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Article

Stochastic Optimization of an Electric Bus Dynamic Wireless Charging System

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Abstract: The Electric Bus Dynamic Wireless Charging (EB-DWC) system is a bus charging system that enables electric buses to receive power wirelessly from ground-based electromagnetic induction devices. In this system, how to optimally configure the charging infrastructures while considering the unpredictable nature of bus movement is a great challenge. This paper presents an optimization problem for an EB-DWC system in urban settings, addressing stochastic elements inherent in the vehicle speed, initial charging state, and dwell time at bus stops. We formulate a stochastic planning problem for the EB-DWC system by integrating these uncertainties and apply Monte Carlo sampling techniques to effectively solve this problem. The proposed method can improve the system's robustness effectively.

Keywords: dynamic wireless charging; electric vehicle; stochastic planning

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1. Introduction

The wireless charging of an electric vehicle (EV) is performed using wireless power transfer (WPT) technology, which does not require any physical contact in the process of transferring electric energy [1]. Wireless charging EV systems can be categorized into stationary and dynamic wireless charging systems [2]. Stationary wireless charging systems are operated to transfer energy when the EV is parked over fixed couplers, e.g., as in [3]. Dynamic wireless charging (DWC) systems involve the use of electromagnetic induction devices to wirelessly transmit electrical energy to the EVs while they are in motion [4,5]. The application of DWC technology enhances the convenience of EV charging and extends the EV's range [6]. Additionally, compared to conventional plug-in charging, DWC can reduce the required battery capacity of the EV for long-distance travel, therefore reducing the on-board battery costs of electric buses. Therefore, many applications of DWC exist across different fields, for instance public transit systems [7], logistics and delivery [6], autonomous vehicles [2], underwater autonomous vehicles [8], etc.

The Electric Bus Dynamic Wireless Charging (EB-DWC) system [7] is a bus charging system that operates on the principle of DWC, enabling electric buses to receive power wirelessly from ground-based electromagnetic induction devices while driving or parked, eliminating the need for traditional plug-in charging. Developed by a research team at the Korea Advanced Institute of Science and Technology (KAIST) [7,9], EB-DWC has undergone pilot implementations in public transportation systems in some cities. The advancement of EB-DWC represents an innovative approach to electric vehicle charging, promising a more convenient and efficient charging method, which could contribute to the widespread adoption and usage of electric vehicles in the future [10].

The charging infrastructure allocation problem is one of the most actively investigated topics for the wireless charging of EVs. For EB-DWC systems, the charging infrastructure allocation problem seeks an optimal location for the charging infrastructure on a given path of the vehicles, which is called the microscopic allocation problem [1]. In addition, the macro-allocation model offers a broader perspective, focusing on scientific insights for wireless charging of EVs and assessing how their integration affects the overall traffic behavior in larger transit systems. The routing of passenger vehicles or commercial trucks is typically dealt with, which have greater flexibility in choosing their routes [2,11,12].

Ref. [13] is the first work on the microscopic allocation problem, and it focused on the distribution of charging infrastructure and the optimization of the battery size for EVs. It utilized a continuous spatial decision space to allocate charging lanes efficiently, balancing battery capacity and the extent of charging infrastructure required. Then, [14] discretized the route into multiple small segments, turning the optimization approach proposed by Ko and Jang (2013) into a discrete problem. Specifically, they proposed an optimization-based approach aimed at identifying the most efficient distribution of wireless charging lanes along a designated route within a closed environment, such as the online electric vehicle (OLEV) system at KAIST, which operates under regulated velocities with minimal traffic interference. Ref. [5] incorporated battery life into the economic analysis, acknowledging the cost implications associated with the frequency and depth of battery charging cycles. Their findings underscored the economic efficiency of installing numerous shorter power tracks as opposed to fewer, longer ones, thus highlighting the critical influence of battery life on long-term infrastructure planning.

The initial focus on single-route transit systems [5,13,14] gradually expanded to encompass more complex, multi-route scenarios [15–17], acknowledging the realities of urban transit networks where routes often intersect. Studies have [18,19] delved into these complexities, exploring optimization models that consider shared route segments, thereby enhancing the efficiency of power track allocation. Particularly noteworthy is the introduction of robust optimization techniques by [19], which account for the uncertainties inherent in energy consumption and travel time, marking a significant leap forward in the strategic planning of EV charging infrastructure. This burgeoning field of research not only highlights the technical and economic considerations involved in the deployment of EV charging systems, but also underscores the critical role of optimization and robust planning in achieving efficient, sustainable urban transit solutions.

However, in all the above-mentioned research, the following assumption were made [2,20]:

1. Each bus line within the system follows a predetermined route.
2. A base station is designated for each bus line, serving as the starting and ending points for the buses' operational loops.
3. After completing a service loop, a bus will be fully recharged at its base station before the next loop.
4. The velocity at which buses travel and the passenger traffic for boarding and alighting at each station are pre-determined.

These four assumptions introduce a gap between the theoretical study and the real-world operation of EB-DWC systems. Although [19] has somewhat relaxed the last assumption above, a significant disparity still exists between the modeled scenarios and the actual system dynamics. In real-world applications, vehicles may not align precisely with the predicted or predefined charging spots, deviating from the expected speed or parking configurations due to diverse factors like driver behavior, traffic conditions, or unforeseen circumstances. Furthermore, in real systems, it cannot be guaranteed that buses will be fully charged before every departure. For instance, delays in arrival times lead to insufficient charging time. Consequently, when transitioning these optimized models to real-world applications, the lack of consideration for vehicle randomness can lead to sub-optimal performance.

