

Article

Demand-Oriented Train Timetabling Integrated with Passenger Train-Booking Decisions

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Abstract: In recent years, with the global energy shortage and severe environmental deterioration, railway transport has begun to attract great interest as a green transportation mode. One of the vital means to realize social sustainable development is to improve railway transportation systems, in which providing a demand-oriented train timetable with a higher service level is the most viable method. A demand-oriented train timetable problem generally deals with passengers' train-choice decisions according to the queue principle, but it is not adapted to rail systems, such as China's, where passengers usually book tickets a few days in advance by telephone or online instead of going to stations. This paper is devoted to modeling and solving the demand-oriented train timetabling problem integrated with passengers' train-booking decisions. Firstly, a bi-level programming model is formulated for their integrated optimization on a rail network. Its upper-level model is to optimize train arrival and departure times at each visited station with the aim of reducing passengers' total travel cost, while its lower-level model aims to determine passengers' train-booking behavior using the user equilibrium theory. Then, a priority-based heuristic algorithm is designed to solve this model. It has two main steps at each iteration: one is to determine the number of passengers booking each train with a given train timetable, and the other is to improve the current train timetable based on the valuable information of passenger train-booking decisions. The performance, convergence, and practicability of the proposed method were analyzed based on the Changsha–Zhuzhou–Xiangtan intercity rail in China. Experimental results show the proposed method can effectively reduce the travel cost for passengers, creating a greater passenger demand for railway travel, which is beneficial to the sustainable development of railway systems and even society.

Keywords: train timetabling; passenger train-booking; demand-oriented; priority-based heuristic algorithm; weighted directed graph

1. Introduction

A train timetable specifies the arrival and departure times of trains at stations on a rail network. It is not only the cornerstone of train organization and station operation, but also the foundation of passengers choosing trains for traveling. Facing the high requirements of passengers at a service-level and the fierce competition with expressway and air transport in the passenger market, railways have become less attractive to passengers. However, the railways are actually an important part of a green transportation system for social sustainable development as they have a large capacity, energy-saving, and environmental protection capabilities. Therefore, it is very important and urgent for rail enterprises to provide passengers with higher service-level train timetables. Firstly, this can attract more passengers to travel by rail instead of road and air, which can effectively reduce environmental pollution and energy consumption. Secondly, with the improvement of the service level of the train timetable, the transport market competitiveness of the railway will be further improved, which is beneficial for the

sound development of railways. Moreover, a better train timetable will contribute to reducing the usage numbers of rail crews, locomotives or electric multiple units. In short, the service-level improvement of train timetables for passengers will necessarily contribute to the sustainable development of railway systems and society.

Nevertheless, most early existing studies about train timetabling have not fully integrated the consideration of the time-dependent origin-to-destination (OD) passenger demands in the train timetabling process, and have usually aimed to optimize a train timetable with the objective of reducing trains' travel time or their delay at stations. This is partly due to the difficulty of determining passengers' train-choices for each OD demand. For example, Higgins et al. [1] only aimed to obtain a train timetable with minimum train delay by reducing train dwell time at stations. Burdett et al. [2] minimized train travel time when optimizing train timetable using a hybrid job shop approach. Zhou et al. [3] built an integer linear programming model with many 0–1 binary and non-negative integer decision variables, and proposed a Lagrangian relaxation decomposition approach to minimize train travel time. Bešinović et al. [4] optimized train timetable with the goal of minimizing train travel time and used heuristic algorithms to solve the problem. Based on the construction of a weighted directed graph, Zhou et al. [5] proposed a multi-path searching model with the aim of minimizing the total travel time of all period-types of trains. As for reducing trains' delay, Goverde [6] used a max-plus system theory to illustrate a rail timetable stability measure, and then further analyzed train delay propagation processes. Corman et al. [7] employed a bi-objective model aiming to minimizing trains' delays for obtaining a set of feasible non-dominated schedules. Xu et al. [8] combined genetic algorithm into travel advanced strategy for finding a train timetable with the lowest delay ratio. In addition, some studies adopted minimizing total operating cost, maximizing total profit, etc. as the objective of train timetabling. For instance, Brannlund et al. [9] aimed to find a profit-maximizing train timetable. Robenek et al. [10] optimized the timetable with the goal of maximizing the revenue of operating company based on passenger satisfaction. Li et al. [11] tried to find a train timetable with the minimum energy consumption. Other studies focused on avoiding conflicts among trains in the process of timetabling; for example, Carey et al. [12] and Caimi et al. [13] developed microscopic models for finding and avoiding conflicts among trains with constraints of various operational needs. Dundar et al. [14] formulated a decision support tool to resolve inter-train conflicts for the single-track railway train re-scheduling problem.

Besides the many studies on rail train timetabling, many scheduling studies on roads, airplane fields, etc. are also worth mentioning. For example, Marguier et al. [15] used mathematical programming method and heuristic search algorithm to optimize the bus schedule under the condition of reduced fleet size. Herbon et al. [16] combined bus operation cost and passenger satisfaction to study the bus departure schedule problem. Yang et al. [17] designed a model to solve the timetable problem for Yizhuang Line of Beijing Metro by using genetic algorithm, which aims to reduce energy consumption. Zheng et al. [18] presented a model for emergency railway wagon scheduling with the aim of minimizing the weighted time of delivering all required supplies to affected areas. Dobson and Lederer [19] presented a model for airline choice of flight schedule and route prices to maximize profit considering competitors decisions. Jiang et al. [20] introduced a dynamic airline scheduling method, which showed that making minor adjustments to flight schedules in the booking period by re-fleeting and re-timing flight legs can significantly improve utilization of capacity and hence increase profit.

In recent years, the demand-oriented train scheduling problem has attracted much attention. This kind of problem, called demand-oriented train timetabling problem (DOTTP), is usually to find an optimal or satisfactory train timetable for minimizing the total cost of passengers such as in-vehicle time, wait time and transfer time. Parbo et al. [21] illustrated that the importance of creating a demand-oriented railway timetable consists in offering the best service for passengers under some limited resources. Therefore, it is necessary to consider the train-choice decisions of passengers in train timetabling. For instance, Yu et al. [22] and Ma [23] employed a bi-level programming model whose objective was to minimize passengers' total travel time with the consideration of the route

choice behaviors of the users. Wang et al. [24] employed a deterministic Frank–Wolfe loading approach to solve the problem of passengers' route choice behavior. Cordone et al. [25] and Parbo et al. [26] presented a mixed integer nonlinear programming model to capture the feedback mechanism between transport offer and passenger demand. These studies on passengers' train-choice usually regarded passengers' train-choices as a feedback of current timetables, and applied it as a useful information for further improving this train timetable, while providing passengers better service based on the improved train timetable. This is particularly crucial nowadays for rail enterprises, which aim to minimize their total operating cost and maximizing total profit when facing the high requirement of passengers on service-level and the fierce competition from expressways and air transport. Therefore, some studies took the interests of both passengers and operators into account to optimize train timetables. For example, Kaspi et al. [27] and Canca et al. [28] presented a scheduling optimization model that takes both the user and the operator points of view into account. Yang et al. [29] focused on designing a passenger-oriented timetable to reduce the passenger travel time and energy consumption simultaneously. Other existing studies on passenger demand-oriented train scheduling mainly aimed to reduce the waiting times or the total travel times, which are two fundamentally important elements for designing timetables. As for reducing the waiting times, for example, Barrena et al. [30] proposed three formulations for the train-scheduling problem with the aim of minimizing the passenger waiting time and a branch-and-cut algorithm to design train schedules under a dynamic demand environment. Niu et al. [31] proposed nonlinear mixed integer optimization models for the train scheduling problem with the objective of minimizing the total passenger waiting time at stations under time-dependent passenger demand and predetermined stop-skipping patterns. As for minimizing the total travel times, for instance, Yang et al. [32] developed a two-stage stochastic integer programming model to minimize the expected travel time and penalty value incurred by transfer activities. Johan et al. [33] proposed combined microscopic simulation and macroscopic timetable optimization approach for minimizing travel time and delays in railway timetables. Zhang et al. [34] formulated two nonlinear non-convex programming models to design timetables with the objective of minimizing the total passenger travel time under the constraints of train operations, passenger boarding and alighting processes and congested conditions.

The problem of scheduling integrated with demands in road, airplane fields, etc. have also attracted more attention of researchers. For instance, Ibarra-Rojas et al. [35] developed a bi-objective, integer programming model to maximize the number of bus passengers benefiting from well-timed transfers and minimize vehicle operating costs. A more complex bi-objective and bi-level approach was presented by Liu et al. [36] and Liu et al. [37], who studied how much the changes on timetable and vehicle scheduling affect users' trip choice behavior. Wang et al. [38] established a demands-matching orientated multiple criteria decision making method for reverse logistics, including weights derivation, rank of collection modes, sensitivity analysis, and quantification of demands matching degree. Jiang et al. [20] introduced the concept of dynamic flight scheduling with the objective is to match the passenger demand and aircraft capacity for all flights. Abdelghany et al. [39] explicitly considered competition among airlines and passengers' demand elasticity by integrating an airline competition analysis model in the scheduling process that captures the passengers' itinerary choice behavior to enable evaluation of the profitability of the developed schedule.

To summarize, most studies deal with the demand-oriented train timetabling problem decided passenger train-choices mainly according to the queue principle, thus these passengers should wait at stations for a train arriving and then get on it one by one until its capacity is reached. For instance, Niu et al. [40] assumed that all passengers boarding a train obey the First-In-First-Out (FIFO) principle; if passengers are not be able to board the next arrival train, then they are forced to wait in queues for the following trains. Barrena et al. [30] obtained the origin–destination matrices based on the available passenger arrival data between stations during each time interval, and then these ordered passengers waited at relative stations for trains to get on without exceeding the capacity of trains. Passengers who are not able to board the current train should wait at this station for the next train. Wang et al. [41]

divided passenger flows into multiple sub-flows so that they can choose different platforms to wait for trains obeying the queue principle. Niu et al. [31] viewed passenger queues in the train timetabling problem as a bulk queue to compute the cumulative number of departure passengers at stations to minimize the total passenger waiting time. Wang et al. [42] considered trains run in a first-in first-out order from one station to another station, thus passengers traveling between two consecutive stations board a train one by one until the train's capacity is achieved. In the accurate passenger flow control strategies Shi et al. [43] proposed, passengers under control are required to queue up outside the entrance facilities. Moreover, no passengers will give up their trips or select other transportation modes. Besides, a few studies distributed all passengers of each interval-based demand on their shortest cost path. For instance, Fu et al. [44] assumed that passengers select their most time-saving path from all the available routes in a train service network.

Obviously, such a condition of passengers arriving at origin station to buy tickets and then waiting in queue there for the earliest arrival available train is adapt to the rail system of many countries such as Japan. However, it does not happen for the rail system in China, where passengers usually book tickets one day or a few days in advance. They usually do not need to go to the station to book tickets, but purchase them using many other methods, such as telephone and online booking. After that, they go to origin stations dozens of minutes in advance for boarding their booked trains on time. Thus, passengers usually do not need to wait for a booked train, and their choices of booking trains depend on not only the total travel time and ticket fare from origin to destination, but also the deviation time, which is the absolute value of the difference between train's departure time and passengers' planned departure time. Generally, they prefer to book a train whose departure time at origin station is close to their planning departure time when having the same other costs. Although train-booking decisions is also a train-choice decision, it is different from that based on queue rules, e.g. it considers the deviation time, rather than the wait time at origin station. Therefore, the existing methods of deciding passenger train-choices based on queuing theory are not efficient for this case as they do not consider passengers' ticket booking decisions.

This paper is devoted to solving the demand-oriented train timetabling problem (DOTTP) integrated with train-booking decisions of passengers in the planning stage. Specifically, we use the user equilibrium theory to simulate passengers' train-booking decisions based on an obtained train timetable, combined with the simulation results, to optimize the train timetable at each iteration. The main contribution of our research is that we propose a method for optimizing train timetable combining with passenger' train-booking decisions, and this method can be effectively adopted in rail systems that allow train booking. Compared to the existing literature, the following two specific contributions are provided in this paper:

1. A bi-level programming model is built for the demand-oriented train timetabling problem, combining it with passenger' train-booking decisions. Its upper-level model is intended to optimize train timetables, while its lower-level model focuses on determining the trains booked by passengers based on a given train timetable.
2. A priority-based heuristic algorithm is designed to solve the proposed bi-level programming model. It has two main steps at each iteration, one is to determine the number of passengers booking each train with a given train timetable, and the other is to improve the current train timetable based on the valuable information of passenger's train-booking decisions.

