Optimal Electric and Heat Energy Management of Multi-Microgrids with Sequentially-Coordinated Operations

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Abstract: We propose an optimal electric and heat energy management for a cooperative multi-microgrid community. The sequentially-coordinated operation for heat energy is proposed in order to distribute the computational burden as an extension of “Optimal Energy Management of Multi-Microgrids with Sequentially Coordinated Operations” and is following the sequentially-coordinated operations for electric energy in it. This sequentially-coordinated operation for heat energy is mathematically modeled and how to obtain the global heat energy optimization solution in the cooperative multi-microgrid community is presented. The global heat energy optimization is achieved for the cooperative community by adjusting the combined electric and heat energy production amounts of combined heat and power (CHP) generators and the heat energy production amount of heat only boilers (HOBs) which satisfy all heat loads, as well as optimize the external electric energy trading in order to minimize the unnecessary cost from the external electric trading, and/or maximize the profit from the external electric trading. To validate the proposed mathematical energy management models, a simulation study is also conducted.

Keywords: heat energy; cooperative multi-microgrids; energy management system (EMS); sequential operation; energy trading; optimal operation

1. Introduction

The concept of microgrids was proposed to make the electricity grid less centralized and provide local consumers a more reliable and cheaper electrical power supply [1]. Nowadays, heat energy becomes as important as electric energy for energy management of microgrids as combined heat and power (CHP) are commonly included in microgrids for heating and hot water services [2]. The microgrid concept can be applied to various electric grid customers, such as a university campus, a research park, a business building, an apartment complex, and/or an island. Energy management of microgrid has received tremendous interest by many researchers. One of the practical solutions for electric energy management of microgrid is multi-agent system-based operation, which has been applied in islanded mode [3,4] and in grid-connected mode [5–7]. This paper, however, concerns the optimal operation of energy management in multi-microgrids as defined in [8]. In this paper, we consider heat energy as well as electric energy for an optimal energy management of multi-microgrids; this paper presents sequentially-coordinated operations to manage electric and heat energy optimally for minimizing operational costs of both electric and heat energy in cooperative multi-microgrids. This paper is a heat energy extension of [9], which optimizes the electric energy management in cooperative multi-microgrids.
The optimal energy management system (EMS) has been studied for a microgrid which operates in islanded mode \cite{10,11} or in grid-connected mode \cite{11–17}. In islanded mode, when the microgrid is isolated from the power grid, an energy optimization for military forward operating base camp was solved by a dynamic programing algorithm to minimize the total daily operation cost in \cite{10} and a resiliency-oriented microgrid optimal scheduling model is proposed to minimize the microgrid load curtailment by scheduling available resources in \cite{11}. While the operation cost minimization problem for energy management of a microgrid has been mostly investigated, the profit maximization problem was also studied in \cite{13}. On the other hand, an online cost minimization scheduling model for co-generation has been developed in \cite{14}. In addition to renewable sources commonly included in a microgrid, controllable energy sources, such as CHP generators, were also considered and managed optimally by controlling their energy production amounts in \cite{11,15,17}.

Heat energy has been also considered along with electric energy in many studies of a microgrid in \cite{18–20}. While the authors \cite{19–21} minimized the operating cost, Bagheria and Tafreshi \cite{18} maximized the profit from trading of electric energy by considering the operation cost. Furthermore, heat energy storage is also considered as a component of the microgrid in \cite{19–21}.

The energy management problem for cooperative multi-microgrids is investigated in \cite{9,22,23}, which targeted minimizing the operation cost of electric energy. For the energy management of cooperative multi-microgrids, electric energy trading was allowed not only internally between microgrids, but also externally with the power grid. Rahbar et al. \cite{22} considered only uncontrollable electric energy sources, where the amount of production cannot be controlled for energy management purposes. On the other hand, Nguyen and Le \cite{23} employed the scenario-based two-stage stochastic optimization approach to deal with the uncertainties of renewable energy resources and load demand in the energy scheduling problem in addition to controllable electric energy sources such as CHPs and diesel generators. Furthermore, Song et al. \cite{9} proposed an optimal electric energy management of a cooperative multi-microgrid community with sequentially-coordinated operations in order to distribute the computational burden.

In this paper, an optimal electric and heat energy management for a cooperative multi-microgrid community is proposed. The proposed sequentially-coordinated operation for heat energy is a heat extension of \cite{9}, which is following the sequentially-coordinated operations for electric energy in \cite{9}. The global heat energy optimization is achieved for the cooperative community by adjusting the combined electric and heat energy production amounts of CHP generators and the heat energy production amount of heat only boilers (HOBs) which satisfy all heat loads in the multi-microgrids, as well as optimize the external electric energy trading. Such adjusting of energy production amounts is performed in order to minimize the unnecessary cost from the external electric trading and/or maximize the profit from the external electric trading. A simulation study is also conducted to validate the proposed mathematical energy management models. In this paper, we did not consider electric heaters which uses electric energy for heat loads. Since electric heaters can satisfy heat loads using electric energy which can be traded externally, they can enable external heat trading by means of external electric trading. Currently, we consider electric heaters in the cooperative multi-microgrid community and are investigating an optimal energy management of the cooperative multi-microgrid community. Its results as an extension of this paper will be published in the near future.

The paper is organized as follows: first, we present a cooperative multi-microgrid community and conceptually describe the sequentially-coordinated operations of energy management for a cooperative multi-microgrid community in Section 2. Next, the mathematical model of the cooperative multi-microgrid operation process for electric energy in \cite{9} is summarized in Section 3. Then, the sequentially-coordinated operation of heat energy optimization is mathematically modeled and how to obtain the optimal heat energy solution is presented in Section 4. Additionally, a simulation study for a cooperative multi-microgrid community with three microgrids is demonstrated in Section 5. Finally, our conclusions and future works are discussed in Section 6.
B explain how to find the optimization solution from the adjusted cost function in Section 4 and offers an illustrated interpretation of the optimization process for heat energy, respectively.


