The Influence of Environmental Constraints on the Water Value †

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Abstract: The establishment of more severe hydrological environmental constraints, usually as seasonal minimum flows \( (\varphi) \) and maximum ramping rates \( (\rho) \), on hydropower operation is a growing trend. This paper presents a study on the influence of \( \varphi \) and \( \rho \) on the water values (WV) of a real hydropower plant that participates in the Spanish day-ahead electricity market. For this purpose, a master-slave algorithm, based on stochastic dynamic programming (SDP) and deterministic mixed integer linear programming (DMILP), is used on a real hydropower plant. The master module, based on SDP, has a yearly planning period with weekly time steps and considers three state variables: stored water volume in the reservoir at the beginning of each week; weekly water inflow; and average weekly energy price. The slave module, based on DMILP, has a weekly planning period with hourly time steps and considers many features of the hydropower plant operation, such as: start-up costs, evaporation, wear and tear costs, etc. The results indicate that WV of a hydropower plant are very sensitive to the presence of these constraints; \( \varphi \) especially during the wettest season and \( \rho \) during the driest one. As the severity of \( \varphi \) and \( \rho \) increase, WV increase and decrease, respectively.

Keywords: water value; minimum environmental flows; maximum ramping rates; stochastic dynamic programming; mixed integer linear programming

1. Introduction

This research article is an expanded study based on the conference paper entitled “Influence of the maximum flow ramping rates on the water value” presented in the 5th International Workshop on Hydro Scheduling in Competitive Electricity Markets celebrated in Trondheim, Norway, during the 17th and the 18th of September 2015 whose proceedings were published in [1].

Water has an economic value in all its competing uses [2]. In the framework of a competitive electricity market, the hydroelectric use of this good, so special [3], may define that value as the marginal change in the hydropower producer’s expected revenue for a marginal change in its available hydro resources [4]. Therefore, in this context, the water value (WV) can be understood as an opportunity benefit which plays an important role between the long- and short-term hydro scheduling models by bringing the consequences of the expected future into the present [5]. WV depends on the time of the year and the water content stored in the reservoir of each hydropower plant [6]. It is usually described as monotonically decreasing when increasing both the stored water volume and the amount of water inflow [7], because of the subsequent growing risk of spill [8].

It has been more than two decades since the United Nations agreed to encourage the sustainable use of renewable natural resources [9], and during this time hydroelectricity has kept its position
as the first source of primary energy among renewable energies [10]. Unfortunately, a plethora of studies [11–19] have found that hydropower plants can yield undesirable effects on the ecosystems where they are located, such as changing evapotranspiration, flow regime, reoxygenation of surface waters, sediment transport, temperature, total gas pressure or water depth, and impeding faunal migration. Hence, many countries in the world have imposed [20] or are considering imposing [21] new constraints on hydropower operation in order to achieve that goal. These environmental constraints are usually: minimum environmental flows (ϕ), seasonal minimum values of water release, and maximum ramping rates (ρ), maximum interhourly increments and decrements of water release. On the one hand, ϕ reduce the water volume and head available to produce electricity, and, on the other hand, ρ limit the hydropower plant’s ability to change power output levels [22]; that is, these ramps introduce some kind of inertia in the plant operation.

Despite the increasingly important contribution of hydroelectricity to the proper operation of power systems as a result of the rise of the other intermittent renewable technologies [23] and the abovementioned environmental policies, [1] is the only work published so far on the influence of ρ on WV to the best of our knowledge. Hence, this study has two objectives: first, to start filling the knowledge gap of how ϕ affects WV, and second, to compare this influence with that caused by ρ described in [1]. This is the reason why a very similar optimisation model and the same case study presented in [1] are used in this study in order to make a fair comparison between the results shown here and those of [1].

In order to compute the WV of the hydropower plant under study, an optimisation model similar to the one proposed in [24] is used. The aim of this model is to solve the annual power generation scheduling of the hydropower plant. The selected case study is a real hydropower plant that participates in the Spanish day-ahead electricity market and is located at the head of its river basin, allowing easy identification of its available water inflows.

This paper on the influence of environmental constraints on WV is organised in the following manner. In Sections 2 and 3, the optimisation model and the case study are described. In Section 4, the analysis (both numerical and analytical) of the new findings (related to ϕ) and the comparison with the results (related to ρ) obtained in [1] are shown. The conclusions of the study are drawn in Section 5, which can be summarised by the following statement: ϕ and ρ have opposite effects on WV, the former increases it and the latter decreases it. Finally, the Appendix 5 contains the applied nomenclature.

2. Optimisation Model

The optimisation model used to understand how ϕ affects WV is practically identical to the one proposed in [1]; the only difference is the inclusion of ϕ. Nevertheless, for the sake of completeness, it is fully described in this section. The optimisation model is a master-slave algorithm whose basic idea is as follows (see Figure 1):

1. The master module, based on stochastic dynamic programming (SDP), decomposes the annual scheduling problem, defined by a specific pair of ϕ and ρ, into weekly subproblems.
2. The slave module, based on deterministic mixed integer linear programming (DMILP), solves each weekly subproblem in the considered week. The weekly subproblems are aimed at maximizing the weekly revenue plus the future expected revenue (given by WV) from a linearized generation characteristic. The process starts from the last week.
3. The master module recalculates, with the real generation characteristic of the plant, the power outputs according to the decisions provided by the slave module and obtains the resulting revenue.
4. The master module calculates WV at the beginning of the week.
5. Second, third and fourth steps are repeated backwards until the beginning of the year for the rest of the weeks using the last-obtained WV.
6. The master module checks if the relative interannual variations of the expected revenue at the beginning of each week converge (see Equation (4)). If the answer is yes, the process stops,
otherwise the process is repeated from the second step but now considering the last-obtained WV at the end of the last week.

![Flowchart of the optimisation model.](image)

**Figure 1.** Flowchart of the optimisation model.

In what follows, the master and slave modules are described.

**2.1. Master Module**

The purpose of the *master module* is to calculate the steady-state policy [25] (p. 107) of a hydropower plant that sells energy in a day-ahead electricity market. Based on SDP, it has a yearly planning period with weekly time steps and its state variables are: stored water volume in the reservoir at the beginning of each week, water inflow of the current week, and average energy price of the current week. According to [26], the first variable is discretised in nine equidistant values from the dead volume to the maximum one. The two latter variables are modelled by means of Markov chains following the approaches of [27,28], respectively.
The order of the Markov chains is selected through the Akaike information criterion [29]. The number of classes of water inflows (five) and energy prices (three) are determined following the recommendation in [30] (p. 51), and according to the length of the available historical series of these variables. As suggested in [31,32], the weekly extreme values of both variables are represented by their respective extreme classes in order to improve the robustness of the plant operation.

