Analytical Modeling of Partially Shaded Photovoltaic Systems

Mohammadmehdi Seyedmahmoudian 1*, Saad Mekhilef 1, Rasoul Rahmani 2, Rubiyah Yusof 2 and Ehsan Taslimi Renani 1

1 Department of Electrical Engineering, University of Malaya, Kuala Lumpur 50603, Malaysia; E-Mails: saad@um.edu.my (S.M.); taslimi.ehsan@siswa.um.edu.my (E.T.R.)
2 Centre for Artificial Intelligence & Robotics, Universiti Teknologi Malaysia, Kuala Lumpur 54100, Malaysia; E-Mails: rahmani@ic.utm.my (R.R.); rubiyah@ic.utm.my (R.Y.)

* Author to whom correspondence should be addressed; E-Mail: smahmodian@siswa.um.edu.my; Tel.: +60-03-26154695; Fax: +60-03-26970815.

Received: 7 November 2012; in revised form: 14 December 2012 / Accepted: 19 December 2012 / Published: 4 January 2013

Abstract: As of today, the considerable influence of select environmental variables, especially irradiance intensity, must still be accounted for whenever discussing the performance of a solar system. Therefore, an extensive, dependable modeling method is required in investigating the most suitable Maximum Power Point Tracking (MPPT) method under different conditions. Following these requirements, MATLAB-programmed modeling and simulation of photovoltaic systems is presented here, by focusing on the effects of partial shading on the output of the photovoltaic (PV) systems. End results prove the reliability of the proposed model in replicating the aforementioned output characteristics in the prescribed setting. The proposed model is chosen because it can, conveniently, simulate the behavior of different ranges of PV systems from a single PV module through the multidimensional PV structure.

Keywords: photovoltaic system; partial shading; multidimensional configuration

Nomenclature:

- $I_{ph}$: Solar-Generated current
- $K_i$: Short-circuit temperature/current coefficient
- $G$: Operating irradiance level (W/m²)
- $G_r$: Nominal irradiance level (W/m²)
- $T_k$: Operating temperature (K)
- $A$: Diode ideality factor
- $Q$: Electron charge constant
- $K$: Boltzmann constant
- $N_s$: Number of series connected cells
- $I_{rs}$: Solar generated current
1. Introduction

The first step to study about an appropriate control method in photovoltaic systems is to know how to model and simulate a PV system attached to the converter and power grid. In general, PV systems present nonlinear Power-Voltage (P-V) and Current-Voltage (I-V) characteristics which tightly depend on the receiving irradiance levels and ambient conditions. The mathematical model of the photovoltaic device is significantly valuable for studying the maximum power point tracking algorithms, doing research about the dynamic performance of converters, and also for simulating photovoltaic components by using circuit simulators [1–3].

Despite the recent advancements in PV cell technology, the effects of certain disruptive environmental factors, which remarkably reduce the efficiency of photovoltaic arrays, still remain an inevitable hurdle. One of these environmental phenomena is partial shading which causes the emergence of multiple peaks in the output power curve and has a huge impact on the efficiency of most of the conventional Maximum Power Point Tracking (MPPT) methods [4–10]. Hence, a comprehensive study on the modeling and simulation of the photovoltaic systems is a necessary effort, so that the designs of possible MPPT schemes and the proper configurations for PV arrays may be simplified.

Regarding the import of photovoltaic technology, there has been expansive research on the modeling and simulation of PV systems exposed to a multitude of temperatures and irradiance intensity levels [11–16]. Villalva and Gazoli [17] presented the basic behavior of photovoltaic devices under different irradiance levels and also introduced a simple method to model and simulate the practical PV array. However, their work is very limited to PV arrays’ behaviors under uniform irradiance levels. While some researchers in [13,18] pursued their investigations to encompass partial shading, their research was, again, restricted to the photovoltaic modules and basic configuration of the PV arrays. Yuncong [19] and Kajihara [20] recommend some useful methods to model and simulate the PV modules under partial shading, but larger size and industrial PV systems have not been discussed in their studies.

