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# An Optimal Management Strategy of Carbon Forestry with a Stochastic Price

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**Abstract:** An analysis for the value of carbon forestry needs to be provided for the successful establishment of the carbon offset market in Korea. We present an optimal management strategy for a forest owner who participates in the offset market. Given a stochastic process of the timber price following a geometric Brownian motion, the profit maximization problem of the forest owner is solved. The model finds an optimal harvest time in the presence of the carbon and timber revenues with opposing time effects. Sensitivity analysis is performed with respect to the volatility rate of the timber price and the discount rate. The presented model is applied to the study of the Korean larch case to identify the threshold timber price above which it is optimal to harvest trees.

**Keywords:** forest valuation; carbon forestry; optimal harvesting time; stochastic price

## 1. Introduction

The anthropogenic carbon dioxide occurred during the last 40 years accounts for about half of the anthropogenic CO<sub>2</sub> accumulated between 1750 and 2010, its direct sources being energy supply, industry, transport and buildings, according to IPCC (Intergovernmental Panel on Climate Change) 5th assessment report. The report concludes that the global average surface temperature by 2100 will have increased from 3.7 °C to 4.8 °C compared to pre-industrial levels without additional mitigating efforts [1]. The policies for the mitigation of temperature rise and the accompanying climate change focus on controlling carbon emissions and utilizing artificial carbon sinks. Although industrial sectors have shown a resistance to emission reductions for the environment believing that they limit profitability and growth, protecting the environment is essential, even for sustainable economic growth. The emission trading scheme is a market where the trade of carbon permits occur between the sellers who reduce carbon emission below the baseline and the buyers who emit excessively, and has been designed to provide economic incentives for industrial sectors to participate in emission reduction and to minimize the economic inefficiency derived by controls. In the emission trading scheme, a carbon offset system in which tradable permits are issued by the amount of carbon offset has a critical role in not only achieving the emission reduction goal by using carbon sinks, but also sustaining economic growth.

New Zealand is one of the countries in which offset schemes are well-established. The forestry sector in New Zealand offsets approximately 32% of its total greenhouse gas emissions, proving that non-energy sectors participating in emission trading schemes make significant contributions to reduce emissions and to avoid market distortion [2].

A total of 65% of Korean territory is covered with forests, with 70% of forests being private. Efficient forest management will offset around 92,300 tC, which implies that forestry will save approximately 41.4 billion won [3]. In terms of forest owners, the carbon offset scheme is attractive

as it provides an opportunity to make a profit while trees grow, in contrast to just making a profit from timber sales at harvesting time. Emission permits started to trade from January 2015 in Korea. The trade of the Korea Credit Unit (KCU), which is its own offset credit, began in April 2015, although the trade volume remains low. Therefore, it is critical to provide incentives to participants in the first stage of the new system to achieve its objective by analyzing the profit structure of forest owners and suggesting ways for efficient forest management.

However, there have been limited studies on carbon forest management in Korea, with few approaches and methods used in the analysis. Kyung-Taek Min (2011) empirically analyzed the optimal harvesting age in Korea, including the benefits from carbon sequestration. The strong assumption of fixed prices was used with the Faustmann model [4]. Sang-Min Lee et al. (2011) evaluated forest values, combining both the economic value from timber harvesting and the environmental value from carbon sequestration. By maximizing the social utility of a function of the amount of timber harvested and carbon sequestered, the optimal amount of timber harvest can be obtained. Although the change of timber volume has been dynamically analyzed, no stochastic variable is permitted [3]. In a study by Hee Sun Jang et al. (2010), the volatility of a future price was considered with the real option approach; carbon price following the geometric Brownian motion. However, that was for A/R CDM (Afforestation/Reforestation Clean Development Mechanism) projects in which the investment of developed countries to developing countries is admitted as its reduction, not for Korean forest owners [5]. Models for carbon forestry reflecting realities, such as the dynamics of timber and carbon stock changes, the uncertainties of price movements, and the possibilities of linked international offset markets, need to be developed. Moreover, it is critical that the models are applied to empirical studies in Korean carbon forestry for the offset market.

Before applying a model, the fundamental method of forest valuation suggested by Faustmann in 1849 should be reviewed. The net present value of a forest is estimated by discounted future cash flows from timber sales. Based on the Faustmann model, many efforts have been made aiming to reflect reality in a model for forest valuation. Brazee, R. and Mendelsohn, R. (1988) considered the volatility of the future price. A stochastic timber price was incorporated, and they concluded that the net present value of a forest is evaluated higher than that based on the Faustmann model [6]. Englin, J. and Cakaway, J. M. (1993) recognized another source of income for a forest owner, considering the carbon-sequestered function of a forest. They integrated the revenue from carbon credit sales, and developed an optimal harvesting rule [7].

Forest management and the optimal cutting time of carbon forestry with a stochastic price have been analyzed in many studies. Chladná, Z. (2007) developed the optimal rotation rules, and provided a sensitivity analysis for the price process types, the structure of incomes and the discount rates. It was assumed that the timber price follows a mean-reverting process while the carbon price follows the geometric Brownian motion [8]. Guthrie, G., and Kumareswaran, D. (2009) used a real option model with a stochastic timber price following a mean-reverting process. The optimal time for cutting trees was suggested, and the options for replanting and abandoning forests were examined [9]. Tee, J. et al. (2014) further investigated the flexible valuation for carbon forestry with two stochastic prices using a real option binomial tree, allowing for joint optimization of the harvest decision [10].

On top of the reviewed literature, the goals of this paper are as follows. (1) Setting up a model to maximize the profit for the forest owners. The carbon credit revenue is added to the traditional income source of timber sales; (2) With a stochastic timber price considered, the optimal time to harvest is investigated in order to maximize the expected returns of forest management; (3) An empirical study is performed with the data of the species of larch in Korea.

