sustainability

MDPI

Article

Is Liquidity Risk Priced? Theory and Evidence

Seok-Kyun Hur [1], Chune Young Chung [1,*] and Chang Liu [2]

[1] School of Business Administration, College of Business and Economics, Chung-Ang University, 84 Heukseok-ro, Dongjak-gu, Seoul 06974, Korea; shur@cau.ac.kr

[2] College of Business, Hawaii Pacific University, 900 Fort Street Mall, Honolulu, HI 96813, USA; cliu@hpu.edu

* Correspondence: bizfinance@cau.ac.kr; Tel.: +82-2-820-5544

Received: 12 April 2018; Accepted: 28 May 2018; Published: 30 May 2018

Abstract: This study studies a recently proposed measure of liquidity premium (or discount). Specifically, the liquidity premium we utilize is defined as a function of a time discount factor, a relative risk aversion parameter, and the expected return and volatility of the asset, given the risk-free rate. Using U.S. stock market data, our empirical results confirm that the proposed liquidity premium measure is largely comparable to that commonly used in existing studies. Our results also imply that a risk factor based on the liquidity premium measure not only explains cross-sectional stock returns, but also time-series excess returns on portfolios sorted on the commonly used liquidity measure. In addition, our study suggests that better understanding the liquidity risk leads to sustainable trading for investors.

Keywords: liquidity premium; uncertainty termination; investment horizon; Amihud's illiquidity ratio; factor models

1. Introduction

A large body of literature attempts to measure various types of risks in financial assets. For example, Allen et al. [1] develop a new measure of risk based on the application of regular vine copulas and apply it to the assessment of composite financial risk. In addition, Yan et al. [2] propose the new empirical method by combining generalized autoregressive score functions and a copula model with high-frequency data to model the conditional time-varying joint distribution of the government bond yields.

An application of regular vine copulas, which are a novel and recently developed statistical and mathematical tool which can be applied in the assessment of composite financial risk. However, liquidity (or illiquidity) risk is not a readily measurable characteristic of financial assets, yet understanding the implications of liquidity on investments results is crucial for the sustainability of investors who face increasingly more investment alternatives that are illiquid (e.g., hedge funds, private equity, and real estate). Existing studies in the literature generally employ an asset's order flow, transaction volume, and the corresponding price impact to measure illiquidity [3–6]. This arises from the conventional wisdom that a transaction's impact on the asset price captures the liquidity premium (illiquidity discount) that a buyer (seller) is willing to pay (offer) to fulfill an order.

In this paper, we study a novel liquidity premium measure based on the equilibrium derived from a dynamic model in Hur and Chung [7], and apply the measure to the US stock market. Their model implies that the liquidity premium of an asset is a function of a time discount factor, the relative risk aversion of the investor, and the expected return and volatility of the asset, given the risk-free rate. Based on reasonable specifications of the parameters, our empirical findings show that the proposed liquidity premium is highly related to Amihud's [8] illiquidity ratio, a measure commonly used in existing studies. More specifically, we find that the cross-sectional variation of the new liquidity premium is significantly explained by Amihud's illiquidity ratio, which supports the validity of the measure.

The model implies that a highly liquid asset should command a price premium while illiquid assets must be offered at price discounts. To examine this implication, we perform two sets of tests based on U.S. data: the first is a test of the cross-sectional relationship between the new liquidity premium and stock returns; and the second is a test of the time-series relationship between a risk factor based on the new liquidity premium and the expected excess returns on portfolios sorted on the Amihud's illiquidity ratio. As predicted, we find a negative relationship in both settings. Overall, the empirical findings corroborate that the liquidity risk factor based on our measure of the liquidity premium is priced in stock/portfolio returns, which further validates the robustness of the proposed liquidity measure.

2. Liquidity Premium

In this section, as in Hur and Chung [7] (who attempted to apply the model's implications to the Korean stock market, and their empirical results are qualitatively similar to ours in this study, hence, we believe that our new measure has compatibility in the global markets.), we derive a closed-form representation of liquidity premium based on a continuous time model.

2.1. Model Setup

We begin by defining the following:

Definition 1. *A random investment horizon is the first time a pre-determined investment goal is attained.*

Definition 2. *An investment goal is the targeted rate of return.*

Definition 3. *An asset's liquidity premium (discount) is the maximum willingness-to-pay that makes an investor indifferent between two consumption options—cashing out and consuming all positions now in the absence of any future liquidation shock, or waiting until the investment horizon terminates before consuming all wealth—while maintaining the same investment strategy with a random horizon.*

The first consumption option in Definition 3 measures utility from holding a financial asset when its liquidity is perfect to the level of money. The second option measures utility when the asset's liquidity is limited. Therefore, the certainty equivalent variation that make these two utilities equal represents the asset's liquidity premium. For brevity and consistency, we focus on locked-in (or locked-out) strategies as these are more convenient when assessing the contributions of holding a designated asset. In contrast, it is challenging to disentangle the contributions of holding an asset from others under a cross-sectional diversification strategy.

We assume that stock price X_t follows a log-normal Brownian motion with drift, as follows:

$$dX_t = \mu X_t dt + \sigma X_t dB_t.$$

A solution to this stochastic differential equation is easily obtained by applying Ito's lemma:

$$X_t = X_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_t\right]$$

This solution for X_t has a drift of $\left(\mu - \frac{1}{2}\sigma^2\right)$, implying that it would grow continuously at the rate of $\left(\mu - \frac{1}{2}\sigma^2\right)$. As a benchmark, we define $\widetilde{X}_t(b)$ to grow at the rate of the risk-free rate r. Note that it is always greater (smaller) than $X_0 \exp[rt]$ by $\exp[b]$ for $b > 0$ ($b < 0$), as follows:

$$\widetilde{X}_t(b) \equiv X_0 \exp[rt + b].$$

More specifically, consider an investor who purchases one share of a stock at time 0. Their investment goal is to outperform a risk-free asset by $\exp[b]$ times. The investor would like to know when this goal will be attained. Such an investment strategy is known by different names, such as buy-and-hold, stop-gains, or locked-in. As these names suggest, the essence of the strategy is to survive in order to achieve the investment goal.

