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Game Optimization Theory and Application in Distribution System Expansion Planning, Including Distributed Generation

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Received: 15 November 2012; in revised form: 12 January 2013 / Accepted: 29 January 2013 / Published: 21 February 2013

Abstract: Based on Game Theory and Multi-objective optimization problems (MOP), Game Optimization Theory (GOT) is discussed in this paper. Optimization Stability Analysis (OSA), Distance Entropy Multi-Objective Particle Swarm Optimization (DEMPSO) and Fuzzy Multi-weights Decision-making Method (FMW) are proposed as well. Game Optimization Theory, which is a comprehensive system, could not only handle multi-objective optimization problems effectively, but also could offset the disadvantages of traditional optimization theories, such as lack of framework and the insufficient consideration of relevant elements. In this paper GOT is used for the first time in solving the distribution systems planning (DSP) issue by implementing distributed generation. The proposed model integrates costs, losses, and voltage index to achieve optimal size and site of distributed generation. The model allows minimizing total system costs, system power losses and maximizing voltage improvement. A detailed DSP example is used for verifying the effectiveness and reasonableness of GOT in this context.

Keywords: game theory; multi-objects optimization; game optimization theory; distributed generation; distribution system planning

Nomenclature:

N total number of system buses
 M total number of load buses

Tu	total number of substation transformers
C_{ij}	the total feeder costs from i to j
C_e	electricity market price
C_{iu}	the costs of potential transformer u in substation i
C_{ri}	the operation costs for DG
T	horizon planning year
β	present worth factor
d	discount rate
P_{iu}	transformer u in substation i dispatched power
S_{DGi}	power generated from distributed generation
pf	system power factor
σ_{ij}	feeder i to j binary decision variable
σ_{iu}	transformer u in substation i binary decision variable
σ_{DGi}	distributed generation binary decision variable
V	bus voltage
Y	the line admittance
θ_{ij}	the angle of admittance connecting i to j
σ_{ji}	the difference of phase of voltage
P_{ij}	the active power flow from node i to j
Q_{ij}	the reactive power flow from node i to j
R	the line resistance
X	the line reactance
S_{ij}	power flow in feeder connecting bus i to j
$S_{ij\max}$	feeder's thermal capacity limit
$S_{DGi\max}$	distributed generation capacity limit
$SS_{\max}$	substation capacity limit

1. Introduction

Game theory, as a branch of applied mathematics, is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. In addition to being used to describe, predict and explain behavior, game theory has also been used to develop theories of ethical or normative behavior. Game theory is mainly used in economics, political science and psychology, as well as logic and biology [1–4].

Multi-objective optimization problems (MOP) are the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. Multi-objective optimization problems can be found in various fields: product and process design, finance, aircraft design, or wherever optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives.

The optimization problems in power system generally belong to MOP, such as Reactive Power optimization, Unit Commitment, and Substation Locating and Sizing optimal planning. The problem of Distributed Generation (DG) optimal planning in distribution systems also belongs to MOP. However, the traditional research achievements concerning DG planning have some deficiencies. In [5], network losses work is selected as objective function and tabu search is applied for finding the optimal allocation of DG, but lacks a comprehensive mathematical model by only using network losses as objective function. In [6], the influences of losses made by implementing DG in a distribution system is analysed; it also lacks a reasonable objective function and a sufficient consideration of the constraints. The above theses choose losses to serve as object function, and cannot formulate a scientific model to assess the effects of implementing DG in distribution systems. In [7–10], the theses construct multi-objective mathematical models, but they convert the multi-objective function into the single objective function, a method that not only omits the relationship among objectives, but also results in the absence of a comprehensive optimization perspective to find the global optimal result. In [11,12], though the theses build reasonable multi-objective functions, they lack a systematic optimization method to handle multi-objectives.

From the above, we can conclude that scientific mathematical models and reasonable optimization methods are needed in solving the DSP by implementing DG. However, traditional optimization methods cannot formulate comprehensive systems to deal with optimal problems. The way the traditional optimization methods generally work is to calculate directly using an optimization algorithm after the objective functions are selected. This mode cannot evaluate the influence of improved approaches, and also they still cannot formulate a general frame to deal with optimal problems. The common methods always convert multi-objective optimizations into single-objective optimizations, however, this approach contains little effective information to obtain the internal relations among objective functions. A tentative solution is called Pareto optimal if there is no other solution yielding at least one better objective without worsening any of the rest. Finding such non-dominant solutions, and quantifying the trade-offs in satisfying the different objectives, is the main problem when setting up and solving MOP.

Combining Game Theory with MOP, Game Optimization Theory (GOT) is formulated to handle MOP in this paper. By presenting Optimization Stability Analysis (OSA) and establishing a new type of game model, GOT constructs a main frame about optimization, it can find non-dominant solutions quickly and qualify the trade-offs to satisfy the different objectives comprehensively. A typical MOP problem in power system concerning DG planning is processed by using GOT, and the results shows that GOT could find a reasonable DSP solution. GOT can be used in other MOP problems in power systems as well. The main content of this paper consists of two parts, one is the introduction of GOT, and the other is the analysis of DSP. The two parts correspond to the two main problems in embedding DG in distribution systems, one is the lack of scientific mathematical models and the other is the lack of comprehensive optimization methods. In this paper, GOT is used to handle DG planning for the first time, and obtain a scientific, reasonable result.

2. Game Theory Concepts

GOT is the development and extension of Game Theory. The application of GOT in power systems expands a new research field, and for this purpose it is necessary some describe some concepts of Game Theory. A normal game consists of a set of players, a set of strategies available to those players, and a specification of payoffs for each combination of strategies [4].

The common situation is n-player game in which the players are numbered 1 to n and an arbitrary player is called player i . S_i denotes the set of strategies available to player i (called i 's strategy space), s_i denotes an arbitrary member of this set ($s_i \in S_i$ indicates that strategy s_i is a member of strategies S_i). $(s_1, \dots, s_n)$ denotes a combination of strategies, u_i denotes players i 's payoff function, $u_i(s_1, \dots, s_n)$ denotes the payoff of the player i which is dependent to all players' chosen strategy.

Definition 1. The normal-form representation of an n-player game specifies the players' strategy spaces $(S_1, \dots, S_n)$ and their payoff functions $(u_1, \dots, u_n)$. We denote this game by $G = \{S_1, \dots, S_n; u_1, \dots, u_n\}$.

Definition 2. In the normal-form game $G = \{S_1, \dots, S_n; u_1, \dots, u_n\}$, let s_i' and s_i'' be feasible strategies for player i , (i.e., s_i' and s_i'' are members of S_i). Strategy s_i' is strictly dominant by strategy s_i'' if for each feasible combination of the other players' strategies, i 's payoff from playing s_i' is strictly less than i 's payoff from playing s_i'' : $u_i(s_1, \dots, s_{i-1}, s_i', s_{i+1}, \dots, s_n) < u_i(s_1, \dots, s_{i-1}, s_i'', s_{i+1}, \dots, s_n)$, for each strategies combination $(s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$ that can be constructed from the other players' strategy spaces $S_1, \dots, S_{i-1}, S_{i+1}, \dots, S_n$.

Each player's predicted strategy must be that player's best response to the predicted strategies of the other players. This prediction is called Nash equilibrium.

Definition 3. In the n-player normal-form game $G = \{S_1, \dots, S_n; u_1, \dots, u_n\}$, the strategies $(s_1^*, \dots, s_n^*)$ are a Nash equilibrium if, for each player i , s_i^* is (at least tied for) player i 's best response to the strategies specified for the n-1 other players $(s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_n^*)$:

$$u_i(s_1^*, \dots, s_{i-1}^*, s_i^*, s_{i+1}^*, \dots, s_n^*) \geq u_i(s_1^*, \dots, s_{i-1}^*, s_i, s_{i+1}^*, \dots, s_n^*).$$

These concepts are the foundations of Game Optimization Theory.