Besides the size of the PV system and qualification of partial shading conditions, the connection and configuration of the PV systems significantly affect the functionality of the whole system under partial shading conditions. In this regard, Petrone and Ramos [21] conducted a precise and comprehensive research, in which a modeling method based on an optimized algorithm for fast computation of PV plant behavior is presented. However, their approach is suited for
the long term evaluation and data collection of the energetic performances of a PV field under mismatching conditions.

The multidimensional (modular) configuration of PV arrays is one of the cost effective forms of PV systems which significantly reduces the hardware cost in the photovoltaic power plant. This configuration is preferred when applying the evolutionary algorithms such as Particle Swarm Optimization (PSO) and Differential Evolutionary (DE) as the main concept of tracking the Maximum Power Point (MPP) [22]. Some researchers have used the multidimensional configuration to prove the effectiveness of their proposed MPPT methods. For example, Keyrouz and Georges [23] used a multidimensional configuration to evaluate the combination of Bayesian Fusion and PSO to track the Maximum Power Point. However, the behavior of the PV system under partial shading was not discussed in their works.

In accordance with the above paragraphs, it might be inferred that, besides the importance of understanding the effects of partial shading on the output of photovoltaic systems, an accurate and user friendly method for modeling and simulation of PV systems is highly required [24]. Such a method which comprehensively covers different scales and configurations of PV systems serves the following statements:

- Being a basic tool for researchers to predict the output characteristics of the photovoltaic systems in both normal and partial shading conditions.
- Having a reliable and robust model is the first requirement for designers who want to analyze the performance and efficiency of different configurations of PV systems before installation.
- It is the first step to study and define the effectiveness of Maximum Power point tracking methods applied in different configuration of a PV system under variable environmental conditions.
- It is an aid for users who want to build actual PV systems without going into the intricate details such as semiconductor physics.

In this paper, the authors pursue the mathematical analysis of the responses of a single module under uniform irradiance levels. Afterwards, in a more practical scheme, by analyzing the effects of the partial shading phenomenon on the output of PV systems, the study is followed up by the modeling of the module and array under partial shading conditions. Finally, the simulation of the outputs for the proposed multidimensional PV arrays configuration correlating to different degrees of partial shading is presented. The considerable advantage of modeling and simulation method in this research is to cover different scales of a PV system under both normal and partial shading conditions, without analyzing the in-depth semiconductor physics definitions.

2. Modeling of Photovoltaic System Parameters

2.1. PV Cell Model

The single-diode circuitry for a photovoltaic cell is represented in Figure 1. Normally, the output of photovoltaic systems corresponds directly to solar irradiance and temperature, so obtaining the maximum power point should involve the most recent values of these factors.
Figure 1. Equivalent circuit of a photovoltaic array.

The mathematical model of PV also varies with the short circuit current ($I_{sc}$) and the open circuit voltage ($V_{oc}$), which are gleaned from the cell manufacturer’s data sheet. Using the General model, while applying Kirchhoff’s law on the common node of the current source, diode and resistances, the PV current can be derived by:

$$I_{pv} = I_{ph} - I_o$$ (1)

In which $I_{pv}$ is the output current to be fed through the load or network grid and $I_o$ represents the diode current which will be discussed later. $I_{ph}$ refers to the solar-generated current; which, as mentioned beforehand, is affected by solar irradiance and temperature, and so can be calculated this way [15,25]:

$$I_{ph}(G) = (I_{sc} + K_i T_{dif}) \frac{G}{G_r}$$ (2)

where $K_i$ is the temperature coefficient, $T_{dif}$ is the deviation of the operating temperature from the reference temperature ($T_{dif} = T_k - T_r$), and $G$ and $G_r$ are the operating and reference irradiances, respectively. Aside from obtaining the open circuit voltage from the PV cell data sheet, one may also procure it by measuring the output voltage when the output current value is assumed zero. Meanwhile, the reverse saturation current ($I_{rs}$) at a certain reference temperature can be calculated as follows [20,26]:

$$I_{rs} = \frac{I_{sc}}{ \exp(\frac{qE_{sc}}{K_b A T_k}) - 1}$$ (3)

wherein $A$ is the diode ideality factor, $q$ is the constant known as the electron charge ($q = 1.602 \times 10^{-19}$ C); $K_b$ is the Boltzmann constant. As stated earlier $I_o$ is the diode current that will be calculated by the Shockley Equation [12]:

$$I_o = I_{o1}\left[\exp\left(\frac{q(V_{pv} + I_{pv} R_p)}{AK_i T_k}\right) - 1\right]$$ (4)