This study addresses the charging-infrastructure-optimization challenge of an EB-DWC system in urban environments by considering the stochastic nature of variables like bus speed, initial charging state, and bus stop dwell times. First, we formulate an optimization model that integrates these random factors, which focuses on maximizing the allocation of charging resources, scheduling charging intervals, and planning routes. Then, by employing the Monte Carlo sampling method, our approach iteratively evaluates the system's performance under various stochastic conditions. This work provides a promising strategy to enhance the EB-DWC bus system's functionality in real-world urban scenarios, ensuring optimized charging and reliable services for passengers while considering unpredictable environmental factors.

2. Model Description

2.1. General Model

In the existing work, the EB-DWC system operates as follows: Several buses operate on a predefined route at constant speeds. The batteries are discharged at a specific rate during their operation. Additionally, when the EBs pass through sections equipped with charging infrastructure, they receive a certain charging power. It is important to note the assumption that the charging and discharging power of the vehicles remain constant, and the amount of energy exchanged is solely dependent on time. Furthermore, due to the buses' constant speed, the charging and discharging times are only related to the location and length of the charging cables. Moreover, when the buses arrive at the primary station, they would be fully charged. The problem of the optimization of the EB-DWC system operating in a closed environment is formulated as follows [14]:

$$\begin{array}{rl} \text{Minimize} & \text{Cable Cost} + \text{Transmitter Cost} + \text{Battery Cost} \\ \text{Subject to} & \begin{array}{l}\text{All online electric vehicles (OLEVs) must complete their services} \\ \text{without depleting the battery's energy during one working day}\end{array} \\ \text{Decision Variables:} & \text{1. Battery Capacity} \\ & \text{2. Power Track Location.} \end{array} \tag{1}$$

The parameters referenced in Equation (1) encompass a wide range of factors, including, but not limited to: route length, number of buses, velocity profile, wireless charging power, the initial state of charge (SOC) of the buses, cable unit cost, battery unit cost, the power transmitter's unit cost. The decision variables include buses' battery capacity and power track locations. It is worth noting that, in the current model, both the velocity profile and the initial SOC of the buses are treated as deterministic, which may not accurately reflect real-world scenarios.

In this paper, we propose a stochastic optimization approach for electric bus wireless charging systems based on the general model in Equation (1), and considering the stochastic nature of vehicle speed profiles and initial battery charge levels. We provide a detailed exposition of our system model in this section.

2.2. System Definition

First, we present the basic model to conceptualize the system of EB-DWC. For ease of reference, comprehensive lists of symbols used in the model are provided in Tables 1–3. We consider a total of K buses circulate on a predetermined route, as illustrated in Figure 1. The route is divided into N segments, as shown in Figure 1 by the short dotted lines. The length of segments, x_n, can be different and predefined by the system operator. Specifically, a segment in the model is defined as the smallest unit for installing charging cables, such as dividing each meter of the route into individual segments. The varying segment lengths takes into account the different road conditions. For instance, if a particular stretch of road is unsuitable for installing charging cables, that entire stretch is designated as a single segment.

Table 1. System parameters.

Symbol	Description	Unit
K	Number of buses	-
k	Index of bus	-
$\mathbf{K}$	$[1, 2, \cdots, K]$, set of all buses	-
N	Number of segments	-
n	Index of segment	-
D	$[1, 2, \cdots, 97]$, set of all bus departures in a working day	-
d	Index of bus departure	-
S	$[1, 2, \cdots, N]$, set of all segments	-
S_r	Set of segments containing regular stations	-
S^+	Segments where charging cables can be installed	-
S^-	Segments that are not allowed to install DWC infrastructures	-
M_k	Number of bus k's departures in a working day	-
D^k	Set of departures of bus k	-
d_m^k	m-th departure of bus k in the working day	-
x_n	Length of segment n	Meters (m)
η	Charging/discharging efficiency of EV battery	-
P_{max}	Upper limit of wireless charging power	kWh/s

Table 2. Random variables and intermediate variables.

Symbol	Description	Unit
$v_d(n)$	Velocity of bus for d-th departure on segment n	m/s
$\bar{v}^t$	Expected speed of bus for the d-th departure	m/s
$\triangle v(n)$	Stochastic error of bus speed on segment n	m/s
σ_v	Standard variance of speed error	m/s
$T^s(n)$	Dwell time at station located in segment n	Seconds (s)
$\bar{T}_s(n)$	Predefined dwell time on the regular station located in segment n	s
$\triangle T_s(n)$	Stochastic error of dwell time on segment n	s
σ_{tx}	Standard variance of dwell time	s
Soc_0^k	Initial SOC of bus k	-
$\underline{soc}_0$	Lower bound of initial SOC	-
$\overline{soc}_0$	Upper bound of initial SOC	-
$D_k^m(n)$	Energy received by the battery on segment n	Kilowatt hours (kWh)
$S_k^m(n)$	Energy requirement on segment n	kWh
$P(v)$	Instantaneous power required to maintain a constant speed v	Watts (W)
Ω	Random variable space	-

Along this route, there are multiple stations, including a primary one referred to as the base station. This base station serves as both the starting point and the termination point of each round trip. It is also the park location when the vehicles are not in operation. To simplify our description, we call all stations except the base station as regular stations. Additionally, during the service time, the bus will make a stop at each regular station for $t^s(n)$ seconds for $n \in S_r$.

Table 3. Decision variables.