We define time T_b to a level of $b \in R$ and $b > 0$ as follows:

$$T_b(\omega) = inf\{t \geq 0; X_t(\omega) = \widetilde{X}_t(b)\}$$

$$T_b(\omega) = inf\{t \geq 0; X_t(\omega) = \widetilde{X}_t(b)\} = inf\{t \geq 0; B_t = \frac{b}{\sigma} - \frac{1}{\sigma}\left(\mu - \frac{1}{2}\sigma^2 - r\right)t\},$$

where a new Brownian motion with drift, $\widetilde{B}_t$, has a drift of $\frac{1}{\sigma}\left(\mu - \frac{1}{2}\sigma^2 - r\right)$ as:

$$\widetilde{B}_t \equiv B_t + \frac{1}{\sigma}\left(\mu - \frac{1}{2}\sigma^2 - r\right)t.$$

From Karatzas and Shreve [9], we have:

$$P^{\left(\frac{1}{\sigma}(\mu - \frac{1}{2}\sigma^2 - r)\right)}[T_b \in dt] = \frac{b}{\sqrt{2\pi\sigma^2 t^3}} \exp\left[-\frac{\left(b - \left(\mu - \frac{1}{2}\sigma^2 - r\right)t\right)^2}{2\sigma^2 t}\right] dt,\ t > 0$$

$$P^{\left(\frac{1}{\sigma}(\mu - \frac{1}{2}\sigma^2 - r)\right)}[T_b \leq t] = \int_0^t \exp\left[\frac{b}{\sigma^2}\left(\mu - \frac{1}{2}\sigma^2 - r\right) - \frac{1}{2\sigma^2}\left(\mu - \frac{1}{2}\sigma^2 - r\right)^2 s\right] P[T_b \in dt]$$

$$P[T_b \in dt] = \frac{b}{\sqrt{2\pi\sigma^2 t^3}} \exp\left[-\frac{b^2}{2\sigma^2 t}\right] dt,\ t > 0.$$

Applying $Ee^{-\alpha T_b} = \mathrm{e}^{-\frac{b}{\sigma}\sqrt{2\alpha}}$, we can calculate the value of $\lim_{t\to\infty} P^{\left(\frac{1}{\sigma}(\mu - \frac{1}{2}\sigma^2 - r)\right)}[T_b \leq t]$:

$$\begin{aligned} P^{\left(\frac{1}{\sigma}(\mu - \frac{1}{2}\sigma^2 - r)\right)}[T_b < \infty] &= e^{\frac{b}{\sigma^2}(\mu - \frac{1}{2}\sigma^2 - r)} E\left[\exp\left(-\tfrac{1}{2\sigma^2}\left(\mu - \tfrac{1}{2}\sigma^2 - r\right)^2 T_b\right)\right] \\ &= \exp\left[\tfrac{b}{\sigma^2}\left(\mu - \tfrac{1}{2}\sigma^2 - r\right) - \tfrac{b}{\sigma^2}\left|\mu - \tfrac{1}{2}\sigma^2 - r\right|\right]. \end{aligned}$$

Assumption 1. $\mu - \frac{1}{2}\sigma^2 > r$

Proposition 1. In an infinite investment horizon, an investor is certain to receive a stochastic cash flow, the present value of which exceeds the present price of an asset by simply holding the asset until time T_b.

Proof. By construction, at $X_t(\omega) = \widetilde{X}_t(b)$ the investor can sell the asset at a price that has continuously grown faster than the speed of r. In addition, T_b is known to be reached in finite time with a probability of 1; that is, $P^{\left(\frac{1}{\sigma}(\mu - \frac{1}{2}\sigma^2 - r)\right)}[T_b < \infty] = 1$, based on Assumption 1.

As b is arbitrary, $P^{\left(\frac{1}{\sigma}(\mu - \frac{1}{2}\sigma^2 - r)\right)}[T_b < \infty] = 1$ implies that the investor will achieve any targeted return in the infinite investment horizon, although the timing is still uncertain. Investors differ in their motives for investment, life cycles, and economic abilities, and these sources of heterogeneity compel them to invest in different time horizons. Thus, $P^{\left(\frac{1}{\sigma}(\mu - \frac{1}{2}\sigma^2 - r)\right)}[T_b < \infty] = 1$ should not be regarded as a sign of arbitrage opportunities.

Proposition 2. The distribution of T_b exhibits first-order stochastic dominance (FOSD) with respect to (b, μ, r), as follows:

$$\frac{\partial}{\partial b} P^{(\frac{1}{\sigma}(\mu - \frac{1}{2}\sigma^2 - r))}[T_b \leq t] \leq 0 \quad (1)$$

$$\frac{\partial}{\partial \mu} P^{(\frac{1}{\sigma}(\mu - \frac{1}{2}\sigma^2 - r))}[T_b \leq t] \geq 0 \quad (2)$$

$$\frac{\partial}{\partial r} P^{(\frac{1}{\sigma}(\mu - \frac{1}{2}\sigma^2 - r))}[T_b \leq t] \leq 0. \quad (3)$$

Proof.

(1) As $b < b'$ and $\widetilde{X}_t(b) < \widetilde{X}_t(b')$, X_t should touch $\widetilde{X}_t(b)$ before it touches $\widetilde{X}_t(b')$. Hence, the distribution of $T_{b'}$ exhibits FOSD over the distribution of T_b.