The master module uses the volume released from the reservoir throughout each week as decision variable [30] (p. 49) and is based on the recursive relationship:

\[
\tilde{z}_{k+1}^{i,a,b} = \max \left\{ \sum_{x} \varepsilon_{k}^{a,x} \cdot \sum_{y} \left( \varepsilon_{k}^{b,y} \cdot \tilde{z}_{k+2}^{x,y} \right) \right\}; \forall \{a,b,i,l,x,y\} \in \Omega_{k} \land \forall k \in K
\]  

Equation (1) is the stochastic Bellman equation in which \(\tilde{z}_{k}^{i,a,b}\) represents the optimum cumulative revenue at \(V_{k}^{i}\) when the weekly water inflow is \(a\) and the weekly average energy price is \(b\) from the week \(k\) to the end of the planning period, \(\tilde{z}_{k}^{i,a,b,l}\) is the revenue corresponding to the decision to go from \(V_{k}^{i}\) to \(V_{k+1}^{l}\) when those inflow and price occur during the considered week, \(\varepsilon_{k}^{a,x}\) signifies the probability that the inflow at the week \(k + 1\) is \(x\) given that the inflow at week \(k\) was \(a\), and \(\varepsilon_{k}^{b,y}\) means the probability that the price at week \(k + 1\) is \(y\) given that the price at week \(k\) was \(b\).

The state-transition equation of the master module is the mass balance equation:

\[
V_{k+1}^{l} = V_{k}^{i} + CF1 \cdot \sum_{i=1}^{168} (W_{t} - e_{t} - q_{t} - qbo_{t} - qsp_{t}); \forall \{i,l\} \in \Omega_{k} \land \forall k \in K
\]

where \(CF1\) is a conversion factor for converting \(m^{3}/s\) into \(Mm^{3}/h\) (0.0036), \(W_{t}\) represents the water inflow to the reservoir during the hour \(t\), \(e_{t}\) refers to the evaporation losses during the hour \(t\), and \(q_{t}, qbo_{t}\) and \(qsp_{t}\), are, respectively, the flows released through the hydro units, the bottom outlets and the spillways during the hour \(t\).

The convergence criterion of the master module is determined by:

\[
\frac{\tilde{z}_{k}^{i,a,b,n} - \tilde{z}_{k}^{i,a,b,n-1}}{\tilde{z}_{k}^{i,a,b,n-1}} \leq 1\%; \forall \{a,b,i\} \in \Omega_{k} \land \forall k \in K
\]

where \(\tilde{z}_{k}^{i,a,b,n}\) represents the optimum cumulative revenue at \(V_{k}^{i}\) when the weekly water inflow is \(a\) and the weekly average energy price is \(b\) from the week \(k\) to the end of the planning period of the iteration \(n\) of the SDP.

Once the iterative process of the SDP converges, the master module calculates the WV through the year given by:

\[
WV_{k}^{j} = \frac{\tilde{z}_{k+1}^{j+1} - \tilde{z}_{k+1}^{j}}{V_{k+1}^{j+1} - V_{k+1}^{j}}; \forall i \in \Omega_{k} \land j = j \land \forall k \in K
\]

where \(WV_{k}^{j}\) represents WV at the end of the week \(k\) along the \(j\)-th portion of the water volume stored in the reservoir (\(j\)-th reservoir segment of the storage-WV curve), \(\tilde{z}_{k+1}^{j+1}\) and \(\tilde{z}_{k+1}^{j}\) are the optimum cumulative revenue at the extremes of that reservoir segment from the next week to the end of the planning period, \(V_{k+1}^{j+1}\) and \(V_{k+1}^{j}\) are the stored volumes corresponding to \(\tilde{z}_{k+1}^{j+1}\) and \(\tilde{z}_{k+1}^{j}\), \(\Omega_{k}\) contains the set of feasible states evaluated during the considered week, \(\Omega_{k}\) signifies the total number of used segments in the storage-WV curve, and \(K\) comprises the set of weeks of the year.

2.2. Slave Module

The purpose of the slave module is to determine the weekly schedule of a hydropower plant that sells energy in a day-ahead electricity market. Based on DMILP, it has a weekly planning period with
hourly time steps. The weekly rate of hourly evaporated water volume per flooded area as well as the hourly water inflows and energy prices are input variables to this module. The disaggregations of these two latter variables from the weekly values considered in the Markov chains of the master module to hourly ones are performed by weighting the historical average weekly profile of each variable in every week by a ratio between the considered value in its respective chain and the mean value of the said average profile (see an example in Figure 2). The slave module uses the flow released from the reservoir as decision variable and its objective function is:

\[
\text{max} \left\{ \sum_{t=1}^{168} (P_t \cdot pow_t) - \alpha \left( pow_t^{inc} + pow_t^{dec} \right) + \beta \cdot \sum_{i \in U} \left( on_t^i + off_t^i \right) + \gamma \cdot \left( qbospt^{inc} + qbospt^{dec} \right) \right\} ; \\
\forall \{a, b\} \in \Omega_k \land \forall k \in K
\]

The first term in Equation (5) represents the revenue obtained in the day-ahead market (hourly energy price, \(P_t\), times hourly generated power, \(pow_t\)). The first term in square brackets is the cost for interhourly power variations (wear and tear cost of hydro units, \(\alpha\), times hourly decrements, \(pow_t^{dec}\), and increments, \(pow_t^{inc}\), in generated power), the second one is the units start-up and shut-down cost (start-up and shut-down cost of hydro units, \(\beta\), times operating state changes of these units, \(on_t^i\) and \(off_t^i\)) and the third one penalises the interhourly variations in the flow released through the spillways and the bottom outlets (penalisation parameter of bottom outlets and spillways, \(\gamma\), times hourly flow decrements, \(qbospt^{dec}\), and increments, \(qbospt^{inc}\), through said elements). The final term refers to the future expected revenue at the end of the week (WW of the \(j\)-th reservoir segment at the end of the week \(k\) given the inflow \(a\) and the average energy price \(b\), \(WV_{k,\max}^{l,\min,k,\min}\), times the stored volume in the \(j\)-th segment at the end of the considered week, \(vwv\)).