3. Game Optimization Theory

Inspired by Game Theory and MOP, GOT is proposed in this paper and used in the field of power system optimal problems for the first time. GOT could assess the value of optimization programming problems and could handle MOP more flexibly than other methods, and it formulates a comprehensive optimization system that could be used in engineering practice. The optimization problems in power system generally belong to MOP, such as that we generally select losses and voltage as objective functions in reactive power optimization, and select costs and losses in substation planning. Therefore, the prospect of the application of GOT in power system is wide-reaching and practicable. The structure of GOT mainly consists of three parts:

- (1) Optimization Stability Analysis (OSA);
- (2) Iterated Elimination of Strictly Dominant Strategies;
- (3) Generate the final strategy.

The above parts compose a whole system and formulate a new optimization theory.

3.1. Optimization Stability Analysis

There is a lack of criteria to evaluate the influence on engineering practice made by improved approaches, and also a lack of mathematical models to judge their value. The usual optimal problem method is to calculate directly with an optimization algorithm after the objective functions are selected. However, this process cannot explain the reasons of the introduction of the improved approach. Therefore, Optimization Stability Analysis is proposed in this paper, as a part of GOT, to offset the insufficient consideration of existing methods.

Optimization Stability is an index to assess the efficiency of an improved approach. A specific system will generate two states, initial state and the final one, after a MOP is established. The initial state is the pre-status before the system is optimized, and final state represents the optimized status. OSA can reach a conclusion to evaluate an improved approach by analyzing the relations between initial state and final state. Evolutionary stable strategy is introduced to analyze the Optimization Stability; the specific process is discussed below.

Evolutionary stable strategy (ESS) was defined and introduced by Maynard Smith and Price in a 1973 *Nature* paper. ESS is a concept belongs to Evolutionary Game Theory. It is a strategy such that, if all the members of a population adopt it, then no mutant strategy could invade the population under the influence of natural selection. An ESS is an equilibrium refinement of the Nash Equilibrium [13,14].

In a specific optimal problem, initial state and final state are similar to two species in a population, where the initial state is the original species and the final state is the mutant one; the improved approach is similar to the mutation of species. When a new game is formulated between two states, we call it Similar Game (SG, the players of a game are two different states for a same participant). If the result of SG is evolutionarily stable, then the Optimization Stability of the optimal problem is stable. Tables 1 and 2 are used to explain the progress of OSA.

Table 1. SG matrix, Optimization Stability Analysis (1).

	<i>O</i>	<i>M</i>
<i>O</i>	$\alpha_{11}; \beta_{11}$	$\alpha_{12}; \beta_{12}$
<i>M</i>	$\alpha_{21}; \beta_{21}$	$\alpha_{22}; \beta_{22}$
	$1 - \varepsilon$	ε

Notes: *O*: original species; initial state; *M*: mutant species; final state; ε : mutant probability; the scale of improved approach.

Table 2. SG matrix, Optimization Stability Analysis (2).

	<i>O</i>	<i>M</i>
<i>O</i>	$\alpha_{11}; \beta_{11}$	$\alpha_{12}; \beta_{12}$
<i>M</i>	$\alpha_{21}; \beta_{21}$	$\alpha_{22}; \beta_{22}$
	ε	$1 - \varepsilon$

Notes: *O*: original species; initial state; *M*: mutant species; final state; ε : mutant probability; the scale of improved approach.

Tables 1 and 2 represent two extreme situations, where one situation assumes the original species is the majority and another is when the mutant species is the majority. In the OSA process, ε is a small positive number, α and β are the payoffs to the original and mutant species when they choose their strategies, α represents the payoffs of the original species, β represents the payoffs of the mutant species. Strategies combination (O,O) denotes that the final state is only the scale expansion of the initial state, there is no improved approach is adopt to the optimal system, α_{11} denotes the payoffs of the initial state under the selected objective functions, β_{11} denotes the payoffs produced by the improved approach (β_{11} is 0 in this strategies combination); strategies combination (O,M) denotes that an improved approach is adopted to the optimal problem, α_{12} denotes the payoffs of the initial state when the improved approach is adopted, β_{12} denotes the payoffs produced by the improved approach; strategies combination (M,O) denotes the interchange of initial state and final state (*i.e.*, this strategies combination is the inverse process of optimization), α_{21} denotes the payoffs of the optimized initial state, β_{21} denotes the payoffs produced by the inversion process; strategies combination (M,M) denotes the initial state is the optimized state, and the final state is the expansion of the initial state, α_{22} is the payoff of the initial state, β_{22} denotes the payoffs produced by the improved approach.

When the original species is a majority in the population as shown in Table 1, the expectation payoffs of the original species and mutant species correspond to A and B :

$$O: \alpha_{11} \times (1 - \varepsilon) + \alpha_{12} \times \varepsilon = A \quad (1)$$

$$M: \alpha_{21} \times (1 - \varepsilon) + \alpha_{22} \times \varepsilon = B \quad (2)$$

If the optimization stability is stable, it should satisfy the condition $B > A$; this result represents that the improved approach will produce better payoffs.

When the mutant species is the majority in the population as shown in Table 2, the expected payoffs of the original species and mutant species correspond to C and D :

$$O: \alpha_{11} \times \varepsilon + \alpha_{21} \times (1 - \varepsilon) = C \quad (3)$$

$$M: \alpha_{21} \times \varepsilon + \alpha_{22} \times (1 - \varepsilon) = D \quad (4)$$

If the optimization stability is stable, it should satisfy the condition $D > C$; this result represents that the initial state cannot produce better payoffs.

In addition to the conclusions made above, another two inequality constraints should be satisfied to guarantee the stability of optimization:

$$\alpha_{12} + \beta_{12} \geq \alpha_{11} + \beta_{11} \quad (5)$$

$$\alpha_{22} + \beta_{22} \geq \alpha_{21} + \beta_{21} \quad (6)$$

The values of α and β are closely related to the object functions, and the OSA process needs a large number of historical data which relate to the optimal problem. From the analysis above, the OSA result could assess the improved approach from a comprehensive viewpoint. Then the general criteria of optimization stability could be summarized as two comparison and two criteria. This criterion formulates a mathematical model to evaluate the improved approach and can be used as guidance for the engineering practice.

3.2. Iterated Elimination of Strictly Dominant Strategies

Iterated elimination of strictly dominant strategies is based on the appealing idea that rational players do not play strictly dominant strategies. Iterated elimination of strictly dominant strategies could generate Pareto non-dominant set completely.

3.2.1. The Establishment of a Game

In mathematic terms, MOP can be written as:

$$\begin{aligned} \text{minimize } y = f(\bar{x}) &= (f_1(\bar{x}), f_2(\bar{x}), \dots, f_m(\bar{x}), \dots, f_M(\bar{x})), \quad m = 1, 2, \dots, M \\ \text{s.t.: } g_i(\bar{x}) &\geq 0 \quad i = 1, 2, \dots, k; h_j(\bar{x}) = 0 \quad j = 1, 2, \dots, l \end{aligned}$$

$f_i(\bar{x})$ is the i -th objective function, $g(\bar{x})$ and $h(\bar{x})$ are the inequality and equality constraints, respectively, and $\bar{x}$ is the vector of optimization or decision variables. Instead of being a unique solution to the problem, the solution to a MOP is a possibly infinite set of Pareto points.