Symbol	Description	Unit
l_n	Installation decision variable of inductive cable on segment n	-
z_n	Installation decision variable of power inverter on segment n	-
L_{cable}	Total length of inductive cable installed	Meters (m)
N_{inv}	Total number of power inverters	-
B	Battery capacity	Kilowatt hours (kWh)
$p^d(n)$	Wireless charging power on segment n	kWh/s
P_{ch}	Set of all charging power decisions in a day	-
D	Decision variable space	-

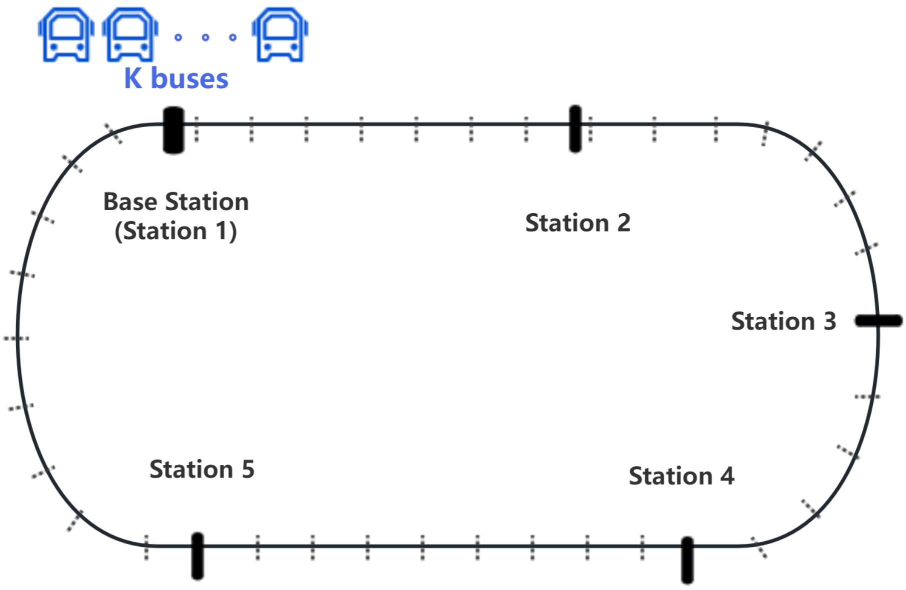

Figure 1. Single-route DWC-EBs.

We assume that the public bus system operates for 16 h each day, from 6:00 to 22:00, with a bus frequency of 10 min. This means that there are a total of 97 bus departures during the working hours of a day. Assume that, at the base station, all buses are serviced in a first-come, first-served (FIFS) manner. As a result, we can obtain the schedule of every bus. Furthermore, there will be one and only one bus in a departure. Then, Equation (2) and (3) hold.

$$\sum_{k \in \mathbf{K}} M_k = 97 \tag{2}$$

$$D^k \bigcap D^j = \emptyset, \forall j \neq k \text{ and } j \in \mathbf{K}, k \in \mathbf{K}. \tag{3}$$

2.3. Power Transmitter Installation Model

As shown in Figure 2, when there are multiple consecutive segments installed with power cables, the first of these segments will be equipped with a power inverter. These consecutively installed cable segments form a cluster. In this scenario, the clustered segments can be interpreted as a single power transmitter.

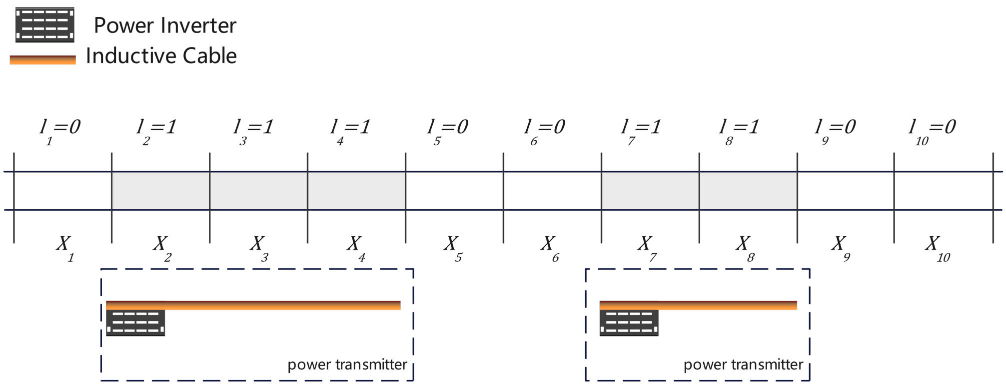

Figure 2. Installation of power transmitter.

Denote l_n as the installation decision variable of the inductive cable on segment n.

$$l_n = \begin{cases} 1 & \text{inductive cable is installed at segment } n \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Denote z_n as the installation decision variable of the power inverter on segment n.

$$z_n = \begin{cases} 1 & \text{if } l_n = 1 \text{ and segment } n \text{ is the first segment of the cluster} \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Moreover, the model incorporates constraints to reflect these installation limitations, ensuring that no charging cables are assigned to segments deemed unsuitable.

$$l_n = 0, \forall n \in S^- \tag{6}$$

Then, the total length of the inductive cable installed in the DWC-EV bus system is $L_{cable} = \sum_{n \in \mathbf{N}} l_n * X_n$ and the total number of power inverters is $N_{inv} = \sum_{n \in \mathbf{N}} z_n$.