## 2. Methods

The value function in forestry is the expected present value of forest management. The total revenue is separated into two parts; that from the carbon credit sales, and that from the timber sales. The carbon revenue is calculated by integrating the future cash flow from the carbon credit sales by

the amount of carbon sequestration of the trees up until the forest is harvested. It is assumed that all the trees are cut and sold as timber when the forest owner determines to harvest and exit the forest market. The costs arise only at harvesting time: The harvesting cost and the exit cost. The exit cost is a type of a penalty for releasing the stored carbon into the atmosphere.

$$\max E_p \left[ \int_0^\tau s\theta\Delta V(t)e^{-\rho t} dt + (X(\tau) - cV(\tau) - K)e^{-\rho\tau} \right]. \quad (1)$$

Equation (1) is the objective function, which will be maximized by considering the optimal timing of harvest. The goal is to determine the optimal harvesting time,  $\tau$ , determined endogenously.  $\int_0^\tau s\theta\Delta V(t)e^{-\rho t} dt$  is the carbon revenue from the sales of the offset credits produced by the amount of annual carbon stock changes. The future revenues should be discounted by the discount rate,  $\rho$ . The offset credit is sold at the fixed carbon price,  $s$ . The carbon stock change is assumed to be proportional to the timber volume change,  $\Delta V(t)$ , which decreases exponentially;  $\Delta V(t) = V_0e^{-r_1t}$ . How much carbon is sequestered relative to the timber growth differs by the species of the trees. Each country tries to develop its own sequestration factors for different species of trees. The sequestration factor,  $\theta$ , is the product of basic wood density, the biomass expansion factor, the root-shot ratio and the carbon fraction. The forest owner gains the carbon sales revenue up until the trees are harvested at the time,  $\tau$ .

The profit from the timber sales is  $(X(\tau) - cV(\tau) - K)e^{-\rho\tau}$ , which is immediately realized at harvesting time,  $\tau$ .  $c$  is a unit harvesting cost, and  $K$  is a lump-sum penalty to exit forestry. By considering the stochastic nature of the revenue, the sales revenue,  $X(t)$ , is assumed to follow the geometric Brownian motion.

$$dX(t) = \mu X(t)dt + \sigma X(t)dz(t). \quad (2)$$

In Equation (2),  $\mu$  is the drift rate of the timber revenue, and  $\sigma$  is the volatility rate.  $dt$  is the increment of time, and  $dz(t)$  represents the Wiener process. The expectation of the Wiener increment is  $E(dz) = 0$ , and the variance is  $Var(dz) = dt$ . The uncertainty of the timber revenue increases as time passes because the variance of  $dz$  is shown to be the time increment. The timber revenue depends on the price of timber,  $p(t)$ , and the timber volume,  $V(t)$ ; that is,  $X(t) = p(t)V(t)$ . It is assumed that the price of timber,  $p(t)$ , follows the geometric Brownian motion with the drift rate of timber price,  $\alpha$ , and the volatility rate,  $\sigma$ , as shown in Equation (3).

$$dp(t) = \alpha p(t)dt + \sigma p(t)dz(t). \quad (3)$$

An exponential timber growth is assumed. In general, the logistic function is appropriate for the growth function of trees. However, based on the assumption that harvesting normally occurs during the time when the trees grow exponentially, exponential timber growth is not regarded as an unreasonable assumption for the sake of simplicity. The timber growth function with the growth rate,  $r_2$ , is defined by  $V(t) = V_0e^{r_2t}$  which provides  $\mu = \alpha + r_2$ . From now on, the timber revenue,  $X(t)$ , is denoted as  $p(t)V(t)$ , and we will follow the stochastic movement of the timber price,  $p(t)$ .

$$\tau = \inf\{\tau \geq 0 | p(t) = a\}. \quad (4)$$

Equation (4) is the condition of the optimal harvesting time. The value function (1) is maximized as the timber price goes beyond the threshold price that will be numerically obtained in the following analysis. Note that  $a > p_0$  holds to avoid the trivial problem where  $p_0$  is the initial price of timber. The optimal time for harvesting,  $\tau$ , is the first passage time when the stochastic timber price,  $p(t)$ , is expected to hit the threshold.

Let  $G(a)$  denote the expected present value of the objective function (1) that is defined as below:

$$G(a) = E_p \left[ \int_0^\tau s\theta\Delta V_0e^{-(\rho+r_1)t} dt + (p(\tau)V_0e^{r_2\tau} - cV_0e^{r_2\tau} - K)e^{-\rho\tau} \right].$$

Note that the harvest time  $\tau$  is subject to stochastic process (3). By applying the expectation operator to the parts endogenously determined, we obtain

$$G(a) = E_p \left[ \int_0^\tau s\theta\Delta V_0 e^{-(\rho+r_1)t} dt \right] + E_p(p(\tau))V_0 E_p(e^{-(\rho-r_2)\tau}) - cV_0 E_p(e^{-(\rho-r_2)\tau}) - KE_p(e^{-\rho\tau}).$$

Since  $p(\tau) = a$ ,

$$G(a) = E_p \left[ \int_0^\tau s\theta\Delta V_0 e^{-(\rho+r_1)t} dt \right] + aV_0 E_p(e^{-(\rho-r_2)\tau}) - cV_0 E_p(e^{-(\rho-r_2)\tau}) - KE_p(e^{-\rho\tau}).$$