Pareto-optimality is used to handle multiple objectives. A decision vector, $\bar{x}^* \in F$ is Pareto-optimal if there does not exist a decision vector, $\bar{x} \neq \bar{x}^* \in F$ that dominates it. An objective vector, $f^*(\bar{x})$, is Pareto-optimal if $\bar{x}$ is Pareto-optimal. The concept of Pareto-optimality, first introduced by Francis Ysidro Edgeworth [15], is named after Vilfredo Pareto.

For a specific MOP, it should be possible to find non-dominant solutions after the OSA step. To find non-dominant solutions, a game must be established and an optimization algorithm must be selected.

For constructing a suitable game model, Similar Mixed Game (SMG) is proposed in this paper; its description is as follows:

Definition 4: In the normal-form game $G = \{S_1, \dots, S_n; u_1, \dots, u_n\}$, suppose $S_i = \{s_{i1}, \dots, s_{ik}\}$. Then a similar mixed strategy for player i is a fuzzy vector $P_i = \{s_{i1}, \dots, s_{ik}\}$, where $k = 1, \dots, K$, and $p_{ik} \leq 1$, P_{ik} is a degree of membership belongs to pure strategy s_{ik} .

Table 3. Optimized classification among three object functions.

		F ₁		
		Low	Middle	High
F ₂	Low	a ₁ ; b ₁ ; c ₁	a ₁ ; b ₂ ; c ₁	a ₁ ; b ₃ ; c ₁
	Middle	a ₂ ; b ₁ ; c ₁	a ₂ ; b ₂ ; c ₁	a ₂ ; b ₃ ; c ₁
	High	a ₃ ; b ₁ ; c ₁	a ₃ ; b ₂ ; c ₁	a ₃ ; b ₃ ; c ₁

The set of strategies for each player is the non-dominant set generated by the optimization algorithm. The payoffs for each player relate to the decision maker's degree of attention. A 3-objective optimization problem works as an example to describe the structure of SMG and the process of generating a non-dominant set.

The object functions are F₁, F₂ and F₃. We use fuzzy mathematics to determine the hierarchical organization for each object function, as Table 3 shows (the level of F₃ is low). (Low; Low; High), (Low, Middle, High), and other strategy pairs are the combinations of strategies. Distance Entropy Multi-Objective Particle Swarm Optimization (DEMPSO) is used to generate a non-dominant set and

Fuzzy Multi-weights Decision-making Method (FMW) is used to quantify the trade-offs in satisfying the different objectives. The trade-offs in this game are called Similar Nash Equilibrium (SNE).

The SMG is suitable for the optimization programming problem in engineering practice, as it converts an optimization procedure into a process of seeking SNE in SMG. The SNE exists in the strategy space of the players in SMG, it is the optimal response and best trade-off for each player, for example, the SNE is the optimal scheme of the size and site of DG in the game of DG optimal planning. SMG, as a part of GOT, could offer integrity strategy space and supply a scientific method to find SNE in a specific game.

3.2.2. Iterated Elimination of Strictly Dominant Strategies

The essence of generating a non-dominant set is the updating of non-dominant solutions; this progress can be reasonably shown in Table 4 and Figure 1.

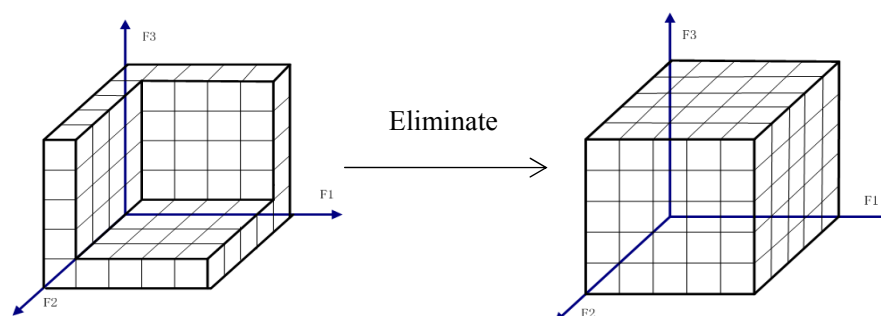
Table 4. Iterated Elimination of Strictly Dominant Strategies.

		F_1		
		Low	Middle	High
F_2	Low	$a_1; b_1; c_1$	$a_1; b_2; c_1$	$a_1; b_3; c_1$
	Middle	$a_2; b_1; c_1$	$a_2; b_2; c_1$	$a_2; b_3; c_1$
	High	$a_3; b_1; c_1$	$a_3; b_2; c_1$	$a_3; b_3; c_1$

↓ Eliminate

		F_1		
		Low	Middle	High
F_2	Low	$a_1; b_1; c_1$	$a_1; b_2; c_1$	$a_1; b_3; c_1$
	Middle	$a_2; b_1; c_1$	$a_2; b_2; c_1$	$a_2; b_3; c_1$
	High	$a_3; b_1; c_1$	$a_3; b_2; c_1$	$a_3; b_3; c_1$

Figure 1. The schematic diagram of iterated elimination of strictly dominant strategies.



An improved Particle Swarm Optimization is used to generate a non-dominant set. Particle Swarm Optimization (PSO) is attributed to Kennedy, Eberhart and Shi and was first intended for simulating social behaviour, as a stylized representation of the movement of organisms in a bird flock or fish school [16,17]. The algorithm was simplified and it was observed to perform optimization. Combining Pareto-optimality with PSO, Coello proposed multi-objective particle swarm optimization (MOPSO) to handle MOP [18]. In this paper Distance Entropy Multi-Objective Particle Swarm Optimization

(DEMPSO) is proposed to improve the performance of MOPSO which turns out that Pareto front can be found quickly.

(1) Adjust Weight

Define the Evolutionary factor:

$$e = \frac{f_{1mean}(t) \times f_{2mean}(t) \times \dots \times f_{mmean}(t)}{f_{1mean}(t-1) \times f_{2mean}(t-1) \times \dots \times f_{mmean}(t-1)} \quad (7)$$

f_{imean} denotes the i -th average fitness of objective function for each particle. From the definition of Evolutionary factor, the values of e reflects the update speed of non-dominant set, when the values of e approach or maintain at 1, it represents that the algorithm is stagnate or the Pareto front is found.

Define the Aggregation factor:

$$g = \frac{f_{1p} \times f_{2p} \times \dots \times f_{mp}}{f_{1mean} \times f_{2mean} \times \dots \times f_{mmean}} \quad (8)$$

f_{ip} denotes the i -th fitness of objective function for ‘best’ particle. From the definition of Aggregation factor, the values of g reflects the degree of aggregation of particle. When the values of g approach or stay at 1, it represents that the algorithm easily sinks into a local optimum.