(2) As $\frac{\partial}{\partial \mu} X_t > 0$, the distribution of T_b shifts leftward as μ increases.

(3) As $\frac{\partial}{\partial r} \widetilde{X}_t(b) > 0$, the distribution of T_b shifts rightward as r rises. □

In contrast, the effect of an increase in σ^2 is ambiguous in that it raises the volatility of X_t, while lowering the drift. The lower drift implies that the distribution of T_b before the change in σ^2 exhibits FOSD over the distribution after the change. However, the increased volatility implies that the distribution of T_b before the change in σ^2 exhibits second-order stochastic dominance (SOSD) over the distribution after the change. Summarizing these two effects, we cannot characterize the shift in the distribution of T_b as either FOSD or SOSD.

The validity of the buy-and-hold (locked-in) strategy is confirmed as a certain investment horizon is guaranteed. The buy-and-hold is a long-term strategy that becomes more favorable as the investment horizon increases. Thus, it appears rational for an investor to constrain risky asset holding to a certain level. The investor can extend the investment horizon effectively by increasing the probability of the investment position, maintaining a positive balance until it reaches the targeted return.

2.2. Explicit Representation of Liquidity Premium

In this section, we derive a probability distribution function for the first passage of time after which any locked-in strategy will attain a given investment goal in continuous time. From Karatzas and Shreve [9], we confirm that any locked-in strategy targeting a positive excess gain will be attained eventually with a probability of 1.

Uncertainty of termination differs from the early resolution of uncertainty. The latter is related to the timing of future uncertainty being revealed. Realization of the future affects an agent's economic interests. Agents can better prepare if they are informed earlier of future uncertainty, in which case their ex ante expected utility may improve.

In contrast, this first passage of time is not related to the timing of uncertainty resolution or to the agent's preference. It only indicates that the effective investment horizon can be random, based on exogenous factors, such as mortality, or on the choice of a locked-in or locked-out investment strategy.

Assumption 2. $b > 0$

Assumption 3. All investors hold a constant relative risk aversion utility $U(W) = \frac{W^{1-\gamma}}{1-\gamma}$, $\gamma > 0$

The previous subsection shows that a locked-in strategy of T_b will be achieved with a probability of 1. In other words, an investor with W_0 will eventually receive $W_0 \exp[b]$ by adopting a locked-in strategy. Hence, in principle, the present value of his/her wealth is equal to the value of a bond that pays $W_0 \exp[rt + b]$ at a randomly chosen maturity.

In a financial market without any frictions and liquidation risks, the present value of the agent's wealth will be equal to $W_0 \exp[b]$. To avoid arbitrage opportunities or violating the law of one price, it should be decreased to at least W_0, which implies that frictions and liquidation risks are present in the current market environment. Hence, we measure the expected utilities based on these two cases.

However, we minimize the investment strategy's liquidity by requiring that an investor liquidate his/her position at a stopping time T_b and consume all the proceeds. The expected utility of this case is compared with that of the previous two, and their differences are measured as the lower and upper bounds respectively, of the liquidity premium (or discount).

First, in the case of perfect liquidity, the current wealth of W_0 increases to $W_0 \exp[rt+b]$, and the corresponding utility is:

$$V^P(W_0) \equiv \frac{{W_0}^{1-\gamma} \exp[b(1-\gamma)]}{1-\gamma}.$$

$$V^L(W_0) \equiv \frac{{W_0}^{1-\gamma}}{1-\gamma} < V^P(W_0).$$

Third, in the case of no liquidity, the utility of the investor with the locked-in strategy of T_b is calculated as follows:

$$V(W_0, b) \equiv \frac{{W_0}^{1-\gamma}}{1-\gamma} \chi_0$$

$$\begin{aligned}\chi_0 \equiv \int_0^\infty \tfrac{b}{\sqrt{2\pi\sigma^2 t^3}} \exp & \left[-\tfrac{\left(b-\left(\mu-\frac{1}{2}\sigma^2-r\right)t\right)^2}{2\sigma^2 t} + (1-\gamma)(rt+b) - \rho t\right] dt \\ &= \int_0^\infty \tfrac{b}{\sqrt{2\pi\sigma^2 t^3}} \exp\left[-\tfrac{\left(b-\left(\mu-\frac{1}{2}\sigma^2-r\right)t\right)^2}{2\sigma^2 t}\right] \exp[(1-\gamma)(rt+b)-\rho t] dt \\ &= e^{(1-\gamma)b}\, E_0[\exp[((1-\gamma)r-\rho)t]].\end{aligned}$$

Considering the properties of the first Brownian motion with drift of μ, we know that $E^{(\mu)}\, e^{-\alpha T_b} \; = \; \exp[\mu b - |b|\sqrt{\mu^2+2\alpha}, \alpha > 0]$. Hence:

$$\chi_0 = \exp\left[\frac{b}{\sigma^2}\left(\mu - \frac{1}{2}\sigma^2 - r\right) - \frac{|b|}{\sigma}\sqrt{\frac{1}{\sigma^2}\left(\mu - \frac{1}{2}\sigma^2 - r\right)^2 + 2(\rho - (1-\gamma)r)} + (1-\gamma)b\right].$$

The upper and lower bounds of the financial asset's liquidity premium are calculated by equating the following inequalities ($0 \leq \varphi_L < \varphi_L$ if $b > 0$, and $\varphi_L > \varphi_L \geq 0$ if $b < 0$.). For $0 < \gamma < 1$:

$$V^P(W_0(1-\varphi_P)) = V(W_0, b)$$

$$V^L(W_0(1-\varphi_L)) = V(W_0, b)$$

$$\varphi_P = 1 - \chi_0^{\frac{1}{(1-\gamma)}} \exp[-b],\ \varphi_L = 1 - \chi_0^{\frac{1}{(1-\gamma)}}.$$