The rest of this paper is organized as follows. The problem definition is introduced in Section 2, a bi-level programming model of train timetabling is built in Section 3, and then its priority-based heuristic search algorithm is designed in Section 4. Moreover, the convergence, effectiveness and practicability of the proposed model and its solving algorithms are tested based on the Changsha–Zhuzhou–Xiangtan intercity railway in Section 5. Finally, conclusions and future studies are summarized in Section 6.

2. Problem Definition

2.1. Assumptions

Our research is based on the following two assumptions:

1. We only concentrate on the problem of train timetabling on a double-track rail network, and suspend the train routing problem at stations; furthermore, stations are viewed as nodes without the restriction of the maximum number of trains there at the same time.
2. The time-dependent demand of each origin–destination is discretized into many granularity-based demands through dividing the planning horizon into multiple time granularities with a length of 10 min or another required time interval, and the planned departure time of each granularity-based demand is the middle time of its corresponding time period. For example, demands of granularity from 8:00 to 8:10 have the same planned departure time of 8:05.

2.2. Inputs and Outputs

The inputs of DOTTP include the rail network, period-based demands, and a set of trains whose arrival and departure times at stations are unknown.

1. The rail network is composed of a set of station nodes and a set of rail one-way sections. Moreover, the operation time-horizon of the trains and some other related parameters of this rail network such as the mileage of rail section are given.
2. Passenger demands are discretized into many period-based demands, and, thus, the origin, terminal, and passenger number of each period-based demand are given.
3. The origin, terminal, and visited stations of each train are given; however, its stops and timetable are also needed for our research. Moreover, operation parameters such as the minimum required travel times in each rail section and dwell time at stations are also given. All symbols defined for expressing the input data of DOTTP are introduced in Table 1.

Table 1. All symbols for expressing the input and the output of demand-oriented train timetabling problem (DOTTP).

Objects	Symbols	Defining Ranges	Descriptions
Rail network	R		The set of origin stations
	S		The set of destination stations
	δ		The time granularities which are obtained by dividing the planning horizon into equivalent time intervals
	t_{δ}		The median time of time period δ
	T_s		The start times of the planning horizon
	T_z		The end times of the planning horizon
Trains	Ω		The set of trains
	E_i	$i \in \Omega$	The set of travel sections of train i
	K_i	$i \in \Omega$	The set of visited stations of train i
	(k, k', i)	$(k, k', i) \in R_p$	The continuous riding segment from station k to station k' traveling with train i on path p . Specifically, passengers are serviced by only one train in a continuous riding segment
	(i, j, k)	$(i, j, k) \in R_p$	The continuous riding segment that passengers transfer from train i to train j at station k on path p
	(r, k, i)	$(r, k, i) \in R_p$	The continuous riding segment from origin r to station k traveling with train i on path p
	$\tau(i, e)$	$i \in \Omega, e \in E_i$	The pure running time of train i in section e
	$\tau'(i, e)$	$i \in \Omega, e \in E_i$	The additional time for starting of train i in section e
	$\tau''(i, e)$	$i \in \Omega, e \in E_i$	The additional time for stopping of train i in section e
	$DT(k, k', k'')$	$(k, k') \in E_i,$ $(k, k'') \in E_j$	The safety departure interval between two trains driving into sections (k, k') and (k, k'') respectively from station k
	$AT(k, k', k'')$	$(k', k) \in E_i,$ $(k'', k) \in E_j$	The safety arrival interval between two trains arriving at station k from sections (k', k') and (k, k'') respectively
	\mathcal{L}_i		The rated passenger capacity of train i

Table 1. Cont.

Objects	Symbols	Defining Ranges	Descriptions
Passengers	$Q_{rs\delta}$	$r \in R, s \in S$	The passenger demand from origin r to destination s in time period δ
	$P_{rs\delta}$	$r \in R, s \in S$	The set of paths chosen by demand $Q_{rs\delta}$. For each path $p \in P_{rs\delta}$, it not only specifies the sequence of stations passengers traversing, but also assigns a specific train for the passengers to take between two neighbor stations.
	R_p	$p \in P_{rs\delta}$	The sequence of continuous riding segments of path p
	$\mathcal{F}_{k,k'}^i$		The discomfort penalty on train i from station k to station k'
	dq_k^i	$i \in \Omega, k \in K_i$	The total number of passengers getting on and off train i at station k
	$cq_{k,k'}^i$		The number of passengers on train i from station k to station k'
	$bq_{rs\delta}^i$		The number of passengers boarding train i at origin r in time period δ
	hq_k^{ij}		The number of passengers transferring from train i to train j at station k
	$md(dq_k^i)$		The minimum required time for passengers to get on and off train i at station k when their total numbers is dq_k^i
Others	λ		A parameter for adjusting the penalty level
	$l_{k,k'}$		The mileage from station k to station k'
	π_i		The passenger price rate per mileage
	β		The passenger average time value

The definitions of the outputs, namely, the decision variables of DOTTP, are given in Table 2.

Table 2. The definitions of the outputs (decision variables) of DOTTP.

Symbols	Defining Ranges	Descriptions
θ_i^k	$i \in \Omega, k \in K_i$	0-1 binary variable, if train i stops at station k , then $\theta_i^k = 1$; otherwise, $\theta_i^k = 0$. $\theta_i^{k'}$ has a similar definition with θ_i^k
x_{ik}	$i \in \Omega, k \in K_i$	The departure time of train i at station k
y_{ik}	$i \in \Omega, k \in K_i$	The arrival time of train i at station k
$f_{rs\delta}^p$	$r \in R, s \in S, p \in P_{rs\delta}$	The passenger number of demand $Q_{rs\delta}$ traveling with path p

3. Bi-Level Programming Model of Demand-Oriented Train Timetabling Problem (DOTTP)

The DOTTP involves the decision-making of both rail enterprises and passengers, and the way they influence each other is shown in Figure 1. First, rail enterprises formulate a timetable with a set of trains based on a given rail network and many safety and operation restrictions, which is regarded as leader decision-making. Then, passengers have to book tickets of one or more specific trains to travel from origins to destinations based on this formulated train timetable, which involves multiple follow-up decisions. Passengers' follow-up decisions can be viewed as feedback to the leader decisions of rail enterprises. Generally, it is very effective for rail enterprises to improve their previous formulated train timetable, i.e., leader decisions, according to the feedback. Accordingly, that adjustment of the leader decisions will further lead to the changes to passengers' follow-up decisions. In reality, the alternate adjustment of the leader and following decisions may last for a long time until passenger service level cannot be improved further, but it also may be terminated for some reason, for example, the constant change of demands. The leader–follower hierarchical relationship of decision-making between railway enterprises and passengers can be described as a bi-level programming model, consisting of an upper-level model for train timetabling detailed in Section 3.1 and a lower-level model for distributing passengers to trains, as introduced in Section 3.2.

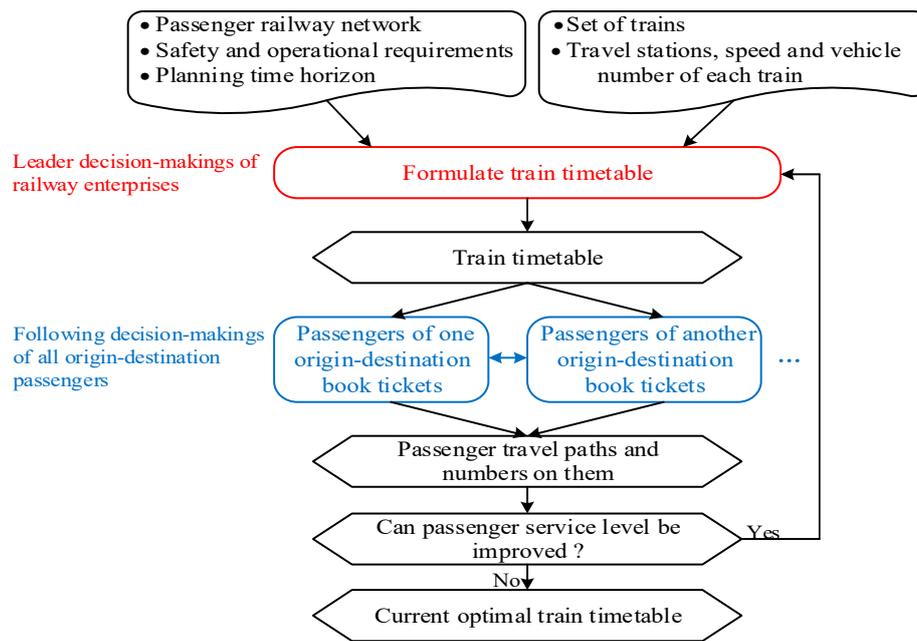


Figure 1. Leader–follower relationship of decision-making between rail enterprises and passengers.

3.1. Upper-Level Model for Train Timetabling

Taking a double-track rail network and a set of trains whose visited stations, speed, and rated passenger capacities are predetermined, the upper-level model (UM) focuses on determining the arrival and departure times of trains at stations in the planning horizon to form a train timetable with the aim of minimizing the total travel time of passengers and the constraints of the safety time-interval of trains.

Based on the notations and decision variables detailed in Tables 1 and 2, respectively, we formulate the objective of UM as minimizing the summation of products of passenger number and travel time on paths, namely,

$$\min UZ = \sum_{r,s,\delta} \sum_{p \in P_{rs\delta}} \left\{ f_{rs\delta}^p \times \left[\sum_{(k,k',i) \in R_p} (y_{ik'} - x_{ik}) + \sum_{(r,k,i) \in R_p} |x_{ir} - t_\delta| + \sum_{(i,j,k) \in R_p} (x_{jk} - y_{ik}) \right] \right\} \quad (1)$$

The first part in the square bracket represents passenger in-vehicle time on path p , the second part expresses the deviation time between the train departure time and passenger planned departure time at origin on path p , and the third part is the transfer time at transfer stations, respectively, on path p . Note that the passengers can book trains that depart earlier or later than their planned departure time, thus the deviation time in the second part needs to be taken as its absolute value.

In addition, the following five types of constraints are considered in UM.

3.1.1. Constraints of Train Travel Time in Sections

The train travel time in each section is composed of pure running time and additional time for starting and stopping depending on whether this train stops at the inbound and outbound stations. That is,

$$y_{ik'} - x_{ik} = \theta_i^k \times \tau'(i, e) + \tau(i, e) + \theta_i^{k'} \times \tau''(i, e), \quad e = (k, k') \in E_i; i \in \Omega \quad (2)$$

3.1.2. Constraints of Train Minimum Dwell Time at Stations

Train dwell time at each station should be sufficient for passengers to get on and off it, namely,

$$x_{ik} - y_{ik} \geq md(dq_k^i), \quad \forall k \in K_i; i \in \Omega \quad (3)$$

For example, Table 3 shows the minimum dwell time of trains at stations with different numbers of passengers on a Japanese high-speed railway. However, it does not indicate that we only take this factor into account, other influencing factors are actually reflected in the stopping time, such as fixed time values. We use the minimum dwelling time without limiting the upper value, if the train needs to carry out technical work at a station, the dwelling time can be extended appropriately.

Table 3. An example of train minimum dwell time with different passenger number at stations.

Number of Passengers Getting on and off the Train	Minimum Dwell Time of Train/Seconds
<100	45
100–200	60
200–300	75
>300	105–120

3.1.3. Safety Interval Constraints between Train Departure Time

The headway between two trains driving into either the same section or two different sections from the same station should not be less than the safety departure interval, that is,

$$|x_{jk} - x_{ik}| \geq DT(k, k', k''), \quad \forall (k, k') \in E_i; (k, k'') \in E_j; i, j \in \Omega \quad (4)$$

3.1.4. Safety Interval Constraints between Train Arrival Time

Similarly, the headway between two trains arriving at a station from either the same section or two different sections should not be less than the safety arrival interval, namely,

$$|y_{jk} - y_{ik}| \geq AT(k', k'', k), \quad \forall (k', k) \in E_i; (k'', k) \in E_j; i, j \in \Omega \quad (5)$$

3.1.5. Value Range Constraints of Train Arrival and Departure Time

The values of train arrival and departure time must be within the given planning horizon. That is,

$$T_s \leq x_{ik} \leq T_z, \quad \forall k \in K_i; i \in \Omega \quad (6)$$

$$T_s \leq y_{ik} \leq T_z, \quad \forall k \in K_i; i \in \Omega \quad (7)$$

As for safety interval constraints (Sections 3.1.3 and 3.1.4), it is worth noting that the headway between two trains driving into two different sections from the same station or arriving at a station from two different sections is not necessary. Only when two trains occupy the same throat zone at a station should the safety headway be taken into consideration; otherwise, the headway is set as 0.

In the above UM, the calculation of objective UZ in Equation (1) needs the passenger numbers of all chosen paths, and the minimum dwell time of trains at stations in safety interval constraints between train departure time (Section 3.1.3) depends on the total numbers of passengers getting on and off them at the stations. Those required passenger numbers in our research are obtained by distributing passengers to trains, which is achieved through solving the following lower-level model.

