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A Pricing Strategy of E-Commerce Advertising Cooperation in the Stackelberg Game Model with Different Market Power Structure

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Abstract: A lot of research work has studied the auction mechanism of uncertain advertising cooperation between the e-commerce platform and advertisers, but little has focused on pricing strategy in stable advertising cooperation under a certain market power structure. To fill this gap, this paper makes a study of the deep interest distribution of two parties in such cooperation. We propose a pricing strategy by building two stackelberg master-slave models when the e-commerce platform and the advertiser are respectively the leader in the cooperation. It is analyzed that the optimization solution of the profits of both parties and the total system are affected by some main decision factors including the income commission proportion, the advertising product price and the cost of advertising effort of both parties' brand in different dominant models. Then, some numerical studies are used to verify the effectiveness of the models. Finally, we draw a conclusion and make some suggestions to the platforms and the advertisers in the e-commerce advertising cooperation.

Keywords: e-commerce advertising cooperation; the pricing strategy; stackelberg game model

1. Introduction

In general, the e-commerce platform provides their advertising positions on their promotion pages to the advertisers by auction. In addition, in much previous literature, most of them demonstrate the e-commerce advertising cooperation mode from the view of auction for mobile advertisers. However, in some cases, they would like to offer the fixed advertising positions for the specific strategic partners. Such specific strategic partners are always big brand advertisers with high reputation and many consumers. Since advertising is very important for the advertisers in demand creation and market expansion, the big advertisers are more than glad to get the fixed ad slot offered by the platform. They do not need to worry about that there is no place to show their products in the promotion page. They can save a lot of time on competing for the advertising space. Advertising with fixed ad slot is better for the advertisers than that without. In addition, they have their own rights to decide what samples are shown in the platform. In addition, moreover, the brand effect of the e-commerce platform can also promote the advertising products. In this case, the strategic cooperation between the e-commerce platform and the big advertisers is different from the temporary auction relationship. It is more stable and deeper than the latter. Thus, the research on the strategic partnership is an important issue to be studied not only for academia but also for industry. Nevertheless, there is little related work on the strategic advertising cooperation with the fixed advertising position. In addition, few researchers have studied the deep interest distribution problem between the two parties from the perspective of the two partners' brand effects and the market power structure.

We study the strategic partnership between the e-commerce platforms and the big advertisers in this paper. The platform provides not only advertising space for the big advertisers but also provides them with relevant advertising publicity to improve the transaction scale. Instead of bidding the advertising slot, the big advertisers would share a proportion of their profits of the advertising products with the platform. Thus, this pricing strategy of e-commerce advertising cooperation transforms into another typical kind of advertising cooperation problem.

In the early researches on advertising cooperation, most of the literatures focus on descriptive analysis of the development trend of advertising industry, relevant regulations and management, or discuss on the advantages and disadvantages of advertising cooperation. Some literatures [1–3] also studied the conflict coordination and control problems caused by the online sales channel. In recent decades, some literatures have gradually adopted the method of establishing relevant mathematical models or empirical studies to solve the longitudinal coordination of advertising cooperation between manufacturers and retailers from a more theoretical perspective. For the advertising cooperation between manufacturers and retailers, Berger [4] is an early scholar who used game theory to establish a game model of advertising arrangement to solve this problem. However, his paper demonstrated the problem of indirect cooperative advertising. The amount of local advertising input of retailers did not directly affect the subsidy scheme of manufacturers, who decided the price discount according to the quantity of retailers' order. Dant and Berger [5] studied the optimal advertising sharing rate of manufacturers and retailers under decentralized decision-making and cooperative decision-making through quantitative analysis when examining the decision-making problem of vertical cooperative advertising cost in franchise environment, and demonstrated that the "win-win" goal could be achieved through cooperative advertising rather than a complete zero-sum game. However, the above-mentioned literature did not consider the impact of manufacturer's advertising on product market demand and did not take the relationship between manufacturer's advertising share rate and retailer's advertising into account. They only conducted a single-cycle game analysis based on the short-term impact of advertising on product sales. Neither the time dynamics nor the long-term effects were considered. Huang and Li et al. [6] built the optimal advertising strategic model and analyzed the change of market power structure contrast between manufacturers and retailers based on their peer-peer relationship with the typical cases of the Wal-Mart and P&G. Yue and Austin et al. [7] modified Huang & Li's model, and put forward a two-layer supply chain advertising cooperation problem in which the manufacturers directly offered discounts to the consumers in combination with the elasticity of demand in the market environment. Lin and Tu [8] presented the game problem of cooperative manufacturers and retailers on one stage, and further analyzed the multi-stage game equilibriums of both channel members. Based on the study of the retailers' advertising behavior influence on brand image, Jørgensen *et al.* [9,10] analyzed the optimal advertising strategy and the transaction efficiency of both parties in the supply chain. These above literatures considered the influence of local advertising of retailers and national advertising of manufacturers on sales volume, but there were many disputes on the correlation between advertising expenditure of manufacturers and retailers. Most of the above literatures carried out the study on the advertising cooperation arrangement and advertising cost sharing in a dynamic environment to prove that advertising cooperation is an effective means to improve marketing channel profits. However, little literature has studied the advertising cooperation strategies of both sides based on the deep analysis of the influence of advertising cost on brand effect, and the modes of the advertising cooperation.