In the meantime, the diode saturation current ($I_{o1}$) fluctuates in accordance with particular environmental changes, and so can be determined by the following mathematical statement [25,26]:

$$I_{o1} = I_{rs} \left(\frac{T_k}{T_r}\right)^3 \exp\left(\frac{qE_{go}}{AK_b} \frac{T_{dif}}{T_k T_r}\right)$$ (5)
In the above equation, the parameter $E_{go}$ refers to the band gap energy for the silicon semiconductor, which should be between 1.1 and 1.2 eV. Finally, by substituting Equation (5) into Equation (1) and considering the slight current through the parallel resistance, we have the following formula for the PV cell’s output current [25]:

$$I_{pv} = I_{ph} - I_{oa} \left[ \exp \left( \frac{q(V_{pv} + I_{pv}R_s)}{AKT_k} \right) - 1 \right] - \frac{(V_{pv} + I_{pv}R_s)}{R_p}$$  (6)

where the term $R_p$ is the parallel resistance which normally has a high resistance and sometimes assumed infinity in the applicable PV module, due to its slight impression. On the other hand the value and variation of series resistance ($R_s$) cannot be ignored according to its impacts on output power. It should be noted that the output current of the PV cell ($I_{pv}$) exists on both sides of the equation; meaning $I_{pv}$ cannot be expressed as a separate function from $V_{pv}$. Thus, the output characteristic of the PV cell can be deduced by solving the following implicit form:

$$F(I_{pv}, V_{pv}, T_k, G) = I_{ph} - I_{pv} - I_{oa} \left[ \exp \left( \frac{q(V_{pv} + I_{pv}R_s)}{AKT_k} \right) - 1 \right] - \frac{(V_{pv} + I_{pv}R_s)}{R_p} = 0$$  (7)

2.2. PV Module Model

From a practical standpoint, the output power of a single solar cell is insufficient for any useful application in this context, so the overall capability of the PV system should be enhanced by connecting the cells either in series or in parallel, in which case, all the cells in the PV module, $N_s$ being their given number, would contribute to the output power. Subsequently, we may calculate the output of the module using this equation:

$$F(I_{pv}, V_{pv}, T_k, G) = I_{ph} - I_{pv} - I_{oa} \left[ \exp \left( \frac{q(V_{pv} + I_{pv}R_s)}{N_sAKT_k} \right) - 1 \right] - \frac{(V_{pv} + I_{pv}R_sN_s)}{R_pN_s} = 0$$  (8)

Figure 2 shows the output of the BP SX PV module considered in this paper, which employs 72 cells connected to provide a power (P) of 150 W at a terminal voltage ($V_{pv}$) of 21.3 V. The detailed information about the electrical parameters is given in Table 1.

<table>
<thead>
<tr>
<th>Electrical Characteristic</th>
<th>BP SX 150s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open circuit voltage</td>
<td>43.5 V</td>
</tr>
<tr>
<td>Short circuit current</td>
<td>4.75 A</td>
</tr>
<tr>
<td>Maximum power voltage</td>
<td>34.5 V</td>
</tr>
<tr>
<td>Maximum power current</td>
<td>4.35 A</td>
</tr>
<tr>
<td>Maximum power</td>
<td>150 W</td>
</tr>
<tr>
<td>Temperature coefficient of $I_{SC}$</td>
<td>$(0.065 \pm 0.015)%/\degree C$</td>
</tr>
<tr>
<td>Temperature coefficient of $V_{OC}$</td>
<td>$-(160 \pm 20) \text{mV} / \degree C$</td>
</tr>
</tbody>
</table>
Figure 2. Output characteristics of PV module at normal condition (a) I-V characteristic; (b) P-V characteristic.