Introduce the Focus Distance ratio factor:

$$k = \frac{MaxDist - MeanDist}{MaxDist} \quad (9)$$

$$MaxDist = \max(\sqrt{\sum_{d=1}^D (x_{best} - x_{id})^2}) \quad (10)$$

$$MeanDist = \frac{(\sum_{i=1}^m \sqrt{\sum_{d=1}^D (x_{best} - x_{id})^2})}{m} \quad (11)$$

where m denotes the number of particles, D denotes the number of dimensions for each particle, x_{best} denotes the best position for all particle at present, x_{id} denotes the value of the i -th particle’s position in dimension d . The Focus Distance ratio factor reflects the degree of aggregation and the diversity of particle swarm from the view of spatial distance [19,20].

e , g , and k reflect the rate of optimal progress from the point of view of the speed of optimization, fitness and spatial distance. Combining these factors, it formulates the Weight adjusted factor:

$$\lambda = e + g + k \quad (12)$$

Then we use λ to adjust weight. The nonlinear expression is shown in Equation (13):

$$\omega = \begin{cases} (\omega_{\max} + |r|/2) \times |\ln \lambda| & |\lambda| > 2.1 \\ \omega_{\max} \times \omega_{\min} + |r|/2 & 0.05 \leq |\lambda| \leq 2.1 \\ (\omega_{\min} + |r|/2) / |\ln \lambda| & |\lambda| < 0.05 \end{cases} \quad (13)$$

(2) The Introduction of Entropy

sc and fc denote the number of consecutive successes and failures, respectively. A success is defined as the present fitness of object functions that dominate the fitness in the last iteration. Equations (14) and (15) are used to adjust the velocity and position of a particle:

$$x_i = (x_i + x_{best}) / 2, sc > \varepsilon_s \quad (14)$$

$$v_i(t+1) = \omega v_i(t) + cr(x_{worst} - x_i(t)), fc > \varepsilon_f \quad (15)$$

where x_{best} denotes the present global best position, x_{worst} denotes the present global worst position. The optimal choice of values for ε_s and ε_f is problem-dependent. The threshold parameters adhere to the following conditions:

$$\begin{aligned} sc(t+1) > sc(t) &\rightarrow fc = 0 \\ fc(t+1) > fc(t) &\rightarrow sc = 0 \end{aligned} \quad (16)$$

The entropy reflects the degree of confusion for a system. The function of sc is to speed up convergence, it is equivalent to the introduction of negative entropy. The function of fc is to introduce positive entropy to the system to improve the possibility of a particle escaping the local optimum.

DEMPSO could effectively prevent the algorithm from falling into a local optimum and find the Pareto front reasonably. Figure 2 shows the DEMPSO process.

Iterated elimination of strictly dominant strategies could generate a non-dominant set. This process could offer an integrated strategy space to the players in SMG. Then the optimization could go to the next step: Generate the final strategy.

3.3. Generation of the Final Strategy

Once a set of Pareto optimal solutions (strategy space) is obtained through DEMPSO, it needs to select one optimum solution (Similar Nash Equilibrium), which satisfies all the goals to some extent. The traditional method is to take fuzzy membership functions to deal with a non-dominant set, but this approach cannot process multi-objectives comprehensively. The Fuzzy Multi-weights Decision-making Method (FMW) is proposed to find SNE in SMG and generate final strategy for an optimal problem. A 3-objective optimization problem works as an example to describe the process of FMW.

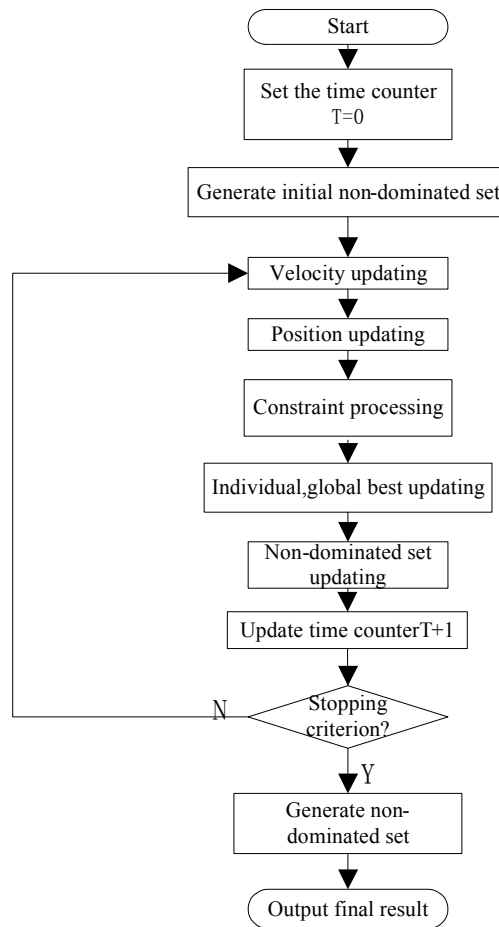
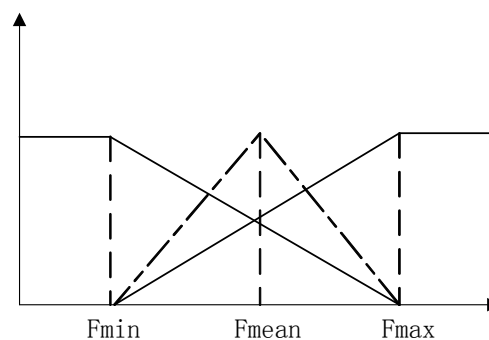
Using fuzzy mathematics to determine hierarchical organization:

$$\mu_l = \frac{F_{\max} - F}{F_{\max} - F_{\min}} \quad (17)$$

$$\mu_m = \begin{cases} \frac{F - F_{\min}}{F_{\max} - F_{\min}} \\ \frac{F_{\max} - F}{F_{\max} - F_{\min}} \end{cases} \quad (18)$$

$$\mu_h = \frac{F - F_{\min}}{F_{\max} - F_{\min}} \quad (19)$$

where $F_{\min}$, $F_{\max}$ and F_{mean} are the lower, mean and upper bounds of the i -th objective function (Figure 3), and their values are evaluated using the results obtained by optimizing each objective separately.

Figure 2. The flowchart of DEMPSO.**Figure 3.** Membership function.

As Table 5 shows, according to the fuzzy membership function of each object function, it is divided into three levels: Low, Middle and High. This method can generate 3×3 ($M \times N$, M denotes the number of object functions, N denotes the number of levels for each object function) weights to evaluate each non-dominant solution.

Then the process of finding SNE could be described as:

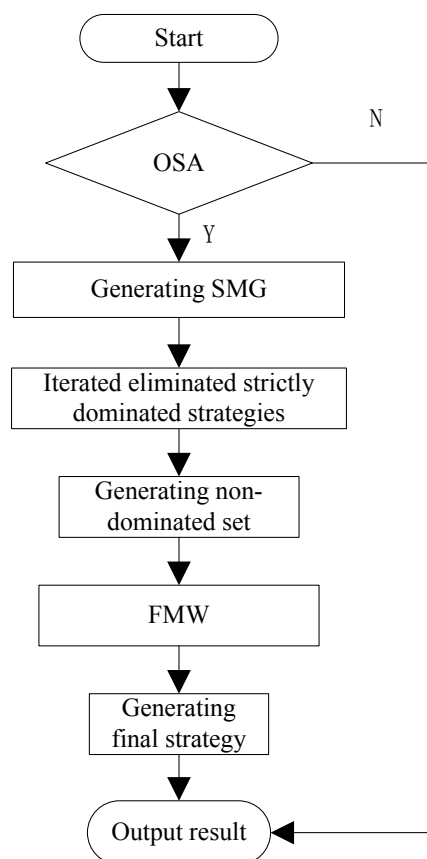
$$\begin{aligned}
 Obj_i &= \mu_i \times (a_i, b_i, c_i) \\
 &= \mu_{1l} \times a_l + \mu_{1m} \times a_m + \mu_{1h} \times a_h + \mu_{2l} \times b_l + \mu_{2m} \times b_m + \mu_{2h} \times b_h + \mu_{3l} \times c_l + \mu_{3m} \times c_m + \mu_{3h} \times c_h
 \end{aligned} \tag{20}$$

The maximum values of Obj_i in all non-dominant solutions correspond to the SNE for a SMG, that is, the final strategy in a MOP. The values of Obj correspond to the values of β in the OSA step, and reflect the payoffs produced by the improved approach. Subjective preference information could have an influence on the final strategy, and FMW could solve this question by using multi-weights to offset this shortcoming. From the analysis made above, the process of GOT can be represented as in Figure 4.

Table 5. Payoffs among object functions table.

	F_1	F_2	F_3
Low	a_l	b_l	c_l
Middle	a_m	b_m	c_m
High	a_h	b_h	c_h

Figure 4. The flowchart of GOT.