At present, the application of closed-loop supply chain in enterprises is increasing and the market demand as well. It has become a new development trend of logistics and supply chain management. Many scholars have conducted deep research on closed-loop supply chain. Giovanni and Zaccour [11] discussed a two-stage closed-loop supply chain game in which the remanufacturer allocated the residual value of the returned item and determined whether to manage the collection of end-of-use products exclusively or outsource it to a retailer or a third-party service provider. They found that the manufacturers tended to outsource the production collection when the outsourcing performed environmentally and operationally better and the manufacturers were generally more sensitive to environmental performance than operational performance. Ma et al. [12] made a study of the closed-loop supply chain under four reverse channel structures, in which the central planner, the manufacturer, the retailer or the third party acted as collectors of second-hand products respectively, and the demand depended on the retailer's marketing efforts. They first achieved supply chain profitability in the centralized and decentralized supply chain management service center, and provided the optimal marketing effort, collection rate and pricing decision for supply chain members. In addition, then they considered the distributional fairness of the retailer and the potential recycle cost advantages by the retailer and the third party as the two directions to extend the manufacturer collection model. The above researches have attracted many people's attention.

Likewise, the cooperation between e-commerce platforms and advertisers proposed in this paper is similar to the game in the supply chain. The e-commerce platform and the advertiser play the different role in the market power structure in such advertising cooperation. One is regarded as the leader and imposes his decisions on the other who is to be the follower. We consider it as a stackelberg master-slave game model. The solution to the certain structure is called the stackelberg equilibrium. On account of the different market power structure in the cooperation, this paper presents two stackelberg master-slave game theoretic models of the strategic ad cooperation considering the brand advertising effect. Therefore, how to deduct a reasonable proportion from the sum of the advertisers' profits is key problem to the e-commerce platform. How to make a rational price for the advertising products is also very important for the advertisers. Due to the brand effect of both parties, we discuss how the brand effect influences the proportion of income commission and the advertising products price under different situations. This paper also analyzes the optimal profits of both parties in different game model combined with the relevant parameters, such as the market demand, the sensitivity coefficient of demand to price and the cost of advertising effort of both parties. Finally, to verify the correctness and validity of the models, we test the models by the numerical analysis and draw the conclusion.

2. Descriptions and Hypothesis

For parsimony, this paper deals with simple advertising mode having only one e-commerce platform and one advertiser. The relevant model description and basic hypothesis are given below:

ξ_0 The original growth contribution rate at which the demand of products enhances due to consumer ' recognition without brand advertising on the e-commerce platform

ξ_1 The derived growth contribution rate at which the demand of products enhances because of the consumers' recognition with brand advertising on the e-commerce platform

ξ The total growth contribution rate at which the demand of products enhances influenced by the consumers' recognition of commodities through brand advertising channels and non-brand advertising channels on the e-commerce platform

q The aggregate demand for advertiser's products without brand advertising on the e-commerce platform

p_a The advertising product price

λ The income commission proportion for e-commerce platform due to advertisers' transaction

B_p The cost of advertising effort of the platform brand

B_a The cost of advertising effort of the advertiser brand

μ The sensitivity coefficient of demand to price

E_p The advertising efficacy of the platform brand

E_a The advertising efficacy of the advertiser brand

γ_p The coefficient of cost to effect of the platform brand

γ_a The coefficient of cost to effect of the advertiser brand

c_a The cost of the advertiser's products

In the game model, the decision variables of the e-commerce platform are λ and B_p, and the decision variables of the advertiser are p_a and B_a. Since λ is the income commission proportion for platform due to advertisers' transaction, it largely determines the profit of the platform. In addition, the cost of advertising effort of the platform brand B_p decides the advertising efficacy of the platform brand. p_a is the advertising product price which directly affects the income of the advertiser. The cost of advertising effort of the advertiser brand B_a determines the advertising efficacy of the advertiser brand.

Hypothesis 1. *According to the law of the market, there is usually an upper limit of market saturation for products. When the upper limit is reached, the demand will not increase. This paper assumes that the maximum sales volume of the advertising products is within the maximum saturation of the market.*

Hypothesis 2. *After the commodity appears in the market, the consumers would gradually accept it even though there is no advertisement on the e-commerce platform. According to the law of the market, there naturally exists the growth contribution rate at which the demand of products enhances due to consumers ' recognition without brand advertising on the e-commerce platform [13]. Here we set this original growth contribution rate as ξ_0. To improve the popularity of products, the merchants are willing to take some measures to promote through brand advertising on the e-commerce platform. Thus, it brings some improvement effects to the sales volume of the advertising products. This is a new derived growth contribution rate which we assume it as ξ_1. The total growth contribution rate of consumers' recognition of products through brand advertising channels and non-brand advertising channels on the e-commerce platform is set as ξ such that $\xi = \xi_0 + \xi_1$.*

Hypothesis 3. *The demand function of advertising products is related to the price and total growth contribution rate of products as shown in Equation (1).*

$$Q(p_a) = \xi \cdot f(p_a) = (\xi_0 + \xi_1) \cdot f(p_a) \tag{1}$$

Hypothesis 4. *$f(p_a)$ shown in Equation (2) reflects the effect of product price on demand where q is the maximum demand without the effect of brand advertising and the sensitivity of demand to price μ are positive constants.*

$$f(p_a) = q - \mu p_a \tag{2}$$

They satisfy the following constraints $0 < p_a < \frac{q}{\mu}$

Hypothesis 5. *For simplicity, the original growth contribution rate ξ_0 of the sales volume of products is $\xi_0 = 1$ when there is no brand advertising on the e-commerce platform.*

Hypothesis 6. *It is assumed that the derived growth contribution rate of sales promotion by brand advertising on the e-commerce platform which is related to the brand effect of the platform and the advertiser such that*

$$\xi_1 = \delta(\mathrm{E}_a, \mathrm{E}_p) \tag{3}$$

where $\delta(\mathrm{E}_a, \mathrm{E}_p)$ is a function of the brand effect of a platform and the brand effect of the advertiser.