As $\varphi_P > \varphi_L$ $(b > 0)$, φ_L and φ_P are regarded as the lower and upper bounds respectively, of the liquidity premium. They are functions of b, μ, σ^2, ρ, γ, and r

For $\gamma > 1$:

$$V^P(W_0(1+\phi_P)) = V(W_0, b)$$

$$V^L(W_0(1+\phi_L)) = V(W_0, b)$$

$$\phi_P = \chi_0^{\frac{1}{(1-\gamma)}} \exp[-b] - 1,\ \phi_L = \chi_0^{\frac{1}{(1-\gamma)}} - 1.$$

As $\phi_P < \phi_L$ $(b > 0)$, ϕ_L and ϕ_P are perceived to be the upper and lower bounds, respectively, of the liquidity discount. They are functions of b, μ, σ^2, ρ, γ, and r.

Proposition 3. The liquidity premium measures the increase in φ_L and φ_P with respect to (b, μ, ρ, γ), but the decrease with respect to (σ^2, r).

Proposition 3 implies that an asset's liquidity premium is inversely related to volatility and the risk-free rate, but is positively linked to the expected return and relative risk-aversion parameter. This result is consistent with the mean–variance trade-off.

Proposition 4. The liquidity discount measures the decrease in ϕ_L and ϕ_P with respect to (b, μ, ρ, γ), but the increase with respect to (σ^2, r).

Proposition 4 implies that an asset's liquidity discount is positively related to volatility and the risk-free rate, but is inversely linked to the expected return and relative risk-aversion parameter. This result is also consistent with the mean–variance trade-off.

The risk premium and discount measures tend to have different values, depending on b. In order to make them independent of b, we devise the following variants by applying L'Hopital's rule in order to obtain explicit representations of the liquidity premium and discount (note that liquidity premium and liquidity discount have the same magnitude, but opposite signs):

$$\varphi_L^* \equiv \lim_{\mathrm{b}\to 0} \frac{\varphi_L}{b} = \frac{1}{(1-\gamma)\sigma}\sqrt{\frac{1}{\sigma^2}\left(\mu - \frac{1}{2}\sigma^2 - r\right)^2 + 2(\rho - (1-\gamma)r)} - \frac{1}{(1-\gamma)\sigma^2}\left(\mu - \frac{1}{2}\sigma^2 - r\right) - 1$$

$$\phi_L^* \equiv \lim_{\mathrm{b}\to 0} \frac{\phi_L}{b} = \frac{1}{(1-\gamma)\sigma^2}\left(\mu - \frac{1}{2}\sigma^2 - r\right) - \frac{1}{(1-\gamma)\sigma}\sqrt{\frac{1}{\sigma^2}\left(\mu - \frac{1}{2}\sigma^2 - r\right)^2 + 2(\rho - (1-\gamma)r)} + 1$$

$$\varphi_P^* \equiv \lim_{\mathrm{b}\to 0} \frac{\varphi_P}{b} = \varphi_L^* + 1, \phi_P^* \equiv \lim_{\mathrm{b}\to 0} \frac{\phi_P}{b} = \phi_L^* + 1.$$

Note that the liquidity premium is simply defined as a function of the mean and variance of an asset's return, together with the risk-free rate, the time discount factor, and the relative risk aversion. In addition, the premium does not depend on the transactional characteristics of the market.

3. Empirical Results

In this section, we empirically estimate the liquidity premiums of stocks. Further, we examine whether the liquidity premium explains the cross-sectional variation in stock returns and whether the expected excess returns on portfolios sorted on the liquidity premium are at least partially represented by the factor loading of the liquidity premium over time. The model implies that a stock with high (low) liquidity should command a high (low) liquidity premium; thus, we not only expect a negative relationship between the liquidity premiums and cross-sectional stock returns, but also a negative relation between the factor based on the liquidity premium and expected excess returns on portfolios sorted on the liquidity premium in the time series.

First, we estimate the liquidity premiums for the sample of all stocks traded on the NYSE, AMEX, and NASDAQ in the U.S. stock market during the period 1980 to 2014. We collect data for daily stock returns from the Center for Research in Security Prices (CRSP), and compute the monthly means and variances of the stock returns. We consider the daily return of the one-month treasury bill rate as the daily risk-free rate. In addition, we set the time discount factor and the relative risk aversion to 0.99 and 0.5 (1.5), respectively, consistent with previous studies on asset pricing (e.g., [10–12]) (the empirical results are not sensitive to the choices of the time discount factor and relative risk aversion, as suggested in the literature. Though unreported for brevity, the results are available upon request). This allows us to calculate the daily liquidity premiums based on the model solution above when the relative risk aversion either falls between 0 and 1 or is greater than 1. Further, we compute the monthly liquidity premium (LIQ) using relative risk aversions of 0.5 and 1.5 as the average daily liquidity premium over a month for each stock, and convert it to the logarithmic form in the empirical analysis to alleviate the outlier effect.

For comparison, we also measure individual stocks' degrees of illiquidity on each day using Amihud's illiquidity ratio [8], which is a widely used measure of a stock's liquidity in the literature.

Amihud's illiquidity ratio is defined as the ratio of the absolute stock return to the dollar value of the trading volume. It captures the price impact of trading and is widely considered a good proxy for the illiquidity of stocks. Amihud [8], Acharya and Pedersen [13], Chordia et al. [14], and Brennan et al. [15] show that the ratio is significant in explaining a cross-section of stock returns. Using daily stock returns and trading volumes obtained from the CRSP, we measure the monthly average of Amihud's daily illiquidity ratio (unlike our model-based measure, Amihud's ratio does not require to consider a specific degree of investor's risk aversion). Further, we calculate the logarithm of this value as follows:

$$\log(Amihud_i) = \log\left(\frac{\sum_{d=1}^{t} \frac{|r_{i,d}|}{Vol_{i,d}}}{D_i}\right),$$

where $r_{i,d}$ is the return of stock i on day d, $Vol_{i,d}$ is the trading volume of stock i on day d (in USD), and D_i is the number of days in the month. A high value of the ratio indicates the stock is less liquid over the month.

