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Statistical Diagnosis of the Best Weibull Methods for Wind Power Assessment for Agricultural Applications

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Abstract: The best Weibull distribution methods for the assessment of wind energy potential at different altitudes in desired locations are statistically diagnosed in this study. Seven different methods, namely graphical method (GM), method of moments (MOM), standard deviation method (STDM), maximum likelihood method (MLM), power density method (PDM), modified maximum likelihood method (MMLM) and equivalent energy method (EEM) were used to estimate the Weibull parameters and six statistical tools, namely relative percentage of error, root mean square error (RMSE), mean percentage of error, mean absolute percentage of error, chi-square error and analysis of variance were used to precisely rank the methods. The statistical fittings of the measured and calculated wind speed data are assessed for justifying the performance of the methods. The capacity factor and total energy generated by a small model wind turbine is calculated by numerical integration using Trapezoidal sums and Simpson's rules. The results show that MOM and MLM are the most efficient methods for determining the value of k and c to fit Weibull distribution curves.

1. Introduction

Energy and environment are the twin major crises in the world [1]. Because of this, both developed and developing countries are becoming increasingly more interested in using pollution free, cost effective and renewable sources of energy [2]. As a renewable and alternative energy, wind is the most common and fastest-growing source of energy in the world [3–5]. The characteristics of wind energy are important in different aspects regarding wind energy exploitation [6,7]. Wind is highly variable, both in space and in time [8]. The importance of this variability becomes critical since it is amplified by the cubic relationship of the available power to the wind speed ($p = 0.5\rho A V^3$). The wind power production faces the fluctuation of the wind velocity [9–12]. Therefore, accurate knowledge about the wind characteristics is needed for planning, design and operation of wind turbines [13–16]. For the proper assessment, the variability of the wind over time can be divided into three distinct time scales. Firstly, the large time scale variability describes the variations of the amount of wind from one year to another, or even over periods of decades or more. Secondly, the medium time scale covers periods up to a year. These seasonal variations of the wind are much more predictable. Finally, the short term time scale variability covers time scales of minutes to seconds, also well known by the term "turbulence" and which is of critical interest in the wind turbine design process [17–23]. For more than half a century the Weibull distribution has attracted the attention of statisticians working on theory and methods as well as various fields of statistics [22,24–27]. Hundreds of papers have been written on this distribution; however the research is still ongoing. Together with the normal, exponential distributions, the Weibull distribution is the most popular model in statistics [28,29]. It is of utmost interest to theory orientated statisticians because of its great number of special features, and to practitioners because of its ability to fit to data from various fields, ranging from life data to weather data or observations made in economics and business administration, in health, in physical and social science, in hydrology, in biology or in the engineering sciences [30–36].

Research is ongoing worldwide on the Weibull distribution to find the most reliable methods for wind energy estimation. The main question is how precisely the values of the Weibull shape factor "*k*" and scale factor "*c*" can be determined [37–39]. For this reason different scientists and engineers have developed different methods to find the Weibull parameters for wind energy assessment. That is why several methods are found in literature to estimate the Weibull factors [38,40–43]. Recently, Mohammadi and Mostafaeipour [44] used two methods (STDM and PDM) for wind data assessment in Zarrineh, Iran. In 2012, Costa Rocha *et al.* [45] dealt with the analysis and comparison of seven numerical methods for the assessment of effectiveness in determining the parameters for the Weibull distribution, using wind data collected for Camocim and Paracuru cities in the northeast region of Brazil. Before that, Chang [41] made a statistical study to compare the performance of six numerical methods in estimating Weibull parameters for wind energy application. Seguro and Lambert [43] concluded that the maximum likelihood method (MLM) performs better than the popularly used graphical method (GM). Akdağ and Dinler [40], Azad and Saha [46], and Azad and Alam [47] reviewed three conventional methods and Akdag and Dinler [40] also proposed a new method, namely the energy pattern factor method, for estimating the Weibull parameters. Chu and Ke [48] and Bhattacharya and Bhattacharjee [49] examined the estimation comparison between the MLM and the least squares method [50]. They found that the least squares method significantly outperforms

the MLM when sample size is the same. Jowder [51] used empirical and graphical methods to analyze the wind power density at 10, 30, and 60 m height in the Kingdom of Bahrain; two Weibull parameters were estimated and compared. From the analysis, it was found that the empirical methods provide more accurate prediction of average wind speed and power density than the GM. Odo *et al.* [52] used the Weibull distribution based model for prediction of wind energy potential in Enugu, Nigeria over a period of 13 years. Oyedepo *et al.* [53] also analyzed south-east Nigeria wind data that spans from 24 years to 37 years and was measured at 10 m height. Abbas *et al.* [54] statistically analyzed the wind speed data in Pakistan to determine the best fitting distribution of wind speed. For this purpose, two parameters Gamma, Weibull, Lognormal and Rayleigh distributions, and three parameters Burr and Frechet distributions were fitted to data and parameters for each distribution were estimated using the MLM [42,50,55–61].

Seguro and Lambart [43] calculated the value of the Weibull parameters by three methods. They recommended that the MLM is useful for time series wind speed data and the modified maximum likelihood method (MMLM) is recommended for use with wind data in frequency distribution format. Philippopoulos and Deligiorgi [62] statistically simulated the wind speed data in Athens, Greece based on the Weibull and autoregressive-moving average (ARMA) method. They found that ARMA methods are superior in simulating the frequency distribution of wind speed. Karpa and Naess [63] also analyzed the extreme value statistics of wind speed by the average conditional exceedance rate (ACER) method. Morgan *et al.* [64] examined the probability distributions for offshore wind speed. They concluded that the two-parameter lognormal distribution performs best for estimating extreme wind speeds, but still gives estimates with significant error. Stathopoulos *et al.* [65] used both numerical and statistical models for wind power prediction. Zhou *et al.* [66] comprehensively evaluated wind speed distribution models for a case study of North Dakota country sites. Wind energy estimation and analysis of wind regions through the Weibull distribution methods is widely used nowadays [35,37,67–80].

This study summarizes the results of the 10-min time series wind speed data measured at 20 m and 30 m height in three windy sites, namely Kuakata, Kutubdia and Sitakunda, located in Bangladesh. Table 1 shows the site information in which DL9210 anemometer used to measure wind speed at a 10-min interval. The anemometers have two sensors each of 32 KB memory and have one/two sockets. Statistical work was involved to find the best method of Weibull distribution with high efficiency. The assessment of wind energy potential has been done by using the best method of Weibull distribution. Section 2 offers a detailed outline of the methodology and associated theories for the statistical analysis. The results and discussions are presented in Section 3. Concluding remarks are presented in Section 4.

Table 1. Site name, location for wind speed measuring at 20 m and 30 m height above surface.

Wind station name	Site name	Latitude (°) N	Longitude (°) E	Height above the sea level	Reference
Station-I	Kuakata	21°54.76'	90°08.24'	3 m	[81]
Station-II	Kutubdia	21°54.71'	91°52.43'	0–4 m	[82]
Station-III	Sitakunda	22°35.68'	91°42.52'	9 m	[83]

2. Outline of the Methodology and Associated Theories

To investigate the feasibility of the wind energy resource at any site, there are basically two ways at present to evaluate wind power. The first and the most accurate method to calculate wind power potential is based on measured values that are recorded at meteorological stations. The second method to assess wind power potential is by using probability distribution functions, namely the Rayleigh distribution, Chi-squared distribution, Normal distribution, Binomial distribution, Poisson distribution and Weibull distribution. In this study, the authors used only the Weibull distribution for wind power assessment as presented and discussed below.

2.1. Weibull Probability Density Function

The Weibull probability density function is a two-parameter function characterized by a dimensionless shape parameter (k) and scale parameter (c in m/s). These two parameters determine the wind speed for optimum performance of a wind conversion system as well as the speed range over which the device is likely to operate [52,84–87] as given in Equation (1):

$$f(v) = \frac{dF(v)}{dv} = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \times e^{-\left(\frac{v}{c}\right)^k} \tag{1}$$

where v, k and c are wind speed (m/s), shape factor (dimensionless) and scale factor (m/s), respectively.