The terms \(\alpha\) and \(\beta\) can be estimated either by means of specific experimental studies in the considered plant or from the information contained in [33,34], respectively, as it was done in this study. \(\gamma\) is a small (0.01) and artificial cost considered in the slave module with the aim of obtaining a more realistic use of the bottom outlets and spillways (this parameter is neglected by the master module during the recalculation of the power outputs with the real generation characteristic of the plant). The objective function of the slave module is subject to the following constraints:

- Water mass balance (Equation (6)):
  \[
v_t = v_{t-1} + CF1 \cdot (W_t - c_t - q_t - qbo_t - qsp_t) ; \forall t \in T
  \]
  This equation is analogous to Equation (2) where \(v_t\) means the stored volume at the end of the hour \(t\) and \(T\) contains the set of hours of the week.

- Initial (Equation (7)), final maximum legal (Equation (8)), hourly maximum physical (Equation (9)) and hourly minimum physical (Equation (10)) stored volume:
  \[
v_0 = V_{k,\max}^i ; \forall t \in \Omega_k \land \forall k \in K
  \]
  \[
v_{168} \leq V_{k,\max}^\text{legal} ; \forall k \in K
  \]
  \[
v_t \leq V_t ; \forall t \in T
  \]
  \[
v_t \geq V_t^\text{dead} ; \forall t \in T
  \]

- Hourly evaporation losses (Equation (11)):
  \[
e_t = CF2 \cdot E_k \cdot (K1_k \cdot v_t + K2_k) ; \forall t \in T \land \forall k \in K
  \]
where $CF$ is a conversion factor for converting $m^3/s$ into $Mm^3/week (1/0.6048)$, $E_k$ signifies the rate of hourly evaporated water volume per flooded area during the week $k$, and $K_{1e}$ and $K_{2e}$ are the coefficients of the linear approximation of the storage-flooded area curve of the reservoir.

- **Use of the bottom outlets and spillways (Equations (12)–(18)):**

  \[
  q_{bos\text{ }inc}^{t+1} - q_{bos\text{ }dec}^{t+1} = q_{bo}^{t+1} + q_{sp}^{t+1} - q_{bo} - q_{sp} ; \forall t \in T \mid t < 168
  \]  
  \[
  q_{bo} \leq K_{1bo} \cdot v_{t} + K_{2bo} ; \forall t \in T
  \]  
  \[
  q_{sp} \leq K_{sp} \cdot v_{asp} ; \forall t \in T
  \]  
  \[
  v_{t} = v_{asp} + v_{bsp} ; \forall t \in T
  \]  
  \[
  v_{bsp} \leq V^{sp} ; \forall t \in T
  \]  
  \[
  v_{asp} \leq (\bar{V} - V^{sp}) \cdot sp_{t} ; \forall t \in T
  \]  

  Equation (12) computes the hourly flow increments, $q_{bos\text{ }inc}$, and decrements, $q_{bos\text{ }dec}$, through the bottom outlets and spillways. Equation (13) limits the maximum water release through the bottom outlets according to the hourly stored volume where $K_{1bo}$ and $K_{2bo}$ are the coefficients of the linear approximation of the storage-maximum bottom outlet flow curve. Equation (14) is analogous to Equation (13) for the case of the spillways where $K_{sp}$ represents the coefficient of the linear approximation of the storage-maximum spillway flow curve and $v_{asp}$ means the stored volume at the end of the hour $t$ above the spillways minimum level. Equations (15)–(18) are used to calculate $v_{asp}$ where $v_{bsp}$ is the stored volume at the end of the hour $t$ below the spillways minimum level, $V^{sp}$ is the stored volume above which the spillways can operate, and $sp_t$ indicates whether (1) or not (0) the stored volume is above $V^{sp}$ during the hour $t$.

- **Maximum plant flow according to the stored volume and minimum technical stored volume for power generation (Equations (19)–(21)):**

  \[
  q_t \leq \sum_{c \in C} \left( Q^{VQ_{c}} \cdot vq_{c}^{t} \right) ; \forall t \in T
  \]  
  \[
  v_{t} \geq \sum_{c \in C} \left( VQ_{c} \cdot vq_{c}^{t} \right) ; \forall t \in T
  \]  
  \[
  o_{1}^{t} \leq qq_{c}^{t} ; \forall t \in T
  \]  

  Equation (19) is analogous to Equations (13) and (14) for the case of the hydro units where $Q^{VQ_{c}}$ is the length (measured in terms of flow) of the $c$-th segment of the storage-maximum plant flow curve and $vq_{c}^{t}$ indicates whether (1) or not (0) the stored volume is above the minimum storage of the $c$-th segment during the hour $t$. It is important to clarify that Equation (19) is added since the maximum plant flow may vary substantially within each week at low reservoir levels. Equations (20) and (21) are used to calculate $vq_{c}^{t}$, where $VQ_{c}$ is the length (measured in terms of volume) of the $c$-th segment of the storage-maximum plant flow curve and $o_{1}^{t}$ indicates whether (1) or not (0) at least one hydro unit is on-line during the hour $t$.

- **Power-discharge piecewise non-concave linear curve as in [35], in terms of the initial and estimated final stored volumes (Equations (22)–(25)):**

  \[
  q_{t} = Q_{1}^{t} \cdot o_{1}^{t} + \sum_{s \in S} q_{s}^{t} ; \forall t \in T
  \]
\[
q_t^\delta = \begin{cases} 
\mathcal{Q} \cdot o_t^\delta; & \forall s \in S \mid s^u \leq s \leq s^{u+1} \land u \in \mathcal{U} \mid 1 \leq u \leq \mathcal{U}^T; \forall t \in T \\
\mathcal{Q} \cdot o_t^\delta; & \forall s \in S \mid s^u \geq s^\mathcal{U}_t; \forall t \in T
\end{cases}
\]

(23)

\[
q_t^\gamma = \mathcal{Q} \cdot o_t^\gamma; \forall s \in S \mid s^u \leq s \leq s^{u-1} \land u \in \mathcal{U}_t; \forall t \in T
\]

(24)

\[
pow_t = \text{POW} \cdot o_t^\delta + \sum_{s \in S} (R^s \cdot q_t^s); \forall t \in T
\]

(25)

In Equation (22), the first term means the minimum flow of a single hydro unit (Q^u) and the second one the sum of the flow in all segments (q_t^\gamma) in which the power-discharge curve is divided. Equations (23) and (24) define the maximum and minimum values of each q_t^\delta. In Equation (25), the first term signifies the minimum power output of one hydro unit (POW) and the second one refers to the sum of the power in all segments of the power-discharge curve, each calculated as the product of q_t^s and the slope of each segment (R^s).