As the definite integral is calculated, we then obtain

$$G(a) = \frac{s\theta\Delta V_0}{\rho+r_1} [1 - E_p(e^{-(\rho+r_1)\tau})] + aV_0 E_p(e^{-(\rho-r_2)\tau}) - cV_0 E_p(e^{-(\rho-r_2)\tau}) - KE_p(e^{-\rho\tau}). \quad (5)$$

The expected discounted factors are derived using the same process as Chang, F. R. (2005) and Harrison, J. M. (1985, p. 42) [11,12]. The expected discounted factors are as follows

$$E(e^{-(\rho+r_1)\tau}) = \left(\frac{p_0}{a}\right)^{\beta_1},$$

$$E(e^{-(\rho-r_2)\tau}) = \left(\frac{p_0}{a}\right)^{\beta_2},$$

$$E(e^{-\rho\tau}) = \left(\frac{p_0}{a}\right)^\gamma$$

where

$$\beta_1 = \frac{-(\alpha - \frac{\sigma^2}{2}) + \sqrt{(\alpha - \frac{\sigma^2}{2})^2 + 2(\rho + r_1)\sigma^2}}{\sigma^2},$$

$$\beta_2 = \frac{-(\alpha - \frac{\sigma^2}{2}) + \sqrt{(\alpha - \frac{\sigma^2}{2})^2 + 2(\rho - r_2)\sigma^2}}{\sigma^2},$$

$$\gamma = \frac{-(\alpha - \frac{\sigma^2}{2}) + \sqrt{(\alpha - \frac{\sigma^2}{2})^2 + 2\rho\sigma^2}}{\sigma^2}.$$

The expected discounted factors  $\beta_1$ ,  $\beta_2$  and  $\gamma$  should be greater than one to have intuitively correct values. Equation (5) with the expected discount factors is given as

$$G(a) = \frac{s\theta\Delta V_0}{\rho+r_1} - \frac{s\theta\Delta V_0}{\rho+r_1} \left(\frac{p_0}{a}\right)^{\beta_1} + aV_0 \left(\frac{p_0}{a}\right)^{\beta_2} - cV_0 \left(\frac{p_0}{a}\right)^{\beta_2} - K \left(\frac{p_0}{a}\right)^\gamma. \quad (6)$$

The first two terms,  $\frac{s\theta\Delta V_0}{\rho+r_1} [1 - (\frac{p_0}{a})^{\beta_1}]$ , represent the fundamental value of carbon revenue. As the threshold timber price,  $a$ , increases, the carbon revenue increases. This is intuitive because the earning period of carbon revenue is extended. A higher threshold of timber price indicates that the time to harvest is delayed as we assumed that the timber price follows the geometric Brownian motion. The other terms in Equation (6),  $aV_0 (\frac{p_0}{a})^{\beta_2} - cV_0 (\frac{p_0}{a})^{\beta_2} - K (\frac{p_0}{a})^\gamma$ , are associated with timber.  $aV_0 (\frac{p_0}{a})^{\beta_2}$ , is the timber sales revenue, and  $cV_0 (\frac{p_0}{a})^{\beta_2} + K (\frac{p_0}{a})^\gamma$  is the costs at harvesting time. When the sales price of timber,  $a$ , increases, which means the time to harvest is postponed, the timber revenue also increases with the more volume of timber sold. The timber revenue in terms of future value grows simply, but the discounted factors lower the timber profit as time passes. We aim to determine the threshold timber price,  $a$ , that maximizes the expected current value of the forest,  $G(a)$ , and the corresponding time,  $\tau$ , as the optimal time to harvest trees. Numerical analysis is performed to determine the maximum point of  $G(a)$  solving the nonlinear function, Equation (6), which is not monotonically increasing or decreasing.

### 3. Results

#### 3.1. Data Description

The larch forest is the most common among the artificial forests in Korea [13]. The timber of larch is multipurpose and of economic significance, and the average carbon stock for a baseline scenario is 51.61 tC/ha, which is a mediocre level among major tree species in Korea [14]. There is an incentive for larch forest owners to participate in the offset market as the owner may increase his profit from carbon credits sales.

The transaction timber price of larch from January 2000–December 2013 was sourced from yearly reports for forestry by the Korea Forest Service (Figure 1) [15–27]. A price,  $p(t)$ , follows the geometric Brownian motion and the drift rate,  $\alpha$ , and the volatility rate,  $\sigma$ , are estimated through log-normal transformation as in Tsay (2002) [28]. Let  $y(t) = \ln p(t) - \ln p(t-1)$ , and  $\Delta$  be the time interval. Let  $\Delta = 1/12$  since we use monthly data. The log difference of price,  $y(t)$ , is normally distributed with a mean  $(\alpha - \sigma^2/2)\Delta$  and variance  $\sigma^2\Delta$ . Let  $\bar{y}$  and  $s_y$  denote the sample mean and standard deviation respectively. Therefore, we obtain the drift rate and the volatility rate as shown in Equations (7) and (8).