Hypothesis 7. *According to the literature [14–16], it is assumed that the cost of advertising effort is a convex function of the brand advertising effect since the cost of advertising effort and the brand advertising effect follow the law of marginal decline [16,17], which satisfies the following Equations (4) and (5):*

$$B_p = \frac{\gamma_p}{2} {\mathrm{E}_p}^2 \tag{4}$$

$$B_a = \frac{\gamma_a}{2} {\mathrm{E}_a}^2 \tag{5}$$

They give (6):

$$\delta(\mathrm{E}_a, \mathrm{E}_p) = \sqrt{\frac{2B_a}{\gamma_a}} + \sqrt{\frac{2B_p}{\gamma_p}} \tag{6}$$

for $\gamma_a > 0$, $\gamma_p > 0$

Based on the above hypothesis, we update the demand function $Q(p_a)$ *to* $Q(p_a, \mathrm{E}_a, \mathrm{E}_p)$ *which the brand effect of the platform and the advertiser is taken into account. It gives the following Equation (7):*

$$Q(p_a, \mathrm{E}_a, \mathrm{E}_p) = (\xi_0 + \xi_1) \cdot f(p_a) = f(p_a) \cdot [1 + \delta(\mathrm{E}_a, \mathrm{E}_p)] \tag{7}$$

Since the brand effect is related to the cost of advertising effort of the e-commerce platform and the advertisers. The demand function $Q(p_a, \mathrm{E}_a, \mathrm{E}_p)$ could be updated again as the following function $Q(p_a, B_a, B_p)$ *in Equation (8):*

$$Q(p_a, B_a, B_p) = (q - \mu p_a)\left(1 + \sqrt{\frac{2B_a}{\gamma_a}} + \sqrt{\frac{2B_p}{\gamma_p}}\right) \tag{8}$$

3. The Stackelberg Game Models

3.1. P-Model: A Stackelberg Game Model with Platform as Leader

The stackelberg game model with platform as leader refers to that, among the participants of the game between the advertising platform and the advertiser, the advertising platform is more powerful in the market power structure and controlling the cooperation between the two parties.

Therefore, the platform as a leader dominates the whole game, while the advertiser is in a subordinate position. It can be considered as a typical stackelberg master-slave game [16]. In addition, we call it as P-model in short. As a leader, the platform determines the proportion of the advertiser's transaction commission, and the cost of brand advertising effort paid by the platform. In addition, the advertiser determines the optimal price of advertising products and the cost of advertising effort paid by the advertiser. To obtain the solution to the structure, the platform's decision problem is solved based on the advertiser's response [16,18].

Therefore, the profit of the e-commerce platform is as the follow (9):

$$\begin{aligned} \prod\nolimits_p &= \lambda p_a Q(p_a, B_a, B_p) - B_p \\ &= \lambda p_a (q - \mu p_a)\left(1 + \sqrt{\frac{2B_a}{\gamma_a}} + \sqrt{\frac{2B_p}{\gamma_p}}\right) - B_p \end{aligned} \tag{9}$$

The profit of the advertiser is calculated as below (10):

$$\begin{aligned} \prod\nolimits_a &= (p_a - c_a)Q(p_a, B_a, B_p) - \lambda p_a Q(p_a, B_a, B_p) - B_a \\ &= [(1-\lambda)p_a - c_a](q - \mu p_a)\left(1 + \sqrt{\tfrac{2B_a}{\gamma_a}} + \sqrt{\tfrac{2B_p}{\gamma_p}}\right) - B_a \end{aligned} \tag{10}$$

The profit of the system is the following (11):

$$\begin{aligned} \prod &= \prod\nolimits_p + \prod\nolimits_a \\ &= (p_a - c_a)(q - \mu p_a)\left(1 + \sqrt{\frac{2B_a}{\gamma_a}} + \sqrt{\frac{2B_p}{\gamma_p}}\right) - B_p - B_a \end{aligned} \tag{11}$$

This stackelberg game model is a typical dynamic game of complete information. For the platform, the decision variables are the income commission proportion λ for platform due to advertisers' transaction and the effort cost of the platform's brand advertising B_p. For the advertiser, the decision variables are the price of the advertising product p_a and the effort cost of the advertiser's brand advertising B_a. It can be solved by the backward induction method to find the stackelberg equilibrium.