- Interhourly variation of the generated power (Equation (26)):

\[
pow_t^{inc} - pow_t^{dec} = pow_{t+1} - pow_t; \forall t \in T \mid t < 168
\]

Equation (26) computes the interhourly power increments, qbospt^{inc}, and decrements, qbospt^{dec}.

- Start-ups and shut-downs of the hydro units as in [36] (Equations (27)–(31)):

\[
q_t \geq Q^1 \cdot o_t^1 + \left(Q^2 - Q^1\right) \cdot o_t^2 + \sum_{u=3}^{\mathcal{U}^T-1} \left(Q^u - Q^{u-1}\right) \cdot o_t^u; \forall t \in T
\]

(27)

\[
q_t \leq Q^2 \cdot o_t^1 + \sum_{u=2}^{\mathcal{U}^T-1} \left(Q^{u+1} - Q^u\right) \cdot o_t^u + \left(Q - Q^1\right) \cdot o_t^\mathcal{U}_t; \forall t \in T
\]

(28)

\[
o_t^u \geq o_t^{u+1}; \forall u \in \mathcal{U}_t \land \forall t \in T
\]

(29)

\[
on_{t+1}^u - off_{t+1}^u = o_t^u; \forall u \in \mathcal{U}_t \land \forall t < 168
\]

(30)

\[
on_{t+1}^u + off_{t+1}^u \leq 1; \forall u \in \mathcal{U}_t \land \forall t < 168
\]

(31)

Equations (27)–(29) are used to calculate the operating state of the hydro units (o_t^u) according to q_t where Q^u means the plant flow above which the u-th hydro unit starts up and \mathcal{U} represents the total number of hydro units of the plant. Equations (30) and (31) compute the changes in the operating state of the hydro units.

- Future expected revenue at the end of the week (Equations (32)–(35)):

\[
v_{168} = V^{dead} + \sum_{j \in J} v_w^{wv^j}
\]

(32)

\[
v_w^{wv^j} \leq VWV^j \cdot wv^j; \forall j \in J
\]

(33)

\[
v_w^{wv^j} \geq VWV^j \cdot wv^{j+1}; \forall j \in J \mid j < \mathcal{J}
\]

(34)

\[
wv^j \geq wv^{j+1}; \forall j \in J \mid j < \mathcal{J}
\]

(35)

where VWV^j is the length (measured in terms of volume) of the j-th segment of the storage-WV curve, v_w^{wv^j} is the stored volume in the j-th segment of the said curve at the end of the week, and wv^j indicates if the stored volume at the end of the week is above the minimum storage of the j-th segment.
• Up and down $\rho$ as in [37] (Equations (36) and (37)):

$$q_{t+1} + q_{b0t+1} + q_{sp1+1} - q_t - q_{b0t} - q_{sp1} \leq \rho_{\text{up}}; \forall t \in T | t < 168$$

$$q_{t+1} + q_{b0t+1} + q_{sp1+1} - q_t - q_{b0t} - q_{sp1} \geq -\rho_{\text{down}}; \forall t \in T | t < 168$$

where $\rho_{\text{up}}$ and $\rho_{\text{down}}$ refer to the maximum interhourly increment and decrement of water release, respectively. The fulfillment of $\rho$ between consecutive weeks is not considered and is one of our ongoing works. The interested reader is referred to [38] where the fulfillment of the interweekly $\rho$ was considered in a deterministic context.

• Seasonal $\varphi$ (Equation (38)):

$$q_t + q_{b0t} + q_{sp1} \geq \min (\varphi_k, W_t); \forall t \in T \land \forall k \in K$$

Equation (38) forces the total water release (through the hydro units, bottom outlets and spillways) to be larger than or equal to the minimum of $\varphi$ and the current water inflow into the reservoir, $W_t$ [39].

![Image](image.png)

**Figure 2.** (a) Historical weekly profiles (grey lines) and their average (black line) of the hourly energy prices of the first week of the water year; (b) Calculation of the weighting factors for disaggregation of the classes of the Markov chain of the energy price of the first week of the water year; (c) Hourly disaggregations of the three classes of the Markov chain of the energy price of the first week of the water year.

3. Case Study

A real hydropower plant, located in the Northwest area of Spain (see Figure 3) was selected to analyse the influence of the two abovementioned environmental constraints on WV. The plant is a
The technical data of the hydropower plant were provided by the company that owns and operates the plant; its main design parameters are included in Table 1. The historical series of daily water inflow and hourly energy price were taken, respectively, from [40] (years: 1963–1965, 1966–2005) and [41] (years: 1998–2005). The weekly rates of hourly evaporated water volume per flooded area were estimated in two steps. First, from the application of the first empirical temperature-based formula proposed in [42] to the average monthly temperatures taken from [43] (years: 1931–1960), the monthly surface evaporations (mm/month) and second the conversion of these values into the said rates. The average values of these inputs are depicted in Figure 4.

Figure 3. (a) Location of the hydropower plant (image taken from Google Earth™); (b) Approximate ground plant of the reservoir water basin.

Figure 4. (a) Average weekly water inflow (1963–1965 and 1966–2005); (b) Average weekly energy price (1998–2005); (c) Average weekly rate of hourly evaporated volume per flooded area (1931–1960).
The magnitudes of the environmental constraints $\varphi$, considered in this study, and $\rho$, evaluated in [1], were selected to cover a wide range of values. As in [44], $\varphi$ is the average of its seasonal values ($\varphi_{1-13}$, $\varphi_{14-26}$, $\varphi_{27-39}$ and $\varphi_{40-52}$) expressed as the percentage of the maximum flow of the hydropower plant, and $\rho$ is expressed as the number of hours necessary for the hydropower plant to “go” from 0 to maximum flow (or vice versa) at a rate equal to the average of its up and down values ($\rho_{\text{up}}$ and $\rho_{\text{down}}$).

The scenarios analysed in this study as well as in [1] are summarised in Table 2.