GOT formulates a complete optimization system, the OSA offers an assessment of the value of optimization, the analysis of SMG generates a non-dominant set and quantifies the trade-offs in satisfying the different objectives. To sum up, GOT supplies a new way of handling MOP, and can be widely used in the field of optimization.

4. Results and Discussion

In this paper GOT is introduced to handle the problem of Distributed Generation (DG) optimal planning in a distribution system. As mentioned above, a scientific mathematical model and reasonable

optimization method are needed in this field. A multi-objective function is constructed in this part, and GOT is used in a detailed case to test the reasonableness of the proposed mathematical model.

4.1. Mathematical Formulation

The problem of DG placement can be formulated as a non-linear optimization problem. The three sets of objectives can be:

- (1) To minimize the investment costs of DG;
- (2) To maximize the reduction in line losses;
- (3) To maximize the improvement in the voltage profile.

Costs, line losses and voltage profile represent three important indexes to assess the performance of a power system. These objectives are contradictory in nature, as the costs are based on the economy viewpoint, the losses are considered from the viewpoint of the system's operation, and the voltage profile is a reflection of the power quality. As a result, we can find that these object functions represent three different views of DG planning, however, these functions also have interactions, and any changes in an objective function will have impact on the others, and the key element is the location and capacity of DG in the distribution system. As a consequence, this is a typical MOP problem of DG optimization planning in a power system. Traditional optimization methods cannot establish the internal relations among those objectives. Therefore, GOT can be used to obtain the comprehensive trade-off solution.

4.1.1. Investment Costs

The investment costs are the key element of the planning of DG, and could decide the DG capacity and the scale of the network transformation, moreover, they will have an influence on the distribution of power flow and affect the performance of other objective functions.

The costs of investment consist of three terms: the costs of the new transformers in substation(s); the costs of DG; the costs of upgrading to new feeders [9,10]. The fixed costs are the investment costs, and the variable costs are the costs of the operation and maintenance of the equipment. The objective function is described as:

$$C = c_1 + c_2 + c_3 \quad (21)$$

$$c_1 = \sum_{i=1}^N \sum_{j=1}^M C_{ij} \delta_{ij} \quad (22)$$

$$c_2 = 8760 \sum_{t=1}^T \beta^t \left(\sum_{i=1}^N \sum_{u=1}^{Tu} C_e P_{iu} \right) + \sum_{i=1}^N \sum_{u=1}^{Tu} C_{iu} \delta_{iu}, \beta = \frac{1}{(1+d)^t} \quad (23)$$

$$c_3 = 8760 \sum_{t=1}^T \beta^t \sum_{i=1}^M C_{ri} \times pf \times S_{DGi} + \sum_{i=1}^M C_{fi} S_{DGi} \delta_{DGi} \quad (24)$$

c_1 denotes the capital costs for upgrading the feeders; c_2 denotes the investment and operation costs for substation expansion; c_3 denotes the investment and operation costs for DG.

4.1.2. Line Losses

This will produce an effect on power flow by embedding DG in the distribution system, and a reasonable location and capacity of DG could reduce network losses. It is a common method to select network losses as an objective function in the optimal planning of DG. The active power losses is an objective function, it can be described as:

$$L = \sum_{i=1}^N \sum_{j=1}^M V_i^2 Y_{ij} \cos(\theta_{ij}) - V_i V_j Y_{ij} \cos(\delta_{ji} + \theta_{ij}) \quad (25)$$

4.1.3. Voltage Profile

Voltage profile is based on the viewpoint of power quality, and it consists of voltage stability index and voltage deviation index. The voltage stability index reflects the voltage stability of a distribution system [21] and the voltage deviation index could assess the quality of the power energy. The corresponding objective function is described as:

$$V = \omega_1 \times v_1 + \omega_2 \times v_2 \quad (26)$$

$$v_1 = \sum_{i=1}^N |V_i - V_{ref}| \quad (27)$$

$$v_2 = \sum_{i=1}^N \sum_{j=1}^M (4(P_{ij}X_{ij} - Q_{ij}R_{ij}) + 4V_i^2(P_{ij}R_{ij} + Q_{ij}X_{ij})) / V_i^4 \quad (28)$$

v_1 denotes voltage deviation; v_2 denotes voltage stability index; ω denotes weight. C , L and V act as three players for a SMG, and the final strategy can be obtained by using GOT.

4.1.4. Constraints

(1) Total Power Conservation. The summation of all incoming and outgoing power should be equal to the total demand at that bus:

$$\sum_{i=1}^N (S_{ij} - \frac{\Delta V_{ij}^2}{|Z_{ij}|}) - \sum_{i=1}^M S_{ji} + S_{DGi} = D_j \quad (29)$$

(2) Distribution Feeder's Thermal Capacity. The feeder has a capacity limit for the total power flow through it during peak loads:

$$0 \leq S_{ij} \leq S_{ij\max} \quad (30)$$

(3) Distribution Substation's Capacity. The summation of total power delivered by the substation's transformers to the distribution system must be within the substation's capacity limit:

$$SS \leq SS_{\max} \quad (31)$$

(4) DG Operation. The DG's generated power must be less than the DG's capacity:

$$0 \leq S_{DGi} \leq S_{DGi\max} \quad (32)$$

(5) Voltage Drop. The voltage range should be satisfied to the maximum permissible voltage drop limit:

$$V_{i\min} \leq V_i \leq V_{i\max} \quad (33)$$

To sum up, the investment costs, losses and voltage profile represent three different views of a power system, and there are inner relationships and contradictions among the objective functions. The objective functions formulate a typical MOP problem and work as three players in a DG optimization similar to a mixed game.

4.2. DG Optimization Similar Mixed Game

The DG optimal planning constructs a SMG, the players are investment costs, losses and voltage profile, the strategy space is obtained by the iterated elimination of strictly dominant strategies and the payoffs are determined by the subjective preference. The DG optimization similar mixed game can be written as: $G_{DG} = \{C_1, C_2, C_3; u_1, u_2, u_3\}$, where C_1 denotes investment costs, C_2 denotes network losses and C_3 denotes voltage profile objective function and u_1, u_2, u_3 denote the payoffs of each player. The process of DG optimization is similar mixed game and can be shown as in Figure 5. The process of the handling the constraints can be illustrated as in Figure 6.

Figure 5. The flowchart of DG optimization similar mixed game.