2.2. Cumulative Distribution Function or the Weibull Function

Cumulative distribution function is the integration of the Weibull density function. It is the cumulative of relative frequency of each velocity interval [31]. The equation of the Weibull Function is given by:

$$F(v) = \int_0^v f(\acute{v})\ d\acute{v} \tag{2}$$

$$or\ F(v) = 1 - e^{-\left(\frac{v}{c}\right)^k} \tag{3}$$

All these distributions are used to determine the probability of occurrence. The nature of the occurrence affects the shape of the probability curve, and in the case of the wind regime, the cumulative curve probability nature mostly fits to the Weibull Function. Several methods to estimate Weibull factors are found in the literature. Some of these methods are:

(1) Graphical method (GM);
(2) Method of moments (MOM);
(3) Standard deviation method (STDM);
(4) Maximum likelihood method (MLM);
(5) Power density method (PDM);
(6) Modified maximum likelihood method (MMLM);
(7) Equivalent energy method (EEM).

2.2.1. GM

The graph is constructed in such a way that the cumulative Weibull distribution becomes a straight line, with the shape factor k as its slope. Taking the logarithm of both sides, the expression of Equation (3) can be rewritten as:

$$\{1 - F(v)\}^{-1} = e^{\left(\frac{v}{c}\right)^{k}} \tag{4}$$

or:

$$-\ln\{1 - F(v)\} = k \ln v - k \ln c \tag{5}$$

The above equation represents a relationship between $\ln(v)$ and $-\ln\{1 - F(v)\}$. Therefore, the horizontal axis of this plot on the Weibull paper is v while the vertical axis is $\ln(1 - F(v))^{-1}$. The result is a straight line with slope k. For $v = c$, one finds $F(v) = 1 - e^{-1} = 0.632$ and t an estimation for the value of c, by drawing a horizontal line at $F(v) = 0.632$. The intersection point with the Weibull line gives the value of c.

2.2.2. MOM

The MOM is another technique commonly used in the field of parameter estimation. If the numbers $v_1, v_2, \ldots, v_n$ represent a set of data, then an unbiased estimator for the n-th origin moment is given by:

$$\widehat{m_n} = \frac{1}{n} \sum_{i=1}^{n} v_i^n \tag{6}$$

where $\widehat{m_n}$ stands for the estimation of m_n. In a Weibull distribution, the nth moment readily follows from Equation (3). With an expression of the Gamma function $\Gamma(x)$, the average wind speed can be expressed as a function of c and k. The integral found cannot be solved, however it can be reduced to a standard integral, the gamma function, as follows:

$$\Gamma(x) = \int_{0}^{\alpha} y^{x-1} \times e^{-y} dy \tag{7}$$

where $y = \left(\frac{v}{c}\right)^k$ and $\frac{v}{c} = y^{x-1}$; $x = 1 + \frac{1}{k}$ and, after a few manipulations:

$$\mu_n = \left(\frac{1}{c^n}\right)^{-\frac{n}{k}} \Gamma\left(1 + \frac{n}{k}\right) \tag{8}$$

After a few manipulations:

$$\mu_n = C \times \Gamma\left(1 + \frac{1}{k}\right) = 0.8525 + 0.0135k + e^{-[2+3(k-1)]} \tag{9}$$

This formula can easily be handled by pocket calculators in energy output calculations. The accuracy of the approximation is within 0.5% for $1.6 < k < 3.5$. Then from Equation (8) we can find the first and the second moments as follows:

$$m_1 = \widehat{\mu_n} = \left(\frac{1}{c}\right)^{\frac{1}{k}} \Gamma\left(1 + \frac{1}{k}\right) \tag{10}$$

and:

$$m_2 = \widehat{\mu_n^2} + \widehat{\sigma_n^2} = \left(\frac{1}{c}\right)^{\frac{2}{k}}\left[\Gamma\left(1+\frac{2}{k}\right) - \left\{\Gamma\left(1+\frac{1}{k}\right)\right\}^2\right] \tag{11}$$

When we divide m_2 by the square of m_1, we get an expression which is a function of the shape factor k only:

$$\frac{\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)} \tag{12}$$

On taking the square roots of the equation, we have the coefficient of variation (*COV*):

$$COV = \frac{\sqrt{\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)}}{\Gamma\left(1+\frac{1}{k}\right)} \tag{13}$$

In this case, this method can be used as an alternative to the MLM. The value of k and c can be easily determined by the following equations:

$$\sigma = c\left[\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)\right]^{1/2} \tag{14}$$

$$\frac{\sigma}{\bar{v}} = \sqrt{\frac{\Gamma\left(1+\frac{2}{k}\right)}{\left[\Gamma\left(1+\frac{1}{k}\right)\right]^2} - 1} \tag{15}$$

After some calculation we can find:

$$k = \left(\frac{0.9874}{\frac{\sigma}{\bar{v}}}\right)^{1.0983} \tag{16}$$

The Weibull scale factor can be calculated by:

$$\bar{v} = c\Gamma\left(1+\frac{1}{k}\right) \tag{17}$$

2.2.3. STDM

In the STDM, the Weibull factors can be obtained as follows:

$$k = \left(\frac{\sigma}{\bar{v}}\right)^{-1.086} \tag{18}$$

and:

$$c = \frac{\bar{v}}{\Gamma\left(1+\frac{1}{k}\right)} \tag{19}$$

where $\bar{v}$ and σ are mean wind speed and standard deviation of wind speed for any specified periods of time respectively, and can be calculated [44,88] as follows:

$$\bar{v} = \frac{1}{n}\sum_{i=1}^{n} v_i \tag{20}$$

By determining the mean wind speed $\bar{v}$, the standard deviation σ of wind speed becomes:

$$\sigma = \sqrt{\left[\int_0^\alpha (v - \bar{v}) \oint v\, dv\right]} \tag{21}$$

or:

$$\sigma = \left[\left(\frac{1}{n-1}\sum_{i=1}^{n}(v - \bar{v})^2\right)\right]^{1/2} \tag{22}$$

One can find next an expression for σ in terms of k and c with $\bar{v} = c \times \Gamma(1 + 1/k)$ and also $\Gamma(x)$ is the gamma function and is defined as:

$$\Gamma(x) = \int_0^\alpha \exp(-u)u^{x-1}dx \tag{23}$$

2.2.4. MLM

Maximum likelihood estimation has been the most widely used method for estimating the parameters of the Weibull distribution. The commonly used procedure of MLM proposed by Cohe [50], Harter and Moore [55] and Gove [89] due to its very desirable properties. Let $v_1, v_2, ..., v_n$ be a random sample of size n drawn from a probability density function $f(v_i, \theta)$, where θ is an unknown parameter. The likelihood function of this random sample is the joint density of the n random variables and is a function of the unknown parameter [89] given as:

$$L = \prod_{i=1}^{n} f_{v_i}(v_i, \theta) \tag{24}$$

Thus, Equation (24) is the likelihood function. The maximum likelihood estimator (MLE) of θ, say $\hat{\theta}$, is the value of θ that maximizes L or, equivalently, the logarithm of L. Often, but not always, the MLE of θ is a solution of $(d\log L)/d\theta = 0$, where solutions that are not functions of the sample values $v_1, v_2, ..., v_n$ are not admissible, nor are solutions which are not in the parameter space. Now, we are going to apply the MLE to estimate the Weibull parameters, namely the shape and the scale parameters [90]. Consider the Weibull probability density function given in Equation (3), and then the likelihood function will be represented as:

$$L(v_i, k, c) = \prod_{i=1}^{n}\left\{\left(\frac{k}{c}\right)\left(\frac{v_i}{c}\right)^{k-1} exp\left[-\left(\frac{v_i}{c}\right)^k\right]\right\} \tag{25}$$

On taking the logarithms of both sides of Equation (25), we obtain the estimating log-likelihood function:

$$\ln(L) = n\ln(k) - nk\ln(c) + (k-1)\sum_{i=1}^{n}\ln(v_i) - \sum_{i=1}^{n}\left(\frac{v_i}{c}\right)^k \tag{26}$$

Differentiating Equation (26) with respect to k and c in turn and equating to zero, we have:

$$\frac{\partial\ln(L)}{\partial c} = -n\left(\frac{k}{c}\right) + \left(\frac{k}{c}\right)\sum_{i=1}^{n}\left(\frac{v_i}{c}\right)^k = 0 \tag{27}$$

$$\frac{\partial\ln(L)}{\partial k} = \left(\frac{n}{k}\right) + \sum_{i=1}^{n}\left(\frac{v_i}{c}\right)^k - \sum_{i=1}^{n}\left(\frac{v_i}{c}\right)^k \ln\left(\frac{v_i}{c}\right) = 0 \tag{28}$$