2.1. Cooperative Multi-Microgrid Community

A cooperative multi-microgrid community is composed of a group of multiple microgrids as in Figure 1 and is a cooperative operation model of electric and heat energy for a group of microgrids from an economic standpoint. Although various types of microgrids can exist according to specific configurations, a cooperative multi-microgrid community having the following configurations and features is assumed and the sequentially-coordinated operations of electric and heat energy for such a cooperative community are dealt with in this paper:

- microgrids are equipped with photovoltaic (PV) systems, CHP generators, HOBs, and solar heat systems, but the production costs of CHP generators are different;
- microgrids can trade electric energy not only internally with other microgrids in the cooperative community but also externally with the power grid;
- microgrids allow only internal trading for heat energy with other microgrids in the cooperative community; this means that all heat loads should be self-supplemented by heat energy sources in the cooperative multi-microgrid community;
- a microgrid energy management system (μEMS) manages electric energy of its microgrid; and
- a central energy management system (central EMS) has a global optimization function to manage energy generators in multi-microgrids and to satisfy both electric and heat energy loads demanded by all multi-microgrids in the cooperative community.

![Figure 1](image-url)  
**Figure 1.** Information and energy flows in cooperative multi-microgrid community. EMS: energy management system; CHP: combined heat and power.

2.2. Operation Process of Cooperative Multi-Microgrids

Our multi-microgrid community has two kinds of EMSs, central EMS and μEMS. A central EMS manages the electric energy globally in the microgrid, while a μEMS in a microgrid manages the electric energy locally. A central EMS and μEMSs operate cooperatively, coordinated with economic viewpoints as described in the Figure 2, and this cooperative operation process of the central EMS and μEMSs consists of the following three steps:

- Step E-1: Local optimization of electric energy in each microgrid by the μEMS;
- Step E-2: Global electric energy trading optimization by the central EMS;
- Step H: Global heat energy optimization by the central EMS.

In this section, the mathematical model of the cooperative multi-microgrid operation process for electric energy in [9] is summarized. Mathematical notations are first listed in Section 3.1, and the mathematical models of the operation process for electric energy are presented sequentially.

3.1. Nomenclature

Mathematical notations for electric energy are listed as follows:

\[ \mathfrak{W} = \text{South Korea Won} \]
\[ t = \text{the identifier of operation interval} \]
\[ T = \text{the number of operation intervals} \]
\[ l = \text{the identifier of microgrid} \]
\[ L = \text{the number of microgrid} \]
\[ j = \text{the identifier of HOB} \]
\[ J = \text{the number of HOBs} \]
\[ e = \text{the identifier of electric energy} \]
\[ C_{\text{CHP}_l} = \text{the electric energy production cost of the CHP in the } l^{\text{th}} \text{ microgrid (won/kWh)} \]
\[ C_{\text{BUY}_l} (t) = \text{the buying price from the power grid in the } l^{\text{th}} \text{ microgrid at } t \text{ (won }/\text{kWh}) \]
\[ C_{\text{SELL}_l} (t) = \text{the selling price to the power grid in the } l^{\text{th}} \text{ microgrid at } t \text{ (won }/\text{kWh}) \]
\[ C_{\text{CHP}_l}^q = \text{the heat energy production cost of the CHP in the } l^{\text{th}} \text{ microgrid (won }/\text{kWh}) \]
\[ C_{\text{HOB}_{l,j}}^q = \text{the cost of the } j^{\text{th}} \text{ HOB in the } l^{\text{th}} \text{ microgrid (won }/\text{kWh}) \]
\[ M_{\text{LOAD}_l}^f (t) = \text{electric energy demand in the } l^{\text{th}} \text{ microgrid at } t \text{ (kWh)} \]
\[ M_{l}^{s+} (t) = \text{the amount of surplus electric energy in the } l^{\text{th}} \text{ microgrid at } t \text{ (kWh)} \]
$M_{t}^{-}(t)$ = the amount of short electric energy in the $l$th microgrid at $t$ (kWh)

$M_{P_{V}}^{-}(t)$ = the output produced from the PV system in the $l$th microgrid at $t$ (kWh)

$M_{t}^{+}(t)$ = the electric energy production amount of the CHP in the $l$th microgrid at $t$ (kWh)

$M_{t}^{-}(t)$ = the decreased electric energy production amount of the CHP in the $l$th microgrid at $t$ (kWh)

$M_{BUY}(t)$ = the amount of the buying electric energy in the $l$th microgrid determined by central EMS at $t$ (kWh)

$M_{SELL}(t)$ = the amount of the selling electric energy in the $l$th microgrid determined by central EMS at $t$ (kWh)

$M_{REC}(t)$ = the received electric energy amount in the $l$th microgrid at $t$ (kWh)

$M_{SEND}(t)$ = the sending electric energy amount in the $l$th microgrid at $t$ (kWh)

3.2. Step E-1: Local Optimization of Electric Energy Operation Process

The cost function of a microgrid in Step E-1 is the total expense occurred by the electric energy for the microgrid when the external trading of the electric energy with the power grid is applied as follows:

$$C^L_t\left(M_{CHP}^L(t)\right) = \left(C_{CHP}^L \times M_{CHP}^L(t)ight) - \left(C_{SELL}^L(t) \times M_{CHP}^{+}(t)\right) + \left(C_{BUY}^L(t) \times M_{CHP}^{-}(t)\right)$$

(1)

Step E-1 is the local optimization process of electric energy by the µEMS in each microgrid. As mentioned in [1], the local electric energy optimization function when external trading of electric energy with the power grid is applied can be expressed as follows:

$$M_{CHP}^L(t) = \arg \min \left\{ C^L_t\left(M_{CHP}^L(t)\right) \right\}$$

subjects to:

$$\min [M_{CHP}^L] \leq M_{CHP}^L(t) \leq \max [M_{CHP}^L]$$

(2)

For $1 \leq t \leq T$, $1 \leq l \leq L$. The constraint to the objective function of a µEMS in Equation (2) implies that a CHP generator should be operated within its operational ranges.


Step E-2 is the global optimization process of electric energy by the central EMS based on local optimization information of electric energy of each µEMS in Step E-1.