2.4. Stochastic Model

In contrast to the previously mentioned general model, this study introduces non-deterministic elements into the model. Specifically, we account for the variability in the velocity, the initial SOC of each EV bus, and the stopping time at regular stations. This incorporation of uncertainty aligns more closely with the real-world dynamics of transportation systems.

2.4.1. Velocity Model

In prior research, the velocity profiles of these buses during transit have been predetermined. However, this assumption often does not hold true in real-world public transit systems, primarily due to two key factors. First, the variability in human driving behaviors makes it challenging for drivers to adhere to preset speeds at all times. Second, the velocity of the buses is frequently influenced by the actions of other vehicles and pedestrians, which adds an unpredictable element to their speed patterns. Then, we model the velocity of the buses for its d-th departure on segment n as Equation (7).

$$v^d(n) = \bar{v}^d + \triangle v(n) \tag{7}$$

To model the error, we employ a normal distribution with a mean of 0 and a variance of σ_v^2 as in Equation (8).

$$\triangle v(n) \sim \mathcal{N}(0, \sigma_v^2). \tag{8}$$

Furthermore, we utilize a time-of-day (TOD) model to characterize the average speed of the vehicles, $\bar{v}^d$, as shown in Figure 3. Specifically, we divide the daily operational hours into peak hours, off-peak hours, and regular hours, each with its own designated average speed, as shown in Equation (9).

$$\bar{v}^d = \begin{cases} 6.1\,\mathrm{m/s} & \text{if } d \in [1,10)\bigcup[86,97], \text{off-peak} \\ 4.7\,\mathrm{m/s} & \text{if } d \in [10,22)\bigcup[64,76), \text{peak} \\ 5.3\,\mathrm{m/s} & \text{otherwise, regular.} \end{cases} \tag{9}$$

It is worth noting that the TOD model defines the expected speed for the entire journey, from when a bus departs from the base station until it returns to the base station.

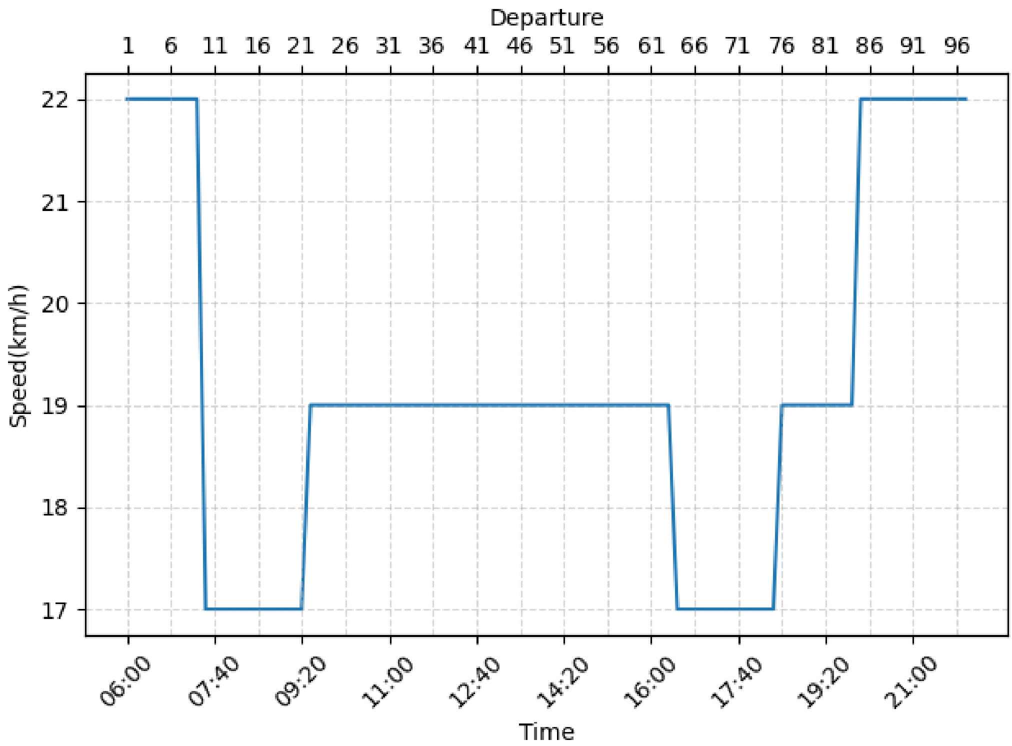

Figure 3. TOD velocity profile.

2.4.2. Dwell Time Model

In the DWC-EV bus system, there is typically a prescribed duration for each station stop, such as 10 s per stop. However, in actual operations, due to factors like passenger flow, the stop duration often becomes an uncertain variable. Therefore, in this paper, we model the stop duration as follows:

$$T_s(n) = \bar{T}_s(n) + \triangle T_s(n), \forall n \in S_r, \tag{10}$$

where $\bar{T}_s(n)$ is the predefined dwell time and $\triangle T_s(n) \sim \mathcal{N}(0, \sigma_{ts}^2)$ is the error of the dwell time, which follows a normal distribution, a mean of 0, and a variance of σ_{ts}^2.

2.4.3. Initial SOC Model

In the existing work, it is usually assumed that, before every trip, the EV battery is fully charged. Taking into account the electricity pressure at the base station or the possibility of unforeseen circumstances that may prevent each bus from starting with a 100% SOC, we model the initial SOC of each bus at the beginning of the working day as a random variable uniformly distributed between $\underline{soc}_0$ and $\overline{soc}_0$ as in (11):

$$Soc_0^k \sim U(\underline{soc}_0, \overline{soc}_0) \tag{11}$$

Generally, we set $\underline{soc}_0 = 0.5$ and $\overline{soc}_0 = 1$.