Proposition 1. *In the stackelberg P-model, the optimal decision variable λ^{p*} of the e-commerce platform is (12):*

$$\lambda^{p*} = 1 + \frac{\mu^2 {c_a}^2}{q\sqrt[3]{3\mu^2 {c_a}^2\left[9q - \sqrt{3\mu^2 {c_a}^2 + 81q^2}\right]}} - \frac{1}{3}\sqrt[3]{3\mu^2 {c_a}^2\left[9q - \sqrt{3\mu^2 {c_a}^2 + 81q^2}\right]} \tag{12}$$

Then, the other optimal decision variables B_p^{p*} is as the follow (13):

$$B_p^{p*} = \frac{\lambda^{p*2}\left[(1-\lambda^{p*})^2 q^2 - \mu^2 {c_a}^2\right]^2}{32\mu^2\gamma_p(1-\lambda^{p*})^4} \tag{13}$$

Proposition 2. *In the stackelberg P-model, the optimal decision variables of the advertiser are (14) and (15):*

$${p_a}^{p*} = \frac{(1-\lambda^{p*})q + \mu c_a}{2\mu(1-\lambda^{p*})} \tag{14}$$

$${B_a}^{p*} = \frac{[(1-\lambda^{p*})q - \mu c_a]^4}{32\gamma_a\mu^2(1-\lambda^{p*})^2} \tag{15}$$

Proposition 3. *In the stackelberg P-model, the e-commerce platform's and advertiser's maximum profits are then, respectively, in the Equations (16) and (17):*

$$\begin{aligned}
{\prod_p}^{p*} = & \left(\frac{\frac{4}{\gamma a}\lambda^{p*3}q^2 - \frac{6}{\gamma p}\lambda^{p*3}q^2 + 4\frac{8}{\gamma a}\mu\lambda^{p*2}qc_a + 16\mu\lambda^{p*2}}{64\mu^2(1-\lambda^{p*})^4}\right. \\
& + \frac{\frac{12}{\gamma p}\lambda^{p*2}q^2 - \frac{12}{\gamma a}\lambda^{p*2}q^2 - \frac{16}{\gamma a}\mu\lambda^{p*}qc_a + \frac{6}{\gamma p}\mu^2\lambda^{p*}{c_a}^2}{64\mu^2(1-\lambda^{p*})^4} \\
& + \frac{16\mu + \frac{4}{\gamma a}\mu^2\lambda^{p*}{c_a}^2 + \frac{8}{\gamma a}\mu q c_a - \frac{6}{\gamma p}\lambda^{p*}q^2 - 32\mu\lambda^{p*}}{64\mu^2(1-\lambda^{p*})^4} \\
& \left. + \frac{\frac{12}{\gamma a}\lambda^{p*}q^2 - \frac{4}{\gamma a}\mu^2{c_a}^2 - \frac{4}{\gamma a}q^2}{64\mu^2(1-\lambda^{p*})^4}\right) \cdot \frac{\left[(1-\lambda^{p*})^2q^2 - \mu^2{c_a}^2\right]}{64\mu^2(1-\lambda^{p*})^4}
\end{aligned} \tag{16}$$

$$\begin{aligned}
{\prod_a}^{p*} = & \left(\frac{\frac{8}{\gamma p}\lambda^{p*2}q^2 - \frac{4}{\gamma p}\lambda^{p*3}q^2 - \frac{4}{\gamma p}\lambda^{p*}q^2 + \frac{6}{\gamma a}\mu^2\lambda^{p*}{c_a}^2}{64\mu^2(1-\lambda^{p*})^3}\right. \\
& + \frac{\frac{12}{\gamma a}\mu q c_a - \frac{24}{\gamma a}\mu\lambda^{p*}qc_a - 32\mu\lambda^{p*} + 16\mu\lambda^{p*2} + 16\mu}{64\mu^2(1-\lambda^{p*})^3} \\
& + \frac{\frac{12}{\gamma a}\mu\lambda^{p*2}q{c_a}^2 + \frac{6}{\gamma a}\lambda^{p*3}q^2 + \frac{12}{\gamma p}\mu^2\lambda^{p*}{c_a}^2 - \frac{18}{\gamma a}\lambda^{p*2}q^2}{64\mu^2(1-\lambda^{p*})^3} \\
& \left. + \frac{\frac{18}{\gamma a}\lambda^{p*}q^2 - \frac{6}{\gamma a}q^2 - \frac{6}{\gamma a}\mu^2{c_a}^2}{64\mu^2(1-\lambda^{p*})^3}\right) \cdot \frac{[(1-\lambda^{p*})q - \mu c_a]^2}{64\mu^2(1-\lambda^{p*})^3}
\end{aligned} \tag{17}$$

And the total profit in the advertising cooperation is calculated by Equation (18):

$$\begin{aligned}\prod^{p*} &= \left[\tfrac{(1-\lambda^{p*})q+\mu c_a}{2\mu(1-\lambda^{p*})} - c_a\right]\cdot\left[q - \tfrac{(1-\lambda^{p*})q+\mu c_a}{2(1-\lambda^{p*})}\right] \\ &\cdot\left[1 + \tfrac{[(1-\lambda^{p*})q-\mu c_a]^2}{4\gamma_a\mu(1-\lambda^{p*})} + \tfrac{\lambda^{p*}\left[(1-\lambda^{p*})^2q^2-\mu^2{c_a}^2\right]}{4\gamma_p\mu(1-\lambda^{p*})^2}\right] \\ &- \tfrac{\lambda^{p*2}\left[(1-\lambda^{p*})^2q^2-\mu^2{c_a}^2\right]^2}{32\gamma_p\mu^2(1-\lambda)^4} \\ &- \tfrac{\left[(1-\lambda^{p*})q^2-\mu c_a\right]^4}{32\gamma_a\mu^2(1-\lambda)^2}\end{aligned} \tag{18}$$

Proof. Firstly, for any given platform's decision variables B_p and λ, the optimal decision variables B_a and p_a of the advertiser are to be determined. Therefore, the profit function of the advertiser on the premise of maximizing his own interests is derived from which the solution to the optimization problem can be obtained as shown in Equation (19).

$$\max_{p_a,B_a}\prod\nolimits_a = [(1-\lambda)p_a - c_a](q-\mu p_a)\left(1+\sqrt{\frac{2B_a}{\gamma_a}}+\sqrt{\frac{2B_p}{\gamma_p}}\right) - B_a \tag{19}$$

$s.t.\ 0 < p_a < \frac{q}{\mu}, B_a > 0$

Because $\prod_a$ is a strict concave function of the price and the cost of advertising effort.