The adjusted saving cost in Step E-2 can be obtained from both main and ancillary internal trading of the electric energy between microgrids in cooperative multi-microgrids as follows:

$$C_{Elec}^{Adj}\left(M_{SEND}^L(t), \ldots, M_{SEND}^L(t)\right) = \left(C_{BUY}(t) - C_{SELL}(t)\right) \times \sum_{l=1}^{L} M_{SEND}^L(t)$$

$$+ \sum_{l=1}^{L} \left(C_{BUY}(t) - C_{CHP}(t)\right) \times M_{CHP}^{+}(t)$$

$$+ \sum_{l=1}^{L} \left(C_{CHP} - C_{SELL}(t)\right) \times M_{CHP}^{-}(t)$$

(3)
Then, the adjusted cost function in Step 2 can be optimized by maximizing the profit resulted by the internal trading of the electric energy as follows:

$$P_{\text{Elec}}^\text{Adj} (t) = \arg \max \left\{ C_{\text{Elec}}^\text{Adj} \left( P_{\text{Elec}}^\text{Adj} (t) \right) \right\}$$

subjects to:

for $l$, such that $M_{i}^{+} (t) > 0$:

$$M_{\text{SEND}_{j}}^e(t) \leq M_{i}^{+} (t) \quad \text{when} \quad M_{i}^{+} (t) > 0$$

for $l$, such that $M_{i}^{-} (t) > 0$:

$$M_{\text{REC}_{i}}^e (t) \leq M_{i}^{-} (t) \quad \text{when} \quad M_{i}^{-} (t) > 0$$

for $l$, such that $M_{i}^{+} (t) > 0$ or $M_{i}^{-} (t) > 0$:

$$\sum_{l=1}^{L} M_{\text{SEND}_{j}}^e(t) = \sum_{l=1}^{L} M_{\text{REC}_{i}}^e (t)$$

for $l$, such that $M_{i}^{+} (t) = M_{i}^{-} (t) = 0$ and $\sum_{l=1}^{L} M_{i}^{+} (t) < \sum_{l=1}^{L} M_{i}^{-} (t)$:

$$M_{C_{i}^{+}}^{CHP_{j}} (t) \leq \max [M_{C_{i}^{+}}^{CHP_{j}} - M_{C_{i}^{+}}^{CHP_{j}} (t)]$$

$$\sum_{l=1}^{L} M_{C_{i}^{+}}^{CHP_{j}} (t) \leq \sum_{l=1}^{L} M_{i}^{-} (t) - \sum_{l=1}^{L} M_{i}^{+} (t)$$

for $l$, such that $M_{i}^{+} (t) = M_{i}^{-} (t) = 0$ and $\sum_{l=1}^{L} M_{i}^{+} (t) > \sum_{l=1}^{L} M_{i}^{-} (t)$:

$$M_{C_{i}^{-}}^{CHP_{j}} (t) \leq M_{C_{i}^{-}}^{CHP_{j}} (t) - \min [M_{C_{i}^{-}}^{CHP_{j}}]$$

$$\sum_{l=1}^{L} M_{C_{i}^{-}}^{CHP_{j}} (t) \leq \sum_{l=1}^{L} M_{i}^{+} (t) - \sum_{l=1}^{L} M_{i}^{-} (t)$$

As a result of the global electric energy trading optimization, the global optimal production amount of the CHP generators located in self-sufficient microgrids have to be changed as follows:

$$M_{C_{i}^{+}}^{CHP_{j}} (t) := M_{C_{i}^{+}}^{CHP_{j}} (t) + M_{C_{i}^{+}}^{CHP_{j}} (t) - M_{C_{i}^{+}}^{CHP_{j}} (t)$$

for $l$, such that $(C_{\text{SELL}_{i}}^{e} < M_{C_{i}^{+}}^{CHP_{j}} (t) < C_{\text{BUY}_{i}}^{e})$ and the buying and selling amount of electric energies to the power grid in a microgrid should be decided by trading the amount of electric energy as follows:

$$M_{\text{BUY}_{i}}^{e} = M_{i}^{-} + M_{\text{REC}_{i}}^e (t) \quad \text{when} \quad M_{i}^{-} (t) > 0$$

$$M_{\text{SELL}_{i}}^{e} = M_{i}^{+} - M_{\text{SEND}_{j}}^e (t) \quad \text{when} \quad M_{i}^{+} (t) > 0$$

Finally, the total operation cost of electric energy in the cooperative multi-microgrid community satisfies all of the electric energy demand and is optimally minimized sequentially in two steps, as follows:

$$C_{\text{TOTAL}}^{e} (t) = \sum_{t=1}^{T} \left\{ \sum_{l=1}^{L} \left( C_{i}^{e} \left( M_{C_{i}^{+}}^{CHP_{j}} (t) \right) \right) \right\} - C_{\text{Elec}}^{\text{Adj}} \left( P_{\text{Elec}}^\text{Adj} (t) \right)$$

In this section, the operation process of heat energy part in the microgrid energy networks (µENet) is mathematically modeled. The proposed sequentially-coordinated operation for heat energy is a heat extension of [9], which is following the sequentially-coordinated operations for electric energy in [9].

Mathematical notations are first defined in Section 4.1, and the mathematical models of the operation process are presented according to in the heat energy part operation process.

4.1. Nomenclature

Before presenting the mathematical models of the cooperative multi-microgrid operation process, mathematical notations necessary for the heat energy models are defined as follows:

- $h$ = the identifier of heat energy
- $\eta_{CHP_i}$ = the heat to power ratio of CHP in the $i^{th}$ microgrid (%)
- $M_{LOAD_i}(t)$ = heat energy demand in the $i^{th}$ microgrid at $t$ (kWh)
- $M_{CHP_i}^+(t)$ = the amount of surplus heat energy in the $i^{th}$ microgrid at $t$ (kWh)
- $M_{CHP_i}^-(t)$ = the amount of short heat energy in the $i^{th}$ microgrid at $t$ (kWh)
- $M_{SH_i}(t)$ = the output produced from the solar heat system in the $i^{th}$ microgrid at $t$ (kWh)
- $M_{CHP_i}(t)$ = the heat energy production amount of the CHP in the $i^{th}$ microgrid at $t$ (kWh)
- $M_{SEND_i}(t)$ = the sending heat energy amount of the CHP in the $i^{th}$ microgrid at $t$ (kWh)
- $M_{CHP_i}^*(t)$ = the capacity of the CHP in the $i^{th}$ microgrid (kWh)
- $M_{REC_i}(t)$ = the received heat energy amount in the $i^{th}$ microgrid at $t$ (kWh)
- $M_{SEND}(t)$ = the sending heat energy amount in the $i^{th}$ microgrid at $t$ (kWh)
- $M_{HOB_{j,i}}^t$ = the heat energy production amount of the $j^{th}$ HOB in the $i^{th}$ microgrid at $t$ (kWh)
- $M_{HOB_{j,i}}^c$ = the capacity of the $j^{th}$ HOB in the $i^{th}$ microgrid (kWh)