4. Results

We report in Table 1 the descriptive statistics for the time-series average of the monthly cross-sectional liquidity premium and Amihud's illiquidity ratio for the period 1980 to 2014. We find that both the premium and illiquidity ratio vary significantly across firms. In addition, the variation of the liquidity premium is consistently evident, regardless of the parameter values of the relative risk aversion. This suggests that the newly proposed measure can appropriately explain the cross-sectional difference in stocks' liquidity.

Table 1. Descriptive statistics.

Variable	Mean	Std. Dev.	Min.	Median	Max.
Liquidity premium ($\gamma = 0.5$; $\rho = 0.99$)	4.76344	0.79846	−2.43841	4.7633	37.14187
Liquidity premium ($\gamma = 1.5$; $\rho = 0.99$)	4.78653	0.78017	0.73592	4.7803	37.14187
Amihud's illiquidity	−2.41855	3.37081	−36.78423	−2.4165	14.59196

Table 2 contains the correlations between the monthly liquidity premium and Amihud's illiquidity measure over the sample period. The results show a significantly negative relationship between the two measures, which is evidence that a higher liquidity premium is required for a more liquid stock. This further confirms that the proposed measure is a viable alternative as a measure of the degree of stock liquidity.

Table 2. Pearson correlation coefficients.

(γ = 0.5; ρ = 0.99)	Liquidity Premium	Amihud's Illiquidity
Liquidity premium	1.000	
Amihud's illiquidity	−0.422 (0.000) ***	1.000
(γ = 1.5; ρ = 0.99)		
Liquidity premium	1.000	
Amihud's illiquidity	−0.420 (0.000) ***	1.000

Notes: p-values are provided in parentheses. *** indicates statistical significance at the 1% level.

Next, we run simple cross-sectional regressions to examine the extent to which a stock's illiquidity (or liquidity) affects its liquidity premium. We use Fama and Macbeth's [16] approach to estimate the model, and compute the coefficients as the time-series averages from the monthly cross-sectional regressions. Table 3 reports the coefficient estimates of regressions of the liquidity premium on Amihud's illiquidity for different values of relative risk aversion. The liquidity premium is negatively and significantly associated with Amihud's illiquidity ratio, suggesting that the liquidity premium reflects the stock's liquidity. Overall, our empirical results validate our measure of liquidity.

Table 3. Effect of stock illiquidity on liquidity premium.

	Intercept		Amihud's Illiquidity		Adj. R^2
	Coefficient	***t*-Stat**	**Coefficient**	***t*-Stat**	
($\gamma = 0.5$; $\rho = 0.99$)	4.5818 ***	206.52	−0.0951 ***	−29.47	0.1682
($\gamma = 1.5$; $\rho = 0.99$)	4.6093 ***	214.03	−0.0925 ***	−29.66	0.1671

Notes: The *t*-statistics are adjusted for Newey–West autocorrelations with three lags and are reported in parentheses. *** indicates statistical significance at the 1% level.

Following Amihud [5], we consider a monthly cross-sectional regression model that relates the liquidity measure to stock returns. In particular, we estimate the model following the Fama and Macbeth [13] method. In each month of year t, stock returns are regressed cross-sectionally on the liquidity measures and on the stock characteristics obtained at the end of year $t-1$. The liquidity measures, including the LIQ and Amihud's illiquidity, are computed as monthly averages over the year (LIQ in Tables 4 and 5 is estimated based on $\gamma = 0.5$, but the results are qualitatively similar to those based on $\gamma = 1.5$. The results with $\gamma = 1.5$ are available upon request). Following Amihud [5], we also control for various stock characteristics. BETA is computed using the Scholes and Williams [17] method. R100 is the buy-and-hold return over the last 100 days of the year, and R100YR is the buy-and-hold return from the beginning of the year to 100 days before its end. SIZE is the logarithm of the market capitalization at year-end. BM is the book-to-market ratio of equity, computed as the book value for the fiscal year ended before the most recent June 30 divided by the market capitalization at year-end. SDRET is the standard deviation of daily returns during the year. DIVYLD is the dividend yield, computed as cash dividends for the fiscal year ended before the most recent June 30 divided by the market capitalization at year-end. The sample period is 198,101–201,412 for the liquidity measures and the stock characteristics, and 198,101–201,512 for the corresponding stock returns. Following Amihud [5], we impose the following data filters on the sample: (1) the stock has return data for more than 200 trading days during the year $t-1$; (2) the stock price is greater than USD 5 at the end of year $t-1$; and (3) the stock has market capitalization data available at the end of year $t-1$ in CRSP (between 2498 and 5345 stocks are included in the cross-sectional regression). Table 4 presents the estimation results and reports the means of the coefficients from the monthly cross-sectional regressions of the stock returns on different liquidity measures and stock characteristics. We find that the LIQs and stock returns are negatively and significantly related, implying that when a stock's liquidity and premium are low, investors require a higher stock return as compensation. Consistent with the findings of Amihud [5], we also find that Amihud's illiquidity is positively related to the cross-sectional stock returns.

Table 4. Cross-sectional regressions of stock return on liquidity and other stock characteristics.