From Equation (27):

$$c = \left[\frac{1}{n}\sum_{i=1}^{n}(v_i)^k\right]^{1/k} \tag{29}$$

When $\hat{k}$ is obtained, then $\hat{c}$ is can be determined. To solve $\hat{k}$ by using the Newton-Raphson method as given below, let $f(k)$ be the same as Equation (28) and taking the first differential of $f(k)$, we have:

$$f'(k) = -\left(\frac{n}{k^2}\right) - \sum_{i=1}^{n}\left(\frac{v_i}{c}\right)^k \ln^2\left(\frac{v_i}{c}\right) \tag{30}$$

Substituting Equation (29) into Equation (28) gives:

$$f(k) = \left(\frac{n}{k}\right) + \sum_{i=1}^{n}\left[\frac{(v_i)}{\left\{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(v_i)_i^k\right\}}\right] - \sum_{i=1}^{n}\left[\frac{(v_i)^k}{\left\{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(v_i)_i^k\right\}}\right]\ln\left[\frac{(v_i)}{\left\{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(v_i)_i^k\right\}^{1/k}}\right] \tag{31}$$

Substituting Equation (29) into Equation (30), we get:

$$f'(k) = -\left\{\left(\frac{n}{k}\right) + \sum_{i=1}^{n}\left[\frac{(v_i)^k}{\left\{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(v_i)^k\right\}}\right]\ln^2\left[\frac{(v_i)}{\left\{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(v_i)^k\right\}^{1/k}}\right]\right\} \tag{32}$$

Therefore, $\hat{k}$ is obtained from the equation below by carefully choosing an initial value for k_i and iterating the process until it converges:

$$k_{i+1} = k_i - \frac{\left(\frac{n}{k}\right) + \sum_{i=1}^{n}\left[\frac{v_i}{\left\{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(v_i)^k\right\}^{1/k}}\right]}{-\left\{\left(\frac{n}{k}\right) + \sum_{i=1}^{n}\left[\frac{(v_i)^k}{\left\{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(v_i)^k\right\}}\right]\ln^2\left[\frac{(v_i)}{\left\{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(v_i)^k\right\}^{1/k}}\right]\right\}} - \frac{\sum_{i=1}^{n}\left[\frac{(v_i)^k}{\left\{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(v_i)^k\right\}}\right]\ln\left[\frac{(v_i)}{\left\{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(v_i)^k\right\}^{1/k}}\right]}{-\left\{\left(\frac{n}{k}\right) + \sum_{i=1}^{n}\left[\frac{(v_i)^k}{\left\{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(v_i)^k\right\}}\right]\ln^2\left[\frac{(v_i)}{\left\{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(v_i)^k\right\}^{1/k}}\right]\right\}} \tag{33}$$

2.2.5. PDM

To obtain the shape factor and scale factor through this method, firstly the energy pattern factor is computed. The energy pattern factor usage is for turbine aerodynamic design. The energy pattern factor is related to the averaged data of wind speed and is defined as a ratio between mean of cubic wind speed to cube of mean wind speed. The energy pattern factor E_{pf} is expressed as [40,45]:

$$E_{pf} = \frac{\text{Total amount of power available in the wind}}{\text{Power calculated by cubing the mean wind speed}}$$

$$\text{or } E_{pf} = \frac{\frac{1}{n}\sum_{i=1}^{n} v_i^3}{\left(\frac{1}{n}\sum_{i=1}^{n} v_i\right)^3} = \frac{\overline{v^3}}{(\overline{v})^3} = \frac{\Gamma\left(1+\frac{3}{k}\right)}{\Gamma^3\left(1+\frac{1}{k}\right)} \tag{34}$$

Once the energy pattern factor is calculated by using the above equation, the Weibull shape factor and scale factor can be estimated from the following formulas:

$$k = 1 + \frac{3.69}{E_{pf}^2} \tag{35}$$

$$c = \frac{\overline{v}}{\Gamma\left(1+\frac{1}{k}\right)} \tag{36}$$

where E_{pf} is the energy pattern factor; and Γ is the gamma function.

2.2.6. MMLM

The MMLM can only be considered if the available data of wind speed are already in the shape of the Weibull distribution. The solution of the equations in the MLM requires some numerical iteration by the Newton-Raphson method [41]. The Weibull parameters are determined by the following equations:

$$k = \left[\frac{\sum_{i=1}^{n} v_i^k ln(v_i)f(v_i)}{\sum_{i=1}^{n} v_i^k f(v_i)} - \frac{\sum_{i=1}^{n} ln(v_i)f(v_i)}{f(v \geq 0)}\right]^{-1} \tag{37}$$

$$c = \left[\frac{1}{f(v \geq 0)}\sum_{i=1}^{n}(v_i)^k f(v_i)\right]^{1/k} \tag{38}$$

where v_i is the wind speed central to bin i; n is the number of bins; $f(v_i)$ represents the Weibull frequency for the wind speed range within bin I; and $f(v \geq 0)$ is the probability for wind speed to equal or exceed zero.

2.2.7. EEM

Consider a random sample of v_1, v_2, ..., v_n by relative frequency of occurrence in a given interval of wind speed. Then a random variable observation discrete W_v associated with wind speed can be obtained from:

$$W_v = f_v(v) \tag{39}$$

This observation random is also related with the Weibull parameters k and c from the equation of the probability of occurrence W. $W(v)$ is the observed frequency of the wind speed for the interval of $(v - 1) \leq V < v$. The mathematical representation of $W(v)$ is:

$$W(v) = Q(v - 1) - Q(v) \tag{40}$$

$$W(v) = \left\{ e^{-\left(\frac{v-1}{c}\right)^k} - e^{-\left(\frac{v}{c}\right)^k} \right\} \tag{41}$$

where $Q(v)$ and the probability of occurrence of wind speeds equal to or higher than v, given by $Q(v) = 1 - F(v)$. Then the random observation Wv can be written using Equations (39) and (41) as:

$$W_v = W(v) + \varepsilon_v = \left\{ e^{-\left(\frac{v-1}{c}\right)^k} - e^{-\left(\frac{v}{c}\right)^k} \right\} + \varepsilon_v \tag{42}$$

The first hypothesis says that "The energy density is a parameter that helps in the determination of parameters of the Weibull distribution for applications in wind energy". The related factor part deterministic must meet the following conditions: (a) be variable with random value expected value equal to 0: $E(\varepsilon) = 0$; (b) be variable random variance with constant: $v(\varepsilon) = \sigma^2$; and (c) the occurrences of ε are non-correlated: $COV(\varepsilon_i, \varepsilon_j)$. To ensure the condition of equivalence initially proposed in the hypothesis, it is the equality between Equation (4) and $v_m^3 = \frac{1}{n}\sum_{i=1}^{n} v_i^3$. The equation resulting from this transaction expresses the parameter c as a function of speed cubed average of observations and the parameter k as:

$$c = \left(\frac{v_m^3}{\Gamma\left(1 + \frac{3}{k}\right)} \right)^{1/3} \tag{43}$$

Substituting Equation (43) into Equation (42) gives:

$$W_v = \left\{ e^{-\left[\frac{(v-1)\left\{\Gamma\left(1+\frac{3}{k}\right)\right\}^{1/3}}{\left(v_m^3\right)^{1/3}} \right]^k} - e^{-\left[\frac{(v)\left\{\Gamma\left(1+\frac{3}{k}\right)\right\}^{1/3}}{\left(v_m^3\right)^{1/3}} \right]^k} \right\} + \varepsilon_v \tag{44}$$

The estimate of the parameter k may be obtained from an estimator of least squares given by the expression [45]:

$$\sum_{i=1}^{n} \left[W_{vi} - e^{-\left[\frac{(v_i-1)\left\{\Gamma\left(1+\frac{3}{k}\right)\right\}^{1/3}}{\left(v_m^3\right)^{1/3}} \right]^k} + e^{-\left[\frac{(v_i)\left\{\Gamma\left(1+\frac{3}{k}\right)\right\}^{1/3}}{\left(v_m^3\right)^{1/3}} \right]^k} \right]^2 = \sum_{i=1}^{n} (\varepsilon_{vi})^2 \tag{45}$$

where W_{vi} is the observed frequency of the wind speed; n is the number of intervals of the histogram of speed; v_i is the value of the upper limit of the i-th speed interval; v_m^3 is the mean of the cubic wind speed; and ε_{vi} is the error of the approximation. Once the value of the parameter k is calculated, the value of c can be obtained directly from Equation (43).