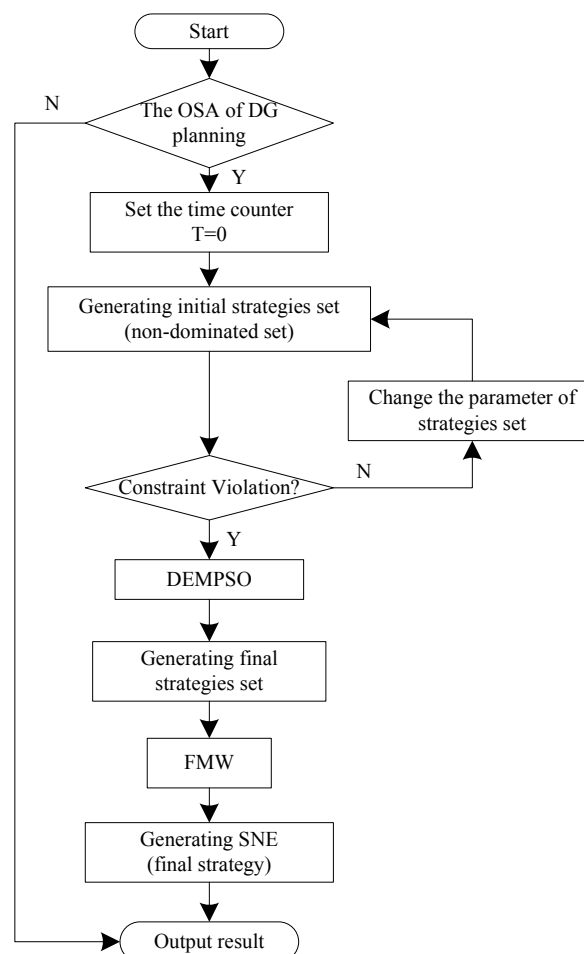
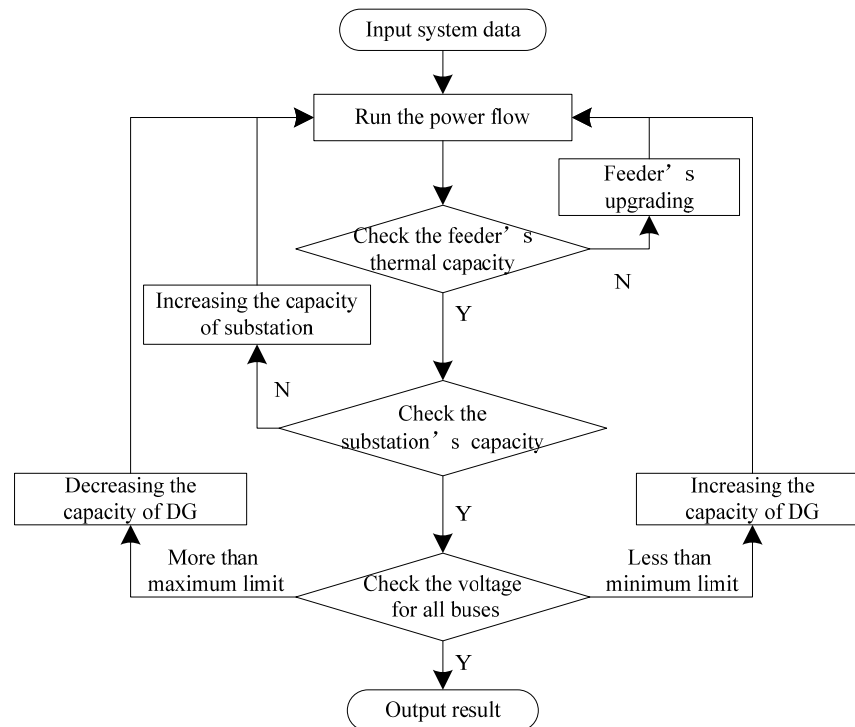


Figure 6. The flowchart of the progress of constraints.

4.3. The OSA of DG Optimization Planning

The embedding of DG in distribution system could reduce investment costs, reduce line losses and improve voltage profiles. DG could reduce environmental emissions, and a large scale DG embedded in a distribution system is beneficial to the operation of the power system [21–25]. Under the selected objectives, a SMG could be formulated, and the progress of OSA for the SMG can be described as in Table 6.

Table 6. The OSA table of DG planning.

	NDG	DG
NDG	3;0	3;1
DG	4;0	4;1

The initial state is without DG, the final state corresponds to the embedding of the DG into the distribution system, and the improved approach is the embedding of the DG. The values of 3 denote the payoffs of the initial state, the values of 1 denote the payoffs produced by the DG. From the OSA result, we can find that the optimization stability has no relation to the values of ε , and we define this situation as Strictly Optimization Stable (SOS). The OSA is the first part of GOT, and it analyses the benefits of embedding DG in the distribution system.

4.4. Case Study

The system under study is shown in Figure 7 and Tables 7 and 8. This system was also considered in [7,9–11], and it represents a typical case of DG optimization planning. It consists of one

132 kV/33 kV substation (40 MVA capacity) at bus 9 to serve eight aggregated loads (33 kV/11 kV service transformers) at buses 1–8 under normal operation conditions. There are four existing distribution feeders with a thermal capacity of 12 MVA and an impedance of $Z = 0.1738 + j 0.2819 \Omega/\text{km}$. Feeders are upgraded to a higher capacity limit of 20 MVA with an impedance $Z = 0.1469 + j 0.2719 \Omega/\text{km}$. There is forecasted load growth of 28 % 4 years after the base year and the power demand will be approximately 51.1 MVA. The kind of DG is assumed to be micro gas turbines. There should be a backup DG unit installed in case of any DG failure and for scheduled maintenance intervals.

Figure 7. System under study.

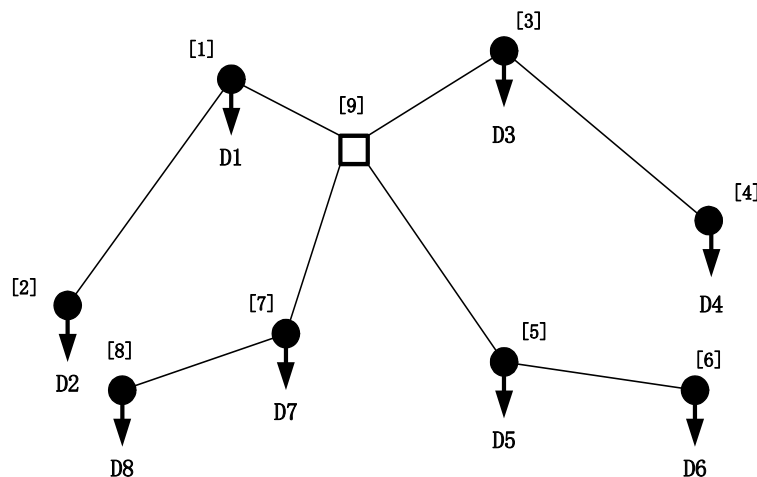


Table 7. System loading.

Bus	Base Year/MVA	Horizon Year/MVA
1	5.98	7.64
2	6.83	8.72
3	5.98	7.64
4	3.13	4.00
5	4.78	6.11
6	4.02	5.14
7	3.59	4.58
8	3.69	7.27

Table 8. The feeder's characteristics.

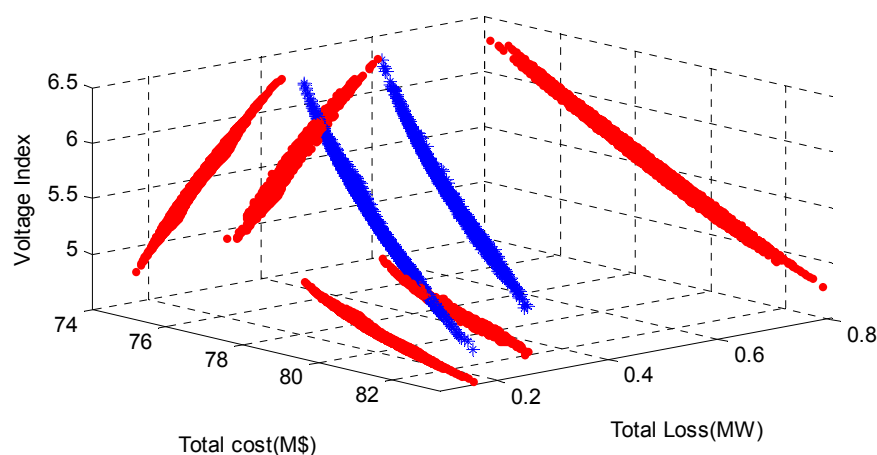
From	To	Resistance/ Ω	Reactance/ Ω	Length/km
9	1	1.390	2.255	8
9	3	2.085	3.383	12
9	5	2.259	3.664	13
9	7	1.738	2.819	10
1	2	2.780	4.510	16
3	4	2.780	4.510	16
5	6	2.433	3.946	14
7	8	2.085	3.383	12

The system power factor is set to be 0.9 and the size of the DG's is multiples of 0.1 MVA. The maximum limit of the DG capacity at each bus is 4 MVA plus the backup DG. The new transformer units used in case of substation expansion are two three phase 10 MVA transformers (132 kV/33 kV). For the cost data, the electricity market price is considered to be 70 \$/MWh for purchasing power from the main grid. The micro gas turbine sets' investment cost is assumed to be 0.5 M\$/MVA and the running cost of the DG is assumed to be 50 \$/MWh. The fixed cost of the new 10 MVA transformer is 0.2 M\$. The cost of upgrading the existing primary distribution feeder with another higher capacity is 0.15 M\$/km. The discount rate is taken as 12.5%.