Differing from [14], we assume that all buses can only be charged at the base station at the end of each working day. Throughout the working day, there is no provision for bus charging at the base station. This assumption holds when considering the minimal construction cost for the primary station. Furthermore, our model can be readily extended to scenarios where buses can be charged at the primary station during the working day.

Define $Soc_{d_m^k}(n)$ as the SOC of bus k on its m-th departure at the start point of segment n. Specifically, $Soc_{d_1^k}(1)$ defines the initial SOC of bus k at the beginning of its' first trip, as shown in (12):

$$Soc_{d_1^k}(1) = Soc_0^k. \tag{12}$$

$Soc_{d_1^k}(1)$ defines the SOC of bus k on its first departure at the base station, which is equivalent to the ending SOC of its preceding trip, as shown in (13).

$$Soc_{d_m^k}(1) = Soc_{d_{m-1}^k}(N) - S_k^{m-1}(N) + D_k^{m-1}(N), \tag{13}$$

where $S_k^{m-1}(N)$ is the energy consumed on segment N of bus k during its m-th departure and $D_k^{m-1}(N)$ is the energy charged on segment N of bus k during the same departure. The model of $D_k^{m-1}(N)$ and $S_k^{m-1}(N)$ will be introduced in the next section.

2.5. Energy Model

We assume that all the batteries equipped on the buses are the same, and the value of the battery capacity, B, is modeled as a decision variable in the model. The SOC of bus k on its m-th departure dynamics is as follows:

$$Soc_{d_m^k}(n+1) = Soc_{d_m^k}(n) + \frac{D_k^m(n) - S_k^m(n)}{B}, \forall n >= 1, \tag{14}$$

where $D_k^{m-1}(n)$ is the energy received by the battery and $S_k^m(n)$ is the energy requirement on segment n, respectively.

As shown in Figure 4, when a bus moves in segment n on the m-th departure in a day, its velocity $v^d(n)$ is modeled as Equation (9). As a common assumption, $v^d(n)$ keeps unchanged in the segment. Therefore, the time on the segment is shown in Equation (15).

$$t^d(n) = \begin{cases} \frac{X_n}{v^d(n)} + t^s(n) & \text{if } n \in S_r \\ \frac{X_n}{v^d(n)} & \text{if } n \in S - S_r. \end{cases} \tag{15}$$

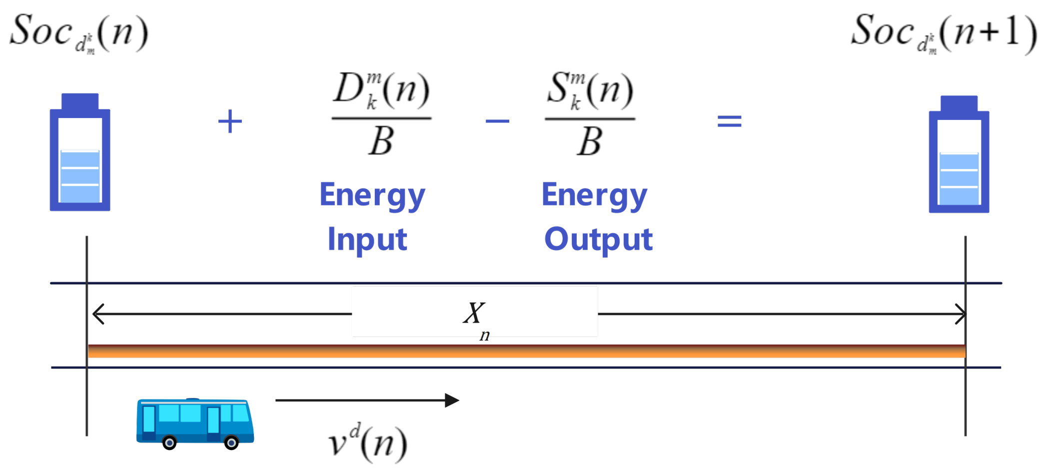

Figure 4. Vehicle moves in segment n.

Suppose the charging power is constant during the traveling on segment $n-1$ and denoted as $p^d(n-1)$. $P_{ch} = \{p^d(n), \forall d \in \mathbf{D}, \forall n \in \mathbf{N}\}$ is the set of all charging power decisions in a day. Then, the amount of energy charged in the battery can be calculated as (16):

$$D_k^m(n-1) = \eta p^d(n-1)t^d(n-1)l(n-1), \tag{16}$$

where η is the charging efficiency and $p^d(n-1)$ should be no more than the power limit of the power transmitter P_{max}, as in (17):

$$0 \le p^d(n-1) \le P_{max} \tag{17}$$

According to [21], the EV's required tractive effort F is formulated as:

$$F = (ma + kv^2 + f_{rl}mg + mgsin\theta), \tag{18}$$

where m is vehicle mass, a is acceleration, k is the aerodynamic resistance constant, v is the vehicle speed, f_{rl} is the rolling resistance constant, $g = 9.8$ is the gravity acceleration, and θ is the roadway grade.

Then, the EV's instantaneous power can be estimated by:

$$P = \frac{rR^2}{K^2}F^2 + vF + mav, \tag{19}$$

where r represents the resistance of the conductor, R is the radius of the tire, K is a constant representing the product of the armature constant, and s the magnetic flux.