2.3. COV

The *COV* is defined as the ratio between mean standard deviation to mean wind speed expressed as a percentage. It demonstrates the mutability of wind speed and can be expressed as [29]:

$$COV\ (\%) = \frac{\sigma}{\bar{v}} \times 100 \tag{46}$$

2.4. Wind Speed Varies with Altitude

Wind energy is indirect solar energy because it is generated by the temperature difference between the equator and the poles which drives the thermal system by solar radiation. It is known that wind speed varies with altitude, however, wind blows relatively slowly at low altitude and wind speed then increases with altitude. Different relationships are found in the literature to calculate wind speed at any height [91–93]. The Weibull factors used for these calculations must be obtained from the best possible method of Weibull distribution. For this purpose, the Weibull factors are initially calculated at desired heights, then wind speed and wind power are obtained [94,95]. The calculation procedures use the following relationships:

$$\eta = 0.37 - 0.0881 \ln c_{10} \tag{47}$$

$$k_h = \frac{k_{10}}{\left[1 - 0.0881 \ln\left(\frac{z_h}{z_{ref}}\right)\right]} \tag{48}$$

$$c_h = c_{10} \left(\frac{z_h}{z_{ref}}\right)^n \tag{49}$$

$$v_h = c_h \Gamma\left(1 + \frac{1}{k_h}\right) \tag{50}$$

$$\bar{P}_h = \frac{1}{2}\rho\bar{v}_h^3 = \frac{1}{2}\rho\bar{v}_h^3 \frac{\Gamma\left(1 + \frac{3}{k_h}\right)}{\Gamma^3\left(1 + \frac{3}{k_h}\right)} = \frac{1}{2}\rho c_h^3 \Gamma\left(1 + \frac{3}{k_h}\right) \tag{51}$$

$$E_D = \frac{1}{2}\rho c_h^3 \Gamma\left(1 + \frac{3}{k_h}\right) \times T \tag{52}$$

The mean energy density over a period of time, *T*, is the product of mean power density and the time period. k_{10} and c_{10} are the shape factor and scale factor at a height of 10 m and η is the power law coefficient. k_h, c_h, v_h, z_h and P_h are shape factor, scale factor, wind speed and wind power at the desired height, respectively. The z_{ref} is the reference height. ρ is the air density and for standard conditions (*i.e.*, at sea level with temperature of 15 °C and pressure of 1 atmosphere) and is equal to 1.225 kg/m³ and *v* is the wind speed (m/s). In this paper the authors used the Weibull factors to find wind speed and wind power at a height of 50 m.

2.5. Statistical Error Analysis/Goodness of Fit

To find the best method for the analysis, some statistical parameters were used to analyze the efficiency of the above mentioned methods. The following tests were used to achieve this goal:

(a) zRelative percentage of error (RPE)

$$RPE = \left(\frac{x_{i,w} - y_{i,m}}{y_{i,m}}\right) \times 100\% \tag{53}$$

(b) Root mean square error (RMSE)

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N}(y_{i,m} - x_{i,w})^2\right]^{1/2} \tag{54}$$

(c) Mean percentage error (MPE)

$$MPE = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_{i,w} - y_{i,m}}{y_{i,m}}\right) \times 100\% \tag{55}$$

(d) Mean absolute percentage error (MAPE)

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{x_{i,w} - y_{i,m}}{y_{i,m}}\right| \times 100\% \tag{56}$$

(e) Chi-square error

$$\chi^2 = \frac{\sum_{i=1}^{N}(y_{i,m} - x_{i,w})^2}{x_{i,w}} \tag{57}$$

(f) Analysis of variance or efficiency of the method

$$R^2 = \frac{\sum_{i=1}^{N}(y_{i,m} - z_{i,\bar{v}})^2 - \sum_{i=1}^{N}(y_{i,m} - x_{i,w})^2}{\sum_{i=1}^{N}(y_{i,m} - z_{i,\bar{v}})^2} \tag{58}$$

where N is the number of observations; $y_{i,m}$ is the frequency of observation or i-th calculated value from measured data; $x_{i,w}$ is the frequency of Weibull or i-th calculated value from the Weibull distribution; $z_{i,\bar{v}}$ is the mean of i-th calculated value from measured data. RPE shows the percentage deviation between the calculated values from the Weibull distribution and the calculated values from measured data. MPE shows the average of percentage deviation between the calculated values from the Weibull distribution and the calculated values from measured data, and MAPE shows the absolute average of percentage deviation between the calculated values from the Weibull distribution and the calculated values from measured data. Best results are obtained when these values are close to zero. R^2 determines the linear relationship between the calculated values from the Weibull distribution and the calculated values from measured data. The ideal value of R^2 is equal to 1 [30,44,45,96,97].

3. Results and Discussion

In this statistical analysis, data from three wind monitoring stations were used to diagnose the best method of the Weibull distribution. The most important results of this analysis, based on hourly, monthly, seasonal and annual figures, are presented. Table 2 shows a lot of information about the sites based on monthly mean as well as annual mean wind speed, number of observations in hours, maximum and minimum wind speed, standard deviation, and *COV*/turbulence intensity at 20 m and 30 m height.

Table 2. Monthly mean wind speed (m/s) data for selected wind stations at 20 m and 30 m height. *COV*: coefficient of variation.

Wind			January	February	March	April	May	June	July	August	September	October	November	December	Yearly mean
Station-I	H = 30 m	*N* (h)	744	672	744	720	744	720	744	744	720	744	720	744	8760
		v_{mean} (m/s)	3.11	3.57	3.54	4.98	4.97	5.07	5.74	5.97	4.95	2.57	2.98	3.22	4.22
		v_{max} (m/s)	5.08	5.79	5.52	8.26	8.28	7.55	9.46	9.29	9.52	3.5	5.77	7.45	7.12
		v_{min} (m/s)	1.66	2.25	2.44	2.52	2.32	2.48	3.23	2.34	1.97	1.5	1.31	1.26	2.11
		σ (m/s)	0.78	0.86	0.82	1.54	1.68	1.56	1.61	2.13	2.14	0.54	0.76	1.39	1.32
		COV (%)	25.08	24.09	23.16	30.92	33.8	30.77	28.05	35.68	43.23	21.01	25.5	43.17	30.37
	H = 20 m	*N* (h)	744	672	744	720	744	720	744	744	720	744	720	744	8760
		v_{mean} (m/s)	2.15	2.62	2.65	3.76	3.81	3.82	4.44	4.75	3.81	1.76	2.12	2.55	3.19
		v_{max} (m/s)	3.65	4.14	4.29	6.03	6.25	5.58	8.15	8.21	7.33	2.77	4.9	7.2	5.71
		v_{min} (m/s)	1.22	1.68	1.78	1.88	1.69	1.77	2.71	1.72	1.53	0.99	0.9	0.97	1.57
		σ (m/s)	0.52	0.69	0.66	1.1	1.24	1.17	1.37	1.83	1.73	0.45	0.65	1.4	1.07
		COV (%)	24.19	26.34	24.91	29.26	32.55	30.63	30.86	38.53	45.41	25.57	30.66	54.9	32.82
Station-II	H = 30 m	*N* (h)	744	672	744	720	744	720	744	744	720	744	720	744	8760
		v_{mean} (m/s)	3.11	2.8	3.15	3.73	4.24	4.53	5.42	4.74	3.39	2.25	2.96	2.65	3.58
		v_{max} (m/s)	5.26	5.69	4.81	6.36	6.95	7.35	7.58	7.58	7.48	3.31	4.15	4.37	5.91
		v_{min} (m/s)	1.64	1.29	1.67	2.22	2.35	2.06	2.82	1.99	1.29	1.25	1.7	1.73	1.83
		σ (m/s)	0.95	1.33	0.69	1.16	1.39	1.52	1.17	1.56	1.74	0.59	0.5	0.64	1.1
		COV (%)	30.55	47.5	21.9	31.1	32.78	33.55	21.59	32.91	51.33	26.22	16.89	24.15	30.87
	H = 20 m	*N* (h)	744	672	744	720	744	720	744	744	720	744	720	744	8760
		v_{mean} (m/s)	1.69	1.79	1.9	2.73	3.3	3.81	4.63	3.95	2.58	1.26	1.45	1.11	2.52
		v_{max} (m/s)	3.51	4.47	3.7	5.1	6.31	6.31	7.26	6.79	7.13	2.41	2.68	2.42	4.84
		v_{min} (m/s)	0.7	0.64	0.76	1.49	1.39	1.38	2.24	1.61	0.87	0.56	0.59	0.5	1.06
		σ (m/s)	0.71	1.25	0.61	1.08	1.42	1.41	1.18	1.45	1.84	0.46	0.43	0.44	1.02
		COV (%)	42.01	69.83	32.11	39.56	43.03	37.01	25.49	36.71	71.32	36.51	29.66	39.64	41.91