After the global electric energy optimization, the amount of heat energy from the CHP generator in a microgrid can be expressed according to the heat and electric energy ratio of the CHP as follows:

$$M_{CHP_i}^h(t) = \eta_{CHP_i} \times M_{CHP_i}^e(t) \quad (15)$$

Then, the amount of surplus heat energy in a microgrid can be calculated as:

$$M_{CHP_i}^+(t) := M_{CHP_i}^h(t) + M_{SH_i}(t) - M_{LOAD_i}(t) \quad \text{when } M_{LOAD_i}(t) \leq M_{CHP_i}^h(t) + M_{SH_i}(t) \quad (16)$$

while the amount of heat energy shortage in a microgrid can be expressed as:

$$M_{CHP_i}^-(t) := M_{LOAD_i}(t) - \left( M_{CHP_i}^h(t) + M_{SH_i}(t) \right) \quad \text{when } M_{LOAD_i}(t) > M_{CHP_i}^h(t) + M_{SH_i}(t) \quad (17)$$

Let us define the set of the heat energy parameters as the internal trading amounts of heat energy between microgrids, the adjusted heat energy generation amounts of CHPs, and the heat energy generation amounts of HOBs to meet all heat demand in the cooperative multi-microgrid community;

$$p_{Heat}^A = (M_{SEND_1}^h(t), \ldots, M_{SEND_L}^h(t), M_{REC_1}^h(t), \ldots, M_{REC_L}^h(t), M_{CHP_1}^h(t), \ldots, M_{CHP_L}^h(t), M_{CHP_1}^- (t), \ldots, M_{CHP_L}^-(t), M_{HOB_1,1}^h(t), \ldots, M_{HOB_{L,L}}^h(t))$$

$$M_{CHP_1}^h(t), \ldots, M_{CHP_L}^h(t), M_{CHP_1}^-(t), \ldots, M_{CHP_L}^-(t), M_{HOB_1,1}^h(t), \ldots, M_{HOB_{L,L}}^h(t))$$
The amounts of heat energy from the solar heat generators are not included since their heat energy production amounts cannot be adjusted. Then, the adjusted cost in Step H to meet all heat demand in the cooperative multi-microgrid community can be defined as the total heat production cost of HOBs, increased energy production of CHPs, and decreased energy production of CHPs as follows:

\[
C_{\text{Heat}}^{\text{Adj}} \left( p_{\text{Heat}}^{\text{Adj}}(t) \right) = \sum_{l=1}^{L} \left\{ \sum_{j=1}^{J} c_{\text{HOB}_{ij}}^{l}(t) \times M_{\text{HOB}_{ij}}^{l}(t) \right\} + C_{\text{CHP}^+}^{\text{Adj}} \left( \sum_{j=1}^{J} M_{\text{CHP}_{ij}}^{l}(t) \right) - C_{\text{CHP}^-}^{\text{Adj}} \left( \sum_{j=1}^{J} M_{\text{CHP}_{ij}}^{l}(t) \right)
\]

(18)

The second term in Equation (18) is the extra cost from the increased energy production of CHPs; the extra production cost, the profit of selling surplus electric energy resulting from increased amount of CHP electric energy production, and the savings from the decreased amount of buying electric energy is as follows:

\[
C_{\text{CHP}^+}^{\text{Adj}} \left( \sum_{j=1}^{J} M_{\text{CHP}_{ij}}^{l}(t) \right) = \sum_{l=1}^{L} \left( \frac{1}{\eta_{\text{CHP}}} c_{\text{CHP}_{ij}}^{l}(t) + c_{\text{CHP}_{ij}}^{l}(t) \right) \times M_{\text{CHP}_{ij}}^{l}(t) - C_{\text{BUY}}^{e} \left( \sum_{l=1}^{L} M_{\text{CHP}_{ij}}^{l}(t), \sum_{l=1}^{L} M_{\text{BUY}_{ij}}^{e}(t) \right) - C_{\text{BUY}}^{e} \left( \sum_{l=1}^{L} M_{\text{CHP}_{ij}}^{l}(t), \sum_{l=1}^{L} M_{\text{BUY}_{ij}}^{e}(t) \right)
\]

(19)

The third term in Equation (18) is the reduced cost from the reduced energy production of CHP, which consists of the reduced production cost, the extra cost of buying electric energy due to shortage caused by the decreased amount of CHP electric energy production, and the reduced profit from decreased selling electric energy, as follows:

\[
C_{\text{CHP}^-}^{\text{Adj}} \left( \sum_{j=1}^{J} M_{\text{CHP}_{ij}}^{l}(t) \right) = \sum_{l=1}^{L} \left( \frac{1}{\eta_{\text{CHP}}} c_{\text{CHP}_{ij}}^{l}(t) + c_{\text{CHP}_{ij}}^{l}(t) \right) \times M_{\text{CHP}_{ij}}^{l}(t) - C_{\text{BUY}}^{e} \left( \sum_{l=1}^{L} M_{\text{CHP}_{ij}}^{l}(t), \sum_{l=1}^{L} M_{\text{BUY}_{ij}}^{e}(t) \right) - C_{\text{BUY}}^{e} \left( \sum_{l=1}^{L} M_{\text{CHP}_{ij}}^{l}(t), \sum_{l=1}^{L} M_{\text{BUY}_{ij}}^{e}(t) \right)
\]

(20)

Note that both the second term and the third term in Equation (18) cannot exist at the same time. Then, the globally-optimized adjusted production amounts of heat energy in Step H can be obtained, as follows:

\[
p_{\text{Heat}}^{\text{Adj}*}(t) = \arg \min \left\{ c_{\text{Heat}}^{\text{Adj}} \left( p_{\text{Heat}}^{\text{Adj}}(t) \right) \right\}
\]

(21)

subject to:

\[
M_{\text{LOAD}_{ij}}^{h}(t) \leq M_{\text{SH}_{ij}}^{h}(t) + M_{\text{CHP}_{ij}}^{h}(t) + M_{\text{CHP}_{ij}}^{h+}(t) - M_{\text{CHP}_{ij}}^{h-}(t) + M_{\text{REC}_{ij}}^{h}(t) - M_{\text{SEND}_{ij}}^{h}(t) + M_{\text{HOB}_{ij}}^{h}(t)
\]

(22)

\[
M_{\text{CHP}_{ij}}^{h+}(t) \leq \max \left[ M_{\text{CHP}_{ij}}^{h-}(t), M_{\text{CHP}_{ij}}^{h}(t) \right]
\]

(23)
The constraints in Equations (23) and (24) are applied to the CHPs, while the constraints in Equation (25) impose that the heat demand in a microgrid has to be satisfied. Inequality in Equation (22) allows heat energy to be produced more than needed so that the total cost of both electric and heat energy can be minimized by adjusting external electric energy trading amounts while wasting heat energy. The constraints in Equations (23) and (24) are applied to the CHPs, while the constraints in Equation (25) are to HOBs. The constraints on the internal trading amounts of heat energy between microgrids are expressed in Equations (26) and (27).