Variable	(1)	(2)	(3)
Amihud	0.090 *** (3.22)		
LIQ ($\gamma = 0.5$)		−0.440 *** (−2.74)	
LIQ ($\gamma = 1.5$)			−0.442 *** (−2.74)
BETA	0.027 (0.22)	−0.084 (−0.69)	−0.084 (−0.69)

Table 4. *Cont.*

R100	0.531 ** (2.53)	0.601 *** (2.90)	0.602 *** (2.90)
R100YR	0.111 (1.22)	0.118 (1.32)	0.118 (1.32)
SIZE	0.051 (1.16)	−0.050 * (−1.84)	−0.050 * (−1.84)
BM	0.001 (0.31)	0.002 (1.36)	0.002 (1.36)
SDRET	−0.273 *** (−4.79)	−0.386 *** (−7.18)	−0.385 *** (−7.17)
DIVYLD	−0.096 (−1.03)	−0.105 * (−1.94)	−0.105 * (−1.94)
R^2	4.87%	4.86%	4.86%

Notes: *t*-statistics are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively (we believe that R^2 could increase by controlling additional factors affecting stock returns).

Given that LIQ explains the cross-sectional stock returns, similar to Amihud's illiquidity, we next investigate the effect of liquidity on excess returns on the portfolios sorted on LIQ and/or Amihud's illiquidity in the time-series. This allows us to directly compare the predictive power of the liquidity factor loading based on LIQ and Amihud's illiquidity in the excess returns. Table 5 presents the estimation results, and reports the mean returns and factor loadings of the portfolios formed on different liquidity measures. At the beginning of each year, stocks are sorted into 10 decile portfolios based on their previous year's liquidity measure. Stocks with the lowest liquidity values are included in decile 1, and those with the highest values are included in decile 10. LIQF is the liquidity factor, constructed as the value-weighted return of the stocks with the lowest 20% liquidity lagged by one month minus the value-weighted return of the stocks with the highest 20% liquidity lagged by one month. Further, MKT, SMB, and HML denote the three factors of Fama and French [18], and MOM denotes the momentum factor defined in the same work. The factor loadings are obtained by regressing the value-weighted portfolio returns on the market factors. Alpha is the intercept from the regression.

In particular, the results in Panel A of Table 5 are based on the portfolios sorted by Amihud's illiquidity and the liquidity factor loadings are formed on Amihud's illiquidity. We find that LIQF is significant, suggesting that the liquidity factor based on Amihud's illiquidity explains the return spread due to Amihud's illiquidity. Alpha remains significant in the five-factor setup, implying that the liquidity factor cannot fully explain the return spread. In Panel B, when the portfolios are sorted by LIQ and the liquidity factor loadings are formed on LIQ, the results show that LIQF is significant, suggesting that the liquidity factor based on LIQ explains the return spread due to LIQ. In addition, we find that alpha is insignificant in the five-factor setup, which suggests that the liquidity factor based on LIQ may fully explain the return spread. Panel C provides the estimation results for the portfolios sorted by Amihud's illiquidity and the liquidity factor loadings formed on LIQ. The results reveal a non-significant role of LIQF, showing that the liquidity factor based on LIQ has some explanatory power on the return spread due to Amihud's illiquidity. However, when the portfolios are sorted by LIQ and the liquidity factor loadings are formed on Amihud's illiquidity in Panel D, we find that the factor loading for LIQF is non-significant. This implies that the liquidity factor based on Amihud's illiquidity cannot explain the return spread due to LIQ. Considered together, our findings highlight that the liquidity factor based on LIQ is priced in the asset pricing test and captures the liquidity risk that Amihud's illiquidity cannot explain. Overall, the empirical results not only validate the robustness of the liquidity premium motivated by liquidity risk, but it also suggests that it plays a complementary role in the widely used Amihud's illiquidity ratio.

Table 5. Portfolio of stocks formed on liquidity lagged by one year.