Let $\frac{\partial\prod_a}{\partial B_a} = 0$

We derive (20):

$${B_a}^{p*} = \frac{1}{2\gamma_a}[(1-\lambda)p_a - c_a]^2(q-\mu p_a)^2 \tag{20}$$

Let $\frac{\partial\prod_a}{\partial p_a} = 0$

We obtain (14):

$${p_a}^{p*} = \frac{(1-\lambda)q+\mu c_a}{2\mu(1-\lambda)}$$

If ${B_a}^{p*}$ is represented in the form of ${p_a}^{p*}$, then we get (15):

$${B_a}^{p*} = \frac{[(1-\lambda)q-\mu c_a]^4}{32\gamma_a\mu^2(1-\lambda)^2}$$

Based on the idea of the backward induction, the platform determines his decision variables according to the optimal decisions of the advertiser, namely, the optimal cost of brand advertising effort and the optimal price of advertising product. Then, to maximize the platform's own interests, the profit function and the solution are derived as follows (21).

$$\max_{\lambda,B_p}\prod\nolimits_p = \lambda p_a(q-\mu p_a)\left(1+\sqrt{\frac{2B_a}{\gamma_a}}+\sqrt{\frac{2B_p}{\gamma_p}}\right) - B_p \tag{21}$$

$s.t.\ 0 < \lambda < 1, B_p > 0$

In the form of ${p_a}^{p*}$ and ${B_a}^{p*}$, $\max\limits_{\lambda,B_p}\prod_p$ can be represented in Equation (22):

$$\begin{aligned}\max_{\lambda,Br_p}\prod\nolimits_p &= \left[\tfrac{\lambda[(1-\lambda)q+\mu c_a]}{2\mu(1-\lambda)}\right]\cdot\left[q-\tfrac{(1-\lambda)q+\mu c_a}{2(1-\lambda)}\right]\cdot \\ &\left[1+\tfrac{[(1-\lambda)q-\mu c_a]^2}{4\gamma_a\mu(1-\lambda)}+\sqrt{\tfrac{2B_p}{\gamma_p}}\right] - B_p\end{aligned} \tag{22}$$

$s.t.\ 0 < \lambda < 1, B_p > 0$

Let $\frac{\partial \Pi_p}{\partial B_p} = 0$
We get (13):

$$B_p{}^{p*} = \frac{\lambda^2 \left[(1-\lambda)^2 q^2 - \mu^2 c_a{}^2 \right]^2}{32\mu^2 \gamma_p (1-\lambda)^4}$$

Let $\frac{\partial \Pi_p}{\partial \lambda} = 0$
We obtain (12):

$$\lambda^{p*} = 1 + \frac{\mu^2 c_a{}^2}{q \sqrt[3]{3\mu^2 c_a{}^2 \left[9q - \sqrt{3\mu^2 c_a{}^2 + 81q^2} \right]}} - \frac{1}{3} \sqrt[3]{3\mu^2 c_a{}^2 \left[9q - \sqrt{3\mu^2 c_a{}^2 + 81q^2} \right]}$$

□

3.2. A-model: A Stackelberg Game Model with Advertiser as Leader

Compared with the advertising platform, the advertiser is much stronger in the market power structure and more effectively controls the cooperation of both sides in this kind of cases. As a result, the e-commerce platform is willing to be in a subordinate position to attract such big advertisers. This is a typical stackelberg master-slave game with advertiser as leader. In addition, we call it as A-model. Since the A-model of advertiser leading brand effect is a complete information dynamic game, for the advertiser, the decision variable is the quality cost of the advertiser's brand advertising and the pricing of the advertiser's advertising products. For the platform, the decision variable is the quality cost of the platform's brand advertising and the proportion of the platform's transaction commission to the advertiser's advertising products. The advertiser, as the leader, first determines the pricing of his own advertising products and the corresponding cost of brand advertising quality. Then the platform determines the proportion of the transaction commission charged by the advertiser and his cost of brand advertising quality.

Proposition 4. *In the stackelberg A-model, the optimal decision variable $p_a{}^{a*}$ of the advertiser is (23):*

$$p_a{}^{a*} = \frac{q}{2\mu} + \frac{c_a}{2} \tag{23}$$

Then, the other optimal variable $B_a{}^{a*}$is as the follow (24):

$$B_a{}^{a*} = \frac{8\left(4 + \frac{6}{\gamma_p}\mu p_a{}^{a*2} + \frac{6}{\gamma_p}c_a q - \frac{6}{\gamma_p}\mu p_a{}^{a*} c_a - \frac{6}{\gamma_p} p_a{}^{a*} q\right)^2}{\gamma_a \left(\frac{16}{\gamma_a} + \frac{18}{\gamma_p}\right)^2} \tag{24}$$