<table>
<thead>
<tr>
<th>Concept</th>
<th>Evaluated Scenarios</th>
<th>Description of Each Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\varphi, \rho)$</td>
<td>(0.5%, 0 h); (1%, 0 h); (2%, 0 h); (3%, 0 h); (4%, 0 h); (5%, 0 h); (8%, 0 h); (5%, 60 h)</td>
<td>$\varphi_{1-13} = 0.75\varphi; \varphi_{14-26} = 1.75\varphi; \varphi_{27-39} = 1.2\varphi; \varphi_{40-52} = 0.3\varphi; \rho_{\text{up}} = 0.75\rho; \rho_{\text{down}} = 1.5\rho$</td>
</tr>
</tbody>
</table>

1 Values proposed by the river basin authority; 2 Scenarios calculated in [1].

### 4. Results and Discussion

The averages across the water content and the seasons of WV in relative terms with respect to WV in the scenario without environmental constraints (i.e., $\text{WV}(\varphi, \rho) - \text{WV}(0\%, 0\%) / \text{WV}(0\%, 0\%)$) are included in Figure 5 for every considered $\varphi$ and $\rho$ (see Table 2). This figure seems to indicate that WV increased (approximately linearly) with $\varphi$ and decreased (approximately quadratically) with $\rho$.

In order to explain the abovementioned opposing effects, the weekly scheduling problem considered in Section 2.2 is reformulated in a simpler manner as follows:

$$
\min \left[ -\sum_{t \in T} (P_t \cdot pow_t(v_t, q_t)) - 2^{168} \right]; 
$$

subject to:

$$
\begin{align*}
v_t - v_{t-1} + CF1 \cdot (q_t - W_t) &= 0; \forall t \in T | t < 168 \land v_0 = V_0 \\
v_t - V &\leq 0; \forall t \in T | t < 168 \\
v_{\text{dead}} - V &\leq 0; \forall t \in T | t < 168
\end{align*}
$$
\[ q_t - Q \leq 0; \forall t \in T \mid t < 168 \]  
(43)

\[ q_k - q_t \leq 0; \forall t \in T \mid t < 168 \]  
(44)

\[ q_t - q_{t-1} - \rho^{up} \leq 0; \forall t \in T \mid t < 168 \]  
(45)

\[ -q_t + q_{t-1} - \rho^{down} \leq 0; \forall t \in T \mid t < 168 \]  
(46)

Equation (39) is the objective function in which \( \sum_{t=0}^{167} \) represents the future expected revenue starting from the volume stored at the final hour of the week (hour 168). The rest are constraints: the water mass balance (Equation (40)), the limits of the stored volume (Equations (41) and (42)), the upper limit of the released flow (Equation (43)), and the environmental constraints (Equations (44)–(46)).

The resulting Lagrangian function of the above-described problem is:

\[
\mathcal{L} \left( q, v, \lambda^v, \mu^V, \mu^\text{vol}, \mu^\text{q}, \mu^\text{w}, \mu^\text{up}, \mu^\text{down} \right) = \sum_{t=0}^{167} \left[ -\left( P_t \cdot \text{pow}(v_t, q_t) \right) + \lambda^v_t \cdot \left[ v_t - v_{t-1} + CF1 \cdot (q_t - W_t) \right] + \mu^V_t \cdot (v_t - \bar{V}) \right] + \mu^\text{vol}_t \cdot (q_t - Q_t) + \mu^\text{q}_t \cdot (\varphi_k - q_t) + \mu^\text{up}_t \cdot (q_t - q_{t-1} - \rho^{up}) + \mu^\text{down}_t \cdot (-q_t + q_{t-1} - \rho^{down}) \right] - \sum_{t=0}^{167} \]  
(47)

In Equation (47), \( \lambda^v \)’s and \( \mu^\text{w} \)’s refer to the Lagrange multipliers of the equality and inequality constraints, respectively. As it is well-known, these multipliers mean the incremental prices of the constraint requirements expressed in units of the objective function [45]. Thus, from the Karush-Kuhn-Tucker conditions, it can be also obtained:

\[
\frac{\partial \mathcal{L}}{\partial q_t} \bigg|_{q_t = q} - P_t \cdot \frac{\partial \text{pow}(v_t, q_t)}{\partial q_t} + \lambda^v_t - \lambda^v_{t+1} + \mu^V_t - \mu^V_{t+1} = 0; \forall t \in T \mid t < 168
\]  
(48)

\[
\frac{\partial \mathcal{L}}{\partial \mu^\text{vol}_t} \bigg|_{\mu^\text{vol}_t = \mu^\text{vol}} + \lambda^v_t + \mu^V_t - \mu^V_{t+1} = 0; \forall t \in T \mid t < 168
\]  
(49)

And from Equations (48) and (49), it is possible to isolate \( \lambda^v \) and demonstrate that it represents water value:

\[
\lambda^v_t = \begin{cases} 
P_t \cdot \frac{\partial \text{pow}(v_t, q_t)}{\partial q_t} - \mu^V_t + \mu^V_{t+1} + \lambda^v_{t+1}; & \forall t \in T \mid t < 168 \\
\frac{\partial \text{pow}(q_t, v_t)}{\partial v_t} - \mu^V_t + \mu^V_{t+1} + \lambda^v_{t+1}; & t = 168
\end{cases}
\]  
(50)

Applying the Karush-Kuhn-Tucker conditions, it can be also obtained:

\[
\frac{\partial \mathcal{L}}{\partial \mu^\text{w}_t} \bigg|_{\mu^\text{w}_t = \mu^\text{w}} - P_t \cdot \frac{\partial \text{pow}(v_t, q_t)}{\partial v_t} + CF1 \cdot \lambda^v_t + \mu^\text{q}_t - \mu^\text{q}_{t+1} - \mu^\text{up}_t - \mu^\text{up}_{t+1} - \mu^\text{down}_t - \mu^\text{down}_{t+1} = 0; \\
\forall t \in T \mid t < 168
\]  
(51)

According to those conditions, the multipliers \( \mu^\text{w} \) are non-negative and only zero if their respective constraints are not active [45]. Then, for a problem with only \( \varphi \), Equation (51) can be re-written as follows:

\[
\lambda^\varphi_t = P_t \cdot \frac{\partial \text{pow}(v_t, q_t)}{\partial q_t} - \mu^\text{q}_t + \mu^\varphi_t = 0 \quad \forall t \in T \mid t < 168
\]  
(52)

Because of Equations (43) and (44) cannot be active at the same time, \( \mu^\text{w} \) and \( \mu^\varphi \) cannot be non-zero at the same moment since those are their respective constraints. Equation (52) demonstrates that water value is related with \( \varphi \) through the corresponding multiplier \( \mu^\varphi \). Considering two problems
with different $\varphi$ but the same input data, generation characteristic and constraints, Equation (52) would indicate that, with a linear generation characteristic, as $\varphi$ increases, water value would also increase, assuming that, in general, $\mu_1^\varphi$ grows as $\varphi$ increases. In a case with a non-linear generation characteristic, the influence of $\varphi$ on water value depends also on the first derivative of the generation characteristic with respect to $q_1$.