Without loss of generality, we assume all the EVs keep a constant speed in every segment and the road is flat. Therefore, $\theta = 0$ and $a = 0$. Then, we can simplify the relationship between P and v as:

$$P = Cv^4 + kv^3 + f_{rl}mgv, \tag{20}$$

where $C = \frac{rR^2}{K^2}k^2$. We substitute the parameters in (20) with real values from [21] and set m = 180,000. In this study, we set the value of m by the average mass of common EBs, omitting the impact of the battery weight on the overall vehicle mass. In future work, we plan to consider more complex mass models.

$$P(v) = 0.000457v^4 + 1.3v^3 + 0.745v^2 + 1058.4V + 303. \tag{21}$$

Then, we can calculate the unit energy consumption in one segment n as:

$$\begin{aligned} S_k^m(n-1) &= \frac{X_{n-1}}{v^d(n-1)}P(v^d(n-1)) \\ &= X_{n-1}(0.000457v^3 + 1.3v^2 + 0.745v + 1058.4 + 303/v), v = v^d(n-1) \end{aligned} \tag{22}$$

In order to simplify the model, we linearize function $S_k^m(n-1)$ for $v^d(n) \in [4.17, 6.94]$ as follows:

$$S_k^m(n-1) = X_{n-1}(1.134v^d(n-1) + 0.004). \tag{23}$$

The unit of $S_k^m(n-1)$ is $kw \cdot s$, and it is observed that the higher vehicle speed leads higher energy consumption per kilometer.

3. Problem Formulation

The optimization problem determines the location to install the power transmitters, the battery capacity of the vehicles, and the wireless charging power as follows:

$$\begin{aligned}
\text{Minimize:} \quad & \mathbf{E}\{C_{cable}L_{cable} + C_{inv}N_{inv} + C_{bat} * K * B\} && (24)\\
\text{Subject to:} \quad & prob(\{Soc_{min} \le Soc_d(n) \le Soc_{max}\}) \ge \alpha, \forall d \text{ and } \forall n && (25)\\
& (6), (12), (13), (14), (17) \\
\text{Random Variables Space:} \quad & \Omega = \{\triangle v(n), \triangle T_s(n), Soc_0^k\} \\
\text{Decision Variables Space:} \quad & D = \{\{l_n, \forall n \in \mathbf{N}\}, P_{ch}, B\}.
\end{aligned}$$

The unit costs of the inductive cable and the inverter are denoted by c_{cable} and c_{inv}, respectively. C_{bat} is the unit cost of the EV bus battery. The requirements on the SOC are formulated as a probabilistic constraint as in (25). This indicates that the probability of the SOC of a bus does not meet its upper and lower bounds of α or more.

4. Model Reformulation and Solution

The DWC-EV-system-optimization problem is an NP-hard problem. As an initial step towards simplifying this issue, it has been observed that the upper bound constraint of the SOC depends solely on the variable P_{ch}, which has no effect on the objective function. This constraint can be effectively satisfied through the following strategy, which can reduce the computational complexity of the original problem.

$$p_t^d(n) = \begin{cases} P_{max}(n) & \text{if } Soc_t^d(n) <= Soc_{max} \\ 0 & \text{otherwise.} \end{cases} \tag{26}$$

In (26), it is important to clarify that $p_t^d(n)$ represents the instantaneous charging power of the vehicle at any given moment t on segment n. (26) signifies real-time monitoring of the vehicle's SOC. Charging is halted immediately once the SOC exceeds Soc_{max}. Otherwise, the vehicle is charged at a power rate of $P_{max}(n)$.

Consequently, the stochastic constraint (17) turns into: $prob(Soc_d(n) \ge Soc_{min}) \ge \alpha, \forall d$. It includes multidimensional numerical integration, and it is difficult to describe it directly in a modeling language or apply optimization software. To effectively address this issue, we first discretize the probability density of random variables by the Monte Carlo Sampling Method (MCSM) [22] and obtain their discrete realizations ω and probabilities π^ω. Then, we show the formulation by the following mixed 0-1 plan with reference to the method of [23]:

$$\begin{aligned}
\text{Minimize:} \quad & C_{cable}L_{cable} + C_{inv}N_{inv} + C_{bat} * K * B && (27)\\
\text{Subject to:} \quad & Soc_{min} + Soc_d(n,\omega) * y_d^n(\omega) \ge Soc_d(n), \forall d \text{ and } \forall n \\
& \sum_{\omega \in \Omega} \pi^\omega y_d^n(\omega) \le 1 - \alpha \\
& y_d^n(\omega) \in \{0,1\} \\
& (6), (12), (13), (14), (26).
\end{aligned}$$

By solving (27), we can obtain the optimal installation of the power transmitters and battery sizing of the DWC-EV bus system. Furthermore, (27) is a typical Mixed-Integer Programming (MIP) problem, which can be solved by existing integer programming algorithms, such as branch-and-bound. Due to space limitation, we omit the details here.

5. Numerical Results

In this section, we present the numerical results obtained from our computational simulations and experiments conducted for evaluating the DWC-EV bus system in urban environments. The settings of the system are as in Table 4.

Table 4. The value of system variables.