Proposition 5. *In the stackelberg A-model, the optimal decision variables of the e-commerce platform are (25) and (26):*

$$\lambda^{a*} = \frac{2\left(\frac{16}{\gamma_a} + \frac{18}{\gamma_p}\right) + \frac{8}{\gamma_a}\left[\frac{6}{\gamma_p}(q - \mu p_a{}^{a*})(p_a{}^{a*} - c_a) - 4\right]}{\frac{6}{\gamma_p} p_a{}^{a*}(q - \mu p_a{}^{a*})\left(\frac{16}{\gamma_a} + \frac{18}{\gamma_p}\right)} \tag{25}$$

$$B_p{}^{a*} = \frac{\frac{2}{\gamma_p}\left[\frac{4}{\gamma_a}(p_a{}^{a*} - c_a)(q - \mu p_a{}^{a*}) + 3\right]^2}{\left(\frac{16}{\gamma_a} + \frac{18}{\gamma_p}\right)^2} \tag{26}$$

Proposition 6. *In the stackelberg A-model, the advertiser's maximum profit is calculated in Equation (27):*

$$\begin{aligned}\prod_a{}^{a*} &= \frac{\frac{4}{\gamma_a\gamma_p}\mu^2 {p_a}^{a*4} + \frac{4}{\gamma_a\gamma_p}{p_a}^{a*2}q^2 + \frac{16}{\gamma_a\gamma_p}\mu c_a {p_a}^{a*2}q}{\frac{4}{\gamma_a}+\frac{9}{2\gamma_p}} \\ &+ \frac{\frac{4}{\gamma_a\gamma_p}q^2{c_a}^2 - \frac{8}{\gamma_a\gamma_p}\mu^2{p_a}^{a*3}c_a - \frac{8}{\gamma_a\gamma_p}c_a{p_a}^{a*}q^2}{\frac{4}{\gamma_a}+\frac{9}{2\gamma_p}} \\ &+ \frac{\frac{4}{\gamma_a\gamma_p}\mu^2{c_a}^2{p_a}^{a*2} - \frac{8}{\gamma_a\gamma_p}\mu{c_a}^2{p_a}^{a*}q - \frac{8}{\gamma_a\gamma_p}\mu{p_a}^{a*3}q}{\frac{4}{\gamma_a}+\frac{9}{2\gamma_p}} \\ &+ \frac{\frac{6}{\gamma_p}\mu c_a{p_a}^{a*} - \frac{6}{\gamma_p}\mu{p_a}^{a*2} + \frac{6}{\gamma_p}{p_a}^{a*}q - \frac{6}{\gamma_p}c_a q - 2}{\frac{4}{\gamma_a}+\frac{9}{2\gamma_p}}\end{aligned} \tag{27}$$

The e-commerce platform's maximum profit is shown in Equation (28):

$$\prod_p{}^{a*} = \frac{\frac{14}{\gamma_p}\left(\frac{4}{\gamma_a}\mu{p_a}^{a*}c_a - \frac{4}{\gamma_a}c_a q + \frac{4}{\gamma_a}{p_a}^{a*}q - \frac{4}{\gamma_a}\mu{p_a}^{a*2} + 3\right)^2}{\left(\frac{16}{\gamma_a}+\frac{18}{\gamma_p}\right)^2} \tag{28}$$

The total profit in the advertising cooperation is calculated by Equation (29):

$$\begin{aligned}\prod{}^{a*} &= ({p_a}^{a*} - c_a)(q - \mu{p_a}^{a*}) \\ &\cdot\left[\frac{\frac{4}{\gamma_a}\left(\frac{6}{\gamma_p}{p_a}^{a*}q - \frac{6}{\gamma_p}\mu{p_a}^{a*2} + \frac{6}{\gamma_p}c_a q - \frac{6}{\gamma_p}\mu{p_a}^{a*}c_a - 4\right)}{\frac{16}{\gamma_a}+\frac{18}{\gamma_p}} + \frac{\frac{2}{\gamma_a}\left[\frac{4}{\gamma_a}{p_a}^{a*}q - \frac{4}{\gamma_a}\mu{p_a}^{a*2} + \frac{4}{\gamma_a}\mu{p_a}^{a*}c_a - \frac{4}{\gamma_a}c_a q + 3\right]}{\frac{16}{\gamma_a}+\frac{18}{\gamma_p}}\right] \\ &- \frac{\frac{2}{\gamma_a}\left[\frac{4}{\gamma_a}{p_a}^{a*}q - \frac{4}{\gamma_a}\mu{p_a}^{a*2} + \frac{4}{\gamma_a}\mu{p_a}^{a*}c_a - \frac{4}{\gamma_a}c_a q + 3\right]^2}{\left(\frac{16}{\gamma_a}+\frac{18}{\gamma_p}\right)^2} \\ &- \frac{\frac{8}{\gamma_a}\left(\frac{6}{\gamma_p}{p_a}^{a*}q - \frac{6}{\gamma_p}\mu{p_a}^{a*2} + \frac{6}{\gamma_p}c_a q - \frac{6}{\gamma_p}\mu{p_a}^{a*}c_a - 4\right)^2}{\left(\frac{16}{\gamma_a}+\frac{18}{\gamma_p}\right)^2}\end{aligned} \tag{29}$$

Proof. The proof process of propositions 4–6 in A-model is similar to that of propositions 1–3 in P-model. Due to the limitation of the length of the paper, the proof of propositions 4–6 in A-model is omitted. □

Thus, these propositions 1–6 of the two stackelberg game models are proved.

4. The Numerical Study and Discussion

Before the experiments, we analyze the case study in the literature [16]. In addition, then, we extend the numerical experiments based on it in this section. According to the experiment results, we make the comparisons of the difference of the optimal equilibriums and the maximum profits of the e-commerce platform and the advertiser in the two different game models.

