Social Science in Chengdu (No. 2020ZXB10), the MOE Layout Foundation of Humanities and Social Sciences (No. 20YJA630079), the Research Project of Sichuan Normal University (No. SNU19J4Z2019-11) and the Undergraduate innovation and entrepreneurship training program of Sichuan Normal University (No. 202010636246).

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** There is a little data involved in the experiments, and detailed are elaborated in Section 5.1, where all experimental data can be found.

**Conflicts of Interest:** The authors declare no conflict of interest.

## Glossary

| | |
|---|---|
| | Indices and main Sets |
| $T$ | Set of decision time periods, that is, the decision time period in rolling horizon, indexed by $t$ and $t \in T = \{1, 2, \ldots, \bar{t}\}$. |
| $S$ | Set of scenarios, the scenarios indicate that the demand may appear in each decision time period, indexed by $s$ and $s \in S = \{1, 2, \ldots, \bar{s}\}$. |
| $I$ | Set of ingots, indexed by $i$ and $i \in I = \{1, 2, \ldots, \bar{i}\}$. |
| $J1$ | Set of furnaces, indexed by $j1$ and $j1 \in J1 = \{1, 2, \ldots, \overline{j1}\}$. |
| $J2$ | Set of hot rolling mill, indexed by $j2$ and $j2 \in J2 = \{1, 2, \ldots, \overline{j2}\}$. |
| $N1$ | Set of event points in the furnace, that divides time points on the time axis of the furnace-based continuous time representation of unit-specific events [68], indexed by $n1$ and $n1 \in N1 = \{1, 2, \ldots, \overline{n1}\}$. |
| $N2$ | Set of event points in the hot rolling mill that divides time points on the time axis of hot rolling-mill-based continuous time representation of unit-specific events [68], indexed by $n2$ and $n2 \in N2 = \{1, 2, \ldots, \overline{n2}\}$. |
| $C$ | Set of ingot types, indexed by $c$ and $c \in C = \{1, 2, \ldots, \bar{c}\}$. |
| $S_t$ | Scenario sets of decision time period $t$. |
| $I_c$ | Set of ingots belonging to the ingot of type $c$. |
| | Parameters |
| $P_{t,s}$ | Probability of the occurrence of scenario $s$ in decision time period $t$. |
| $TP1_c$ | Processing time of the type $c$ ingot in the furnace. |
| $TM$ | Maximum residence time of the ingot in the furnace. |
| $TP2_c$ | Processing time of the type $c$ ingot in the hot rolling mill. |
| $TC$ | The time of the ingot transferred from the furnace to the hot rolling mill. |
| $H_c$ | Weight of the type $c$ ingot. |
| $M1$ | Maximum capacity of the furnace. |
| $M2$ | Maximum capacity of the hot rolling mill. |
| $B$ | The maximum number of ingots per batch that can be heated in a furnace |
| $D_{t,s,c}$ | Demand for the type $c$ ingot in scenario $s$ in decision time period $t$. |
| $SC$ | Scheduling cycle period. |
| | Variables |
| $W1_{t,s,i,j1,n1}$ | 1 if ingot $i$ is assigned to event point $n1$ in furnace $j1$ for processing in scenario $s$ in decision time period $t$, 0 otherwise. |
| $W2_{t,s,i,j2,n2}$ | 1 if ingot $i$ is assigned to event point $n2$ in hot rolling mill $j2$ for processing in scenario $s$ in decision time period $t$, 0 otherwise. |
| $V1_{t,s,j1,n1}$ | 1 if event point $n1$ in furnace $j1$ in scenario $s$ in decision time period $t$ is idle, 0 otherwise. |
| $V2_{t,s,j2,n2}$ | 1 if event point $n2$ in hot rolling mill $j2$ in scenario $s$ in decision time period $t$ is idle, 0 otherwise. |
| $TS1_{t,s,i,j1,n1}$ | The start time of the ingot $i$ at event point $n1$ in furnace $j1$ based on scenario $s$ in decision time period $t$. |
| $TF1_{t,s,i,j1,n1}$ | The completion time of ingot $i$ at event point $n1$ in furnace $j1$ based on scenario $s$ in decision time period $t$. |

| | |
|---|---|
| $TS2_{t,s,i,j2,n2}$ | The start time of ingot $i$ at event point $n2$ in hot rolling mill $j2$ based on scenario $s$ in decision time period $t$. |
| $TF2_{t,s,i,j2,n2}$ | The completion time of ingot $i$ at event point $n2$ in the hot rolling mill $j2$ based on scenario $s$ in decision time period $t$. |
| $MS_{t,s}$ | The makespan based on scenario $s$ in decision time period $t$. |

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