C_{cable}	The unit cost of cable (USD/m)	60
C_{inv}	The unit cost of inverter (USD)	5000
$C_{battery}$	The battery cost per unit capacity (USD/kwh)	400
k	No of EV buses in the system	5
N	No. of segments	560
S_r	The segments where there is a regular station	[100, 200, 300, 400, 500]
S^-	No segment that has forbidden cable installation	$\emptyset$
x	Length of every segment (km)	0.0005
Soc_{min}	The minimal requirement on SOC	0.2
Soc_{max}	The maximum requirement on SOC	0.8
η	The charging efficiency [24]	0.9
P_{max}	The maximum wireless charging power (kw)	100

For the random variable, we set the following parameters: $\sigma_v = 5$, $\sigma_{ts} = 0.0001$, $\underline{soc}_0 = 0.5$, and $\overline{soc}_0 = 1$. We then use MCSM with 10,000 samples and 20 bins to discretize the probability distribution of $\triangle v$, $\triangle T_s$ and Soc_0^k. An example of the discretization results are shown in Figure 5.

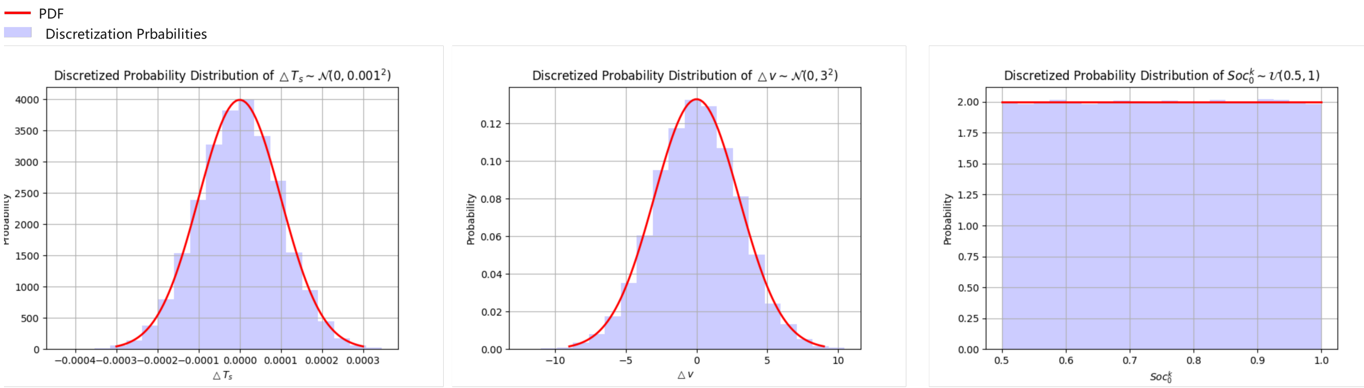

Figure 5. The examples of the probability distribution discretization of $\triangle v$, $\triangle T_s$, and Soc_0^k.

Then, we use *CPLEX* as the solver to solve (27) and obtain the results of power transmitter placement, as shown in Table 5.

Table 5. Optimal installation of power transmitters in the DWC-EV bus system.

Battery Capacity	3 kwh
No. of power transmitters	6
1st transmitter	330–700 m
2nd transmitter	930–1000 m
3rd transmitter	1400–1500 m
4th transmitter	1850–2000 m
5th transmitter	2380–2500 m
total length of power cable	810 m
total cost	USD 79,600

The results indicate that five power transmitters are needed to meet the requirements and the total cable length is 835 m, which covers almost 30% of the total route. Furthermore, we can see that all the segments with regular stations are installed with power transmitters, which is an efficient way to install the cables. Because at the regular stations, the buses will be charged with more energy. Compared with the result in [14], the results show that more transmitters including a longer cable are installed to improve the robustness of the system. In Figure 6, we show the expected value of the SOC under different velocity settings. With the improvement of the velocity, the charging energy decreases and the energy consumption increases, which brings risk to the system. We can see that, even with the largest velocity, the SOC requirement is always fulfilled.