Table 2. *Cont.*

| Wind			Month of the year												Yearly mean
			January	February	March	April	May	June	July	August	September	October	November	December	
Station-III	H = 30 m	N (h)	744	672	744	720	744	720	744	744	720	744	720	744	8760
		v_{mean} (m/s)	2.72	3.17	3.36	4.35	4.46	4.81	5.36	5.15	10.09	2.25	2.7	2.32	4.23
		v_{max} (m/s)	4.16	6.21	5.14	7.36	7.55	8.16	7.59	13.76	39.2	3.48	3.69	3.67	9.16
		v_{min} (m/s)	1.16	1.24	1.96	2.09	2.63	2.14	2.91	2.31	1.49	1.23	1.86	1.11	1.84
		σ (m/s)	0.84	1.43	0.82	1.42	1.4	1.79	1.14	2.58	13.07	0.48	0.43	0.71	2.18
		COV (%)	30.88	45.11	24.4	32.64	31.39	37.21	21.27	50.1	129.53	21.33	15.93	30.6	39.2
	H = 20 m	N (h)	744	672	744	720	744	720	744	744	720	744	720	744	8760
		v_{mean} (m/s)	1.96	2.47	2.64	3.72	3.84	4.26	4.79	4.87	5.97	1.43	1.82	1.66	3.29
		v_{max} (m/s)	3.36	5.61	4.35	6.62	6.86	7.34	6.89	14.35	18.58	2.6	2.67	2.7	6.83
		v_{min} (m/s)	0.66	0.79	1.33	1.65	2.01	1.56	2.31	1.73	1.28	0.76	0.93	0.86	1.32
		σ (m/s)	0.73	1.45	0.82	1.42	1.42	1.79	1.09	2.88	5.33	0.42	0.36	0.52	1.52
		COV (%)	37.24	58.7	31.06	38.17	36.98	42.02	22.76	59.14	89.28	29.37	19.78	31.33	41.32

A sample data frequency distribution and cumulative frequency distribution has been presented in Table 3.

Table 3. Frequency distribution and cumulative frequency distribution for the selected sites in June.

Wind speed (m/s)	Station-I		Station-II		Station-III	
	Frequency (%)	Cumulative frequency (%)	Frequency (%)	Cumulative frequency (%)	Frequency (%)	Cumulative frequency (%)
0–1	0.403	0.403	0.269	0.269	0.134	0.134
1–2	2.151	2.554	1.882	2.151	0.806	0.94
2–3	5.242	7.796	3.629	5.78	5.645	6.585
3–4	9.005	16.801	14.247	20.027	17.339	23.924
4–5	17.742	34.543	19.086	39.113	24.059	47.983
5–6	20.43	54.973	24.328	63.441	18.414	66.397
6–7	18.011	72.984	18.28	81.721	17.742	84.139
7–8	12.634	85.618	11.559	93.28	8.199	92.338
8–9	8.602	94.22	5.242	98.522	3.898	96.236
9–10	4.032	98.252	1.344	99.866	2.016	98.252
10–11	1.613	99.865	0	99.866	1.344	99.596
11–12	0.134	100	0.134	100	0.269	99.865
12–13	-	-	-	-	0.134	100

The hourly variation of wind speed at the selected altitude has been presented in Figure 1. Wind speed is higher during the daytime than the nighttime at every site. The *COV*/turbulence with the time series is at its minimum during the higher windy periods.

Figure 1. Hourly mean wind speed at 20 m and 30 m height for the selected sites.
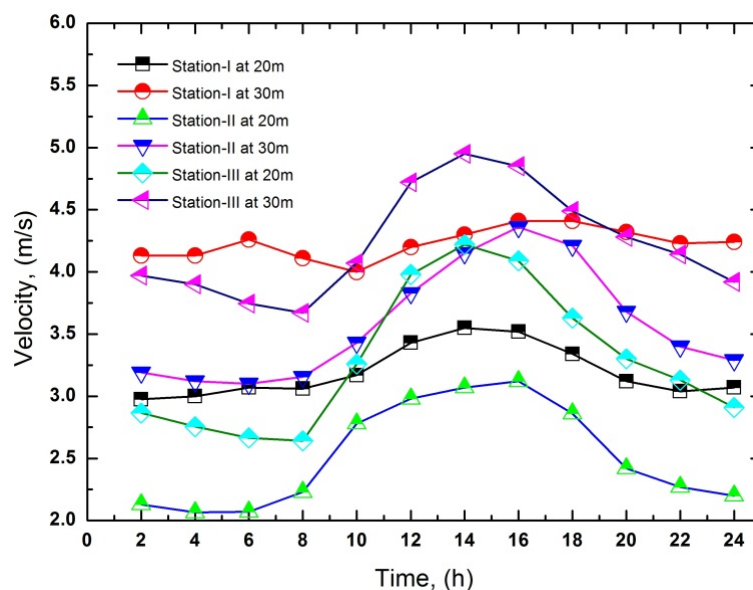
The hourly mean *COV* is presented in Figure 2. An important issue which has been clearly shown in Figure 2 is that turbulence is comparatively low at 30 m height than at 20 m height. Therefore, at the higher level the wind velocity stream is more uniform, *i.e.*, the *COV* is lower.

Figure 2. Hourly mean *COV* of turbulence at 20 m and 30 m height.
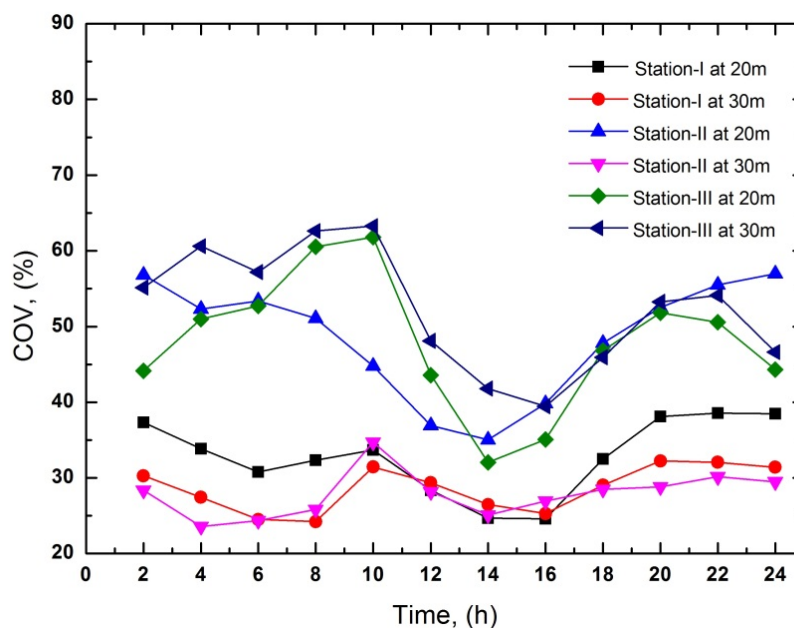
From the sample frequency distribution (Table 3), it can be clearly seen that more than 50% of the frequency is between 5 m/s and 13 m/s of wind speed at every station. Table 3 can be used to clearly identify the total number of hours at certain wind speed available in a month. Anyone can find monthly wind speed scenario at a glance from the Table. The results are similar for other months at each station. Figures 1 and 2, shows the hourly mean variations of wind speed and coefficient of variation for the period of 24 h as a mean of a year. It has been shown that the wind speed is low until 8:00 AM to 10:00 AM, and after that it increases during the day until the maximum value is gained from 2:00 PM to 4:00 PM, and afterwards it decrease again until the end of the day. The same wind characteristics have been found at 20 m and 30 m height at every station. The maximum and minimum values of hourly mean wind speeds at 30 m height are 4.95 m/s at Station-III and 3.10 m/s at Station-II. The variation of wind speed at Station-I is lower than the others. The *COV* is lower when the wind speeds become a maximum. Its values are very high at nighttime. From Table 3 and Figures 1 and 2, it is clear that 30 m height wind speed data at Station-I shows better results than the other sites. To find the best Weibull method, the statistical analysis of the Weibull distribution at 20 m and 30 m height is presented in Tables 4–9.