Experiment 1: *the corresponding variables are given as below.*

$$q = 1000,\ c_a = 100,\ \mu = 8,\ \gamma_p = 60,\ \gamma_a = 40$$

After calculation, the optimal equilibrium and profit of the platform and the advertiser respectively are obtained in the different models and as shown in the following Table 1.

Table 1. Optimal equilibriums and profits of the platform and the advertiser in two models in Exp. 1.

The Decision Variables	P-Model	A-Model	Comparisons
λ^*	0.1082	0.0325	$\lambda^{p*} > \lambda^{a*}$
$p_a{}^*$	118.5665	112.5000	$p_a{}^{p*} > p_a{}^{a*}$
$B_p{}^*$	3633.2000	1114.6000	$B_p{}^{p*} > B_p{}^{a*}$
$B_a{}^*$	1089.9000	5975.9000	$B_a{}^{p*} < B_a{}^{a*}$
$\Pi_p{}^*$	9167.9000	7801.9000	$\Pi_p{}^{p*} > \Pi_p{}^{a*}$
$\Pi_a{}^*$	4634.8000	15,584.0000	$\Pi_a{}^{p*} < \Pi_a{}^{a*}$
Π^*	13,803.0000	23,386.0000	$\Pi^{p*} < \Pi^{a*}$

Experiment 2: *the corresponding variables are given as below.*

$$q = 1000,\ c_a = 100,\ \mu = 8,\ \gamma_p = 40,\ \gamma_a = 60$$

The optimal equilibriums and profits of the two parties in the different models are calculated and displayed in Table 2.

Table 2. Optimal equilibriums and profits of the platform and the advertiser in two models in Exp. 2.

The Decision Variables	P-Model	A-Model	Comparisons
λ^*	0.1082	0.0214	$\lambda^{p*} > \lambda^{a*}$
$p_a{}^*$	118.5665	112.5000	$p_a{}^{p*} > p_a{}^{a*}$
$B_p{}^*$	5449.8000	725.5922	$B_p{}^{p*} > B_p{}^{a*}$
$B_a{}^*$	726.6296	8741.3000	$B_a{}^{p*} < B_a{}^{a*}$
$\Pi_p{}^*$	9359.7000	5079.1000	$\Pi_p{}^{p*} > \Pi_p{}^{a*}$
$\Pi_a{}^*$	5896.3000	15,570.0000	$\Pi_a{}^{p*} < \Pi_a{}^{a*}$
Π^*	15,256.0000	20,649.0000	$\Pi^{p*} < \Pi^{a*}$

Experiment 3: *the corresponding variables are given as below.*

$$q = 1000,\ c_a = 200,\ \mu = 4,\ \gamma_p = 60,\ \gamma_a = 40$$

Then the optimal equilibriums and profits of the platform and the advertiser are shown in Table 3.

Table 3. Optimal equilibriums and profits of the platform and the advertiser in two models in Exp. 3.

The Decision Variables	P-Model	A-Model	Comparisons
λ^*	0.1082	0.0321	$\lambda^{p*} > \lambda^{a*}$
$p_a{}^*$	237.1331	225.0000	$p_a{}^{p*} > p_a{}^{a*}$
$B_p{}^*$	14,533.0000	4354.4000	$B_p{}^{p*} > B_p{}^{a*}$
$B_a{}^*$	4359.8000	24,700.0000	$B_a{}^{p*} < B_a{}^{a*}$
$\Pi_p{}^*$	35,351.0000	30,480.0000	$\Pi_p{}^{p*} > \Pi_p{}^{a*}$
$\Pi_a{}^*$	17,949.0000	60,941.0000	$\Pi_a{}^{p*} < \Pi_a{}^{a*}$
Π^*	53,300.0000	91,421.0000	$\Pi^{p*} < \Pi^{a*}$

By the comparisons of the results in the above experiments, we make some analysis as below:

(1) The income commission proportion for e-commerce platform due to advertisers' transaction would be higher in the P-model than that in the A-model. Because in the P-model, the e-commerce platform plays a leading role in decision-making and is willing to enhance the proportion to maximize his own interest.

(2) The price of the advertiser's product is higher in the P-model than that in the A-model. Because in the P-model, the advertiser is in the passive position. To ensure his own profit, it should make a relative high price since the platform makes a higher commission proportion in P-model than in A-model.

(3) The e-commerce platform and the advertiser invest more in their own brand effect in their respective dominant model than the other side's investment. Because both intend to pay more cost on building their own brand image to enhance their own brand effect when they are respectively taking the leadership in the game model.
(4) The profit of the platform is greater in the P-model than that in the A-model. Because the platform is in the dominant position in the stackelberg P-model, he aims at maximizing his own profit. The platform would increase the decision variables: the proportion of advertiser's income commission and the cost of advertising effort of the platform brand which could help to obtain much higher profit than that in the A-model.
(5) The profit of advertiser is greater in the A-model than that in the P-model. Since the advertiser plays a leading role in the stackelberg A-model, it would make the optimal decision variables: the advertising product price and the cost of advertising effort of the advertiser brand, to achieve his own optimal income, which is better than that in the P-model when the platform as the leader.
(6) The total profit is greater in the A-model than that in the P-model. Because both parties are benefited by the advertiser's large income in e-commerce transaction and the profit of the advertiser accounts for a larger proportion of the total profit, so the total profit is much greater in the A-model than that in the P-model.