$$\alpha = \frac{\bar{y}}{\Delta} + \frac{s_y^2}{2\Delta} \quad (7)$$

$$\sigma = \frac{s_y}{\sqrt{\Delta}} \quad (8)$$

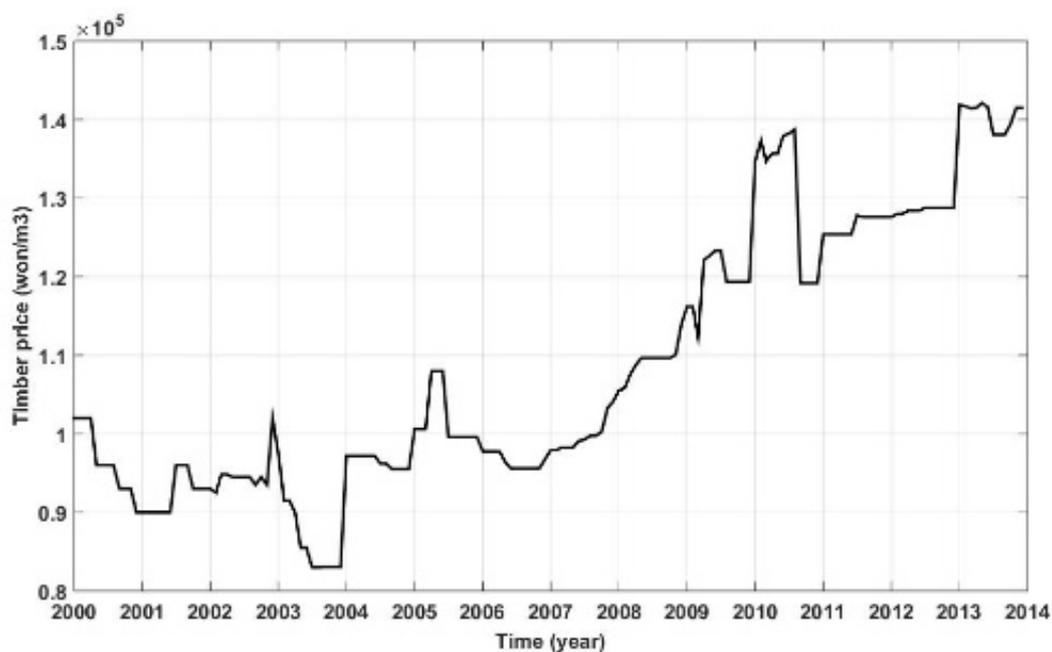


Figure 1. Timber Price, January 2000–December 2013.

The sample mean,  $\bar{y}$ , is 0.00196, and standard deviation,  $s_y$ , is 0.028382. The estimate of the drift rate is given as  $\alpha = 0.02$ , and volatility rate as  $\sigma = 0.09$ .

The timber volume data was sourced from the Korea Forest Service and the Korea Forest Research Institute (Figure 2) [29]. The original data uses 5-year time intervals, showing a polynomial function; therefore, the data is reproduced as annual data by polynomial interpolation.

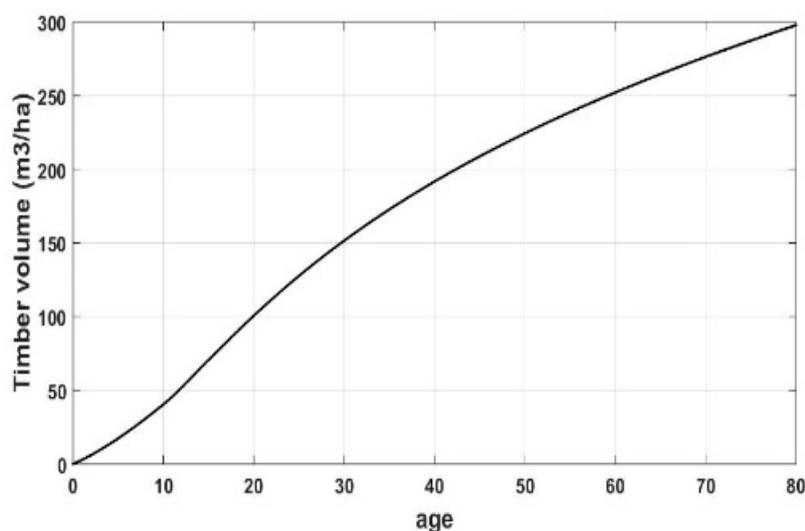


Figure 2. Timber volume for site index 16.

The model in Section 2 assumed an exponential timber growth for calculation simplicity and to follow existing theories of forestry. The hypothetical increase rate of timber volume,  $r_2$ , is estimated as 0.0196. In order to reflect the characteristics of the data, the final results will be adjusted.

It is assumed that offset credits issued yearly are sold during the same year. The carbon stock change is based on the species-specific sequestration factor and the timber volume change, as shown in Equation (9).

$$\text{Carbon stock change (tCO}_2\text{/ha/yr)} = \Delta V(t) \times D \times BEF \times (1 + R) \times CF \times \frac{44}{12}. \quad (9)$$

Table 1 summarizes the carbon sequestration factors of larch developed by the Korea Forest Service [30]. The sequestration factor,  $\theta$ , for larch is calculated to be 1.4261 by Equation (9).

Table 1. Carbon sequestration factors for larch.

Basic Wood Density (D) (tdm/m <sup>3</sup> )	Biomass Expansion Factor (BEF)	Root-Shoot Ratio (R)	Carbon Fraction (CF)
0.45	1.34	0.29	0.5

It is assumed that the annual change of timber volume and carbon stock decrease exponentially. As shown in Figure 3, the decreasing rates of the timber volume change and the carbon stock change show no difference because the carbon stock changes are the result of the scalar multiplication of the timber volume change by  $\theta$ . The decreasing rate,  $r_1$ , is estimated as 0.018.

The discount rate is assumed to be 5%. The sensitivity analysis with regards to the discount rate is presented below. Let the price of the offset credit, KCU (Korea Credit Unit) be constant at 10,000 won/tCO<sub>2</sub>. Although the amount of KCU transactions is quite small, it is reported by the Korea Environment Institute that the prices are approximately 10,000 won/tCO<sub>2</sub> [31]. Harvesting cost per unit volume,  $c$ , is 62,500 won according to a project design document (PDD) reported by the Korea Forest Service. A penalty to exit,  $K$ , is assumed to be zero as the penalty now is zero to intrigue more foresters to join in the offset system in Korea. All the parameters are summarized in Table 2.