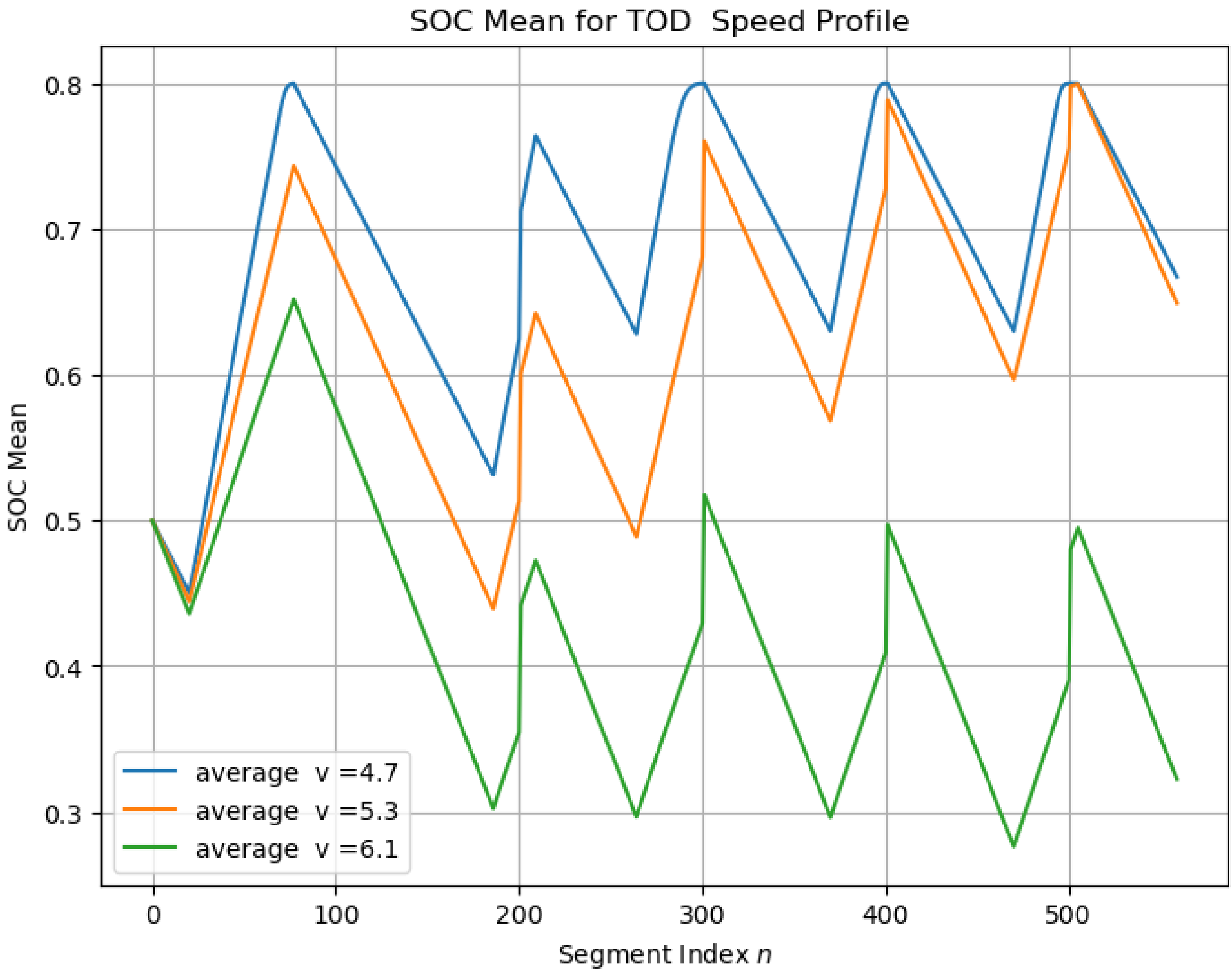

Figure 6. Mean values of SOC dynamics under different speed profiles.

From Figure 7, we can observe the trend of system cost and cable length as the number of buses changes. With an increase in the number of buses, the system cost shows an upward trend, while the cable length remains relatively constant. This is because the amount of electricity required for each bus operation is relatively stable, and this portion of electricity needs to be replenished directly through DWC infrastructures. The total amount of wireless charging energy is only related to the length of the charging cable. Therefore, with a predefined path, the optimal cable length required remains constant. On the other hand, when the number of buses decreases, the total cost spent on EV batteries reduces, allowing for the installation of larger capacity batteries in each bus. Large batteries can reduce the frequency of charging throughout the trip, which means fewer inverters need to be installed. This is the fundamental reason why system costs decrease as the number of buses decreases.

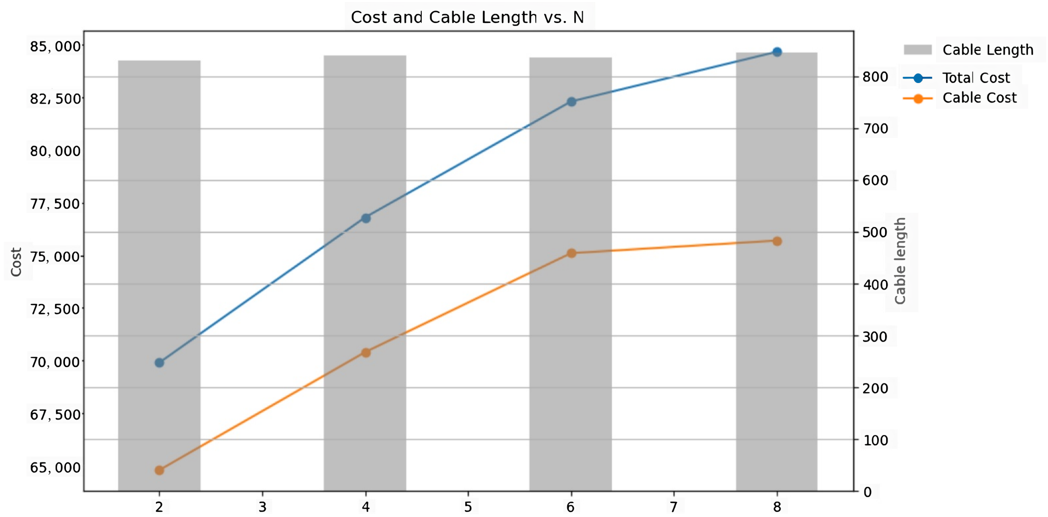

Figure 7. Total cost and cable cost under different k.

6. Conclusions

This study proposes a stochastic optimization framework for the EB-DWC system within urban environments, aiming to tackle inherent stochastic elements such as fluctuations in vehicle speed, initial charging state, and dwell time at bus stops. We formulated a stochastic planning problem for the EB-DWC system by merging these uncertainties. We employed Monte Carlo sampling techniques to efficiently solve the problem. The method we propose demonstrates that a bit more spend on the charging cables brings a significant enhancement in the robustness of the system. Based on our experimental results, we can also conclude that there is a tradeoff between the system's battery capacity and cable configuration: an increase in battery capacity does not directly reduce the costs associated with the cable length, but instead, has a direct impact on the number of inverters required.

Based on the findings of this paper, we plan to explore the stochastic optimization problem of the DWC-EB system in a multi-route environment as our future research direction. Furthermore, the impact of battery degradation and the influence of battery weight on energy consumption and the optimization of DWC infrastructure allocation will also be considered in our future work.

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