(A) Scoring Variable = Amihud's illiquidity, Factor = Amihud's illiquidity												
	Four-Factor Model						Five-Factor Model					
Portfolio	**Mean Return (%)**	**Alpha (%)**	**MKT**	**SMB**	**HML**	**MOM**	**Alpha (%)**	**LIQF**	**MKT**	**SMB**	**HML**	**MOM**
1 (Low)	2.633	1.632	0.644	0.881	0.057	0.194	0.987	−0.362	0.728	0.585	−0.039	0.295
2	2.527	1.442	0.776	0.706	0.279	0.092	0.905	−0.301	0.846	0.459	0.199	0.176
3	2.435	1.347	0.795	0.804	0.235	0.085	1.108	−0.134	0.826	0.695	0.199	0.123
4	2.418	1.308	0.888	0.808	0.127	0.089	1.177	−0.073	0.905	0.748	0.107	0.110
5	2.302	1.216	0.880	0.783	0.119	0.063	1.121	−0.053	0.893	0.740	0.105	0.078
6	2.226	1.148	0.923	0.599	0.155	0.019	1.062	−0.048	0.934	0.559	0.143	0.033
7	2.039	0.961	0.966	0.522	0.153	−0.012	0.902	−0.033	0.973	0.495	0.145	−0.003
8	2.051	0.965	0.976	0.534	0.110	0.019	1.023	0.033	0.968	0.560	0.119	0.009
9	1.916	0.810	1.042	0.332	0.142	0.002	0.826	0.009	1.040	0.339	0.145	−0.001
10 (high)	1.492	0.569	0.968	−0.149	−0.052	−0.014	0.590	0.012	0.965	−0.139	−0.049	−0.018
10 − 1 spread	−1.141 ***	−1.063 ***	0.324 ***	−1.030 ***	−0.109 **	−0.209 ***	−0.397 **	0.373 ***	0.238 ***	−0.725 ***	−0.010	−0.313 ***
	(−5.17)	(−7.02)	(9.06)	(−20.39)	(−2.01)	(−6.18)	(−2.43)	(8.08)	(6.80)	(−12.01)	(−0.20)	(−9.22)
(B) Scoring Variable = LIQ, Factor = LIQ												
	Four-Factor Model						Five-Factor Model					
Portfolio	**Mean Return (%)**	**Alpha (%)**	**MKT**	**SMB**	**HML**	**MOM**	**Alpha (%)**	**LIQF**	**MKT**	**SMB**	**HML**	**MOM**
1 (Low)	2.949	1.956	1.481	0.960	−0.979	−0.100	1.082	0.587	1.152	0.191	−0.519	0.091
2	2.520	1.530	1.355	0.614	−0.636	−0.113	0.999	0.357	1.155	0.146	−0.356	0.004
3	2.242	1.223	1.278	0.497	−0.418	−0.089	0.828	0.265	1.130	0.150	−0.210	−0.003
4	1.975	0.932	1.196	0.313	−0.202	−0.056	0.706	0.152	1.111	0.114	−0.082	−0.007
5	1.750	0.637	1.151	0.102	0.031	0.015	0.564	0.049	1.123	0.038	0.070	0.031
6	1.706	0.658	1.057	−0.062	0.145	−0.033	0.735	−0.052	1.086	0.006	0.104	−0.050
7	1.581	0.518	0.991	−0.182	0.249	0.026	0.658	−0.094	1.043	−0.060	0.176	−0.004
8	1.485	0.487	0.884	−0.165	0.269	0.010	0.679	−0.129	0.956	0.003	0.168	−0.032
9	1.371	0.470	0.724	−0.175	0.309	−0.010	0.696	−0.152	0.809	0.024	0.190	−0.059
10 (high)	1.306	0.505	0.526	−0.074	0.282	0.028	0.747	−0.163	0.617	0.139	0.154	−0.025
10 − 1 spread	−1.643 ***	−1.452 ***	−0.955 ***	−1.034 ***	1.261 ***	0.128 **	−0.335	−0.750 ***	−0.535 ***	−0.053	0.672 ***	−0.116 **
	(−3.36)	(−5.11)	(−14.24)	(−10.92)	(12.38)	(2.02)	(−1.63)	(−20.45)	(−10.42)	(−0.65)	(8.74)	(−2.53)
(C) Scoring Variable = Amihud's illiquidity, Factor = LIQ												
	Four-Factor Model						Five-Factor Model					
Portfolio	**Mean Return (%)**	**Alpha (%)**	**MKT**	**SMB**	**HML**	**MOM**	**Alpha (%)**	**LIQF**	**MKT**	**SMB**	**HML**	**MOM**
1 (Low)	2.633	1.632	0.644	0.881	0.057	−0.014	1.446	0.125	0.574	0.718	0.155	0.235
2	2.527	1.442	0.776	0.706	0.279	0.002	1.441	0.001	0.776	0.705	0.280	0.092
3	2.435	1.347	0.795	0.804	0.235	0.019	1.354	−0.005	0.798	0.811	0.231	0.084
4	2.418	1.308	0.888	0.808	0.127	−0.012	1.268	0.027	0.873	0.773	0.148	0.098
5	2.302	1.216	0.880	0.783	0.119	0.019	1.194	0.015	0.872	0.764	0.131	0.068
6	2.226	1.148	0.923	0.599	0.155	0.063	1.165	−0.012	0.929	0.614	0.146	0.016
7	2.039	0.961	0.966	0.522	0.153	0.089	0.956	0.004	0.964	0.517	0.156	−0.011
8	2.051	0.965	0.976	0.534	0.110	0.085	0.912	0.035	0.956	0.487	0.138	0.030
9	1.916	0.810	1.042	0.332	0.142	0.092	0.728	0.055	1.012	0.260	0.185	0.019
10 (high)	1.492	0.569	0.968	−0.149	−0.052	0.194	0.521	0.032	0.950	−0.191	−0.027	−0.004
10 − 1 spread	−1.141 ***	−1.063 ***	0.324 ***	−1.030 ***	−0.109 **	0.209 ***	−0.925 ***	−0.093 ***	0.376 ***	−0.909 ***	−0.182 ***	−0.239 ***
	(−5.17)	(−7.02)	(9.06)	(−20.39)	(−2.01)	(6.18)	(−5.96)	(−3.36)	(9.75)	(−14.77)	(−3.14)	(−6.91)

Table 5. *Cont.*

(D) Scoring Variable = LIQ, Factor = Amihud's illiquidity												
		Four-Factor Model					Five-Factor Model					
Portfolio	Mean Return (%)	Alpha (%)	MKT	SMB	HML	MOM	Alpha (%)	LIQF	MKT	SMB	HML	MOM
1 (Low)	2.949	1.956	1.481	0.960	−0.979	−0.100	1.858	−0.055	1.494	0.915	−0.994	−0.085
2	2.520	1.530	1.355	0.614	−0.636	−0.113	1.350	−0.101	1.378	0.531	−0.663	−0.084
3	2.242	1.223	1.278	0.497	−0.418	−0.089	1.145	−0.044	1.288	0.461	−0.430	−0.077
4	1.975	0.932	1.196	0.313	−0.202	−0.056	0.795	−0.077	1.214	0.250	−0.222	−0.035
5	1.750	0.637	1.151	0.102	0.031	0.015	0.551	−0.048	1.162	0.063	0.019	0.028
6	1.706	0.658	1.057	−0.062	0.145	−0.033	0.573	−0.048	1.068	−0.101	0.132	−0.020
7	1.581	0.518	0.991	−0.182	0.249	0.026	0.437	−0.046	1.001	−0.220	0.237	0.039
8	1.485	0.487	0.884	−0.165	0.269	0.010	0.543	0.032	0.877	−0.139	0.277	0.001
9	1.371	0.470	0.724	−0.175	0.309	−0.010	0.427	−0.024	0.730	−0.195	0.303	−0.003
10 (high)	1.306	0.505	0.526	−0.074	0.282	0.028	0.459	−0.025	0.532	−0.095	0.275	0.035
10 − 1 spread	−1.643 ***	−1.452 ***	−0.955 ***	−1.034 ***	1.261 ***	0.128 **	−1.399 ***	0.030	−0.962 ***	−1.010 ***	1.269 ***	0.119 *
	(−3.36)	(−5.11)	(−14.24)	(−10.92)	(12.38)	(2.02)	(−4.25)	(0.32)	(−13.63)	−8.29	12.10	(1.74)