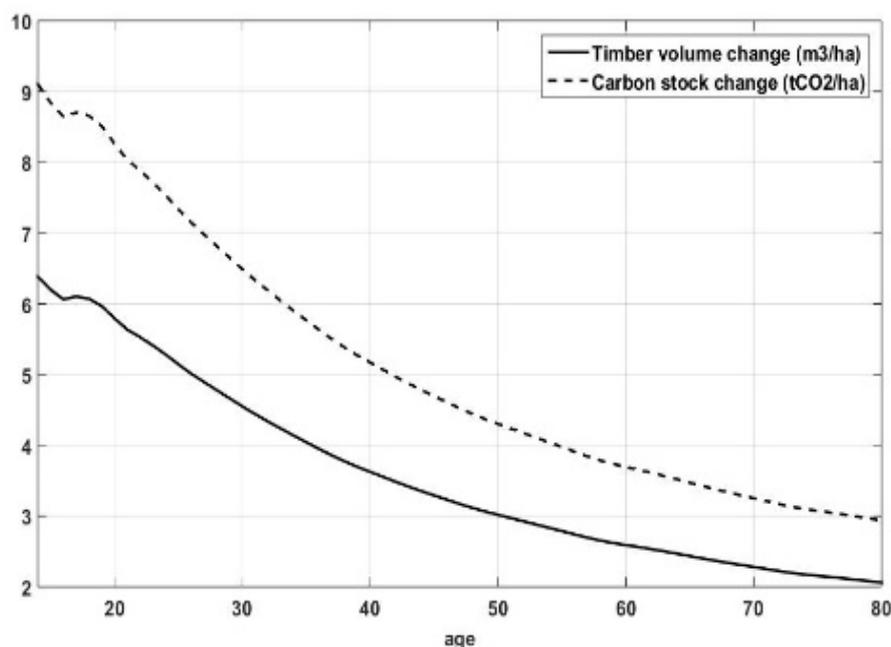


Figure 3. Timber volume change and Carbon stock change.

Table 2. Parameter estimates.

Parameter	Estimates
The drift rate, $\alpha$	0.02
The volatility rate, $\sigma$	0.09
The negative growth rate of $\Delta V(t)$ , $r_1$	0.018
The growth rate of $V(t)$ , $r_2$	0.196
The sequestration factor, $\theta$	1.4261
The discount rate, $\rho$	0.05
The offset credit price, $s$	10,000 won
The initial timber volume, $V(0)$	64.5 m <sup>3</sup> /ha
The initial change of timber volume, $\Delta V(0)$	6.38 m <sup>3</sup> /ha/year
The initial price of timber, $p(0)$	102,000 won
The harvesting cost, $c$	62,500 won/m <sup>3</sup>
The penalty, $K$	0 won

### 3.2. Empirical Result

Firstly, whether a forest owner has enough incentive to participate in the offset market was investigated. Figure 4 compares the profit from both carbon and timber sales with the profit from only timber sales. Carbon forestry yields a higher profit at any threshold timber price of  $a$ ; that is, forest owners can earn more money if they participate in the offset market, regardless of the timber price at which they determine to sell.

The optimal threshold price will provide the maximum value of the forest. Figure 5 represents that threshold price, which converges to about 260,000 won/m<sup>3</sup>. This implies that the forest owner needs to decide to harvest the trees when the timber price reaches 260,000 won/m<sup>3</sup> in order to maximize his profits, including both carbon credits and timber sales.

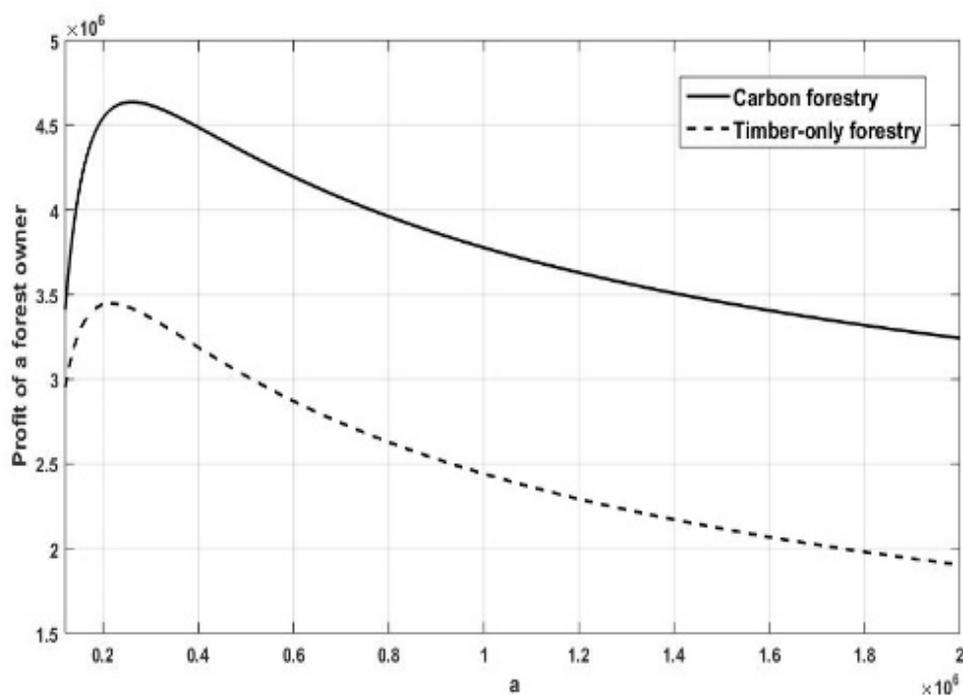


Figure 4. Profits of carbon forestry and timber-only forestry.