![Graph](image)

**Figure 5.** (a) Average relative WV in terms of $\varphi$; (b) Average relative WV in terms of $\rho$.

For a problem with only $\rho$, and considering that $q_t$ is not at its limits, Equation (51) can be re-written as follows:

$$\lambda_t^\rho = \frac{P_t^{\rho_{up}}(\rho_{up}) - P_t^{\rho_{down}}(\rho_{down})}{\rho_{up} - \rho_{down}} - \frac{(\mu_{t+1}^{\rho_{up}} - \mu_t^{\rho_{up}}) - (\mu_{t+1}^{\rho_{down}} - \mu_t^{\rho_{down}})}{\rho_{up} - \rho_{down}};$$

(53)

Equation (53) indicates that water value is related with $\rho_{up}$ and $\rho_{down}$ through the corresponding multipliers. Considering two problems with different $\rho$ but the same input data, generation characteristic and constraints, Equation (53) would indicate that, with a linear generation characteristic, water value would increase if $(\mu_{t+1}^{\rho_{up}} - \mu_t^{\rho_{up}})$ increases or $(\mu_{t+1}^{\rho_{down}} - \mu_t^{\rho_{down}})$ decreases.

In order to anticipate the variation in $(\mu_{t+1}^{\rho_{up}} - \mu_t^{\rho_{up}})$ or $(\mu_{t+1}^{\rho_{down}} - \mu_t^{\rho_{down}})$ between these latter problems, the reader is referred to Figure 6, where the hypothetical resulting water release $q_t$ of a set of problems with different $\rho_{up}$ is depicted. From this figure, it might be deduced that $(\mu_{t+1}^{\rho_{up}} - \mu_t^{\rho_{up}})$ increases and $(\mu_{t+1}^{\rho_{down}} - \mu_t^{\rho_{down}})$ decreases as the severity of $\rho$ decreases, and therefore water value would grow. In a case with a non-linear generation characteristic, the influence of $\rho$ on water value depends also on the first derivative of the generation characteristic with respect to $q_t$. An analogous conclusion might be deduced for a set of problems with different $\rho_{down}$.

The abovementioned opposing effects can be also understood, in a more intuitive manner, from the storage-head relationship. Suppose two separate problems aimed at maximising the future revenue of a hydropower plant starting from two different stored volumes ($V^1$ and $V^2$). Regardless of the future values of the inputs (evaporations, inflows and prices), the trajectory (in a time-volume diagram) followed by the reservoir from $V^2$ would have a higher average head than the one corresponding to $V^1$, and therefore the impact of $\varphi$ would be higher for $V^1$ than for $V^2$ (i.e., WV would increase) since the detraction of water ($-\Delta V$) represents a greater reduction of head for $V^1$ than for $V^2$ (see the example in Figure 7). The greater $\varphi$ (i.e., $-\Delta V$), the greater its impact on WV. As regards $\rho$, its impact would be
larger for \( V^2 \) than for \( V^1 \) (and therefore WV would decrease) because the average head is bigger at \( V^2 \) than \( V^1 \). The greater \( \rho \), the greater its impact on WV.

The increasing trend of WV with \( \varphi \) can be to some extent inferred also from the few numerical evidences of WV reported in the literature [6–8,46,47], where WV was shown for different reservoir levels and periods within the hydrological year (usually weeks). In said references, it can be seen that WV increases as the reservoir level or water inflow decrease. \( \varphi \) can be understood as a reduction both in the reservoir level or water inflow, and consequently, the increasing trend of WV with \( \varphi \) might be anticipated. However, to the authors’ opinion, the quantification of the impact of \( \varphi \) on WV is nowadays a relevant contribution for the hydropower sector.

It is interesting to highlight that the trends of the annual WV observed in Figure 5 are in agreement with the trends of the average revenue losses obtained in [44]. Nonetheless, the impact of both \( \varphi \) and \( \rho \) on WV is smaller than that on the average revenue losses obtained in [44]. Without the aim of dismissing the results presented in [44], the results presented in this paper can be considered more realistic since they were obtained as a result of a stochastic approach, whereas those presented in [43] were obtained as a result of a deterministic one.

Returning to Figure 5, it can also be observed that the effect of \( \varphi \) was more significant in spring and winter, whereas \( \rho \) had more influence in spring and summer. In order for these results not to make the reader draw misleading conclusions, it is important to note that Figure 5 shows relative values but that as shown in Figure 8, WV in absolute terms is higher in spring and summer than in autumn.

Figure 6. Example with different up \( \rho \).

Figure 7. Reservoir storage curve.
and winter. On the other hand, the reader should bear in mind that $\varphi$ has its largest value during spring and winter (see Table 2). Even though $\varphi$ usually varies seasonally, Figure 9a,b show for the sake of clarity the same results as Figures 5a, 6a, 7a and 8a but considering $\varphi$ constant along the year (all seasonal values are identical). In addition, Figure 9c shows the impact of $\varphi$ constant along the year on WV (i.e., WV("constant $\varphi$", 0 h)-WV(0%, 0 h)) in k€/Mm$^3$. As regards $\rho$, its higher impact on WV in spring and summer is to a certain extent expected, since the lower the water inflow, the higher the economic value of the the operational flexibility.

Figure 8. (a) Average WV in terms of $\varphi$; (b) Average WV in terms of $\rho$.

Figure 9. (a) Average relative WV in terms of $\varphi$ constant along the year; (b) Average WV in terms of $\varphi$ constant along the year; (c) Difference in average WV in terms of $\varphi$ constant along the year.
The results in Figures 5 and 7 would indicate that the use in the short-term scheduling of WV determined considering $\varphi$ or $\rho$ would result in an operation with a higher or lower average reservoir level. These effects are coherent as regards the risk of spillage since it decreases as the severity of $\varphi$ increases and increases as the severity of $\rho$ increases.

In Figure 10, the level curves of WV according to the water content across the year are represented for the scenario without environmental constraints, with $\varphi = 5\%$, with $\rho = 60$ h, and with $\varphi = 5\%$ and $\rho = 60$ h (values proposed by the river basin authority). As it can be seen in the figure, on the one hand, the lowest average absolute values appeared in winter and the highest ones in summer, and, on the other hand, the lowest relative values of each week were given at the maximum stored volumes in winter whereas these ones during summer were located at the minimum stored volumes. It also shows how the effects of $\varphi$ and $\rho$ on WV approximately counteracted each other.