3. Characteristics of the PV System under Partial Shading

In any outdoor environment, the whole or some parts of the PV system might be shaded by trees, passing clouds, high building, etc., which result in non-uniform insolation conditions as in Figure 3. During partial shading, a fraction of the PV cells which receive uniform irradiance still operate at the optimum efficiency. Since current flow through every cell in a series configuration is naturally constant, the shaded cells need to operate with a reverse bias voltage to provide the same current as the illuminated cells [6,24,27,28]. However, the resulting reverse power polarity leads to power consumption and a reduction in the maximum output power of the partially-shaded PV module. Exposing the shaded cells to an excessive reverse bias voltage could also cause “hotspots” to appear in them, and creating an open circuit in the entire PV module. This is often resolved with the inclusion of a bypass diode to a specific number of cells in the series circuit [29].
Figure 3. PV system under partially shaded conditions caused by passing cloud.

3.1. Effect of Bypass and Blocking Diodes on PV Characteristics

Figure 4 depicts $n$ PV modules with their bypass diodes connected in series inside an array. It is important to note that the characteristics of an array with bypass diodes differ from the one without these diodes. Since the bypass diodes provide an alternate current path, cells of a module no longer carry the same current when they are partially shaded. Therefore, the power-voltage curve develops multiple maxima, shown in Figure 5. This figure shows how the extractable maximum power point differs in PV array with and without bypass diodes. However, presenting multiple maxima in the P-V characteristic is a crucial issue and most of the conventional MPPT algorithms may not distinguish between the local and global maxima.

Figure 4. Circuit model of array consist of $n$ series connected array.
If the generated current ($I_{ph}$) of $i^{th}$ module decreases to less than the current generated by the whole array, the bypass diode restricts the reverse voltage to be less than the breakdown voltage of the PV cells. In other words, the $i^{th}$ bypass diode shown in Figure 4 begins to conduct when Equation (9) is satisfied:

$$I_{pva} > I_{ph(i)}$$

These diodes can be mathematically modeled as one resistance with regards to measured solar generated current of the PV module. As shown in Equation (9), a bypass diode is represented as a high resistance ($10^{10} \, \Omega$) when it is reverse biased and low resistance ($10^{-2} \, \Omega$) while it is forward biased:

$$R_{by} (I_{ph}) = \begin{cases} 10^{-2} & D_{by} On \\ 10^{10} & D_{by} Off \end{cases}$$

(10)

### 3.2. Partially Shaded Module

A partially shaded module can be modeled by two groups of PV cells connected in series inside a module. Each group receives different level of irradiance. Let’s assume no bypass diode for the cells inside a module, so Figure 6 shows the circuit model for a partially shaded module. The module is composed of $r$ series connected cells in which $s$ shaded cells receive irradiance $G1$ and $(r-s)$ shaded cells receiving irradiance $G2$. The PV parameters can be represented as:

$$I_{ph1} = I_{ph}(G_1), I_{ph2} = I_{ph}(G_2), N_{S1} = sN_{S1}, N_{S2} = (r-s)N_{S2}$$

(11)

where, the subscripts 1 and 2 refer to the cells receiving irradiance of $G1$ and $G2$, respectively.

In accordance to the presence of single bypass diode for whole PV module, the output current and voltage of PV array are equal to following mathematical statements:

$$I_{pva} = \text{Min}(I_{pv1}, I_{pv2})$$

$$V_{pva} = \sum V_{pv(i)}$$

(12)
3.3. Partially Shaded Array

In experiments, the solar arrays inclusive of several PV modules are used to generate a higher level of electrical output power. The output power of the array may lead to a complex form when facing partial shading conditions. The mathematical analysis on the output characteristics of the PV array consisting of several modules in an array is presented in this section. Figure 7 shows $K$ series connected modules in an array. If single bypass diode is assumed for each PV module the following steps should be considered to find the output characteristics of the array:

- Calculate the solar irradiance received by each individual PV module and determine the irradiance matrix. This point must be mentioned that in accordance with assuming a single bypass diode for each PV module, if the partial shading occurs in the PV module, the lowest irradiance level will be considered.
- Compute the $I_{ph}$ and $Ns$ of each module using Equation (11) and define the $I_{phs}$ $Ns$ matrix respective to their solar irradiances.
- Rearrange $I_{ph}$ matrix from the highest toward the lowest value.
- Calculate the output current of array ($I_{pva}$) using Equation (13) in which the $I_{pvm(i)}$ is the output current of $i^{th}$ module.
- Calculate the output PV voltage ($V_{pva}$) using Equation (13) in which the $V_{pvm(i)}$ is the output voltage of $i^{th}$ module:

$$I_{pva} = I_{pvm(i)} \quad I_{pva} \geq I_{ph(i+1)}$$

$$V_{pva} = \sum V_{pvm(i)}$$

(13)

The flowchart shown in Figure 8 can be used for coding purposes.
**Figure 7.** Series connected configuration of modules in an array.