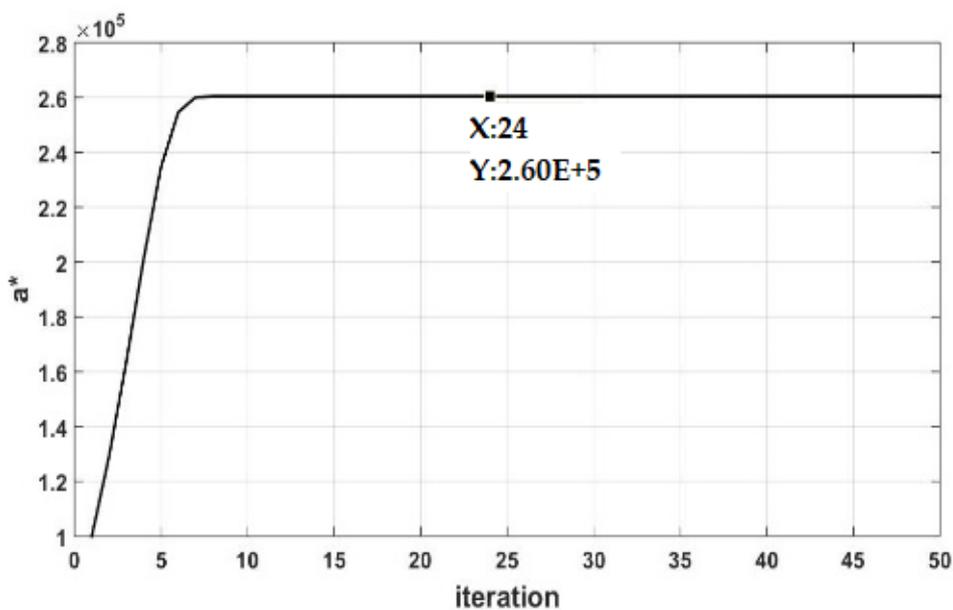


Figure 5. The threshold price of timber,  $a^*$ .

The corresponding first hitting time can be obtained by:

$$E(\tau^*) = \begin{cases} \frac{1}{\alpha - 0.5\sigma^2} \ln\left(\frac{p^*}{p_0}\right) & \text{if } \alpha > 0.5\sigma^2 \\ \infty & \text{if } \alpha \leq 0.5\sigma^2. \end{cases} \tag{10}$$

The optimal time at which the owner should determine harvest is 45.1 years after the trees are planted.

The threshold price and the corresponding optimal time to harvest are affected by the volatility of the price and the discount rate. The sensitivity analyses of the threshold price and the optimal

harvesting time with respect to the volatility rate of the timber price and the discount rate are shown below.

Figure 6 illustrates that the converging point of the threshold price becomes higher as the rate of price volatility increases. The relatively high risk of business with a price fluctuation increases the threshold of the timber price.

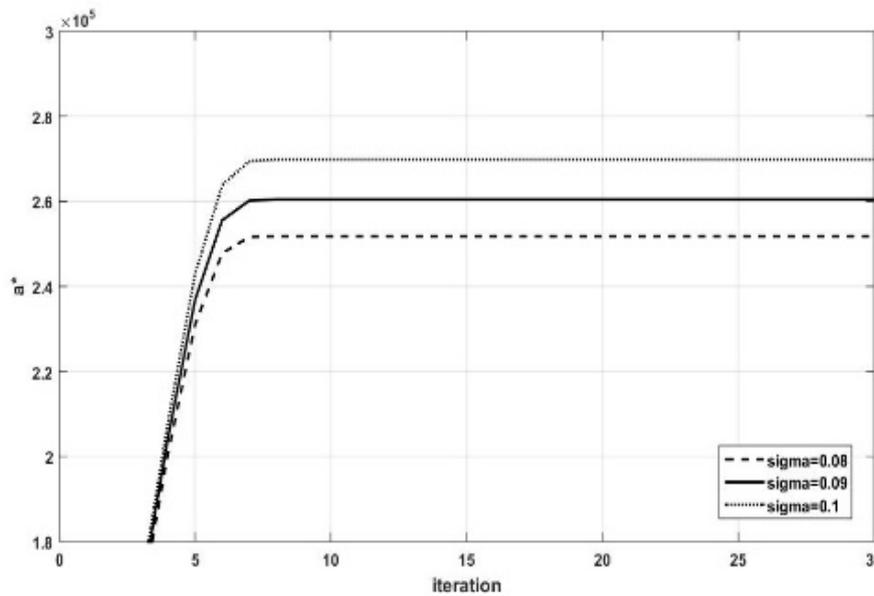


Figure 6. Sensitivity analysis of the threshold price (the volatility rate).

Figure 7 shows an inverse relationship between the threshold price and the discount rate. The higher the discount rate, the lower the evaluated present value of carbon forestry. It is optimal for a forest owner to determine to harvest when the timber price is relatively low at a higher discount rate.