Figure 10. (a) WV level curves without environmental constraints; (b) WV level curves with $\varphi = 5\%$; (c) WV level curves with $\rho = 60$ h; (d) WV level curves with $\varphi = 5\%$ and $\rho = 60$ h.
These results are only partially in agreement with the few numerical evidences of WV reported in the literature. As discussed above, according to said evidences WV increases as the reservoir level or water inflow decrease. However, as it can be seen in Figure 10, during summer WV increases as the reservoir level increases. This interesting phenomenon which, to the authors’ knowledge, had not been reported in the literature yet until [1], occurs not only because of the lack of risk of spill during summer, but also because of the subsequent prominence of the plant generation characteristic. In order to better understand this effect, we invite the reader to think about another theoretical problem aimed at maximising the future revenue of a hydropower plant starting from three different stored volumes ($V_1$, $V_2 = V_1 + \Delta V$, $V_3 = V_2 + \Delta V$), assuming that future expected water inflows are null. In this extremely dry scenario, it is obvious that the difference in revenue between $V_1$ and $V_2$, and between $V_2$ and $V_3$, is due to both the plant generation characteristic and the difference in the initial available water volume ($\Delta V$), whereas the difference in WV depends only on the former.

With the aim of supporting this discussion, the WV was calculated in five additional cases, each with a constant throughout the year weekly profile of both hourly water inflow and price, and with a different water inflow (very dry, dry, normal, wet and very wet). The five weekly profiles of hourly water inflow were selected by clustering the historical series. The weekly profile of hourly energy price was formed with the average hourly prices of the historical series. It is easily seen in Figure 11 that the trend reported in the literature (increasing WV with decreasing reservoir level) disappears and even inverts as the water inflow decreases.

![Figure 11. Average profiles of WV for five different, and constant through the year, weekly water inflows (Very dry = lowest cluster; Dry = second lowest cluster; Normal = medium cluster; Wet = second highest cluster; Very wet = highest cluster).](image)

**5. Conclusions**

This paper has offered an analysis of the influence of the most common environmental constraints (seasonal minimum flows and maximum ramping rates) on the water value of a real hydropower plant that sells energy in the Spanish day-ahead electricity market. For this purpose, an annual scheduling model, based on stochastic dynamic programming and deterministic mixed integer linear programming, has been developed.

The empirical results of the case study point out that water value is very sensitive to the presence of the considered environmental constraints, as well as their magnitudes. According to said results, water value would follow an upward (approximately) linear trend and a downward (approximately) quadratic one as the severity of the seasonal minimum flows and the maximum ramping rates increase,
respectively. These opposing trends have been analytically deduced from the Karush-Kuhn-Tucker conditions, and inferred in a more intuitive manner, from two simple problems and the storage-head curve of the reservoir. Finally, it has been also found that the impacts of these environmental constraints on water value counteract each other, and that significantly vary throughout the year.

To conclude, the following line of work is proposed in order to continue the research presented in this paper: to study the usefulness of considering the seasonal minimum flows and maximum ramping rates to estimate water value, by calculating the weekly generation schedule and revenue of a hydropower plant throughout a long enough set of consecutive weeks, using the water values determined with and without considering said environmental constraints.

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Author Contributions: This work belongs to the future doctoral thesis of Ignacio Guisández. The thesis is being supervised by Juan I. Pérez-Díaz and José R. Wilhelmi.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ϕ</td>
<td>Minimum environmental flow(s) (%)</td>
</tr>
<tr>
<td>ρ</td>
<td>Maximum ramping rate(s) (h)</td>
</tr>
<tr>
<td>DMILP</td>
<td>Deterministic mixed integer linear programming</td>
</tr>
<tr>
<td>SDP</td>
<td>Stochastic dynamic programming</td>
</tr>
<tr>
<td>WV</td>
<td>Water value(s)</td>
</tr>
</tbody>
</table>

Appendix

The nomenclature used throughout the paper is presented next:

Indexes

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Weekly water inflow.</td>
</tr>
<tr>
<td>b</td>
<td>Average weekly energy price.</td>
</tr>
<tr>
<td>c</td>
<td>Segment of the storage-maximum plant flow curve.</td>
</tr>
<tr>
<td>i</td>
<td>Initial stored volume.</td>
</tr>
<tr>
<td>k</td>
<td>Week of the year.</td>
</tr>
<tr>
<td>l</td>
<td>Final stored volume.</td>
</tr>
<tr>
<td>n</td>
<td>Iteration of the SPD.</td>
</tr>
<tr>
<td>j</td>
<td>Segment of the storage-WV curve.</td>
</tr>
<tr>
<td>s</td>
<td>Segment of the power-discharge curve.</td>
</tr>
<tr>
<td>su</td>
<td>First segment of the power-discharge curve of the u-th hydro unit in ascending order of flow.</td>
</tr>
<tr>
<td>t</td>
<td>Hour within the week.</td>
</tr>
<tr>
<td>u</td>
<td>Hydro unit of the plant.</td>
</tr>
<tr>
<td>x</td>
<td>Weekly water inflow in the next week.</td>
</tr>
<tr>
<td>y</td>
<td>Average weekly energy price in the next week.</td>
</tr>
</tbody>
</table>

Constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF1</td>
<td>Conversion factor (0.0036 (Mm³/h)/(m³/s)).</td>
</tr>
<tr>
<td>CF2</td>
<td>Conversion factor (1/0.6048 (Mm³/week)/(m³/s)).</td>
</tr>
</tbody>
</table>

Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Wear and tear costs of hydro units due to variations in the generated power (€/MW).</td>
</tr>
<tr>
<td>β</td>
<td>Start-up and shut-down costs of hydro units (€/ud).</td>
</tr>
<tr>
<td>γ</td>
<td>Virtual cost of the bottom outlets and spillways due to variations in the released flow (€/m³/s).</td>
</tr>
</tbody>
</table>
\( \varepsilon_{k}^{x} \)
Probability that the inflow at the week \( k+1 \) is \( x \) given that the inflow at the week \( k \) was \( a \).

\( \varepsilon_{k}^{y} \)
Probability that the average energy price at the week \( k+1 \) is \( y \) given that the average energy price at the week \( k \) was \( b \).

\( E_{k} \)
Rate of hourly evaporated water volume per flooded area during the week \( k \) \((\text{Mm}^{3}/\text{km}^{2})\).

\( \varphi_{k} \)
\( \varphi \) during the week \( k \) \((\text{m}^{3}/\text{s})\).