Since the adjusted cost function in Step H has a min() function, it has to be linearized to find the global heat energy optimization solution as in Appendix A; first the linearized adjusted cost function has to be optimized by CPLEX for each case, and then the global heat energy optimization solution has to be selected as the optimization solution for the case which results in the minimum adjusted cost among the five cases, as in Appendix A. Please refer to Appendix B, which gives an illustrated interpretation of the heat energy optimization process for typical cases.

After the global heat energy optimization is completed, the external trading amount of electric energy with the main grid has be re-arranged again, as follows:

\[
M_{CHP_i}^h(t) \leq M_{CHP_j}^h(t) - \min[M_{CHP_j}^h]
\]  
(24)

\[
M_{HOB_i}^h(t) \leq M_{HOB_j}^h, \quad 1 \leq j \leq J
\]  
(25)

\[
M_{SEND_i}^h(t) \leq M_{CHP_1}^h(t) + M_{CHP_j}^h(t) - M_{CHP_i}^h(t) + \sum_{j=1}^{J} M_{HOB_i}^h(t)
\]  
(26)

\[
\sum_{l=1}^{L} M_{SEND_i}^h(t) = \sum_{l=1}^{L} M_{REC_i}^h(t)
\]  
(27)

for \(1 \leq l \leq L, \quad 1 \leq t \leq T\). The first constraint to the global heat energy optimization in Equation (22) imposes that the heat demand in a microgrid has to be satisfied. Inequality in Equation (22) allows heat energy to be produced more than needed so that the total cost of both electric and heat energy can be minimized by adjusting external electric energy trading amounts while wasting heat energy. The constraints in Equations (23) and (24) are applied to the CHPs, while the constraints in Equation (25) are to HOBs. The constraints on the internal trading amounts of heat energy between microgrids are expressed in Equations (26) and (27).

After the global heat energy optimization is completed, the external trading amount of electric energy with the main grid has be re-arranged again, as follows:

when \(\sum_{l=1}^{L} M_{SEL_i}^e(t) > 0\):

\[
\sum_{l=1}^{L} M_{SEL_i}^e(t) := \sum_{l=1}^{L} M_{SEL_i}^e(t) + \frac{1}{\eta_{CHP_i}} \sum_{l=1}^{L} M_{CHP_j}^h(t)
\]  
(28)

for \(\frac{1}{\eta_{CHP_i}} \sum_{l=1}^{L} M_{CHP_j}^h(t) \geq 0\):

\[
\sum_{l=1}^{L} M_{SEL_i}^e(t) := \sum_{l=1}^{L} M_{SEL_i}^e(t) - \frac{1}{\eta_{CHP_i}} \sum_{l=1}^{L} M_{CHP_j}^h(t)
\]  
(29)

for \(\frac{1}{\eta_{CHP_i}} \sum_{l=1}^{L} M_{CHP_j}^h(t) \leq M_{SEL_i}^e(t)\) when \(\sum_{l=1}^{L} M_{CHP_j}^h(t) > 0\):

\[
\sum_{l=1}^{L} M_{BUY_i}^e(t) := \frac{1}{\eta_{CHP_i}} \sum_{l=1}^{L} M_{CHP_j}^h(t) - \sum_{l=1}^{L} M_{SEL_i}^e(t), \quad \text{and} \quad \sum_{l=1}^{L} M_{BUY_i}^e(t) := 0
\]  
(30)

for \(\frac{1}{\eta_{CHP_i}} \sum_{l=1}^{L} M_{CHP_j}^h(t) > \sum_{l=1}^{L} M_{SEL_i}^e(t)\) when \(\sum_{l=1}^{L} M_{CHP_j}^h(t) > 0\) and when \(\sum_{l=1}^{L} M_{BUY_i}^e(t) > 0\):

\[
\sum_{l=1}^{L} M_{BUY_i}^e(t) := \sum_{l=1}^{L} M_{BUY_i}^e(t) + \frac{1}{\eta_{CHP_i}} \sum_{l=1}^{L} M_{CHP_j}^h(t)
\]  
(31)

for \(\sum_{l=1}^{L} M_{CHP_j}^h(t) > 0\):

\[
\sum_{l=1}^{L} M_{BUY_i}^e(t) := \sum_{l=1}^{L} M_{BUY_i}^e(t) - \frac{1}{\eta_{CHP_i}} \sum_{l=1}^{L} M_{CHP_j}^h(t)
\]  
(32)
for $\frac{1}{\eta_{\text{CHP}}} \sum_{l=1}^{L} M^h_{\text{CHP}} (t) \leq \sum_{l=1}^{L} M^e_{\text{BUY}} (t)$ when $\sum_{l=1}^{L} M^h_{\text{CHP}} (t) > 0$:

$$\sum_{l=1}^{L} M^e_{\text{SELL}} (t) := \frac{1}{\eta_{\text{CHP}}} \sum_{l=1}^{L} M^h_{\text{CHP}} (t) - \sum_{l=1}^{L} M^e_{\text{BUY}} (t) \quad \text{and} \quad \sum_{l=1}^{L} M^e_{\text{BUY}} (t) = 0$$ (33)

for $\frac{1}{\eta_{\text{CHP}}} \sum_{l=1}^{L} M^h_{\text{CHP}} (t) > \sum_{l=1}^{L} M^e_{\text{BUY}} (t)$ when $\sum_{l=1}^{L} M^h_{\text{CHP}} (t) > 0$.

Furthermore, the total heat energy produced by the CHP generator in a microgrid can be re-arranged, as follows:

$$M^h_{\text{CHP}} (t) := M^h_{\text{CHP}} (t) + M^h_{\text{CHP}} (t) \quad \text{when} \quad M^h_{\text{CHP}} (t) > 0$$ (34)

$$M^h_{\text{CHP}} (t) := M^h_{\text{CHP}} (t) - M^h_{\text{CHP}} (t) \quad \text{when} \quad M^h_{\text{CHP}} (t) > 0$$ (35)

4.3. Total Operation Costs

Finally, the total optimum operation cost can be expressed with the objective functions defined earlier, as follows:

$$C^*_\text{TOTAL} (t) = \sum_{l=1}^{L} \left( C^e_{\text{CHP}} (t) \times M^e_{\text{CHP}} (t) \right) - C_{\text{Adj}} ^{\text{Elec}} \left( p^{\text{Adj}} ^{\text{Elec}} (t) \right) + C_{\text{Adj}} ^{\text{Heat}} \left( p^{\text{Adj}} ^{\text{Heat}} (t) \right)$$ (36)

The total operation cost of the cooperative multi-microgrid for electric energy and heat energy can be reduced significantly by performing all of the energy optimization processes sequentially in the three steps as shown in Sections 3 and 4.