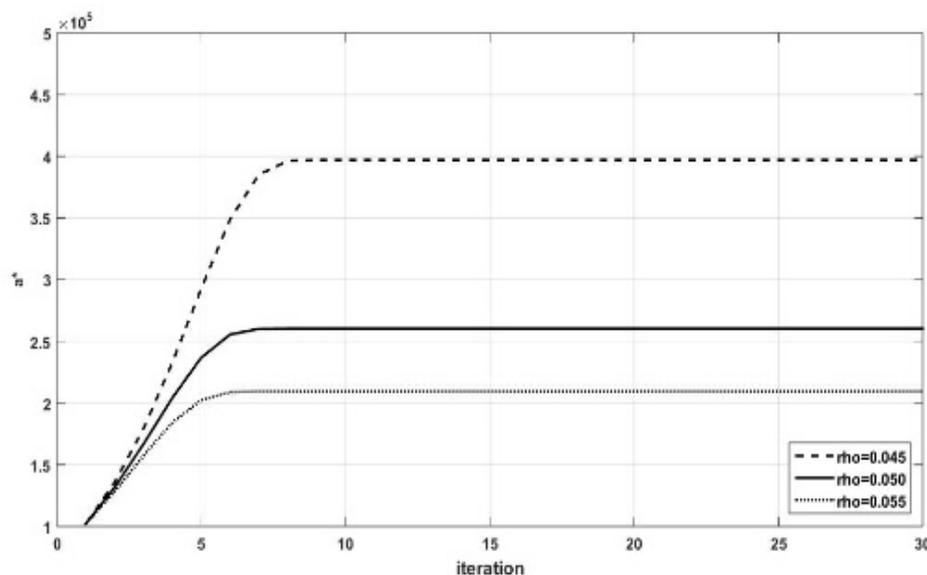


Figure 7. Sensitivity analysis of the threshold price (the discount rate).

The overall sensitivity analysis of the optimal harvesting time with respect to the volatility rate and the discount rate is provided in Figure 8. The higher threshold price means the time for harvest

needs to be postponed. The optimal harvesting time is delayed by an increase in the price volatility and a decrease in the discount rate.

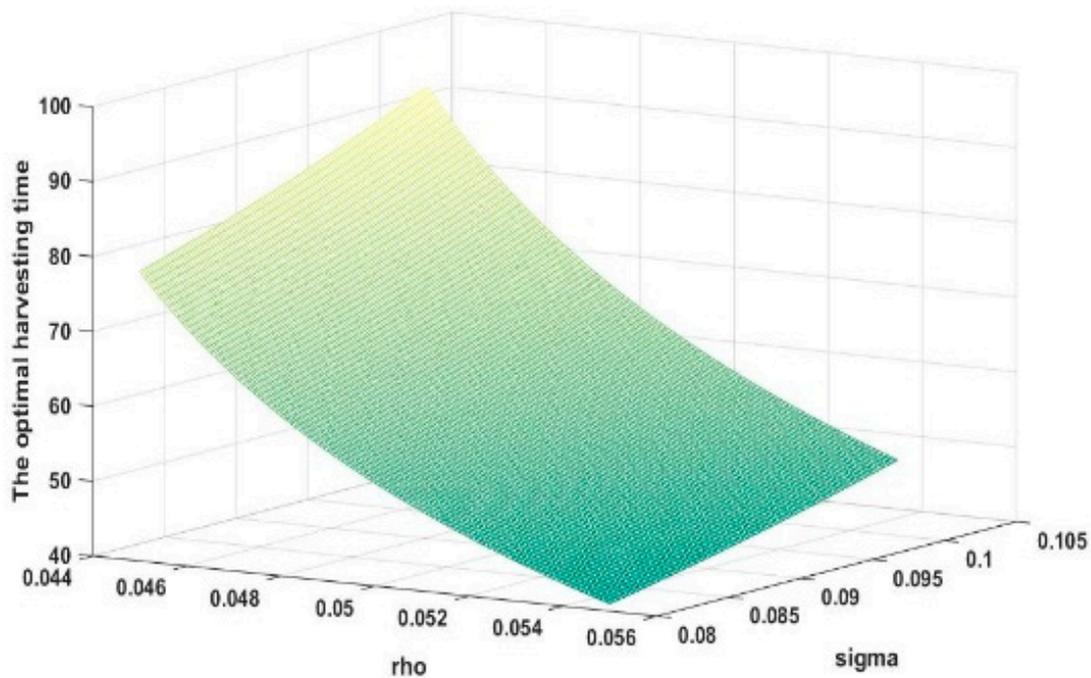


Figure 8. Sensitivity analysis of the optimal harvesting time (the discount rate & the volatility rate).

However, this result is based on the assumption of exponential timber growth. As shown in Figure 9, a gap exists between the timber volume used in the model and the actual timber volume data. An adjustment process is needed for the tree growth to be shown as a polynomial function in the data. The cubic polynomial function of timber growth,

$$\overline{V}(t) = m_1t + m_2t^2 + m_3t^3 \tag{11}$$

was estimated as in Ref. [8], and the results are as below in Table 3.

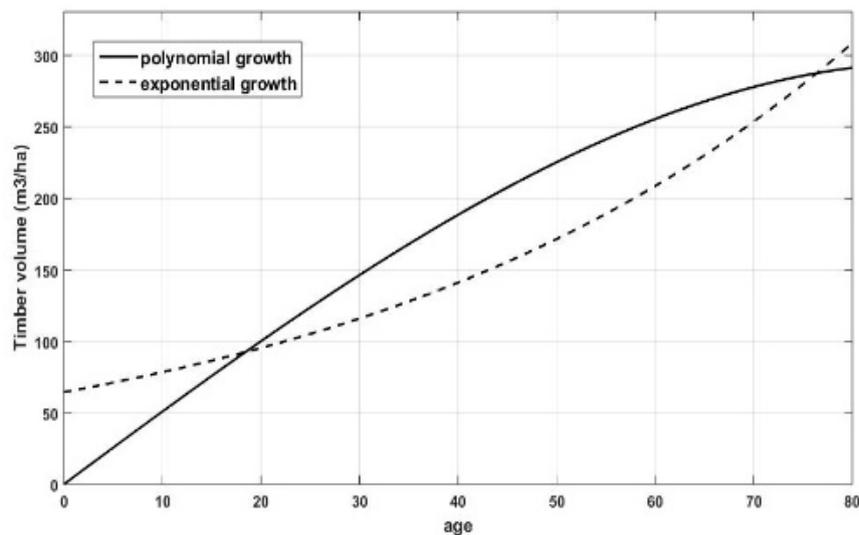


Figure 9. Timber growth function.