5. Conclusions

In view of the e-commerce advertising cooperation between the strategic partners, this paper considers the different market power structure as a fundamental factor for the pricing problem of the cooperation. It builds two game models of master-slave led by the platform and advertisers, respectively. To maximize the profit of the both sides or the whole system, it optimizes the corresponding decision variables. Through the research and comparison of the two decentralized decision game models, the paper analyzes the influence of different market power structures on the income commission proportion for e-commerce platform due to advertisers' transaction, the advertising product price, the cost of advertising effort of the platform brand, the cost of advertising effort of the platform brand, and the profits of both parties and the whole system profit. From the different perspectives of the e-commerce platform and the big advertiser, some suggestions are given to guide the platform or the advertisers to make relevant adjustments, and to find the most suitable partners to achieve their goal of maximizing profits and the coordination of the system. Then numerical examples are used to verify the effectiveness of the two decision-making models.

This paper draws the following conclusions.

Firstly, when the platform plays a leading role in the model, the income commission proportion for e-commerce platform due to advertisers' transaction is independent on the advertising product price, the cost of advertising effort of the advertiser brand and some other factors. The platform takes the lead in determine the proportion of advertiser's income commission just according to the goal of maximizing his own interests. It always makes a higher income commission proportion than that of the other model with the advertiser as a leader. In the stackelberg game model led by the advertiser, the platform is in a follow-up position. When the income commission proportion for e-commerce platform due to advertisers' transaction is to be determined, it is necessary for the platform to consider the price of the advertising products set by the advertiser and the cost of the brand advertisements. Therefore, the platform would be more cautious to appropriately reduce the income commission proportion a little bit lower to attract the big advertisers to cooperate with him.

Secondly, the price of advertisers' advertising products is higher in the platform-dominant game model than that in the advertiser-dominant model. When the platform dominates, he will formulate a higher income commission proportion than that in the advertiser-dominated model aiming to maximize his profit. Because the advertiser is in a passive position, to ensure his own profit, it is necessary to take into account the high income commission proportion of the platform to increase the price of the advertising product, just like "a rising tide raises all boats". Thus, the price of advertising products is relatively higher than that in the stackelberg game model led by the advertiser. When the advertiser is a dominant force in the market structure, he is strong enough to let the platform to subordinate to him. Therefore, the advertiser can independently determine the relatively appropriate price of the advertising product to achieve the goal of maximizing his own interests according to the factors such as the maximum market prospects of his own advertising products, the sensitivity coefficient of demand to price and the cost of advertising products.

Thirdly, the platform and the advertiser will invest more in their own brand effect in their respective leading model. It shows that in the platform-dominant game model, the platform has a dominant position in the market power structure. He has much stronger consciousness of master home court and has more responsibility to promote the brand advertising effect. Thus, the platform is more willing to invest more in improving the quality of the brand advertising than that in the advertiser-dominant game model. While in the advertiser-dominant model, the advertiser would invest more to improve his own brand effect than in the other game model as well.

Fourthly, in the platform-dominant game model, the platform gains more profit than that in the advertiser-dominant game model because all the decision variables are formulated with the goal of maximizing the platform's own interests. Aiming to obtain the optimal profit, the platform would like to determine a high income commission proportion from the advertiser's transaction. Although the cost of the advertising effort of the platform's brand is higher than that in the advertiser-dominant game model, the return is also generous while the platform's brand effect has a positive impact on the e-commerce transaction. Thus, the overall profit of the platform in the stackelberg game model led by platform is higher than that of the other game model. In addition, in the advertiser-dominant model, the advertiser obtains more income than that in the platform-dominant game model. On account of maximizing his own profit, the advertiser is willing to give an optimal price to attract the consumers. Though the advertiser put much more investment to promote his brand, the better brand promotion effect is very helpful to the e-commerce transaction. On the contrary, in the platform-dominant game model, the advertiser is in a passive status, he cannot control the market power. The price of advertising product is greatly influenced by the income commission proportion decided by the platform. The higher the income commission proportion, the higher the advertising product's price. However, the increasing of the price correspondingly has a negative impact on the transaction. Thus, the profit of advertisers will be lower in the platform-dominant game model than that of advertisers in the advertiser-dominant game model. That is to say, the profit of the platform and the advertiser in the respective dominant model is greater than that in the other party's leadership role model. So it shows that in order to achieve the goal of maximizing their own profits, both parties are more suitable to find the partners who are willing to be their follower in the cooperation, and they can make the best profit strategy based on their dominance in the market power structure.

At last, from the perspective of total system profit, the system profit of the model with advertiser as the leader is greater than the system profit of the model with platform as the leading role. We know, the profits of all parties are derived from the advertiser's e-commerce transaction. The advertiser's profit supports a larger part of the total profit of the system. Since in the advertiser-dominant model, advertiser's profit is far greater than that in platform-dominant model. As consequence, the corresponding total system profit in the advertiser-dominant model is greatly higher than that of platform-dominant model. Therefore, if we want to get high total profit, selecting the advertiser-dominant model is better choice.

This paper studies the game model dominated by the platform and advertisers respectively, which belongs to the decentralized decision-making model. In both models, the platform and the advertiser optimize their decision variables with the goal of maximizing their own profit rather than the whole system profit. However, in some cases, the whole system profit is quite more important than the individual's in the cooperation. Consequently, we will consider the complete cooperation between the e-commerce platform and the advertiser to establish a centralized decision-making model. In this complete cooperative game model, both parties contribute their own brand effects to the system, and no longer consider their own interests and gains. When making decisions, they just jointly take the maximization of the whole system profit as the first goal. This is our future work.

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