3.2. Lower-Level Model for Distributing Passengers to Trains

Based on a train timetable determined by the above UM, passengers generally book the trains with the lowest generalized cost to travel. The user equilibrium distribution theory (see Mahmassani [45] and Watling [46]) shows us that passenger choices of train booking finally reach an equilibrium state, in which passengers of the same granularity-based demand have equal generalized costs regardless of which trains they have booked, and their generalized costs are not more than those of un-booked trains. More importantly, the number of passengers on each booked train in the equilibrium state can be obtained by solving an equivalent mathematical programming model that forms our lower-level model (LM).

Note that this paper actually assigns passengers on the travel network consisting of all trains instead of a pre-determined railway network. Each OD-pair passenger can book tickets of one or more specific trains to travel from its origin to destination. In other words, the different combination of trains booked by passengers corresponds to different travel paths for these passengers, and, thus, there are actually multiple paths for each OD-pair to choose.

Before constructing model LM, we first have to detail the composition and calculation of passenger generalized cost. Besides in-vehicle time, deviation time between a train’s departure time and passengers’ planned departure time at origin, and transfer time considered in the objective function of UM, passengers also have to bear ticket price and an in-vehicle discomfort penalty caused by the congestion when the number of passengers on trains exceed the rated passenger capacity. The first three costs correspond to the three parts in the square bracket of Equation (1), and ticket price in each continuous riding segment is the product of the total mileage and price rate per mileage. The passenger in-vehicle discomfort penalty can be viewed as an additional cost for the discomfort caused by congestion, and it increases sharply with the continual increase of passenger numbers. However, if the passenger number is less than the rated passenger capacity, we assume that passengers have no congestion discomfort and their discomfort penalty is set as 0. Moreover, the longer passengers stay in a congested train, the more their discomfort penalty is. The following piecewise function is defined to describe the changing relationship between discomfort penalty and passenger number.

$$\mathcal{F}_{k,k'}^i(cq_{k,k'}^i) = \begin{cases} 0 & cq_{k,k'}^i \leq \mathcal{L}_i \\ \lambda \times (y_{ik'} - x_{ik}) \times \left[e^{(cq_{k,k'}^i - \mathcal{L}_i)} - 1 \right] & cq_{k,k'}^i > \mathcal{L}_i \end{cases} \quad (8)$$

The greater the value of λ , the greater the passenger discomfort penalty is.

Now, we can formulate the model LM as follows to assign passengers to trains to obtain detailed passenger numbers for trains in sections and at stations.

$$\begin{aligned} \min LZ = & \sum_{i \in \Omega} \sum_{(k,k') \in E_i} \int_0^{cq_{kk'}^i} \left[(y_{ik'} - x_{ik}) + \frac{l_{k,k'} \times \pi_i}{\beta} + \mathcal{F}_{k,k'}^i(cq_{k,k'}^i) \right] dq \\ & + \sum_{r,\delta} \sum_{i:r \in K_i} |x_{ir} - t_\delta| b q_{r\delta}^i + \sum_k \sum_{i,j:k \in K_i \cap K_j} (x_{jk} - y_{ik}) h q_k^{ij} \end{aligned} \quad (9)$$

$$\text{s.t. } \sum_{p \in P_{rs\delta}} f_{rs\delta}^p = Q_{rs\delta}, \quad \forall r, s, \delta \quad (10)$$

$$f_{rs\delta}^p \geq 0, \quad \forall p \in P_{rs\delta}; \forall r, s, \delta \quad (11)$$

$$cq_{kk'}^i = \sum_{r,s,\delta} \sum_{p: p \in P_{rs\delta}, (k,k',i) \in R_p} f_{rs\delta}^p, \quad \forall (k,k') \in E_i; i \in \Omega \quad (12)$$

$$bq_{r\delta}^i = \sum_s \sum_{p: p \in P_{rs\delta}, (r,k,i) \in R_p} f_{rs\delta}^p, \quad \forall r, \delta; i : r \in K_i \quad (13)$$

$$hq_k^{ij} = \sum_{r,s,\delta} \sum_{p \in P_{rs\delta}} \sum_{(i,j,k) \in R_p} f_{rs\delta}^p, \forall k \in K_i \cap K_j; \forall i, j \quad (14)$$

In the above LM, the objective in Equation (9) is to minimize the total generalized cost for passengers, which consists of three parts. The first part represents the cost on trains including in-vehicle time, ticket price, and discomfort penalty; the second part expresses the deviation time between the train's departure time and passengers' planned departure time at origin; and the third part is the transfer time at transfer stations respectively.

The constraint in Equation (10) assures that the summation of passengers on chosen paths between each OD is equal to its total demand, and the constraint in Equation (11) ensures that the values of passenger numbers on chosen paths are non-negative.

Equation (12) gives the calculation method of the total number of passengers on train i from station k to station k' , Equation (13) determines the number of passengers of time period δ boarding train i at origin r , and Equation (14) gives the calculation approach of the total number of passengers transferring from train i to train j at station k .

4. Priority-Based Heuristic Algorithm

The common passenger train timetable problem (PTTP) has proven to be an NP-hard problem, and adding the consideration of passenger train booking into it, consequentially, further exacerbates its difficulty. A general optimization algorithm cannot solve our proposed bi-level programming model within an acceptable computing time. Therefore, we design a priority-based heuristic algorithm to solve it in this section.

The solving procedure and the relations between the priority-based heuristic Algorithm (A1) and its sub-algorithms are shown in Figure 2:

1. Initialize a train timetable. Schedule trains one by one based on a random order. Specifically, for each train, set its departure time at its origin as the middle time of the departure time range, and then compute its arrival and departure time at each visited station according to this origin departure time and train minimum travel time in sections and dwell time at stations.
2. Equilibrium assignment of passenger flow. Based on the initial timetable, use a path-based passenger distribution Algorithm (A2) to assign passengers to trains, namely, to determine the number of passengers booking each train. The specific steps are illustrated in Section 4.1.
3. Update criterion of the optimal solution and judge the termination rules as follows; use criteria similar to the metropolis criteria to determine whether the new solution is accepted. If $f(s') - f(s) < 0$, s' is accepted as the new current solution s , otherwise s' is accepted as the new current solution s with the probability $\exp(-(f(s') - f(s))/kf(s))$. $f(x)$ is the objective function, s is the current solution, and s' is the new solution.

The following three rules are set to control the termination of the algorithm.

1. The iteration count reaches its maximum value n_{max} .
2. The objective UZ of model UM cannot be improved for continuous ζ_{max} time.
3. The difference in passenger costs between the last two iterations satisfies

$$\frac{\sum_{r,s} \sum_{\delta} \left| \left[\sum_{p \in P_{rs\delta}(n)} \left[f_{rs\delta}^p(n) \times c_{rs\delta}^p(n) \right] - \sum_{p \in P_{rs\delta}(n-1)} \left[f_{rs\delta}^p(n-1) \times c_{rs\delta}^p(n-1) \right] \right| \right|}{\sum_{r,s} \sum_{\delta} Q_{rs\delta}} \leq \xi \quad (15)$$

where $P_{rs\delta}(n)$ represents the set of paths chosen by demand $Q_{rs\delta}$ in the n th iteration, and $f_{rs\delta}^p(n)$, $c_{rs\delta}^p(n)$ are, respectively, the number and generalized cost of time period δ for passengers traveling on path p from origin r to destination s in the n th iteration. In addition, ξ is a given small positive number.

4. Improve the current train timetable. Rearrange trains' arrival and departure times to improve the current timetable based on the present passenger train-booking decisions and obtain a new train timetable.

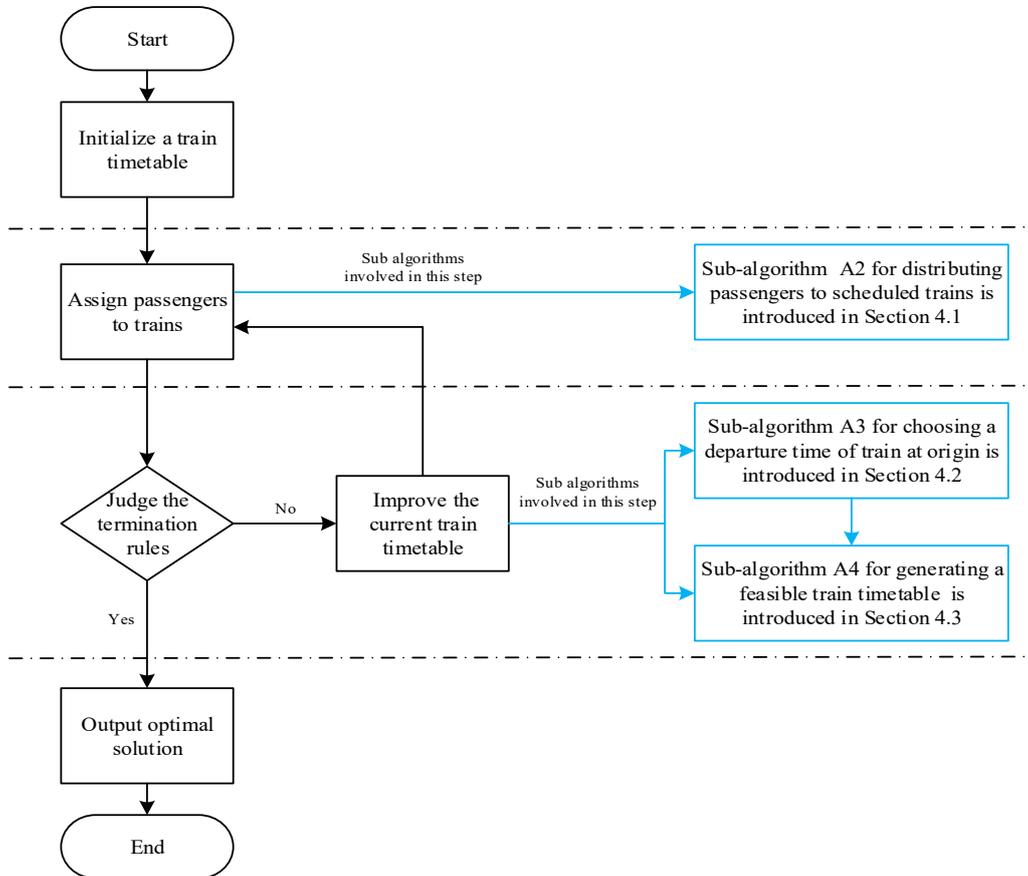


Figure 2. The flowchart of the priority-based heuristic Algorithm (A1).

In summary, the frame of the priority-based heuristic is given in Algorithm 1 (A1), and more detailed explanations for some steps and sub-algorithms are introduced afterwards.

Algorithm 1 A priority-based heuristic Algorithm (A1) for DOTTP combining passenger train-booking decisions.

Input: $R, S, \delta, T_s, T_z, \Omega, E_i, K_i, DT, AT, \mathcal{L}_i, Q_{rs\delta}, \lambda, l_{k,k'}, \pi_i, \beta$.

Output: $\theta_i^k, x_{ik}, y_{ik}, J_{rs\delta}^p$ upper-level objective function value (UO), lower-level objective function value (LO)
Rank trains based on the decreasing order of their speeds, and sort those with the same speed in descending order of their travel mileages. Trains in the front have higher priority than those in the back.

For i^* in Ω :

 Generate the set TS of time segments based on the departure time of scheduled trains at the origin station of train i^* to split the planning horizon.

 Find train i^* with the highest priority from all unscheduled trains, and set its departure time at origin as the middle time of the biggest time segments in set TS .

 Calculate the arrival and departure times of train i^* at stations based on its departure time at origin and its travel time in sections. Here the train dwell time at stations is set as 0 because there are no passengers getting on and off it.

End

Assign passengers to trains based on the current train timetable using the passenger distribution

Sub-Algorithm (A2) introduced in Section 4.1.

While termination rules 1,2,3 detailed before this algorithm are not met:

 Rank trains in by the descending order of passenger numbers on them.

 Clear the current train timetable and set all trains as unscheduled.

 For i^* in Ω :

 Choose a departure time of train i^* at origin using the Sub-Algorithm (A3) introduced in Section 4.2.

 Flag = 1.

 While Flag = 1:

 Calculate the arrival and departure times of train i^* at stations based on its departure time at origin, its travel time in sections, and minimum dwell time at stations, and then add them into the train timetable composed of all scheduled trains. The added arrival and departure times of train i^* may not satisfy the safety interval constraints, which are temporarily shelved when calculating them.

 Make train arrival and departure times satisfy safety interval constraints by finely adjusting them with the Sub-Algorithm (A4) described in Section 4.3 if they do not satisfy these constraints.