\( J \)
Total number of segments in the storage-WV curve.

\( K_{1bo}, K_{2bo} \)
Coefficients of the linear approximation of the storage-maximum bottom outlet flow curve \((\text{m}^{3}/\text{s})/\text{Mm}^{3}; \text{m}^{3}/\text{s})\).

\( K_{1e}, K_{2e} \)
Coefficients of the linear approximation of the storage-flooded area curve \((\text{km}^{2}/\text{Mm}^{3}; \text{m}^{3})\).

\( K_{sp} \)
Coefficient of the linear approximation of the storage-maximum spillway flow curve \((\text{m}^{3}/\text{s})/\text{Mm}^{3})\).

\( \rho_{\text{down}} \)
Down \( \rho \) \((\text{m}^{3}/\text{s})/\text{h})\).

\( \rho_{\text{up}} \)
Up \( \rho \) \((\text{m}^{3}/\text{s})/\text{h})\).

\( P_{t} \)
Energy price during the hour \( t \) \((€/\text{MW})\).

\( \text{POW} \)
Minimum power output of the power-discharge curve (MW).

\( \mathcal{Q} \)
Maximum plant flow \((\text{m}^{3}/\text{s})\).

\( \mathcal{Q}_{s} \)
Maximum flow of the \( s \)-th segment of the power-discharge curve \((\text{m}^{3}/\text{s})\).

\( Q_{u} \)
Plant flow above which the \( u \)-th hydro unit starts up \((\text{m}^{3}/\text{s})\).

\( Q_{VQ}^{c} \)
Plant flow corresponding to the \( c \)-th interval of the storage-maximum plant flow curve \((\text{m}^{3}/\text{s})\).

\( R_{s}^{\prime} \)
Slope of the \( s \)-th segment of the power-discharge curve \((\text{MW}/(\text{m}^{3}/\text{s}))\).

\( \mathcal{U} \)
Total number of hydro units of the plant.

\( V_{\text{dead}} \)
Dead reservoir volume \((\text{Mm}^{3})\).

\( V_{k}^{i} \)
Stored volume \( i \) at the beginning of the week \( k \) \((\text{Mm}^{3})\).

\( V_{k}^{\text{egal}} \)
Maximum legal stored volume at the end of the week \( k \) \((\text{Mm}^{3})\).

\( V_{\text{sp}}^{v} \)
Stored volume above which the spillways can operate \((\text{Mm}^{3})\).

\( V_{Q}^{c}^{\text{vq}} \)
Minimum stored volume of the \( c \)-th interval of the storage-maximum plant flow curve \((\text{Mm}^{3})\).

\( V_{WV}^{j} \)
Stored volume of the \( j \)-th segment of the storage-WV curve \((\text{Mm}^{3})\).

\( W_{t}^{V} \)
Water inflow to the reservoir during the hour \( t \) \((\text{m}^{3}/\text{s})\).

\( W_{k}^{V_{j}} \)
\( W_{V_{j}}^{k} \)
WV of the \( j \)-th reservoir segment of the storage-WV curve at the end of the week \( k \) \((€/\text{Mm}^{3})\).

\( W_{V_{j}}^{k,a,b} \)
WV of the \( j \)-th reservoir segment of the storage-WV curve at the end of the week \( k \) given the inflow \( a \) and the average energy price \( b \) \((€/\text{Mm}^{3})\).

\textbf{Sets}

\( C \)
Intervals of the storage-maximum plant flow curve.

\( J \)
Segments of the storage-WV curve.

\( K \)
Weeks of the year.

\( S \)
Segments of the power-discharge curve.

\( T \)
Hours of the week.

\( \Omega_{k} \)
Feasible states within the state diagram of the \textit{master module} during the week \( k \).

\textbf{Binary variables}

\( \text{off}_{t}^{u} \)
=1 if the \( u \)-th hydro unit is shut-down during the hour \( t \).

\( \text{on}_{t}^{u} \)
=1 if the \( u \)-th hydro unit is started-up during the hour \( t \).

\( \text{on}_{t}^{u} \)
=1 if the \( u \)-th hydro unit is on-line during the hour \( t \).

\( \text{sp}_{t} \)
=1 if the stored volume is above \( V_{Q}^{v} \) during the hour \( t \).

\( \text{vq}_{t}^{c} \)
=1 if the stored volume is within the \( c \)-th interval of the storage-maximum plant flow curve during the hour \( t \).

\( \text{wv}_{t}^{j} \)
=1 if the stored volume is above the minimum storage of the \( j \)-th segment of the storage-WV curve.

\textbf{Non-negative variables}

\( e_{t} \)
Flow of evaporation during the hour \( t \) \((\text{m}^{3}/\text{s})\).

\( \lambda_{t} \)
Lagrange multipliers of the equality constraints during the hour \( t \) \((€)\).
µ<sub>t</sub>  
Lagrange multipliers of the inequality constraints during the hour <i>t</i> (€).

pow<sub>t</sub>  
Generated power during the hour <i>t</i> (MW).

pow<sub>inc</sub><sup>dec</sup>  
Decrease in generated power between the hours <i>t</i> and <i>t</i> + 1 (MW).

pow<sub>inc</sub><sup>inc</sup>  
Increase in generated power between the hours <i>t</i> and <i>t</i> + 1 (MW).

q<sub>t</sub>  
Plant flow during the hour <i>t</i> (m<sup>3</sup>/s).

q<sub>s</sub><sup>i</sup>  
Plant flow corresponding to the <i>s</i>-th segment of the power-discharge curve during the hour <i>t</i> (m<sup>3</sup>/s).

q<sub>b0</sub><sup>dec</sup><sup>t</sup>  
Decrease in released flow through the bottom outlets during the hour <i>t</i> (m<sup>3</sup>/s).

q<sub>b0</sub><sup>inc</sup><sup>t</sup>  
Increase in released flow through the bottom outlets between the hours <i>t</i> and <i>t</i> + 1 (m<sup>3</sup>/s).

q<sub>sp</sub><sup>dec</sup><sup>t</sup>  
Decrease in released flow through the spillways during the hour <i>t</i> (m<sup>3</sup>/s).

q<sub>sp</sub><sup>inc</sup><sup>t</sup>  
Increase in released flow through the spillways between the hours <i>t</i> and <i>t</i> + 1 (m<sup>3</sup>/s).

v<sub>t</sub>  
Stored volume at the end of the hour <i>t</i> (Mm<sup>3</sup>).


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