In the statistical analysis, seven methods were used to determine the shape parameter *k* and scale parameter *c* of the Weibull distribution. For comparison of these seven methods to each other and to find out the efficiency of the methods, six statistical tools were used, *i.e.*, relative percentage error (RPE), RMSE, MPE, MAPE, chi-square error (χ^2), and analysis of variance or efficiency of the method (R^2). It is important to note that only one column is required to rank the methods, since the above six criteria all gave the same relative results. For a more precisely diagnosis, the authors used these six statistical tools to rank the methods. The statistical analysis results for the seven numerical methods and the six statistical test results are shown in Tables 4–9 for the three stations at 20 m and 30 m height, respectively. The results show that the method of moments (MOM) and MLM give better results than other methods, where the value of *k* and *c* becomes almost the same by these two methods. For MLM, RPE and RMSE becomes zero and the value of MPE becomes negative, because, by this method, the calculated

value becomes greater than the measured one. But the most important statistical test gives a chi-square error of $\chi^2 = 0.0010$ and the efficiency of the method is $R^2 = 0.9997$, where the best results are obtained when these values are close to zero and unity, respectively. For Station-II, in Table 7, MLM and PDM methods show better performance than others. The common factor is that the MLM gives the better performance at every site. Therefore, it can be said that the MLM is the fisrt and MOM is the second most efficient method for wind data assessment at 30 m height.

Table 4. The Weibull distribution analysis for Station-I at 20 m height. GM: graphical method; MOM: method of moments; STDM: standard deviation method; MLM: maximum likelihood method; PDM: power density method; MMLM: Modified maximum likelihood method; and EEM: equivalent energy method.

Statistical methods	The Weibull parameters				Statistical test efficiency			
	k (-)	c (m/s)	RPE (%)	RMSE	MPE (%)	MAPE (%)	χ^2	R^2
GM	3.63	3.55	0.0785	0.0500	0.8283	4.2571	0.1432	0.9577
MOM	3.58	3.54	−0.0523	0.0408	−0.0394	0.0394	0.0001	0.9999
STDM	3.99	4.46	26.6736	0.9219	27.9826	27.982	2.2593	0.1542
MLM	3.5	3.55	0.0000	0.0000	−0.0438	0.3700	0.0005	0.9998
PDM	3.11	3.53	−0.7845	0.1581	−0.6649	0.6648	0.0285	0.9917
MMLM	3.54	3.55	0.0784	0.0500	0.0723	0.1885	0.0002	0.9999
EEM	3.37	3.54	−0.2876	0.0957	0.1507	2.4687	0.0451	0.9866

Note: Dimensionless shape factor (k) and scale factor (c) in m/s, relative percentage error (RPE), root mean square error (RMSE), mean percentage error (MPE), mean absolute percentage error (MAPE), chi-square error (χ^2), analysis of variance or efficiency of the method (R^2).

Table 5. The Weibull distribution analysis for Station-I at 30 m height.

Statistical methods	The Weibull parameters		Statistical test efficiency					
	k (-)	c (m/s)	RPE (%)	RMSE	MPE (%)	MAPE (%)	χ^2	R^2
GM	2.71	4.79	0.8880	0.1930	0.6250	1.7040	0.0350	0.9900
MOM	3.87	4.68	−0.5530	0.1530	−0.5420	0.5890	0.0023	0.9990
STDM	2.64	4.78	1.7370	0.2710	2.3310	3.7180	0.2380	0.9320
MLM	3.78	4.68	0.0000	0.0000	−0.0560	0.3440	0.0010	0.9997
PDM	2.58	4.66	−1.2040	0.2250	−1.1870	1.2390	0.0150	0.9960
MMLM	3.18	4.67	−0.8090	0.1850	−0.9210	0.9540	0.0064	0.9980
EEM	3.26	4.73	0.4930	0.1440	0.6290	1.6220	0.0290	0.9920

From Table 4, it can be clearly seen that MOM ($\chi^2 = 0.0001$, $R^2 = 0.9999$), MLM ($\chi^2 = 0.0005$, $R^2 = 0.9998$) and MMLM ($\chi^2 = 0.0002$, $R^2 = 0.9999$) give very close results and show better performance than other methods. In Table 6, MOM and PDM methods show better results than others. In the case of Station-III, MLM and PDM methods show better results than others in Table 8. Summarizing these results, it can be easily said that MOM and MLM methods were applicable for every height at any location wind data assessment. The PDM method is better for wind data assessment at the lower height, but not perfect for the higher height.

Table 6. The Weibull distribution analysis for Station-II at 20 m height.

Statistical methods	The Weibull parameters		Statistical test efficiency					
	k (-)	c (m/s)	RPE (%)	RMSE	MPE (%)	MAPE (%)	χ^2	R^2
GM	3.11	2.70	−3.8742	0.3120	−4.1546	4.5644	0.1336	0.9787
MOM	2.82	2.81	0.0662	0.0408	0.0422	0.0422	0.0002	0.9999
STDM	3.4	3.74	34.2053	0.9278	40.0035	40.003	2.8453	0.3654
MLM	2.84	2.9	3.0463	0.2768	6.7823	7.0891	0.3063	0.9475
PDM	2.65	2.82	0.1324	0.0577	0.1170	0.2672	0.0003	0.9999
MMLM	2.84	2.85	1.5894	0.2000	3.4823	3.6671	0.0795	0.9865
EEM	2.88	2.76	−1.8212	0.2140	−1.9377	2.1966	0.0337	0.9945

Table 7. The Weibull distribution analysis for Station-II at 30 m height.

Statistical methods	The Weibull parameters		Statistical test efficiency					
	k (-)	c (m/s)	RPE (%)	RMSE	MPE (%)	MAPE (%)	χ^2	R^2
GM	4.17	3.88	−1.8150	0.2550	−1.5198	2.7444	0.0610	0.9790
MOM	3.99	3.94	−0.5820	0.1440	−0.5930	0.6490	0.0074	0.9974
STDM	4.47	4.83	22.6440	0.9004	23.8630	23.863	1.9020	0.1780
MLM	3.92	3.96	−0.0930	0.0580	−0.0920	0.5220	0.0020	0.9990
PDM	3.25	3.99	−0.2560	0.0960	−0.2560	0.2560	0.0004	0.9998
MMLM	3.59	3.98	−6.5160	0.4830	−8.3160	8.5270	2.3420	0.2280
EEM	4.23	4.39	11.1940	0.6330	11.7710	11.771	0.5180	0.7970

Table 8. The Weibull distribution analysis for Station-III at 20 m height.

Statistical methods	The Weibull parameters		Statistical test efficiency					
	k (-)	c (m/s)	RPE (%)	RMSE	MPE (%)	MAPE (%)	χ^2	R^2
GM	3.04	3.66	0.0254	0.0289	−0.8825	4.4901	0.1568	0.979
MOM	3.14	3.62	−0.2536	0.0913	−0.1711	0.1711	0.0031	0.9996
STDM	3.43	4.95	35.1255	1.0743	34.5353	34.535	5.7001	0.0331
MLM	3.13	3.65	0.1775	0.0764	0.0826	0.4453	0.0011	0.9998
PDM	2.77	3.65	−0.0507	0.0408	−0.0311	0.0311	0.0001	0.9999
MMLM	3.14	3.64	−1.1666	0.1958	−0.8205	1.4034	0.0924	0.9877
EEM	2.91	3.66	0.0000	0.0000	−0.449	2.2702	0.041	0.9945

Table 9. The Weibull distribution analysis for Station-III at 30m height.

Statistical methods	The Weibull parameters		Statistical test efficiency					
	k (-)	c (m/s)	RPE (%)	RMSE	MPE (%)	MAPE (%)	χ^2	R^2
GM	2.1	4.71	−0.5910	0.1580	−0.5250	2.8630	0.0640	0.9950
MOM	3.76	4.45	0.0790	0.0580	0.1320	0.1650	0.0010	0.9999
STDM	2.3	3.73	−21.540	0.9540	−11.410	13.920	20.680	0.3470
MLM	3.66	4.5	−0.4730	0.1414	−0.3095	0.5340	0.0140	0.9990
PDM	2.32	4.74	1.4980	0.2520	−0.0730	3.4630	0.3080	0.9740
MMLM	2.99	4.62	0.1580	0.0820	−0.4510	1.9760	0.0580	0.9950
EEM	3.03	4.09	−11.845	0.7080	−6.0510	7.4310	5.9690	0.5632

Table 10 summaries the test results of the seven methods and ranking of the methods according to their performance and efficiency in wind data assessment. The rankings were done by considering minimum error and maximum efficiency according to first to seventh positions respectively. Regarding this test, six statistical tools have been used and considered four decimal places of each value by numerical iteration methods. One test is enough to rank the methods, but for more precise analysis, the authors used more tools which helped to verify the discussion about the best method. In this statistical research work, it has been found that the MOM achieved the first position and the MLM took the second position in rank. Although the PDM got the third position, this method has better performance for low height wind data assessment. At increased height, PDM was a less efficient method than others. But MOM and MLM methods were applicable at any altitude with minimum error and maximum efficiency. Our first goal has been satisfied by the above statistical analysis where we identified the best methods to determine the Weibull distribution. Another goal is to select the best wind site by using these best methods, which has been analyzed below.