**Table 3.** The result of OLS estimation for the polynomial timber growth.

Variable	Coefficient	Std. Error
AGE	5.133933	0.110198
AGE2	−0.002411	0.004242
AGE3	−0.000203	0.026278

The exponential growth function of timber in the model is given as

$$V(t) = V_0 e^{r_2 t} \tag{12}$$

In addition, the threshold of the timber price,  $a^*$ , and the corresponding optimal harvesting time,  $\tau$ , were calculated. Plugging these endogenously determined variables into the value function (1), the maximized profit,  $G_\tau$ , can be calculated as

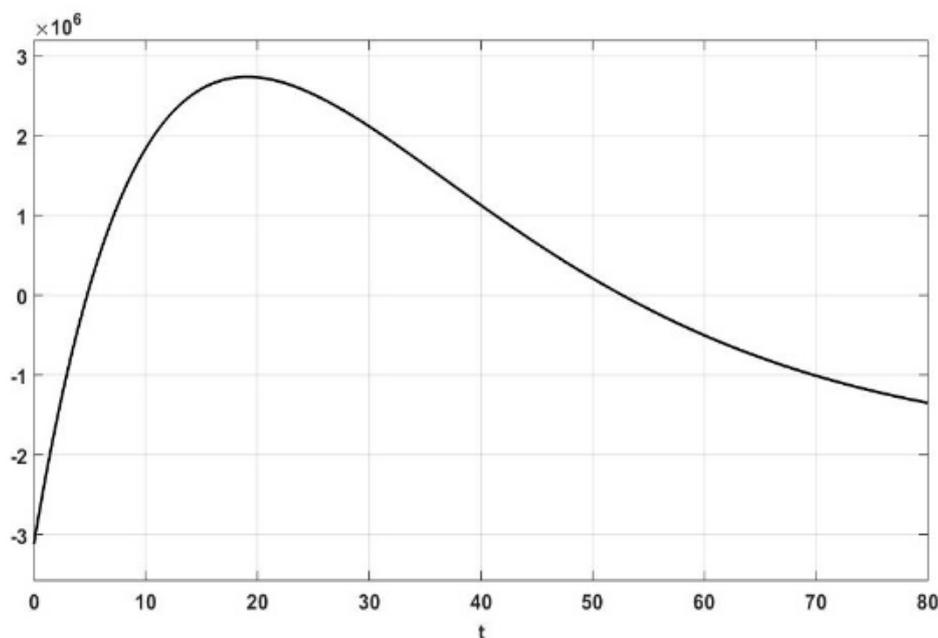
$$G_\tau = \frac{s\theta\Delta V_0}{\rho + r_1} [1 - e^{-(\rho+r_1)\tau}] + [a^*V(\tau) - cV(\tau) - K]e^{-\rho\tau} \tag{13}$$

The final objective is to find the actual optimal harvesting time  $\bar{\tau}$  corresponding to the timber data showing polynomial growth. The profit calculated with the polynomial timber growth function (11),  $\bar{G}$ , should be equal to  $G_\tau$ .

$$\bar{G} = \frac{s\theta\Delta V_0}{\rho + r_1} [1 - e^{-(\rho+r_1)\tau}] + [a^*\bar{V}(\tau) - c\bar{V}(\tau) - K]e^{-\rho\tau} \tag{14}$$

$\tau$  will be adjusted to  $\bar{\tau}$  by numerically equating (13) to (14).

The gap between  $\bar{G}$  and  $G_\tau$  was plotted in Figure 10. At the point where the function  $(\bar{G} - G_\tau)$  equals zero, the new optimal harvest time is determined. While the previous optimal harvest time,  $\tau$ , was 45.1 assuming exponential timber growth, the adjusted harvest time,  $\bar{\tau}$ , is delayed for 7 years to 52.1 by applying polynomial timber growth. A total of 14 years should be added as the first 14 years of data when the change of timber volume increased had been excluded in the analysis. The last adjusted optimal time is 66.7.



**Figure 10.** The gap between  $\bar{G}$  and  $G_\tau$ .

#### 4. Discussion

In this study, the threshold timber price and the optimal time for harvest are estimated, while allowing a stochastic timber price and carbon revenue from offset credit sales in the case of larch in Korea. Carbon revenues are generated from the amount of the annual change of carbon stock until a forest owner decides to harvest. Timber sales revenue arises only at the time of harvest. Carbon forestry apparently provides more profit for forest owners than timber-only forestry. Combining both revenue sources, the optimal harvest time is determined endogenously to maximize the forest owner's profit, involving the time effect between the carbon revenue and the timber revenue.

The sensitivity analyses for the volatility rate of the price and the discount rate are also provided. The threshold price increases as the volatility rate increases, and the sensitivity analysis of the threshold price with respect to the discount rate showed the opposite result; the threshold price increases with a lower discount rate. In the model, an exponential timber growth is assumed while the timber volume data shows a polynomial growth function. The first estimate of the optimal harvest time needs to be adjusted by applying a polynomial timber growth. The adjusted optimal harvest time is delayed to yield the maximized profit.

A limitation of this study is the assumption of single rotation while many other studies on forest valuation are based on an infinite rotation model. For simplicity, here, what happens after harvesting is not considered. Moreover, the carbon price is fixed in the model because carbon price data has not been sufficiently accumulated in Korea for research purposes until now. As a suggestion for a more flexible future study on this issue, it may be interesting to allow for an infinite rotation or to consider a land conversion option with two stochastic prices.

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