4.4.1. Iterated Elimination of Strictly Dominant Strategy

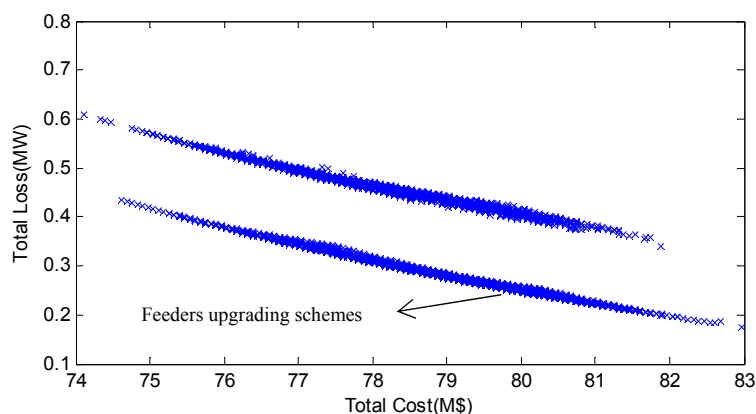
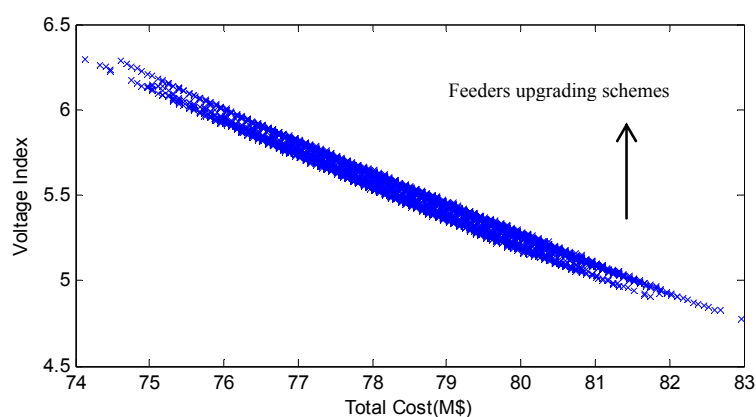
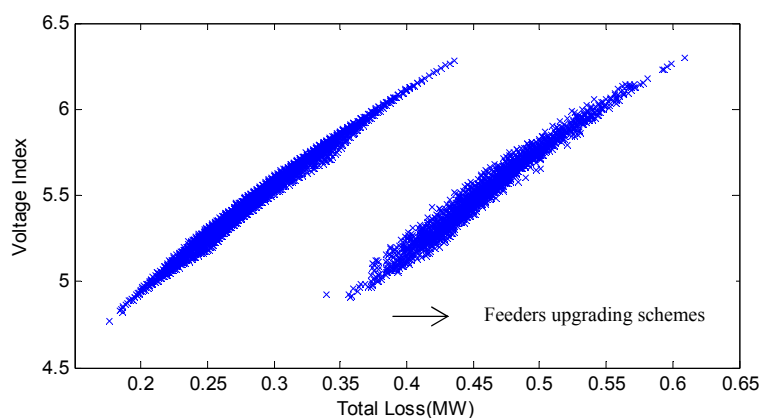
The process of optimization goes into the second step after OSA. Using DEMPSO to eliminate strictly dominant strategies, we set the number of particles at 100 and iterations at 400. The optimization result is shown in Figure 8. The non-dominant set obtained by the iterated elimination of strictly dominant strategies is shown in Figure 8, and it is the second step of GOT. Figure 8 shows the DSP optimization result, where the blue parts are the Pareto front, which consists of 4,394 non-dominant solutions.

Figure 8. Pareto front distribution graph.



Each non-dominant solution involves the information of the DG's location and capacity, the non-dominant set serves as the strategies space for each player in SMG. The Pareto front shown in Figure 8 consists of two lines, one is the feeders upgrading schemes, and the other one is consists of the schemes only embedding DG into the distribution system.

Figure 9 shows that the feeders upgrading schemes could reduce line losses more when the investment costs are equivalent. Figure 10 shows that the introduction of DG is beneficial to the voltage profile when the investment costs are same. Figure 11 shows that the feeder upgrading schemes have better performance in voltage profile when the total line losses are similar. From the analysis we can know that the feeder upgrading has an influence on the distribution of the two lines in the Pareto front.

Figure 9. Costs, losses projection graph.**Figure 10.** Costs, voltage index projection graph.**Figure 11.** Losses, voltage index projection graph.

4.4.2. Generate Final Strategy

According to the subjective preference information, we use FMW to find SNE in SMG when a non-dominant set is generated. It can find SNE through the analysis of the strategy space, and five schemes are compared in this paper, Scheme 1 is obtained by the traditional decision-making method, Scheme 2 to Scheme 5 correspond to different subjective preferences, and each scheme has specific

characteristics. From the results of comparison, we could deduce that FMW could handle subjective preference flexibility and find SNE scientifically:

Scheme 1: The final strategy is proposed by fuzzy method [26];

Scheme 2: The final strategy is proposed by FMW, the subjective preference is total costs;

Scheme 3: The final strategy is proposed by FMW, the subjective preference is total losses;

Scheme 4: The final strategy is proposed by FMW, the subjective preference is voltage profile;

Scheme 5: The final strategy is proposed by FMW, there is no subjective preference among the objectives.

Table 9 shows the performances in total costs among the five schemes, where Schemes 2, 3 and 5 are feeder upgrading schemes. Scheme 1 cannot reflect the influence of subjective preference information. Scheme 4 has the best performance in voltage profile, however, the DG embedded capacity approaches 35.5 MW, it is larger than the other schemes, so this configuration could reduce substation operation costs but increase DG investment costs. Scheme 2 has preference on total costs, it takes an improved feeder upgrading approach to reduce costs, and it has the minimum DG embedded capacity (26.6 MW). Scheme 3 has preference on line losses; this scheme requires the upgrading of feeders to reduce line losses, and the DG embedded capacity is 29.9 MW. Scheme 5 is a comprehensive strategy compared with Scheme 1, it has lower total costs and the DG embedded capacity is 29.6 MW.

Table 9. Costs of five schemes.

Economical parameter	Scheme1	Scheme2	Scheme3	Scheme4	Scheme5
Number of Feeders Upgrades	0	1	1	0	1
Feeders Fixed Cost/M\$	0	1.2	1.2	0	1.2
Number of New Transformers	0	0	0	0	0
Substation Total Cost/M\$	34.21	39.92	36.71	31.58	37.01
DG Capacity/MVA	32.7	26.6	29.9	35.4	29.6
DG Total Cost/M\$	45.62	35.34	40.90	50.16	40.39
Total Planning Cost/M\$	79.83	76.46	78.81	81.74	78.60

Figure 12 shows the relations between bus voltage amplitude and DG embedded capacity. The DG embedded capacity is approaching 4 MVA at the terminal of feeders in each scheme. Schemes 2, 3 and 5 are feeder upgrading schemes, and the upgrading of feeders could improve the voltage amplitude, such as the amplitude of voltage is improved from 32.2 kV to 32.9 kV in node 2 after the feeder is upgraded. The five schemes could satisfy the voltage requirement of the power system's operation.