Table 10. Ranking of the methods by statistical test results.

Statistical methods	Station-I		Station-II		Station-III		Discussion
	20 m	**30 m**	**20 m**	**30 m**	**20 m**	**30 m**	
GM	Sixth	Sixth	Fifth	Fourth	Sixth	Fourth	-
MOM	First	Second	First	Third	Third	First	The First choice
STDM	Seventh	Seventh	Seventh	Seventh	Seventh	Seventh	-
MLM	Third	First	Sixth	Second	Second	Second	The second choice
PDM	Fourth	Fourth	Second	First	First	Fifth	The third choice
MMLM	Second	Third	Fourth	Sixth	Fifth	Third	-
EEM	Fifth	Fifth	Third	Fifth	Fourth	Sixth	-

The procedure mentioned in this study is not only applicable in case study sits, it can be applied in any climatic conditions at any site in any countries in the world. For example, Mohammadi and Mostafaeipour [44] used STDM and PDM for wind turbine utilization in Zarriuneh at Kurdistan mountainous province in Iran. Chang [41] used moment method, empirical method, GM, MLM, MMLM and energy pattern factor method at three wind farms (Dayuan, Hengchun and Penghu) experiencing different weather conditions in Taiwan. Costa Rocha *et al.* [45] used EEM, moment method, MLM, *etc.* for wind energy generation in coastal area of the State of Ceara, located in the northeast region of Brazil. Dorvlo [98] used moment method, Chi-square method, empirical method and regression method for estimation of wind energy in four weather stations (Marmul, Masirah, Sur and Thumrait) in Oman. Many authors have already used the methods at different geographical locations for wind energy estimation [30,43,99]. The paper presented more generalized form of the Weibull distribution methods which is validated by widely acceptable error correction methods for practical application. This procedure is applicable at any geographical location; at any weather condition (*i.e.*, summer, winter, spring *etc.*) at any altitude in any country in the world. The main outcome of the study is the development of the processes for identifying best methods for wind power generation which is applicable to any location.

Table 11 presents the variation of wind speed during different months of the year and the annual mean values. From the average values, the variation of the maximum and minimum wind speed occurred

in April to September and October to March respectively at every station. At Station-I, the maximum and minimum wind speed occurred in June and October with values of 5.07 m/s and 2.57 m/s, respectively. The *COV* and standard deviation at this site varies between 21.01%–35.68% and 0.54–2.13, respectively. On the other hand, Station-III has wind speeds similar to Station-I, but the *COV* and standard deviation varies between 15.93%–129.53% and 0.71–13.07, respectively. In August and September, the *COV* is 50.1% and 129.53%, which means that these months have some irregular wind behavior, *i.e.*, gusty wind exists during these months. Station-II has wind behavior (wind speed) performance lower than other stations. Therefore, Station-I has shown better wind characteristics than the other Stations, hence it is the selected wind site. The mean available power and energy in the wind at 30 m height at Station-I is analyzed below.

Table 11. Monthly mean wind speed, power law coefficient (η), the Weibull shape factor (*k*) and scale factor (*c*) at 20 m, 30 m and 50 m height.

Month	Power law coefficient η	Measured value						Extrapolated value		
		At 20 m height			At 30 m height			At 50 m height		
		v_{20}	k_{20}	c_{20}	v_{30}	k_{30}	c_{30}	v_{50}	k_{50}	c_{50}
January	0.262	2.15	4.33	2.35	3.11	4.42	3.40	3.55	4.63	3.89
February	0.25	2.62	4.18	2.89	3.57	4.41	3.91	4.05	4.62	4.44
March	0.251	2.65	4.12	2.91	3.54	4.41	3.87	4.02	4.62	4.4
April	0.219	3.76	3.86	4.16	4.98	3.58	5.53	5.57	3.75	6.18
May	0.219	3.81	3.45	4.25	4.97	3.30	5.55	5.58	3.46	6.21
June	0.218	3.82	3.95	4.23	5.07	3.84	5.63	5.69	4.02	6.29
July	0.207	4.44	3.41	4.93	5.74	3.82	6.34	6.39	4.00	7.05
August	0.203	4.75	2.91	5.34	5.97	3.17	6.69	6.66	3.32	7.42
September	0.218	3.81	2.4	4.31	4.95	2.52	5.60	5.56	2.64	6.26
October	0.28	1.76	4.36	1.94	2.57	5.65	2.79	2.98	5.92	3.22
November	0.266	2.12	3.06	2.34	2.98	3.70	3.26	3.37	3.87	3.73
December	0.256	2.55	1.99	2.89	3.22	2.49	3.63	3.68	2.61	4.14

The monthly mean available power is analyzed using both MOM and MLM methods in Table 12. From this table, it can be seen that the available power from April to September was above 100 W/m^2 and the maximum and minimum power occurred in August, 179.36 W/m^2, and October, 11.8 W/m^2, respectively. Only two statistical tools (RMSE and Chi-square error) have been used for comparing the MOM and MLM methods. Here, the MLM method gives better performance with minimum error. Using the MLM method, the Weibull shape factor and scale factor vary between 2.52–5.65 and 2.79–6.69 m/s. respectively.

Therefore, the six months from April to September show more potential wind generated power than other months in the year. In this research work, the Weibull parameters at heights of 30 m and 20 m were determined.

Table 13 presents the seasonal analysis of wind speed, Weibull parameters and available power at 20 m and 30 m height. It also presents the extrapolated values at 50 m. The months in each season can be classified as winter (November, December and January), spring (February, March and April), summer (May, June and July), and autumn (August, September and October).

Table 12. Monthly mean power based on measured data and calculated data by statistical methods.

Month	Power (W/m²)	MOM					MLM				
		k (-)	c (m/s)	P (W/m²)	$RMSE$	χ^2	k (-)	c (m/s)	P (W/m²)	$RMSE$	χ^2
January	18.75	4.50	3.41	21.84	1.7578		4.42	3.4	21.67	1.7088	
February	27.87	4.71	3.90	32.63	2.1817		4.41	3.91	33.24	2.3173	
March	26.71	4.92	3.86	31.40	2.1656		4.41	3.87	31.96	2.2913	
April	73.11	3.58	5.53	97.57	4.9457		3.58	5.53	97.57	4.9457	
May	71.47	3.25	5.55	101.46	5.4763		3.3	5.55	101.04	5.4378	
June	75.87	3.6	5.63	102.85	5.1942	0.9904	3.84	5.63	101.21	5.0339	0.9724
July	110.11	3.98	6.34	143.45	5.7741		3.82	6.34	144.7	5.8813	
August	123.88	3.06	6.68	181.11	7.5651		3.17	6.69	179.36	7.4485	
September	70.61	2.48	5.58	118.12	6.8927		2.52	5.6	117.25	6.8293	
October	10.05	5.47	2.78	11.70	1.2845		5.65	2.79	11.8	1.3229	
November	16.21	4.42	3.27	19.38	1.7804		3.7	3.26	19.82	1.9000	
December	20.81	2.48	3.63	32.46	3.4132		2.49	3.63	32.37	3.4000	

Table 13. Seasonal mean wind speed in (m/s), the Weibull shape factor (-), scale factor (m/s) and power density (W/m²) at 30 m and 50 m height.

Seasons	η	Measured data								Extrapolated data			
		At 20 m height				At 30 m height				At 50 m height			
		v_{20}	k_{20}	c_{20}	P_{20}	v_{30}	k_{30}	c_{30}	P_{30}	v_{50}	k_{50}	c_{50}	P_{50}
Winter	0.24	3.19	3.5	3.55	26.21	4.22	3.78	4.7	59.1	4.76	3.96	5.27	83.6
Spring	0.24	3.01	4.05	3.32	20.2	4.03	4.13	4.4	48.2	4.55	4.33	5.01	68.7
Summer	0.21	4.02	3.6	4.47	48.64	5.26	3.65	5.8	108.24	5.89	3.83	6.52	157.2
Autumn	0.23	3.44	3.22	3.86	34.1	4.5	3.78	5.1	72.3	5.07	3.96	5.63	100.67

The mean velocity, shape factor, scale factor, *etc.* at 50 m were extrapolated using Equations (44)–(48) and these extrapolated results are presented in Table 14.