**Figure 8.** Flowchart diagram for modeling the output characteristic of an array.

1. Divide array to substring cells or modules involved for each Bypass diode
2. Calculate G and produce the irradiance matrix
3. Calculate $I_{ph}$ for each module and create the $I_{ph}$ and $N_s$ matrix equivalent with G matrix
4. Arrange $I_{ph}$ matrix from highest to lowest values
5. Define the step size to vary the Voltage
6. Set the voltage to its min value $n = 0$

   - $I_{pva} = I_{ph}(1)$
   - $i = i + 1$
   - $I_{pva} > I_{ph}(1)$
   - $I_{pva} = I_{pva(i)}$ when $V_{pva} = n*steppsize$
   - $I_{pva} = 0$

**END**
4. Numerical Examples

4.1. Partially Shaded Module Series with Fully Illuminated Module

Figure 9 shows the series configuration of two PV modules in an array receiving irradiance levels of $G_1$ and $G_2$ respectively. Under the uneven insolation, $G_1$ is given as 1000 W/m$^2$ and $G_2$ is assumed to be 500 W/m$^2$. Taking the presence of a single bypass diode for each PV module into consideration, the output current and voltage at the array terminal can be obtained by solving the following formulae:

$$I_{pv} = \begin{cases} I_{ph}(G_1) - I_{01} \left[ \exp \left( \frac{q(V_{pvm1} + I_{pv}R_s)}{N_sAKT_k} \right) - 1 \right] \frac{(V_{pvm1} + I_{pv}R_s)}{R_pN_s} & I_{pv} > I_{ph1} \\ I_{ph}(G_2) - I_{01} \left[ \exp \left( \frac{q(V_{pvm2} + I_{pv}R_s)}{N_sAKT_k} \right) - 1 \right] \frac{(V_{pvm2} + I_{pv}R_s)}{R_pN_s} & I_{pv} < I_{ph2} \end{cases}$$ \hspace{1cm} (14)

$$V_{pv} = \begin{cases} V_{pvm1} & I_{pv} > I_{ph1} \\ V_{pvm1} + V_{pvm2} & I_{pv} < I_{ph2} \end{cases}$$ \hspace{1cm} (15)

**Figure 9.** PV array consisting two series connected modules (a) Circuitry diagram; (b) Block diagram.

In this case, the modeling of the PV array is divided into two zones defined with respect to the photonic generated current of the shadowed module as compared to the output current of the illuminated module. Within the first zone, the output current of the entire PV array is identical to the current generated by the well-lit module. In this zone, the generated current of the second PV module is less than total array output current; therefore, by assuming node A as common point between two modules, Equation (9) will be satisfied for the second module and bypass diode 1 will become ON. The array current will be equivalent to the current generated by the un-shaded module until its value reaches the same value as the generated current of the shaded module and enters the second zone. In the second zone, PV module 2 starts to generate the power.
Applying Equations (8) and (12) for the output of unshaded and shaded modules, and Equations (14) and (15) for the current and voltage of the entire PV array, the results represented in Figure 10 are obtained. As observed, there are multiple peaks in the output Power-Voltage characteristics as a result of the different irradiance levels and presence of bypass diodes.

**Figure 10.** Output characteristics of partially shaded PV array (a) I-V characteristic; (b) P-V characteristic.

4.2. Multidimensional PV System

Figure 11a shows a multidimensional PV system in which each PV array is controlled by its individual DC/DC converter. Despite the disadvantageous need for numerous sensors and transducers, this system can minimize the effects of partial shading while meeting the load demand. One solution is considering a centralized controller which would supply the required alternating patterns of each converter just as well. Therefore, the configuration shown in Figure 11b is proposed as a compromise, to increase the output power of the PV system and reduce the number of sensors needed at the same time. The general advantages of the multidimensional PV systems can be briefly stated as below:

- The lower number of sensors and transducers which significantly reduces the overall system cost.
The lower space required for the control unit, even in a large scale PV system.
High flexibility of this configuration helps the designers to develop the system without increasing the control units. Only some amendments in programming are required.