Figure 13 shows the voltage stability index of the five schemes, where the small values of voltage stability index reflect a better voltage stability performance, reaching the static voltage stability limit when the values are beyond 1. Scheme 4 has the best voltage stability index because large scale DGs are embedded in the system. All schemes have good voltage stability index performance.

Figure 14 shows the line losses in each feeder. The values of total losses in the five schemes are 0.425, 0.363, 0.281, 0.357 and 0.298 MW, respectively. Comparing Scheme 1 with Scheme 2, it can be found that the upgrading of feeders could reduce line losses, as the losses in scheme 2 reduce to 0.17 MW in feeder 1 compared with Scheme 1. From the analysis we can conclude that the embedding of DG could reduce line losses to a great extent.

Figure 12. Voltage performance of five schemes.

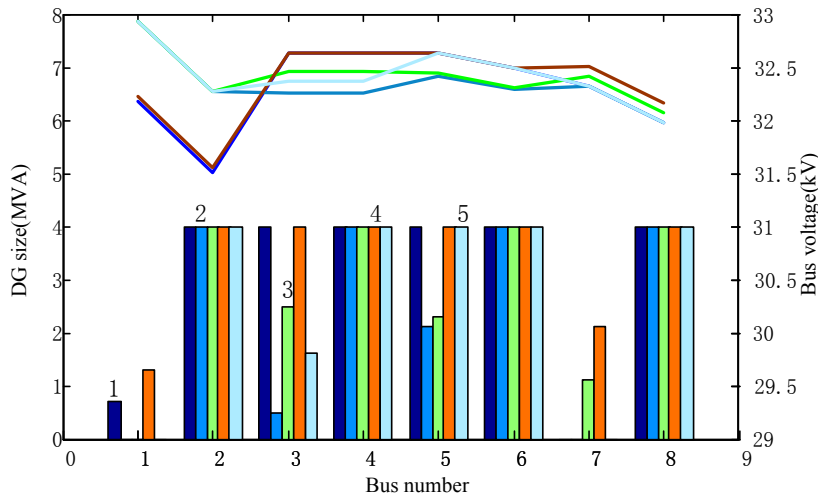


Figure 13. Voltage stability index performance of five schemes.

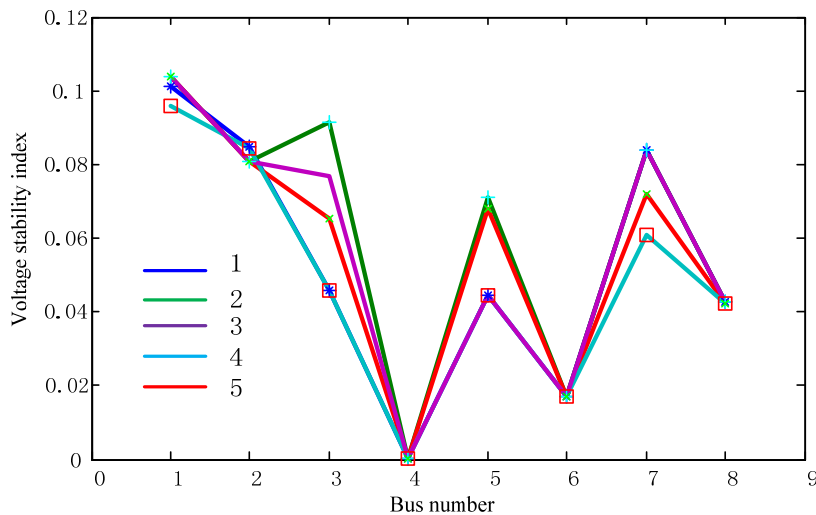
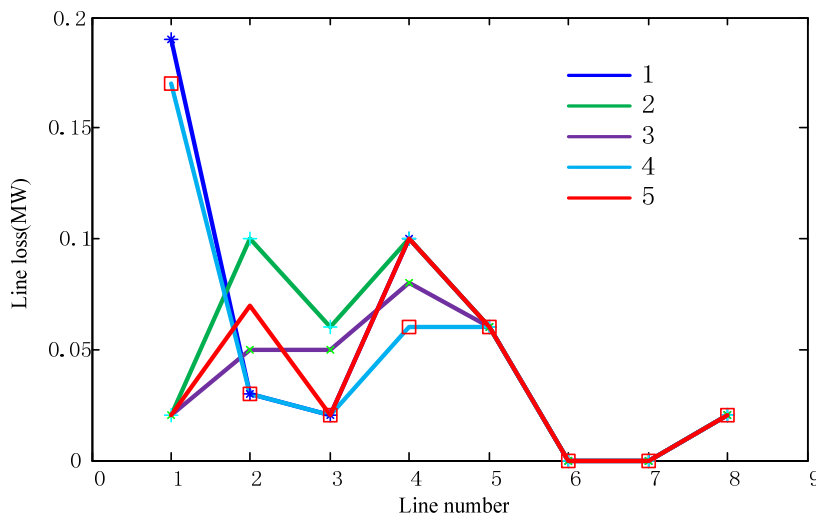


Figure 14. Branch losses performance of five schemes.



The scheme proposed by FMW is SNE in SMG under different subjective preference information. The FMW can find SNE comprehensively through the use of multi-weight factors to evaluate non-dominant solutions, and it could offset the shortcomings of traditional decision-making techniques. The DSP result shows that GOT can handle MOP effectively and find internal relations between objectives reasonably.

Compared with the current optimization theory, GOT has three advantages, the OSA could assess the value of improved approach, the iterated elimination of strictly dominant strategies could offer complete strategy space, and the FMW could handle subjective preferred reasonably and find SNE in SMG scientifically. The traditional optimization theory lacks an assessment of improved approaches and the decision-making method has limitations in processing subjective preferences. GOT formulates an optimization system and could offset the drawbacks of the traditional method which calculates directly with an optimization algorithm after the object functions are selected for an optimal problem. Furthermore, in the field of DG optimal planning, a comprehensive mathematical model is proposed in this paper, which could evaluate the influence of embedding DG in a distribution system reasonably. In conclusion, a novel optimization theory which is called GOT and a new multi-objective mathematical model for DG planning are proposed in this paper, offering a scientific solution to the problem of optimal DG planning in power systems.

5. Conclusions

Combined Game Theory and MOP, Game Optimization Theory is proposed in this paper, and used to handle DSP in a power system for the first time. From the analysis we can obtain the following conclusions:

- (1) The establishment of GOT supplies a new method to explain MOP, and GOT is used for the first time in solving the distribution system planning problem by implementing DG. The results obtain a suitable final DSP scheme.
- (2) Optimization Stability Analysis offsets the disadvantage of calculating directly with an optimization algorithm after the object functions are selected for an optimal problem. The OSA process could offer an assessment of the value of the optimization and improved measures.
- (3) The concept of SMG constructs a new type of game to handle the MOP and enriches the concepts of Game Theory. The DEMPSO could find the Pareto front quickly and generate a non-dominant set reasonably. The FMW could weaken the influences produced by subjective preference information by taking multi-weight into consideration in the evaluation of non-dominant solutions.
- (4) A mathematical model consisting of costs, line losses and voltage profile, is constructed in this paper. This model could assess comprehensively the influence of implementing DG in a distribution system. The result of optimization shows that the scheme of DG embedded in distribution system could bring benefits to the operation of the power system, reduce power losses, improve the amplitude of voltage and improve the voltage stability of feeders, and this method could dominate the substation expansion scheme.

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