5. Simulation Study

A simulation study has been conducted for a cooperative multi-microgrid community to show the optimal electric and heat energy management with sequential operation processes and its results, especially for the global heat energy optimization, are presented in this section.

In our simulation study, a cooperative multi-microgrid community is composed of three microgrids having different CHPs and HOBs. Note that when a CHP produces 1 kWh electric energy, $\eta_{\text{CHP}}$ kWh heat energy is produced; the unit production cost of combined electric and heat energy of a CHP can be defined as the combined production cost of 1 kWh electric energy and $\eta_{\text{CHP}}$ kWh heat energy; that is, $(C^e_{\text{CHP}} (t) + \eta_{\text{CHP}} C^h_{\text{CHP}} (t))$ as shown in Table 1. For simplicity, we assume that $C^e_{\text{CHP}} (t) = C^h_{\text{CHP}} (t)$. The production costs $(C^h_{\text{HOB}} (t))$ of HOB A, B, and C are 240, 230, and 240, respectively.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>CHP A</th>
<th>CHP B</th>
<th>CHP C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined electric and heat (E and H) energy</td>
<td>90</td>
<td>120</td>
<td>165</td>
</tr>
<tr>
<td>1 kWh Electric energy $(C^e_{\text{CHP}} (t))$</td>
<td>42.86</td>
<td>53.33</td>
<td>66</td>
</tr>
<tr>
<td>$\eta_{\text{CHP}}$ kWh Heat energy $(C^h_{\text{CHP}} (t))$</td>
<td>47.14</td>
<td>66.67</td>
<td>99</td>
</tr>
<tr>
<td>Heat and power ratio $(\eta_{\text{CHP}})$</td>
<td>1.1</td>
<td>1.25</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The external trading prices of electric energy are designed as time of use (TOU) plan having off-peak, non-peak, and peak hours for 24 h of a day as in Table 2; the buying price is always set higher than the selling price. The combined E and H production costs of CHPs are compared to external trading prices by the TOU plan in Figure 3.
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The main focus of this paper, electric energy optimization results in Tables 3 and 4 will not be discussed in this study. The focus will be on the simulation of CHPs and the determination of optimal operation costs through the use of Eqs. (1) and (2) in the study. Tables 3 and 4 show the results of the local and global optimal operation of electric energy from Steps E-1 and E-2. The local optimal operation involves the steps of the CHPs and the optimization results of electric energy from Steps E-1 and E-2. Since the heat energy optimization process is the main focus of this paper, electric energy optimization results in Tables 3 and 4 will not be discussed here (please refer to [9] for a better understanding of the electric energy optimization process).

Table 2. External trading prices by time of use (TOU) plan.

<table>
<thead>
<tr>
<th>Price</th>
<th>Off-Peak</th>
<th>Non-Peak</th>
<th>Peak</th>
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</thead>
<tbody>
<tr>
<td>Buying price</td>
<td>57</td>
<td>105</td>
<td>130</td>
</tr>
<tr>
<td>Selling price</td>
<td>47</td>
<td>85</td>
<td>110</td>
</tr>
</tbody>
</table>

The simulation results for the cooperative multi-microgrids are arranged in Tables 3–5 for Steps E-1, E-2, and H, respectively. First, the local and global optimal operation results of electric energy from Steps E-1 and E-2 are arranged in Tables 3 and 4. Since the heat energy optimization process is the main focus of this paper, electric energy optimization results in Tables 3 and 4 will not be discussed here (please refer to [9] for a better understanding of the electric energy optimization process).

Table 3. Local optimization of electric energy in Step E-1.

<table>
<thead>
<tr>
<th>T</th>
<th>Microgrid A</th>
<th>Microgrid B</th>
<th>Microgrid C</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>MLOAD</td>
<td>MCHP</td>
<td>MPV</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<tr>
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<tr>
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<td>450</td>
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<tr>
<td>9</td>
<td>371</td>
<td>450</td>
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</tbody>
</table>

Figure 3. Unit production cost of combined E and H energy of CHPs.
### Table 4. Global optimization of electric energy in Step E-2.

<table>
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<tr>
<th>T</th>
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<th>M\textsubscript{CHP}, M\textsubscript{SEND}, M\textsubscript{REC}</th>
<th>M\textsubscript{CHP}, M\textsubscript{SEND}, M\textsubscript{REC}</th>
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### Table 5. Global optimization of heat energy in Step H.

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<th>M\textsubscript{LOAD}, M\textsubscript{SH}, M\textsubscript{SH}</th>
<th>M\textsubscript{LOAD}, M\textsubscript{SH}, M\textsubscript{SH}</th>
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</tbody>
</table>
First, Case_1 (\( \sum_{l=1}^{T} M_{\text{CHP}}^h (t) = \sum_{l=1}^{T} M_{\text{CHP}}^h (t) = 0 \)) occurs for Time = 8. Since all three CHPs already produce their maximum capacities in Step E-2 and there is still a heat energy shortage, this heat energy shortage is supplemented by the heat energy produced from HOB B in Step H. Since the production cost of HOB B is lower than other HOBs, and higher than unit production costs of combined E and H energy of all CHPs, only HOB B produces 113 kWh heat energy to fulfill the heat energy shortage in Microgrid A and, thus, achieve global heat energy optimization in Step H. The energy shortage in Microgrid A (161 kWh) is supplemented by receiving 141 kWh heat energy from Microgrid B and 20 kWh heat energy from Microgrid C.

Now, Case_2 (\( \sum_{l=1}^{T} M_{\text{CHP}}^h (t) > 0 \) and \( \sum_{l=1}^{T} M_{\text{CHP}}^h (t) = 0 \)) occurs for Time = 1–5. CHP A for Time = 1–4 increases its production but the heat energy shortage in Microgrid A is supplemented by other CHPs, while CHP B for Time = 5 and CHP C for Time = 4 also works similarly to Example-1 in Figure B1. On the other hand, CHP B for Time = 1–5 increases its production and its surplus heat energy is sent to other microgrids while CHP C for Time = 1–4 also works similarly just like Example-2 in Figure B2.