In this structure, each PV array consists of two PV modules, which are connected in series and controlled by both DC/DC converters and a centralized controller.

**Figure 11.** Multidimensional PV system (a) Controlled by multiple controllers; (b) Controlled by centralized controllers.

To analyze the influence of partial shading on the output of the proposed configuration, a numerical case study is considered. In the conventional scheme, the controllers generate single switching patterns for the individual converters while in the cost-effective multidimensional scheme, the centralized controller is required to provide two appropriate duty cycles for the two individual converters. Finding an accurate output characteristic of PV system with this configuration, in a simple procedure, is very important with which the functionality of the MPPT method can be evaluated. At first, the system is divided into the separate PV arrays connected to separate DC/DC converters. Afterwards, the effects of partial shading must be analyzed through Equations (9) till (12) for each of these separate PV arrays. Finally the output results of all PV arrays will be considered to define the output result of the whole PV system.

For example, for defining the output characteristics of the PV system shown in Figure 11(b), the system must be divided into two separate PV arrays inclusive of two series connected PV modules. The mathematical model of each PV array has been discussed in Sections 3.2–4.1. It is assumed that the modules inside the both PV array receive irradiance levels of $G_{A1} = 1,000 \text{ W/m}^2$, and $G_{A1} = 500 \text{ W/m}^2$. While inside the second PV array receive $G_{A2} = 1,000 \text{ W/m}^2$, and $G_{A2} = 700 \text{ W/m}^2$. The output current of each PV array would be computed using Equation (13). Consequently the output characteristic of each PV array results to the similar characteristic shown in Figure 10. As mentioned earlier in the Cost-effective configuration, the central controller is tracking the MPP for both PV arrays. So the output characteristic will result in three dimensional characteristic shown in Figure 12. In this figure, the X axis shows the output voltage of the first array, the Y axis
shows the output voltage of the second array and the Z axis represents the output current [in Figure 12(a)] and power [in Figure 12(b)] of the whole PV system.

Figure 12 also shows how the output curves of two PV arrays are seen in the output characteristic of the whole PV system. The zones indicated in this figure are exactly the ones described in Figure 10 for each PV array. The combination of these zones will result into four regions in output characteristics of PV system shown in Figure 7, in which the regions are defined by:

- Region 1 = Contribution of PV arrays in Zone2A2 + Zone1A1
- Region 2 = Contribution of PV arrays in Zone2A2 + Zone2A1
- Region 3 = Contribution of PV arrays in Zone1A2 + Zone1A1
- Region 4 = Contribution of PV arrays in Zone2A2 + Zone2A1

The graph presented in Figure 13 shows four distinct regions marked by differences in the output’s characteristics created by each individual array. It also proves that MPP occurs inside the region in which the contribution of individual array to generate the power is maximized.

**Figure 12.** Output characteristics of partially shaded multidimensional PV system with centralized controller (a) I-V characteristic (b) P-V characteristic.
5. Conclusions

As discussed in the analytical modeling of photovoltaic systems, the model should consider all the parameters of the photovoltaic system under normal conditions and also partial shading. The latter condition was simulated in a step by step analysis, using a range of irradiance levels for $G_1$ and $G_2$, and a model encompassing PV modules connected in series with individual bypass diodes. The study then proceeded by modeling the multidimensional structure of PV systems which is known as a cost-effective configuration for PV arrays. This system significantly reduces the number of sensors and transducers utilized in control system. The cost effective aspect of this configuration can be even more apparent when the large scale PV farm is the case. The final results show how the output characteristics of PV system are the combination of output behavior of each PV array involved in the whole system. The study shows even the mathematical modeling of large n-dimensional PV system in which the central controller tracks the MPP for all the PV arrays, can be achieved by only analyzing the characteristic of the PV arrays inside the system.

Acknowledgments

The authors would like to thank Ministry of Higher Education of Malaysia and University of Malaya for providing financial support under the research grant No. UM.C/HIR/MOHE/ENG/16001-00-D000024.

References