 If a feasible train timetable is obtained:

 Flag = 0;

 Else:

 Look for another departure time for train i^* at origin using the Sub-Algorithm (A3).

End

End

Assign passengers to trains based on the current train timetable using the passenger distribution

Sub-Algorithm (A2) introduced in Section 4.1.

End

In the above Algorithm (A1), the initial train timetable obtained may not be feasible because train arrival and departure times may not satisfy the safety interval constraints, which are not considered. However, the improved train timetable obtained by the Sub-Algorithm (A4) must be feasible.

4.1. Sub-Algorithm (A2) for Distributing Passengers to Scheduled Trains

The Sub-Algorithm (A2) of the above priority-based heuristic Algorithm (A1) is to distribute passengers to scheduled trains. Thus, this section focuses on detailing a sub-algorithm for assigning passengers to scheduled trains. The passenger distribution methods are generally divided into link-based and path-based types. Compared with the link-based approach, the path-based method will not only provide more path information about passengers including their travel trains and each

part of the cost on the paths, but also have much higher solving efficiency (see Jayakrishnan [47] and Chen [48]). Hence, we prefer to apply a path-based passenger distribution algorithm to solve model LM.

The following two notes should be emphasized before detailing Sub-Algorithm (A2).

1. Update the passenger number of chosen paths and the current lowest-cost path as follows:

$$f_{rs\delta}^p(n) = \max \left\{ f_{rs\delta}^p(n-1) - \frac{\alpha_n}{w_n(p)} [d_n(p) - d_n(p_{rs\delta}^*(n))], 0 \right\}, p \in P_{rs\delta}(n) / \{p_{rs\delta}^*(n)\} \quad (16)$$

$$f_{rs\delta}^{p'}(n) = Q_{rs\delta} - \sum_{p' \in P_{rs\delta}(n) / \{p_{rs\delta}^*(n)\}} f_{rs\delta}^{p'}(n), p = p_{rs\delta}^*(n) \quad (17)$$

where $d_n(p)$ and $w_n(p)$ are, respectively, the first and second derivatives of objective LZ to passenger number of path p , and $p_{rs\delta}^*(n)$ is the lowest-cost path in the n th iteration. Moreover, $\alpha_n = \alpha_0/n$ is the updating step, and α_0 is a given fixed value. Obviously, α_n becomes smaller with the increase of iteration n . If the passenger number of chosen path is updated as 0, then we delete it from the set of chosen paths as it is not chosen by any passengers.

2. The termination rules of Sub-Algorithm (A2). where η is a given small tolerance value.

$$\max \left\{ \sum_{p \in P_{rs\delta}(n) / \{p_{rs\delta}^*(n)\}} \frac{f_{rs\delta}^p(n) (d_n(p) - d_n(p_{rs\delta}^*(n)))}{Q_{rs\delta} d_n(p)} \mid \forall r, s, \delta \right\} \leq \eta \quad (18)$$

In summary, we detail Sub-Algorithm (A2) for distributing passengers to trains in Algorithm 2.

Algorithm 2 Sub-Algorithm (A2) for distributing passengers to scheduled trains.

Input: $R, S, \delta, \mathcal{L}_i, \mathcal{F}_{k,k'}^i, Q_{rs\delta}, \lambda, l_{k,k'}, \pi_i, \beta$, the current train timetable.

Output: $f_{rs\delta}^p$, upper-level objective function value (UO), lower-level objective function value (LO).

Initialize iteration count $n = 0$.

Search the lowest cost path $p_{rs\delta}^*(n)$ for each granularity-based demand.

Distribute all passengers $Q_{rs\delta}$ to the lowest-cost path $p_{rs\delta}^*(n)$, and initialize the set of chosen paths $P_{rs\delta}(n) = \{p_{rs\delta}^*(n)\}$.

While the inequality in Equation (18) is not satisfied:

 Update iteration count $n = n + 1$.

 Count passenger number $cq_{kk'}^i, bq_{r\delta}^i$ and hq_k^{ij} with Equations (12)–(14), respectively.

 Update the cost of paths in set $P_{rs\delta}(n)$. Re-search the lowest cost path $p_{rs\delta}^*(n)$, and set

$P_{rs\delta}(n) = P_{rs\delta}(n) \cup \{p_{rs\delta}^*(n)\}$ if $p_{rs\delta}^*(n) \notin P_{rs\delta}(n)$.

 For each path $p \in P_{rs\delta}(n) - \{p_{rs\delta}^*(n)\}$, update its passenger number $f_{rs\delta}^p(n)$ by Equation (16).

 For each path $p = p_{rs\delta}^*(n)$, update its passenger number $f_{rs\delta}^p(n)$ with Equation (17).

 For each path $p \in P_{rs\delta}(n)$, delete it from set $P_{rs\delta}(n)$ if $f_{rs\delta}^p(n) = 0$.

End

Calculate the upper-level objective function value (UO) and the lower-level objective function value (LO) according to Equations (1) and (9) respectively.

4.2. Sub-Algorithm (A3) for Choosing a Departure Time of a Train at Origin

Generally, the departure time of a train at origin has a great influence on passengers' deviation time and transferring time, but little effect on passengers' in-vehicle time as a train's travel time is usually only allowed to vary within a small range. For simplicity, we first determine the departure time of the train at origin with the objective of minimizing the total deviation and transferring times of

its distributed passengers, and then calculate its arrival and departure times at other visited stations. Therefore, we first have to calculate the deviation and transferring times of passengers traveling partially or wholly with the current scheduling train before detailing the algorithm. The following two notes should be emphasized when calculating them.

1. We do not need to calculate those deviations and transferring times that are unaffected by the current scheduling train. For instance, we do not calculate the deviation time of passengers who first board a scheduled train and then transfer to the current scheduling train as their deviation time is determined by the former train not the latter one.
2. For unscheduled trains, we calculate the travel times between two stations based on their travel times in sections and minimum dwell times at stations. Especially for the current scheduling train, we can determine arrival and departure times at stations from a given departure time at origin based on its travel times in sections and minimum dwell times at stations.

Set train i as the current scheduling train and denote decision variable \bar{x}_i as its departure time at origin. Then, we calculate the deviation and transferring times of passengers traveling with it in two sub-trips, respectively. The first sub-trip is before traveling with train i , and the other is after traveling with train i .

Passenger deviation and transferring times in their first sub-trip are calculated with the following three types of passengers, respectively:

4.2.1. Type 1: Passengers Whose First Travel Train Is just Train i

As illustrated in Figure 3, r_i and r_p are the origin stations of train i and passengers respectively, $\Gamma_i(r_i, r_p)$ is the travel time of train i from station r_i to station r_p , and t_p is the planned time of passengers. For this type of passengers, their transferring time before traveling with train i is set as 0 as they don't need to transfer, and their deviation time at origin denoted as $\phi_p(\bar{x}_i)$ is calculated by

$$\phi_p(\bar{x}_i) = |\bar{x}_i + \Gamma_i(r_i, r_p) - t_p| \tag{19}$$

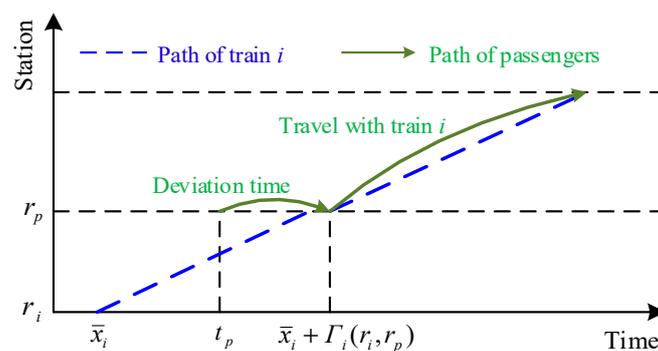


Figure 3. A trip example of the first type of passengers.

4.2.2. Type 2: Passengers Whose Traveling Trains before Train i Are Unscheduled

This type of passengers has to first travel with some unscheduled trains, and then transfer to train i , as shown in Figure 4, in which \hat{k}_p^i is the station passengers transferring to train i , and $Y_p(r_p, \hat{k}_p^i)$ is passenger total in-vehicle time from station r_p to station \hat{k}_p^i traveling with one or more unscheduled trains. For this type of passengers, their transferring time before traveling with train i is set as the minimum transferring time t_r , and their deviation time can be determined as

$$\phi_p(\bar{x}_i) = |\bar{x}_i + \Gamma_i(r_i, \hat{k}_p^i) - Y_p(r_p, \hat{k}_p^i) - t_r - t_p| \tag{20}$$

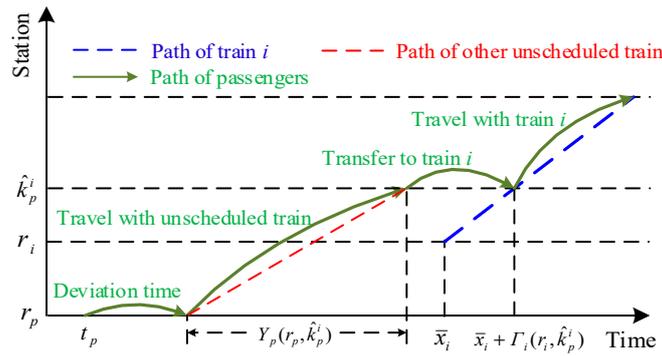


Figure 4. A trip example of the second type of passengers.

4.2.3. Type 3: Passengers Whose Traveling Trains Exist at least One Scheduled Train before Train *i*

Assume train *j* as the last one among these scheduled trains that passengers travel with before train *i*, as shown in Figure 5. Obviously, their deviation and transfer times before train *j* are irrelevant to train *i*, and only those passengers who get off train *j* to get on train *i* are influenced by train *i*, and their transferring times are calculated by

$$\varphi_p(\bar{x}_i) = \begin{cases} \bar{x}_i + \Gamma_i(r_i, \hat{k}_p^i) - Y_p(\check{k}_p^j, \hat{k}_p^i) - y_{j, \check{k}_p^j} & \text{if } \bar{x}_i \geq y_{j, \check{k}_p^j} - \Gamma_i(r_i, \hat{k}_p^i) + Y_p(\check{k}_p^j, \hat{k}_p^i) \\ M & \text{else} \end{cases} \quad (21)$$

where $\varphi_p(\bar{x}_i)$ represents passenger total transferring time before traveling with train *i*, \check{k}_p^j is the station passenger getting on train *j*, and *M* is set as a given big value, which can be viewed as a penalty value when passengers cannot transfer to train *i* as it departs earlier.

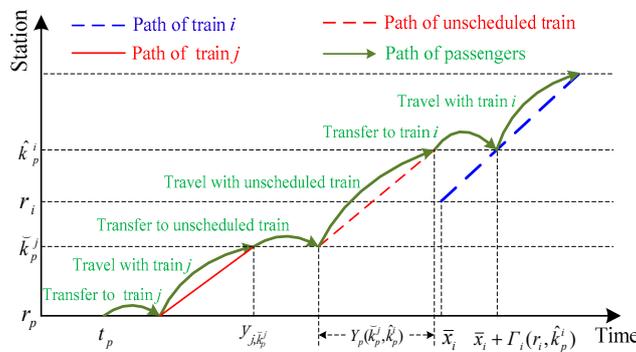


Figure 5. A trip example of the third type of passengers.

Next, we further calculate passenger transfer times in the sub-trip after train *i*. They are set as 0 in the following two cases: (1) train *i* is the last train passengers traveling; and (2) all traveled trains after train *i* are not scheduled. Only when at least one scheduled train passenger traveling after train *i* exists, as shown in Figure 6, and train *j* is assumed as the first one among these scheduled trains passengers traveling after train *i*, we should calculate passenger transfer time denoted as $\varphi'_p(\bar{x}_i)$ from getting off train *i* to getting on train *j* as follows:

$$\varphi'_p(\bar{x}_i) = \begin{cases} x_{j, \check{k}_p^j} - Y_p(\check{k}_p^i, \hat{k}_p^j) - \Gamma_i(r_i, \check{k}_p^i) - \bar{x}_i & \text{if } \bar{x}_i \leq x_{j, \check{k}_p^j} - Y_p(\check{k}_p^i, \hat{k}_p^j) - \Gamma_i(r_i, \check{k}_p^i) \\ M & \text{else} \end{cases} \quad (22)$$

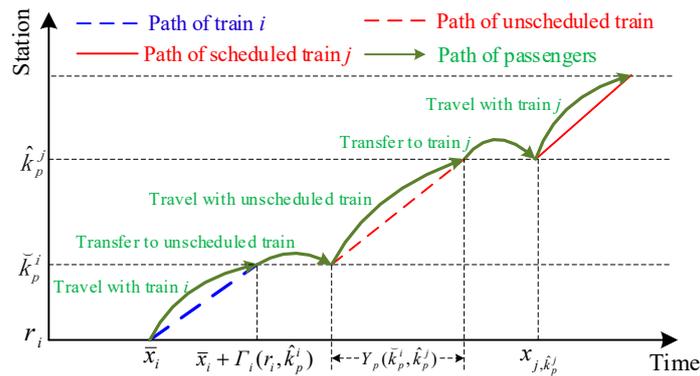


Figure 6. An example of passenger partial trip after train i .