Finally, Case_3 (\( \sum_{l=1}^{T} M_{\text{CHP}}^h (t) = 0 \) and \( \sum_{l=1}^{T} M_{\text{CHP}}^h (t) > 0 \)) occurs for Time = 6, 7, 9–24. CHP C for Time = 7, 9–11, 14, 15, 17, 22–23 decreases its production and the heat energy shortage in Microgrid C is supplemented by other CHPs; CHP B for Time = 18, 22 also works similarly to Example-3 in Figure B3. Just like Example-4 in Figure B4, all three CHPs for Time = 24 reduce their production and, yet, have heat energy wasted. Similarly, CHP B for Time = 12, 13, 19, and CHP C for Time = 19–21 also waste heat energy even after reducing energy production of the CHP. For Time = 12 and 13, CHP A produces its maximum energy since its unit production cost of combined E and H energy is lower than the selling price of electric energy; CHP B produces the amount of energy just enough not to buy or sell any electric energy since its unit production cost of combined E and H energy is between the buying and selling prices of electric energy; and CHP C produces its minimum energy since its unit production cost of combined E and H energy is higher than the buying price of electric energy. Note the simulation results for Time = 12 and 13: the total decreased amount of electric energy in Step H is the same as the total selling amount of electric energy after global electric optimization in Step E-2.

### 6. Conclusions

In this paper, we considered heat energy in addition to electric energy for a cooperative multi-microgrid community and studied an optimal energy management problem with
sequentially-coordinated operations to satisfy electric loads, as well as heat loads. The sequentially-coordinated operation for heat energy in this paper is following the sequentially-coordinated operations for electric energy in [9] and, thus, is a heat energy extension of [9]. First, we modeled this sequentially-coordinated operation for heat energy mathematically and presented how to obtain the global heat energy optimization solution in the cooperative multi-microgrid community. The global heat energy optimization is achieved for the cooperative community by adjusting the combined electric and heat energy production amounts of CHP generators and the heat energy production amount of HOBs; these adjusted energy production amounts satisfy all heat loads in the multi-microgrids, as well as optimize the external electric energy trading in order to minimize the unnecessary cost from the external electric trading and/or maximize the profit from the external electric trading.

As a further study, we are now considering electric heaters in this cooperative multi-microgrid community with sequentially-coordinated operations to investigate external heat trading through electric heaters, which are running by electric energy form the main grid. Its interesting result, as an extension of this paper, will be published in the near future.

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**Author Contributions:** The paper was a collaborative effort between the authors. The authors contributed collectively to the theoretical analysis, modeling, simulation, and manuscript preparation.

**Conflicts of Interest:** The authors declare no conflict of interest.

**Appendix A Linearization of the Adjusted Cost Function in Step H**

The adjusted Equation (18) of Step H is as follows:

\[
C_{\text{Adj Heat}} \left(p_{\text{Heat}}^\text{Adj} (t) \right) = \sum_{i=1}^L \left\{ \sum_{j=1}^L \left( C_{\text{HOB},j} (t) \times M_{\text{HOB},j}^h (t) \right) \right\} + C_{\text{CHP}^+} \left( \sum_{i=1}^L M_{\text{CHP},i}^+(t) \right) - C_{\text{CHP}^-} \left( \sum_{i=1}^L M_{\text{CHP},i}^-(t) \right)
\]

since the last two terms in Equation (18) have a min() function as expressed in Equations (19) and (20), CPLEX (IBM ILOG) cannot be utilized directly to find the optimization solution of the adjusted cost function for heat energy extension in Step H.

In Appendix A, the adjusted cost function in Step H will be linearized by removing min() in it depending on the conditions for \( \sum_{i=1}^L \frac{1}{\eta_{\text{CHP},i}} M_{\text{CHP},i}^h (t) \) and \( \sum_{i=1}^L \frac{1}{\eta_{\text{CHP},i}} M_{\text{CHP},i}^h (t) \). This linearization process of the adjusted cost function in Step H enables us to utilize CPLEX and, thus, find the global heat optimization solution.

First, let us consider when CHPs do not change their production amounts in Step H, which implies as follows:

\[
\frac{1}{\eta_{\text{CHP},i}} \sum_{i=1}^L M_{\text{CHP},i}^h (t) = 0 \quad \text{and} \quad \frac{1}{\eta_{\text{CHP},i}} \sum_{i=1}^L M_{\text{CHP},i}^h (t) = 0
\]

Thus, the adjusted cost function in Step H includes only the total heat production cost of HOBs as follows:

**Case_1:** \( C_{\text{Adj Heat}} \left(p_{\text{Heat}}^\text{Adj} (t) \right) = \sum_{i=1}^L \left\{ \sum_{j=1}^L \left( C_{\text{HOB},j} (t) \times M_{\text{HOB},j}^h (t) \right) \right\} \quad (A1)\)

Secondly, let us consider when CHPs increase energy production in Step H, which implies as follows:

\[
\sum_{i=1}^L M_{\text{CHP},i}^h (t) > 0 \quad \text{and} \quad \sum_{i=1}^L M_{\text{CHP},i}^h (t) = 0
\]
In this situation, the following two cases have to be considered due to the min() function in Equation (19):

Case_2 - 1 : \[ 0 < \frac{1}{\eta_{CHP_l}} \sum_{i=1}^{L} M_{CHP_l}^h(t) \leq \sum_{i=1}^{L} M_{BUY_i}(t) \]

Case_2 - 2 : \[ \frac{1}{\eta_{CHP_l}} \sum_{i=1}^{L} M_{CHP_l}^h(t) > \sum_{i=1}^{L} M_{BUY_i}(t) \]