Notes: *t*-statistics are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

5. Conclusions

This paper highlights the empirical application of an explicit representation of liquidity premium (or discount) of financial assets by parameterizing the return process in Hur and Chung [3]. They suggest that the derived measure of liquidity is confirmed to be a function of a time discount factor, a relative risk aversion parameter, and the expected return and volatility of the asset, given the risk-free rate.

Our empirical analysis particularly based on the US data suggests that the proposed measure of liquidity premium is highly comparable to the existing measures in the literature. This implies that the proposed liquidity premium can be used as an alternative to conventional measures of liquidity (e.g., it can be used in empirical asset pricing studies to investigate the liquidity betas of stocks). We also find a negative relationship between the liquidity premium and cross-sectional stock returns, as well as a negative relationship between the risk factor based on the liquidity premium and the expected excess returns on portfolios sorted on the liquidity premium in the time series. This suggests that the risk factor captured by the proposed liquidity premium measure is priced in stock and portfolio returns, which further corroborates the theoretical foundation of the new measure in Hur and Chung [3].

In future research, we would like to introduce an additional coefficient that governs the liquidity premium (or discount) in order to alleviate the burden on the risk-aversion parameter. In a related study, Epstein and Zin [19] adopt the power utility version of Kreps and Porteus [20] and separate risk aversion from the intertemporal elasticity of substitution in a similar manner.

Author Contributions: The authors contributed equally to this work.

Acknowledgments: We would like to thank the four reviewers for their helpful comments and suggestions. All authors contributed equally to this work.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Allen, D.E.; McAleer, M.; Singh, A.K. Risk measurement and risk modelling using applications of vine copulas. *Sustainability* **2017**, *9*, 1762. [CrossRef]
2. Yang, L.; Ma, J.Z.; Hamori, S. Dependence structures and systemic risk of government securities markets in central and eastern europe: A CoVaR-Copula approach. *Sustainability* **2018**, *10*, 324. [CrossRef]
3. Amihud, Y.; Mendelson, H. Dealership market: Market-making with inventory. *J. Financ. Econ.* **1980**, *8*, 31–53. [CrossRef]
4. Easley, D.; O'Hara, M. Price, trade size, and information in securities markets. *J. Financ. Econ.* **1987**, *19*, 69–90. [CrossRef]
5. Keim, D.B.; Madhavan, A. The upstairs market for large-block transactions: Analysis and measurement of price effects. *Rev. Financ. Stud.* **1996**, *9*, 1–36. [CrossRef]
6. Kyle, A.S. Continuous auctions and insider trading. *Econometrica* **1985**, *53*, 1315–1335. [CrossRef]
7. Hur, S.K.; Chung, C.Y. A novel measure of liquidity premium: Application to the Korean stock market. *Appl. Econ. Lett.* **2018**, *25*, 211–215. [CrossRef]
8. Amihud, Y. Illiquidity and stock returns: Cross-section and time-series effects. *J. Financ. Mark.* **2002**, *5*, 31–56. [CrossRef]
9. Karatzas, I.; Shreve, S. *Brownian Motion and Stochastic Calculus*, 2nd ed.; Springer: Berlin, Germany, 1991.
10. Guvenen, F. A parsimonious macroeconomic model for asset pricing. *Econometrica* **2009**, *77*, 1711–1750.
11. Mankiw, N.G.; Zeldes, S.P. The consumption of stockholders and nonstockholders. *J. Financ. Econ.* **1991**, *29*, 97–112. [CrossRef]
12. Mehra, R.; Prescott, E.C. The equity premium: A puzzle. *J. Monetary Econ.* **1985**, *15*, 145–161. [CrossRef]
13. Acharya, V.V.; Pedersen, L.H. Asset pricing with liquidity risk. *J. Financ. Econ.* **2005**, *77*, 375–410. [CrossRef]
14. Chordia, T.; Goyal, A.; Sadka, G.; Sadka, R.; Shivakumar, L. Liquidity and the post-earnings-announcement drift. *Financ. Anal. J.* **2009**, *65*, 18–32. [CrossRef]
15. Brennan, M.; Huh, S.W.; Subrahmanyam, A. An analysis of the Amihud illiquidity premium. *Rev. Asset Pricing Stud.* **2013**, *3*, 133–176. [CrossRef]

16. Fama, E.F.; MacBeth, J.D. Risk, return, and equilibrium: Empirical tests. *J. Polit. Econ.* **1973**, *81*, 607–636. [CrossRef]
17. Scholes, M.; Williams, J. Estimating betas from nonsynchronous data. *J. Financ. Econ.* **1977**, *5*, 309–327. [CrossRef]
18. Fama, E.F.; French, K.R. Common risk factors in the returns on stocks and bonds. *J. Financ. Econ.* **1993**, *33*, 3–56. [CrossRef]
19. Epstein, L.G.; Zin, S. Substitution, risk aversion and the temporal behavior of consumption and asset returns: An empirical framework. *J. Polit. Econ.* **1991**, *99*, 263–286. [CrossRef]
20. Kreps, D.M.; Proteus, E.L. Temporal resolution of uncertainty and dynamic choice theory. *Econometrica* **1978**, *46*, 185–200. [CrossRef]