In conclusion, the objective of choosing departure time for train i at origin is described as

$$CZ(\bar{x}_i) = \sum_{p \in P_i} f_p \times [\phi_p(\bar{x}_i) + \varphi_p(\bar{x}_i) + \varphi'_p(\bar{x}_i)] \tag{23}$$

where P_i is the path set of passengers traveling with train i wholly or partially, and f_p is the passenger number on path $p \in P_i$.

The objective function $CZ(\bar{x}_i)$ is a piecewise linear function of train departure time \bar{x}_i as all of its component functions in Equations (19)–(22) are piecewise and their all segments are linear. The segregating times of function $CZ(\bar{x}_i)$ are those of Equations (19)–(22). As illustrated in Figure 7, the green and blue dashed lines display two piecewise functions that form the objective function $CZ(\bar{x}_i)$ shown with red solid lines. Function $CZ(\bar{x}_i)$ is piecewise and contains three linear segments separated by time points 25 and 70 in the planning horizon from 0 to 100, and the best departure time is time point 70.

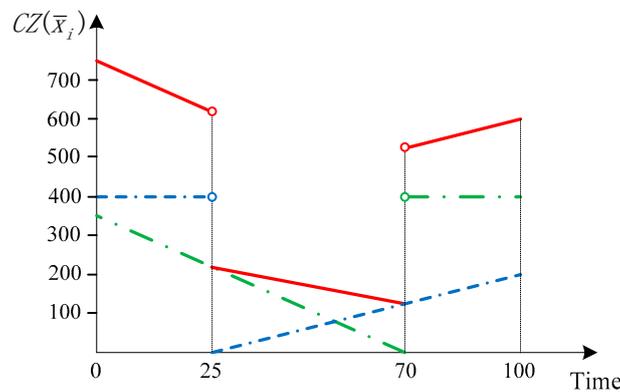


Figure 7. An example of the piecewise objective function $CZ(\bar{x}_i)$.

Based on the segmentation characteristics of the objective function $CZ(\bar{x}_i)$, Algorithm (A3), intended to choose the departure time at origin for train i , is shown in Algorithm 3.

Algorithm 3 Sub-Algorithm (A3) for choosing the departure time at origin for train i .

Input: $P_{rs\delta}, R_p, f_{rs\delta}^p, M$, the current train timetable.

Output: a set of departure time of train i^* at origin.

Find passenger paths consisting of train i , and determine the segregating times from the calculation of Equations (19)–(23) of deviation and transferring times on them.

Sort all segregating times as well as departure times of all scheduled trains at the origin station of train i , and then divide the planning horizon into multiple time segments using them as splitting times.

Find the optimal time for each time segment, and then rank them based on the increasing order of their corresponding values of $CZ(\bar{x}_i)$.

Choose the best, next best, or other required time as the departure time of train i at origin.

In Step 3 of Sub-Algorithm (A3), we can easily determine the optimal time of each time segment according to the continuous and monotonic function $CZ(\bar{x}_i)$. That is, if $CZ(\bar{x}_i)$ increases with time \bar{x}_i , then the optimal departure time is determined as the left border of this segment; otherwise, it is chosen as the right one.

4.3. Sub-Algorithm (A4) for Generating a Feasible Train Timetable Based on Given Departure Times

As train arrival and departure times calculated in Algorithm (A1) may not satisfy safety interval requirements, this subsection focuses on making them satisfy these requirements through finely adjusting them with the aid of a weighted directed graph. Their adjustment must obey the follows rules.

1. It is not permitted to generate new conflicts. A conflict represents the condition that two involved trains do not satisfy the safety interval.
2. The increment of passengers' deviation time should be limited to a given small range, but its decrement is welcome. For example, a train departing at 10:10 before adjustment has been chosen by passengers of time 10:00, and then its departure time is only allowed to adjust in the range from 10:00 to 10:15 as 5 min is set as the maximum increment of passengers' deviation time. Similarly, the increments of passenger transfer time and dwell times at stations also must be limited to a small range.

Before detailing this algorithm, we first build a weighted directed graph corresponding to the train timetable as follows:

4.3.1. Nodes of Graph

Five types of nodes are defined in Table 4.

Table 4. The definition of nodes.

Name	Weight	Whether the Weight Can Be Changed
Train departure node	Corresponding train departure time	Yes
Train arrival node	Corresponding train arrival time	Yes
Passenger planned departure node	Planned departure time of the corresponding passengers	No
Start node of planning horizon	Start time of planning horizon	No
End node of planning horizon	End time of planning horizon	No

It should be noted that the start and end nodes of the planning horizon are defined for each station, and, thus, their numbers are just the number of stations. A weight is defined for each node, and ε_v is denoted as the weight of node v . Moreover, only the weights of train departure and arrival nodes can be adjusted while other nodes are not allowed to be adjusted.

4.3.2. Directed Arcs of the Graph

Seven types of directed arcs are constructed between nodes shown in Figure 8 to restrict the adjustment of train arrival and departure times, and their detailed definitions, requirements, and weights are introduced in Table 5.

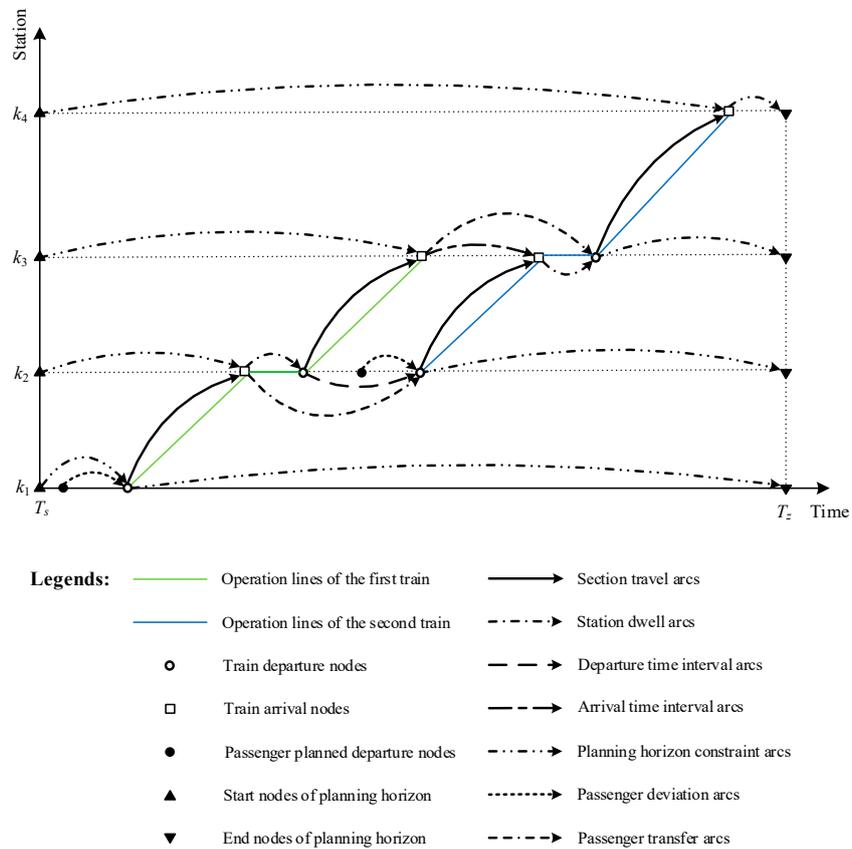


Figure 8. The definitions of nodes and arcs in an example weighted directed graph.

Table 5. The definition of directed arcs.

Name	Start Node	Terminal Node	Requirements	Minimum Weight	Maximum Weight
Section travel arc	Train departure node	Train arrival node	Start and terminal nodes belong to a same train and section	The sum of train pure running time and additional times for starting and stopping	The sum of train pure running time and additional times for starting and stopping
Station dwell arc	Train arrival node	Train departure node	Start and terminal nodes belong to a same train and station	Minimum time for passenger getting on and off train	Passenger maximum allowed dwell time at a station
Departure time interval arc	Train departure node	Train departure node	Start and terminal nodes belong to two different trains but to a same station	Safety headway between two train departures	Positive infinity
Arrival time interval arc	Train arrival node	Train arrival node	Start and terminal nodes belong to two different trains but to a same station	Safety headway between two train arrivals	Positive infinity
Planning horizon constraint arc	Start node of planning horizon	Train arrival or departure node	Start and terminal nodes belong to a same station	0	Time length of planning horizon
	Train arrival or departure node	End node of planning horizon	Start and terminal nodes belong to a same station	0	Time length of planning horizon
Passenger deviation arc	Passenger planned departure node	Train departure node	Start and terminal nodes belong a same station	0	Passenger maximum allowed deviation time at origin
Passenger transfer arc	Train arrival node	Train departure node	Start and terminal nodes belong to two different trains but to a same station	Minimum time for transferring including times for getting off one train, boarding another train and walk between them	Passenger maximum allowed time per transfer

Take $\rho(u, v) = \varepsilon_v - \varepsilon_u$ as the weight of directed arc (u, v) , $\rho_{min}(u, v)$ and $\rho_{max}(u, v)$ as its minimum and maximum values, respectively. The weight of each arc should range between its minimum and maximum.

In the construction process of directed arcs, the following four notes should be emphasized:

1. As the number of departure (arrival) time interval arcs is very large when they are constructed between any two trains' departure (arrival) nodes of the same station, we only construct these between two neighbor trains' departure (arrival) nodes as shown with the red dotted lines in Figure 9a for reducing their number. However, if two trains cross in a section, then their interval arcs are constructed in two cases, as shown with the red dotted lines in Figure 9b,c, respectively.
2. Similarly, planning horizon constraint arcs are only defined for the first and last trains' departure or arrival nodes at a station for reducing their number.
3. For passenger deviation and transfer arcs, they are constructed only when there are passengers traveling from their start nodes to their terminal nodes. For example, if passengers are boarding Train 1 rather than Train 2 at their origin, then we should construct the passenger deviation arc from these passengers' departure node to the departure node of Train 1, not to that of Train 2 as they do not board it.
4. The maximum increments of passenger dwell time, deviation time, and transfer time are pre-specified as inputs of the algorithm. Generally, the smaller they are, the smaller the increase of passenger travel time, and the higher the difficulty of obtaining a feasible train timetable is. Thus, we can set them as smaller values at first when the number of trains is considerably below the capacity of the rail line, namely, the maximum of scheduled trains on the rail line, and then gradually increase them as the train numbers increase.

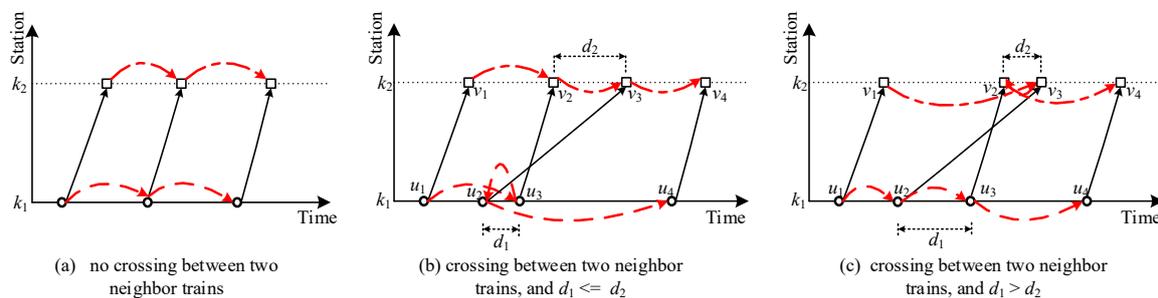


Figure 9. The construction of interval arcs in different cases.

Now, with the help of the weighted directed graph of the train timetable, we introduce how to finely adjust train arrival and departure times to make them satisfy all constraints of model UM.

First, we find the set of all conflict arcs whose weights are less than their minimum weights and denote them as A_c . According to the construction method of the initial train timetable given in Algorithm (A1), we ensure that all section travel arcs, station dwell arcs, passenger deviation, and transfer arcs satisfy their minimum and maximum weight requirements, and only the departure time interval arcs, arrival time interval arcs, and planning horizon constraint arcs may become conflict arcs.