Then, the adjusted cost functions in Step H for the above two cases become as follows:

\[
\begin{align*}
\text{Case}_2 - 1 : & \quad C_{Adj}^\text{Heat}\left(p_{Adj}^\text{Heat}(t)\right) = \sum_{i=1}^{L} \left\{ \sum_{j=1}^{J} \left( C_{HOB,ij}^\text{Heat}(t) \times M_{HOB,ij}^\text{Heat}(t)\right) \right\} \\
& + \sum_{i=1}^{L} \left( \frac{1}{\eta_{CHP_l}} C_{CHP_l}^\text{Heat}(t) + c_{CHP_l}^h(t) \right) \times M_{CHP_l}^h(t) - \sum_{i=1}^{L} \frac{1}{\eta_{CHP_l}} M_{CHP_l}^h(t) \tag{A2} \\
\text{Case}_2 - 2 : & \quad C_{Adj}^\text{Heat}\left(p_{Adj}^\text{Heat}(t)\right) = \sum_{i=1}^{L} \left\{ \sum_{j=1}^{J} \left( C_{HOB,ij}^\text{Heat}(t) \times M_{HOB,ij}^\text{Heat}(t)\right) \right\} \\
& + \sum_{i=1}^{L} \left( \frac{1}{\eta_{CHP_l}} C_{CHP_l}^\text{Heat}(t) + c_{CHP_l}^h(t) \right) \times M_{CHP_l}^h(t) \\
& - C_{BUY}^\text{Heat}(t) \left( \sum_{i=1}^{L} \frac{1}{\eta_{CHP_l}} M_{CHP_l}^h(t) - \sum_{i=1}^{L} M_{BUY_i}(t) \right) - C_{BUY}^\text{Heat}(t) \times \sum_{i=1}^{L} M_{BUY_i}(t) \tag{A3} \\
\end{align*}
\]

Finally, let us consider the situation when CHPs decrease energy production in Step H, which implies as follows:

\[ \sum_{i=1}^{L} M_{CHP_l}^h(t) = 0 \quad \text{and} \quad \sum_{i=1}^{L} M_{CHP_l}^h(t) > 0 \]

In this situation, there are following two cases due to the min() function in Equation (20):

Case_3 - 1 : \[ 0 < \frac{1}{\eta_{CHP_l}} \sum_{i=1}^{L} M_{CHP_l}^h(t) \leq \sum_{i=1}^{L} M_{SELL_i}(t) \]

Case_3 - 2 : \[ \frac{1}{\eta_{CHP_l}} \sum_{i=1}^{L} M_{CHP_l}^h(t) > \sum_{i=1}^{L} M_{SELL_i}(t) \]

Then, the adjusted cost functions in Step H for the above two cases become as follows:

\[
\begin{align*}
\text{Case}_3 - 1 : & \quad C_{Adj}^\text{Heat}\left(p_{Adj}^\text{Heat}(t)\right) = \sum_{i=1}^{L} \left\{ \sum_{j=1}^{J} \left( C_{HOB,ij}^\text{Heat}(t) \times M_{HOB,ij}^\text{Heat}(t)\right) \right\} \\
& - \sum_{i=1}^{L} \left( \frac{1}{\eta_{CHP_l}} C_{CHP_l}^\text{Heat}(t) + c_{CHP_l}^h(t) \right) \times M_{CHP_l}^h(t) + C_{BUY}^\text{Heat}(t) \times \sum_{i=1}^{L} \frac{1}{\eta_{CHP_l}} M_{CHP_l}^h(t) \tag{A4} \\
\text{Case}_3 - 2 : & \quad C_{Adj}^\text{Heat}\left(p_{Adj}^\text{Heat}(t)\right) = \sum_{i=1}^{L} \left\{ \sum_{j=1}^{J} \left( C_{HOB,ij}^\text{Heat}(t) \times M_{HOB,ij}^\text{Heat}(t)\right) \right\} \\
& - \sum_{i=1}^{L} \left( \frac{1}{\eta_{CHP_l}} C_{CHP_l}^\text{Heat}(t) + c_{CHP_l}^h(t) \right) \times M_{CHP_l}^h(t) \\
& + C_{BUY}^\text{Heat}(t) \left( \sum_{i=1}^{L} \frac{1}{\eta_{CHP_l}} M_{CHP_l}^h(t) - \sum_{i=1}^{L} M_{SELL_i}(t) \right) + C_{SELL}^\text{Heat}(t) \times \sum_{i=1}^{L} M_{SELL_i}(t) \tag{A5} \\
\end{align*}
\]

Appendix B  Illustrated Interpretation of Optimization Operation for Heat Energy

For Case_2-1, the increased production amount of electric energy in Step H \( \frac{1}{\eta_{CHP_l}} \sum_{i=1}^{L} M_{CHP_l}^h(t) \) cuts off the electric energy purchase in Step E-2 if there is electric energy purchase \( \sum_{i=1}^{L} M_{BUY_i}(t) > 0 \) due to an electric energy shortage. In such a case, it will be sold to the main grid \( \sum_{i=1}^{L} \frac{1}{\eta_{CHP_l}} M_{CHP_l}^h(t) - \sum_{i=1}^{L} M_{BUY_i}(t) \).

On the other hand, the heat energy shortage in a microgrid is supplemented, first, by the increased heat energy of its CHP generator, and then by the received heat energy from other microgrids with
surplus heat energy, as in Figure B1. However, even when there is a heat energy surplus in a microgrid, the microgrid can increase its CHP production and sends its surplus heat energy to other microgrids as in Figure B2.

Figure B1. Example-1 when $\sum_{d=1}^{L} M_{\text{CHP}}^h (t) > 0$.

Figure B2. Example-2 when $\sum_{d=1}^{L} M_{\text{CHP}}^h (t) > 0$.

For Case_3-1 and Case_3-2, the decreased production amount $\left( \frac{1}{H_{\text{CHP}}} \sum_{d=1}^{L} M_{\text{CHP}}^h (t) \right)$ of electric energy in Step H first cuts off the electric energy selling in Step E-2 if $\sum_{d=1}^{L} M_{\text{SELL}}^e (t) > 0$ and then can be supplemented by new electric energy purchase $\left( \sum_{d=1}^{L} \frac{1}{H_{\text{CHP}}} M_{\text{CHP}}^h (t) - \sum_{d=1}^{L} M_{\text{SELL}}^e (t) \right)$.

On the other hand, the heat energy shortage in a microgrid is supplemented by the received heat energy from other microgrids with surplus heat energy, as in Figure B3. However, the surplus heat energy can be wasted even after internal heat trading, as in Figure B4. Such heat wasting can happen when the buying price of electric energy is higher than the unit combined $E$ and $H$ production price of a CHP, since producing electric and heat energy by the CHP would save money instead of buying electric energy even though the produced heat is wasted. Furthermore, when the selling price of electric energy is higher than the unit combined $E$ and $H$ production price of a CHP, the CHP has to produce its maximum electric and heat energy since selling electric energy produced by the CHP is profitable even after the produced heat energy is wasted.
Figure B3. Example-3 when $\sum_{d=1}^{L} M^{h}_{\text{CHP}}(t) > 0$.

Figure B4. Example-4 when $\sum_{d=1}^{L} M^{h}_{\text{CHP}}(t) > 0$.

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