Table 14. Wind power classification.

Power class	Potential	Power density and wind speed at 10 m (33 ft)		Power density and wind speed at 30 m (98 ft)		Power density and wind speed at 50 m (164 ft)	
		Power (W/m²)	Speed (m/s)	Power (W/m²)	Speed (m/s)	Power (W/m²)	Speed (m/s)
Class-I	Poor	$P_{10} \leq 100$	≤4.4	$P_{30} \leq 160$	≤5.1	$P_{50} \leq 200$	≤5.6
Class-II	Marginal	$P_{10} \leq 150$	≤5.1	$P_{30} \leq 240$	≤6.0	$P_{50} \leq 300$	≤6.0
Class-III	Moderate	$P_{10} \leq 200$	≤5.6	$P_{30} \leq 320$	≤6.5	$P_{50} \leq 400$	≤7.0
Class-IV	Good	$P_{10} \leq 250$	≤6.0	$P_{30} \leq 400$	≤7.0	$P_{50} \leq 500$	≤7.5
Class-V	Very good	$P_{10} \leq 300$	≤6.4	$P_{30} \leq 480$	≤7.5	$P_{50} \leq 600$	≤8.0
Class-VI	Excellent	$P_{10} \leq 400$	≤7.0	$P_{30} \leq 640$	≤8.2	$P_{50} \leq 800$	≤8.8
Class-VII	Excellent	$P_{10} \leq 1000$	≤9.4	$P_{30} \leq 1600$	≤11.0	$P_{50} \leq 2000$	≤11.9

Table 15 also includes yearly or annual mean values. According to this table, the maximum and minimum values of wind speed and wind power have been observed in summer and spring with the values of 5.26 m/s and 4.03 m/s and the power is 108.24 W/m² and 48.16 W/m² respectively at 30 m

height. In the literatures, Elliot *et al.* [100], Yu *et al.* [101], Ilinca *et al.* [102] and Zhou *et al.* [103] classified the wind power in seven categories which are shown in Table 14. Considering these wind power classes, it has been found that Station-I has poor wind power at every height (*i.e.*, 20 m, 30 m and 50 m). For a more realistic analysis, the available energy in the wind of this site was determined by using numerical iteration methods and is presented in Table 15.

Table 15. The most frequent wind velocity (V_{Fmax}), velocity contributing the maximum energy (V_{Emax}), energy density (E_D) and total energy intensity (E_{DT}).

Seasons	V_{Fmax} (m/s)	V_{Emax} (m/s)	Energy density, E_D (kW/m^2)	Total energy intensity, E_{DT} (kW/m^2)
Winter	4.31	5.24	0.0586	129.408
Spring	4.15	4.88	0.0492	105.082
Summer	5.35	6.58	0.1148	253.531
Autumn	4.64	5.63	0.0728	160.667
Yearly mean	4.31	5.24	0.0586	513.412

The energy estimation of wind regimes by the Weibull based approach has been presented in Table 15. It also shows the most frequent wind speed, velocity contributing the maximum energy, energy density and total energy intensity. From the above table, it can be clearly seen that the energy density and total energy intensity are higher in summer than during autumn seasons and also are a minimum in spring. Therefore, summers have more wind potential than any other seasons. To find out the energy generated by a wind turbine, a small model wind turbine (NACA 4418 turbine power curve) with rated power of 20 kW or 0.02 MW, rated speed of 8 m/s and hub height of 30 m has been used for this calculation. Values of capacity factor and total energy output were obtained using numerical integration methods of Trapezoidal Sums and Simpson's 1/3 Rule and the values were verified to four decimal places. From Table 16, it can be seen that, in summer, 13.2938 MW·h of energy can be extracted by the suggested model wind turbine. This means that only 147.67 kW/day of power can be generated by the wind turbine in summer and the value becomes 50% less in the rainy season. Therefore, it can be finally concluded that the analytical results show that Station-I does not have the necessary available wind power potential for large turbines but has sufficient wind power for small wind turbine both for electricity generation and pumping water for irrigation.

Table 16. Energy generated by the wind turbine and the capacity factor by numerical integration. The turbine rated power is 20 kW or 0.02 MW, rated speed is 8 m/s and hub height at 30 m.

Seasons	Numerical integration			
	Trapezoidal sums		Simpson's rule	
	Capacity factor, C_F	Total energy output, E_T (MW·h)	Capacity factor, C_F	Total energy output, E_T, (MW·h)
Winter	0.1238	5.4689	0.1238	5.4689
Spring	0.0920	3.9283	0.0920	3.9283
Summer	0.3010	13.2937	0.3010	13.2938
Autumn	0.1698	7.5005	0.1698	7.5005
Yearly mean	0.1238	21.6973	0.1238	21.6974

4. Conclusions

In this work, statistical diagnosis of the best Weibull distribution methods for wind data analysis is presented. By using the available wind data, the values of shape factor *k* and scale factor *c* were determined using seven methods and were then investigated as to how efficiently the methods can estimate the Weibull factors with minimum error. To satisfy the main objectives of this work, six statistical tools were used to find the best method of Weibull distribution. It is important to note that any one of these statistical tools, namely RPE, RMSE, MPE, MAPE, chi-square error (χ^2), and analysis of variance or efficiency of the method (R^2) is good enough to rank the methods, however, analysis using all of them was done to rank more precisely. The results show that the MOM and MLM are the most efficient methods for determining the value of *k* and *c* to fit the Weibull distribution curves. The PDM is more efficient for low altitude wind data but is not efficient for higher altitude wind data. MOM and MLM methods are more efficient with less error and are applicable for any altitude. Other methods such as MMLM, EEM, GM and STDM are the least efficient methods to fit the Weibull distribution curves for the assessment of wind speed data. Another objective of this work was to find the best wind site using the best Weibull distribution methods and calculate available wind power. Monthly mean wind speed was found to be relatively higher in Station-I than that of the other sites. As a result this is our selected site. The MLM method has shown better results than MOM in calculating monthly mean wind power at Station-I. The value of shape factor, scale factor, wind speed and power was determined at 50 m height using extrapolation of numerical equations to satisfy the wind power classes as discussed in this paper. The poor class wind power has been found in this site in each altitude. Furthermore, energy density and total energy intensity per unit area has been analyzed by numerical iteration methods. Finally, the energy extracted by a small model wind turbine has been analyzed by using numerical integration methods of Trapezoidal Sums and Simpson's 1/3 Rule. This study offers a new pathway on how to evaluate feasible locations for wind energy assessment which is applicable at any windy sites in any country in the world.

Author Contributions

The major part of the works was done by corresponding author, however, co-authors also contributed significantly. The first co-author Mohammad Golam Rasul contributed in outlining the paper, methodology and associated theories and related description. He helped also in data some analysis. The second co-author Talal Yusaf contributed in format of the paper and its alignment with themes of the journal and revisions of the manuscript. Corresponding author is grateful to co-authors for their fruitful contribution in this work.

Nomenclature

P	Total power, W/m^2
A	Area, m^2
P_e	Practically extractable power, W/m^2
k	Dimensionless shape parameter
c	Scale parameter (m/s)

$f(v)$	Weibull probability density function
$F(v)$	Cumulative distribution function
C	Constant
L	Likelihood function
COV	Coefficient of variation
v_i	Random sample of wind speed central to bin i
n	Number of sample or bin
$f(v_i)$	Weibull frequency for wind speed ranging within bin i
$f(v \geq 0)$	Probability for wind speed ≥ 0
W_{vi}	Observed frequency of the wind speed
v_m^3	Mean of the cubic wind speed, m/s
ε_{vi}	Error of the approximation
z_{ref}	Reference height, m
k_h	Weibull shape factor at desired height
c_h	Weibull scale factor at desired height, m/s
v_h	Wind speed at desired height, m/s
P_h	Power at desired height, W/m^2
N	Total number of observations
$y_{i,m}$	i-th calculated value from measured data
$x_{i,w}$	i-th calculated value from the Weibull distribution
$z_{i,\bar{v}}$	Mean of i-th calculated value from measured data
R^2	Analysis of variance
Γ	Gamma function
σ	Standard deviation of wind speed, m/s
ρ	Air density, kg/m^3
θ	Unknown parameter for maximum likelihood function
η	Power law coefficient
χ^2	Chi-square error

Conflicts of Interest

The authors declare no conflict of interest.

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