Second, we increase the weights of conflict arcs one by one. For the conflict arc $(u, v) \in A_c$, its weight can be increased by either reducing the weight of node u or increasing the weight of node v . Denote $V(u)$ and $V(v)$ as the sets of associate adjusted nodes when adjusting the weights of nodes u and v , respectively. Specifically, we only decrease the weights of nodes in set $V(u)$ simultaneously for reducing the weight of node u , and similarly only increase the weights of nodes in set $V(v)$ synchronously to add the weight of node v . For example, as illustrated in Figure 10a, we can increase the weight of the conflict arc (v_5, v_6) shown with the red dotted line by reducing the weight of node v_5 and increasing the weight of node v_6 , and set $V(v_5)$ is determined as $\{v_5, v_7\}$ shown with the blue color

because the weight of the dwell arc (v_7, v_8) is less than its maximum value 2, while set $V(v_6)$ is decided as $\{v_6, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$ shown in the green color.

The detailed steps of searching set $V(u)$ are given in Algorithm 4, and steps for set $V(v)$ are similar.

Algorithm 4 The detailed steps for searching set $V(u)$.

Input: conflict arcs (u, v) , the weighted directed graph, DT, AT .

Output: $V(u)$.

Initialize set $V(u) = \{u\}$.

For u' in $V(u)$:

 Find all outbound arcs A_o and all inbound arcs A_i for node u' .

 For (u', u'') in A_o :

 If $\rho(u', u'') \geq \rho_{max}(u', u'')$:

 Add node u'' into set $V(u)$.

 For (u'', u') in A_i :

 If $\rho(u'', u') \leq \rho_{min}(u'', u')$:

 Add node u'' into set $V(u)$.

End

The maximum weight decrement $\Delta\bar{\rho}(V(u))$ of node set $V(u)$ is calculated by

$$\Delta\bar{\rho}(V(u)) = \min \left\{ \begin{array}{l} \min\{\rho(u'', u') - \rho_{min}(u'', u') | u' \in V(u), u'' \notin V(u)\} \\ \min\{\rho_{max}(u', u'') - \rho(u', u'') | u' \in V(u), u'' \notin V(u)\} \end{array} \right\} \quad (24)$$

and the maximum weight increment $\Delta\bar{\rho}(V(v))$ of the node set $V(v)$ is determined as

$$\Delta\bar{\rho}(V(v)) = \min \left\{ \begin{array}{l} \min\{\rho_{max}(v'', v') - \rho(v'', v') | v' \in V(v), v'' \notin V(v)\} \\ \min\{\rho(v', v'') - \rho_{min}(v', v'') | v' \in V(v), v'' \notin V(v)\} \end{array} \right\} \quad (25)$$

Then, their actual adjustments are determined separately by

$$\Delta\rho(V(u)) = \min\{\Delta\bar{\rho}(V(u)), \rho_{min}(u, v) - \rho(u, v)\} \quad (26)$$

$$\Delta\rho(V(v)) = \min\{\Delta\bar{\rho}(V(v)), \rho_{min}(u, v) - \rho(u, v) - \Delta\rho(V(u))\} \quad (27)$$

According to them, we can update the weights of nodes in set $V(u)$ and $V(v)$ as follows.

$$\varepsilon_{u'} = \varepsilon_{u'} - \Delta\rho(V(u)), \quad u' \in V(u) \quad (28)$$

$$\varepsilon_{v'} = \varepsilon_{v'} + \Delta\rho(V(v)), \quad v' \in V(v) \quad (29)$$

Correspondingly, the weights of arcs whose start or end nodes belong to set $V(u)$ or $V(v)$ are updated as follows.

$$\rho(u'', u') = \rho(u'', u') - \Delta\rho(V(u)), \quad \forall (u'', u') | u' \in V(u), u'' \notin V(u) \quad (30)$$

$$\rho(u', u'') = \rho(u', u'') + \Delta\rho(V(u)), \quad \forall (u', u'') | u' \in V(u), u'' \notin V(u) \quad (31)$$

$$\rho(v', v'') = \rho(v', v'') - \Delta\rho(V(v)), \quad \forall (v', v'') | v' \in V(v), v'' \notin V(v) \quad (32)$$

$$\rho(v'', v') = \rho(v'', v') + \Delta\rho(V(v)), \quad \forall (v'', v') | v' \in V(v), v'' \notin V(v) \quad (33)$$

For example, both the maximum weight decrement $\Delta\bar{\rho}(V(v_5))$ and increment $\Delta\bar{\rho}(V(v_6))$ are 1 in Figure 10a, and their actual adjustments are also determined as 1. After updating the weights of all relevant nodes and arcs, we can obtain the weighted directed graph as shown in Figure 10b. If we want to further reduce the weight of node v_5 , we should add nodes v_8 and v_9 into set $V(v_5)$ to

simultaneously adjust for the weight of dwell arc (v_7, v_8) reaching its maximum. Similarly, if we continue to increase the weight of node v_6 , we still have to add node v_{18} into set $V(v_6)$.

After the above adjustment of node and arc weights, if conflict arc (u, v) satisfies $\rho(u, v) = \rho_{min}(u, v)$, then stop and choose another conflict arc to adjust its weight; otherwise, update sets $V(u)$ and $V(v)$, and repeat the above adjustment process of weights.

In the adjustment process of weights, if set $V(u)$ contains a node whose weight is not allowed to be changed, then we cannot decrease the weights of nodes in set $V(u)$ any further. Similarly, if set $V(v)$ includes a constant-weight node, then we cannot further increase the weights of nodes in set $V(v)$. Therefore, if the weights of both nodes in sets $V(u)$ and $V(v)$ cannot be further adjusted and the weight $\rho(u, v)$ of the conflict arc (u, v) is still less than its minimum $\rho_{min}(u, v)$, it means that we cannot obtain a feasible train timetable from the initial train timetable. In this case, we have to construct another initial train timetable based on a new departure time and adopt the proposed algorithm in this subsection again to find a feasible train timetable. For instance, we cannot further increase the node weights of set $V(v_6)$ in Figure 10b as it contains node v_{18} , so we only continue to decrease the node weights of set $V(v_5)$ to further increase the weight of conflict arc (v_5, v_6) , and then update the weighted directed graph, as shown in Figure 10c, in which the weight of the arc (v_5, v_6) reaches its minimum value of three, and its corresponding train timetable is feasible.

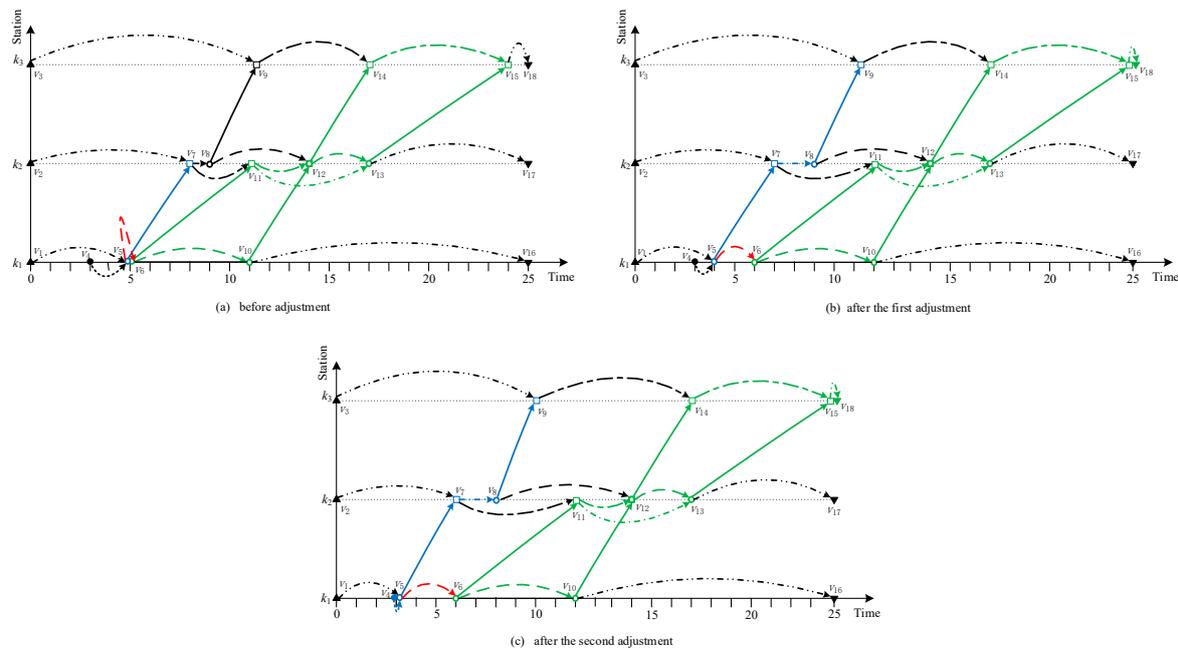


Figure 10. An example of increasing the weight of a conflict arc.

In summary, the steps of Sub-Algorithm (A4) for making train departure and arrival times feasible from an initial train timetable are detailed in Algorithm 5.

Algorithm 5 Sub-Algorithm (A4) for forming a feasible train timetable based on given departure times.

Input: the current train timetable, DT, AT .

Output: a feasible train timetable.

Generate the weighted directed graph based on the given initial train timetable.

Find the conflict arc set A_c , fixed nodes set A_f .

While $A_c \neq \emptyset$:

Choose a conflict arc (u, v) from set A_c , find sets of associate adjusted nodes $V(u)$ and $V(v)$.

Calculate the maximum weight adjustment decrement $\Delta\bar{\rho}(V(u))$ and increment $\Delta\bar{\rho}(V(v))$ with Equations (24) and (25), respectively.

Decide the actual adjustments $\Delta\rho(V(u))$ and $\Delta\rho(V(v))$ with Equations (26) and (27).

Adjust the weights of relevant nodes and arcs with Equations (28)–(33).

If $\rho_{min}(u, v) \geq \rho(u, v)$:

Update set $A_c = A_c / \{(u, v)\}$.

Else:

Update sets $V(u)$ and $V(v)$.

If $((V(v) \cup V(u)) \cap A_f) \neq \emptyset$:

Stop the algorithm without finding a feasible train timetable;

End

5. Numerical Analysis

To test the validity of our method, this section takes the Changsha–Zhuzhou–Xiangtan intercity rail network to test our proposed approach, and analyzes its performance, convergence and effectiveness based on the obtained optimization results.

5.1. Inputs and Parameter Setting

The Changsha–Zhuzhou–Xiangtan (CZT) intercity railway network consists of 24 stations, of which 23 stations are in operation now, as shown in Figure 11. It has the mileage of 104.36 km, and provides fast, punctual and convenient transportation services among Changsha, Zhuzhou and Xiangtan cities. About 80% of passengers on this network are recently distributed among the ten busiest stations, including Chang-sha West, Xian-feng, Ban-tang, Xiang-tan, and Tian-xin East. For simplicity, we retain only these stations with many departures and arrivals, and remove those with fewer passengers visiting in CZT rail network to construct a “Y”-shaped test network consisting of 10 stations and 9 sections to test our proposed method.

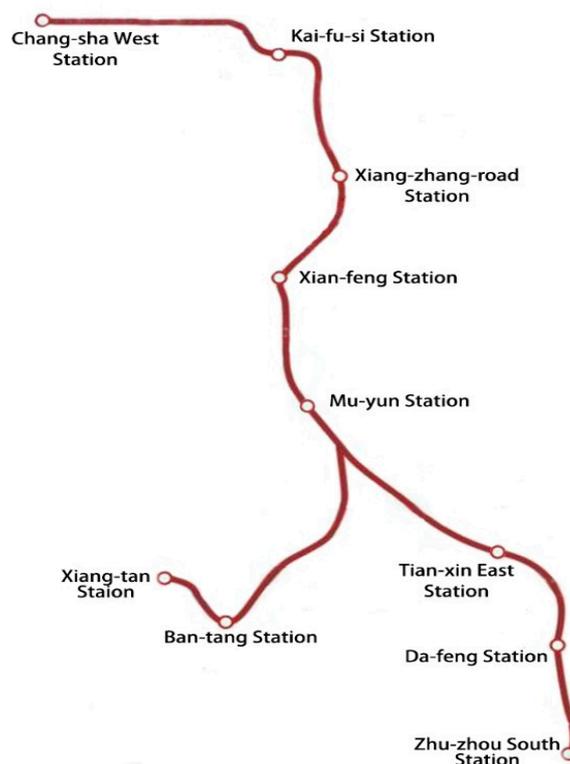


Figure 11. Changsha–Zhuzhou–Xiangtan intercity rail network.

As all sections have two one-way tracks in our test network, the numerical example only focuses on scheduling trains from Changsha to Zhuzhou, or Changsha to Xiangtan. The stations visited by trains of the two paths are shown in Table 6.

Table 6. Stations passed by two paths of CZT intercity.

Path of Trains	Number of Stations	Sequence of Vistied Stations
Changsha to Zhuzhou	8	(1) Chang-sha West Station; (2) Kai-fu-si Station; (3) Xiang-zhang-road Station; (4) Xian-feng Station; (5) Mu-yun Station; (8) Tian-xin East Station; (9) Da-feng Station; (10) Zhu-zhou South Station
Changsha to Xiangtan	7	(1) Chang-sha West Station; (2) Kai-fu-si Station; (3) Xiang-zhang-road Station; (4) Xian-feng Station; (5) Mu-yun Station; (6) Ban-tang station; (7) Xiang-tan Station

The actual operation time of CZT intercity rail network now is from 07:00 to 23:00, and it is also applied as our optimization time horizon of trains. Currently, there are 43 trains operating in this time horizon, of which 22 trains are bounded from Chang-sha West Station to Zhu-zhou South station, and 21 trains are bounded from Chang-sha West to Xiang-tan station. More detailed information can be obtained from the website: www.12306.com [49]. Obviously, these operating trains and the volume of service passengers are small, and are far less than the service capacity of this network. For better testing our method, we operate the corresponding number of passenger trains based on the predicted number of passengers sent out in 2020 (see Shao [50] and Pan [51]). Therefore, there are 100 trains operating all day, of which 60 trains are from Chang-sha West station to Zhu-zhou South station and 40 trains are from Chang-sha West station to Xiang-tan station. The parameters of operating trains on two paths are shown in Table 7.

Table 7. Operation parameters of Changsha–Zhuzhou–Xiangtan intercity railway.

Paths	Number of Stations	Paths Mileage (km)	Train Number	Train Capacity	Technical Velocity (km/h)
Changsha to Zhuzhou	8	86	60	1000	160
Changsha to Xiangtan	7	70	40	1000	160

In the whole rail network, there are 39 pairs of OD passenger flows in total. The passengers' origin station, terminal station, and demand quantity of each OD pair are shown in Table 8.

Table 8. Travel demand quantity of each OD pair.

OD Sequence Number	Origin–Terminal Station	Passenger Volume	OD Sequence Number	Origin–Terminal Station	Passenger Volume
O-D 1	1–2	1200	O-D 21	3–7	1200
O-D 2	1–3	3600	O-D 22	3–8	1200
O-D 3	1–4	4800	O-D 23	3–9	2400
O-D 4	1–5	1200	O-D 24	3–10	2400
O-D 5	1–6	4800	O-D 25	4–5	1200
O-D 6	1–7	6000	O-D 26	4–6	2400
O-D 7	1–8	4800	O-D 27	4–7	3600
O-D 8	1–9	3600	O-D 28	4–8	2400
O-D 9	1–10	6000	O-D 29	4–9	2400
O-D 10	2–3	1200	O-D 30	4–10	3600
O-D 11	2–4	2400	O-D 31	5–6	1200
O-D 12	2–5	1200	O-D 32	5–7	2400
O-D 13	2–6	3600	O-D 33	5–8	1200
O-D 14	2–7	6000	O-D 34	5–9	1200
O-D 15	2–8	2400	O-D 35	5–10	3600
O-D 16	2–9	3600	O-D 36	6–7	7200
O-D 17	2–10	3600	O-D 37	8–9	3600
O-D 18	3–4	1200	O-D 38	8–10	3600
O-D 19	3–5	1200	O-D 39	9–10	7200
O-D 20	3–6	2400	Total		118,800

Taking five minutes as unit time interval, the operating time of whole day can be divided into 192 time periods; each pair of OD passenger flow is assigned to the corresponding time period. Suppose the passengers' travelling ratio coefficient of every time period in morning rush hours (7:00–9:00) and evening rush hours (17:00–19:00) are 0.0084, while that of every time period in other non-peak hours is 0.0041. The minimum safety departure interval and minimum safety arrival interval in various transit sections are set as 3 min, and the other parameters for the algorithm are set as shown in Table 9.

Table 9. Some parameters of our algorithm.

Parameter Name	Parameter Values	Unit
T (operating cycle)	960	minute
G (number of time period)	192	per
ε (calculation accuracy)	0.01	-
$keep_n$ (times of keeping unchanged)	30	times
max_n (total times of iterations)	300	times

The above two algorithms were both solved by MATLAB 2016a, using the hardware of a Lenovo C5030 desktop computer corresponding to 2 GHz Intel (TM) Core (TM) i3-5005U CPU, 8 GB (4) memory, and Windows 10 specialized operating system.

5.2. Convergence Analysis

Figure 12 (left) shows the change of upper-level objective function value (UO) along with the increasing number of iterations in the proposed algorithm, while the change of lower-level objective function value (LO) is shown in Figure 12 (right).

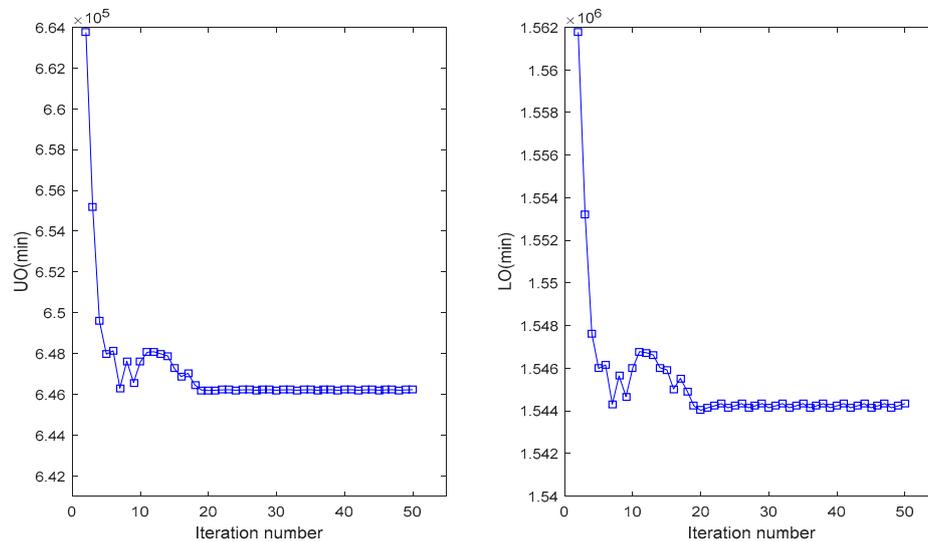


Figure 12. The relationship between UO/LO and the number of iterations.

Figure 12 demonstrates that, with the continuous increase of the iteration number, the objective function value UO and LO decrease rapidly at first, and then decrease slowly until the algorithm converges smoothly. In the convergence process of UO, the number of iterations needed for the algorithm to reach convergence is about 50, but, in fact, the convergence solution was obtained in 20 iterations. It indicates that our proposed algorithm performs well in convergence.

5.3. Equilibrium Analysis

To judge whether the OD passenger flow distribution in each time-period has reached an equilibrium, this section aims to analyze the behaviors of passengers' train-booking in the optimized results obtained by our proposed algorithm. Due to the huge number of granularity-based demands, it is hard to enumerate their chosen paths in detail one by one, thus we only select the optimized results of path-choice in two time periods (13:00–13:05 and 18:00–18:05) as examples to assess the effect of passengers assignment. The paths chosen or not chosen by passengers and its passenger generalized cost are shown in Figure 13.

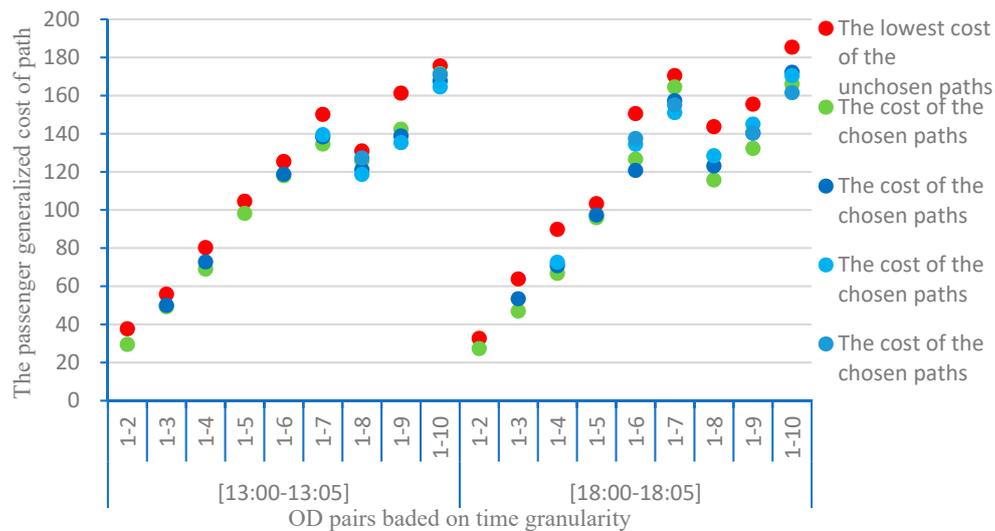


Figure 13. The paths chosen or not chosen by passengers and its passenger generalized cost.

Figure 13 shows that the passengers of each OD-pair can choose at least one path to travel, and the generalized travel cost on their chosen paths are almost equivalent with each other. Furthermore, for every OD-pair, the generalized travel cost on chosen paths (green dots, dark blue dots, light blue dots, and cyan dots) are all less than the minimum among other unchosen paths (red dots). Therefore, it proves that the passenger assignment has achieved an equilibrium. For example, for OD 1–8 in time period 13:00–13:05, the total number of passengers corresponding to this OD is 20, and they are assigned to four different paths with the number of passengers being 5, 5, 3 and 7, respectively. The relevant generalized travel cost on these four paths are accordingly 125.65, 121.00, 118.60 and 127.25. It is obvious that the difference among them are limited in a small range, and all of them are no more than the minimum among other paths not chosen by passengers of this OD-pair.

5.4. The Benefits of Scheduling Trains Considering Passenger Train-Choosing

To show the benefits of scheduling trains integrating passenger train-choosing, this section first optimizes train schedules with the above input using a passenger-free algorithm, and then optimizes it with our proposed algorithm. By comparing these results obtained by two algorithms, respectively, we analyze the benefits of our algorithm regarding with passenger service level. Note that our algorithm optimizes the train timetable combining with passenger' train-booking, while passenger-free algorithm only schedules trains by minimizing the summation of train travel time subject to some operation headway constraints, and the service level of the train timetable obtained by the latter is determined by distributing passengers to trains at last. Figure 14 gives the train diagram optimized by our proposed algorithm. Table 10 gives the objectives, the average deviation times of passengers at origins and the average travel speeds of two algorithms.

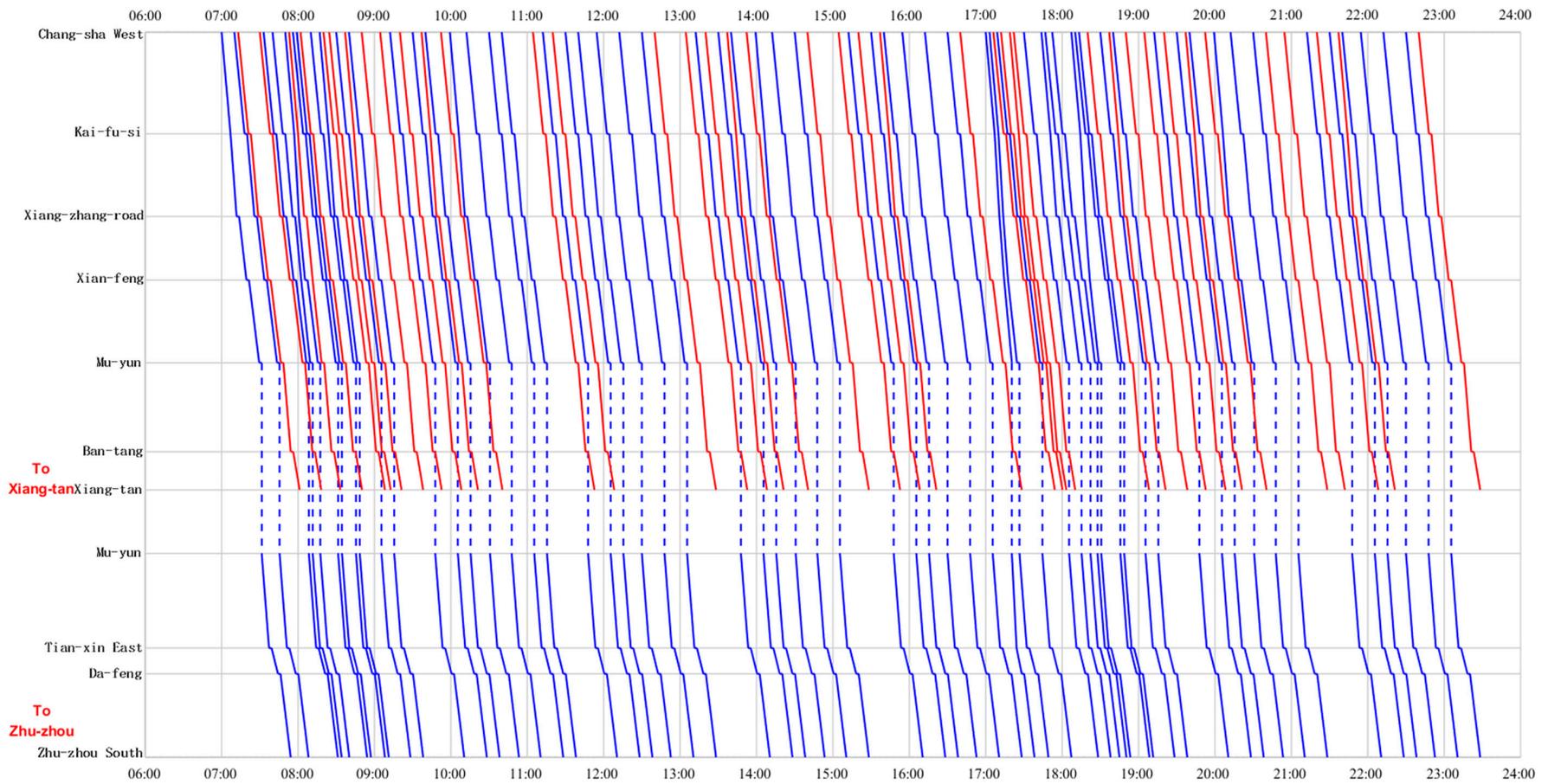


Figure 14. The full-day train diagram of Changsha–Zhuzhou–Xiangtan Intercity Railway obtained by our algorithm.

Table 10. The indicators of two algorithms.

Algorithms	UO (min)	LO (min)	Per Capita Deviation Time (min)	Per Capita Traveling Speed (km/h)
The proposed algorithm	2,646,186	66,544,031	9.89	104.35
Passenger-free algorithm	5,306,730	158,946,562	14.10	93.57

Table 10 demonstrates that values of indicators in our proposed algorithm are obviously better than those obtained in the passenger-free algorithm. On the one hand, per capita deviation time of passengers in our algorithm is reduced by 29.86% compared to that in passenger-free algorithm, which is because we have taken train-booking of passengers into account so that making optimized timetable is well matched with the actual passenger trip rules. On the other hand, as the stop plan was optimized in our algorithm, per capita travelling speed is increased by 10.34% compared with that in passenger-free algorithm.

To illustrate the performance of two algorithms specifically, more indicators are presented to evaluate and analyze comparatively between two algorithms.

5.4.1. Average Travel Speed of Each OD Pairs

Based on the results shown in Figure 15, the average travel speed of each OD passenger flow in our algorithm is significantly faster than that in passenger-free algorithm. Specifically, in our algorithm, the average travel speed is 104.35 km/h, while it is 93.57 km/h in the passenger-free algorithm. Moreover, the percent of OD pairs whose average travel speed exceeds 100 km/h is 53.85% in our algorithm compared with only 20.51% in passenger-free algorithm. Thus, the train schedule obtained by our algorithm can give passengers a much faster travel speed, which improves the travel efficiency of passengers in urban region and ensures the timeliness of travel.

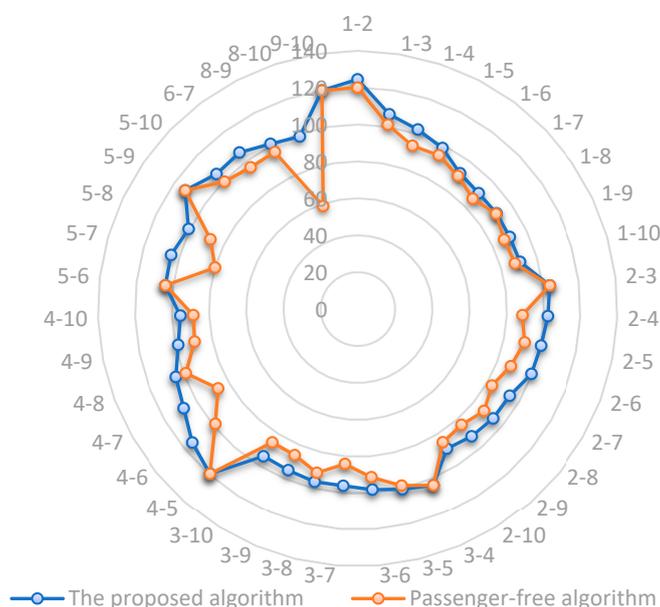


Figure 15. Comparison diagram of the OD pairs' average travel speed between two algorithms.

5.4.2. Percentage of Passengers Corresponding to Different Deviation Time at Origin

As illustrated in Table 11, the train timetable obtained by the algorithm in this paper can significantly reduce the deviation time of passengers at the origin station through detailed data. Specifically, the percentage of passengers in our algorithm are all higher than passenger-free algorithm when their deviation time is shorter than 15 min, while they are all lower than passenger-free algorithm

when their deviation time is longer than 15 min. For example, on the train timetable obtained by our algorithm, 47.35% of the passengers have a deviation time of less than 5 min at origin, compared with 39.98%. Similarly, the former only has 3.58% of passengers whose deviation time is more than 30 min at origin, while the latter is 12.95%.

Table 11. The percentage of passengers in different range of deviation time at origin.

Algorithms	The Range of Deviation Time at Origin (min)						Other
	[0,5]	(5,10]	(10,15]	(15,20]	(20,25]	(25,30]	
The proposed algorithm	47.35%	22.79%	14.57%	4.35%	3.86%	3.50%	3.58%
Passenger-free algorithm	39.98%	14.07%	14.42%	7.26%	5.20%	6.12%	12.95%

5.4.3. The Changing Trend of Passengers' Demand and Running Trains' Number

To analyze the matching degree between train capacity and passenger flow in different periods, their matching curves are presented in Figure 16. Due to the huge number of granularity-based demands, we integrate data in units of 1 h. The red line denotes the percentage of passenger flow in each period, while the blue and green lines represent the percentage of train number in each period optimized by our proposed algorithm and passenger-free algorithm, respectively. According to the changing trend of three lines, we can draw the conclusion that the changing trend of train number is more consistent with passenger travel demand in our algorithm, while there is no obvious matching relationship in passenger-free algorithm.

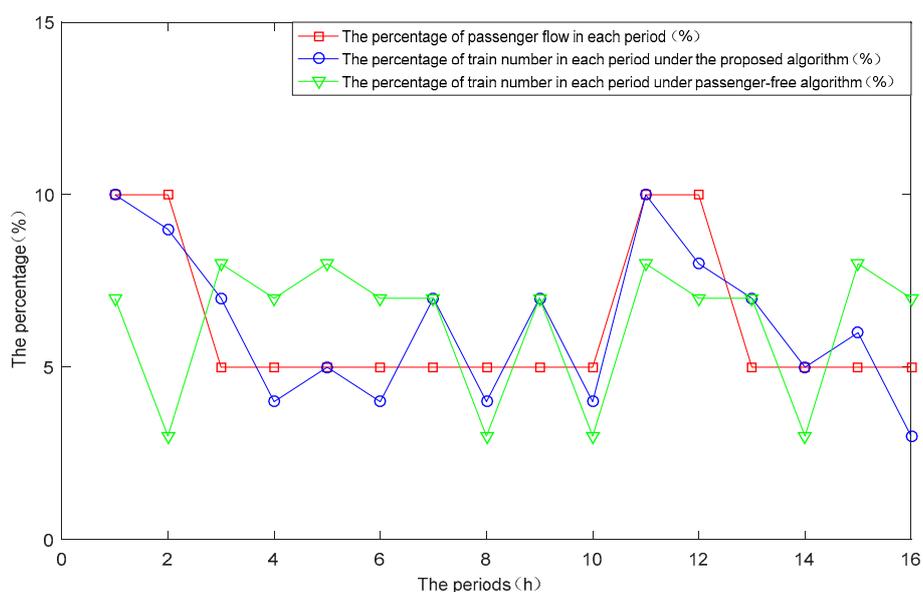


Figure 16. Matching curves between train number and passenger flow in different periods.

5.4.4. Service Frequency of Each OD Pair

A satisfactory train schedule should not only minimize the total travel time and travel cost of passengers, but also optimize the service frequency between OD pairs along the railway line. In the process of optimizing the arrival and departure times at stations of trains, our algorithm also takes the optimization of train stops into consideration. On the train schedule obtained by our algorithm, the service frequency of each OD pair of CZT Intercity Railway is shown in Figure 17. As shown in Figure 16, the service frequency of each OD pair effectively match with its travel demand, so that trains' total travel time is cut down, and the satisfaction of passengers is improved greatly.

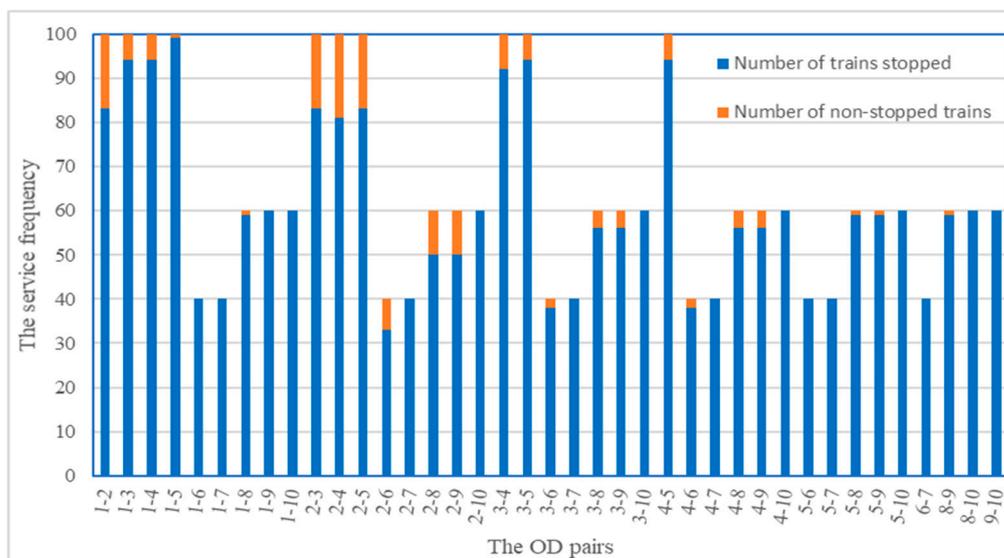


Figure 17. The service frequency of each OD pair in the operation period under our algorithm.

6. Conclusions and Further Study

To apply the feedback of passenger train-booking decisions to guide the optimization of the demand-oriented train timetable, this paper combines passengers' equilibrium assignment to trains into train timetable optimization and proposes a priority-based heuristic approach for demand-oriented train timetabling. The train timetable obtained by the proposed method has a higher service level. It can reduce the travel cost of passengers, thereby attract more passengers to choose the more energy-efficient and environmentally friendly railway and actively promote the sustainable development of society. The main conclusions of this paper are as follows:

1. A bi-level programming model is designed for a demand-oriented train timetabling problem integrated with the train-booking decisions of passengers, and a priority-based heuristic algorithm is designed to solve this model. The proposed algorithm consists of three sub-algorithms for distributing passengers to trains, finding trains' original departure times based on their priorities and distributed passengers, and generating a feasible train timetable, respectively.
2. Experimental results from the Changsha–Zhuzhou–Xiangtan intercity rail network show that the train timetable obtained by our proposed method has higher service level compared with that of a passenger-free algorithm. On the one hand, the per capita deviation time of passengers in our algorithm is reduced by 29.86% compared to that in the passenger-free algorithm, and the per capita traveling speed is increased by 10.34% compared with that in the passenger-free algorithm. On the other hand, as the train stop plan was optimized in our algorithm, the number of train stops is reduced by 6.11%, which reduces the energy consumption and time waste caused by the stops. Moreover, the optimized train timetable is well-matched with the actual passenger trip rules, which effectively improves the load factor of the trains.

The following are worth further study in the future.

1. In this paper, each station is simplified as a node with parking capacity, but the train's travel path at a station is not considered. Therefore, future studies should consider combining the train routing problem at stations with the train timetabling problem to obtain a more practical train timetable.
2. Due to the competition between expressways and intercity railways, the service level of the train timetable has a great influence on its passenger demand. Thus, it is necessary to optimize the train schedule further considering the elastic change relationship between passenger demand and train service level.

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