

Article

## Dispatch Method for Independently Owned Hydropower Plants in the Same River Flow

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**Abstract:** This paper proposes a coexistence model for two independent companies both operating hydropower plants in the same river flow, based on a case study of the Cetina river basin in Croatia. Companies are participants of the day-ahead electricity market. The incumbent company owns the existing hydropower plants and holds concessions for the water. The new company decides to build a pump storage hydropower plant that uses one of the existing reservoirs as its lower reservoir. Meeting reservoir water balance is affected by decisions by both companies which are independently seeking maximal profit. Methods for water use settlement and preventing of spillage are proposed. A mixed-integer linear programming approach is used. Head effects on output power levels are also considered. Existences of dispatches that satisfy both companies are shown.

**Keywords:** cascade hydro system; day-ahead; deterministic model; mixed-integer linear programming; pumped storage hydropower plant; symbiosis; water trading

### Nomenclature

HPP	<u>H</u> ydro <u>P</u> ower <u>P</u> lant.
PSHPP	<u>P</u> umped <u>S</u> torage <u>H</u> ydro <u>p</u> ower <u>P</u> lant.
HC	<u>H</u> ydro <u>C</u> ompany.
PC	<u>P</u> ump <u>C</u> ompany that owns PSHPP 6.
$T$	Set of indices of the steps of the optimisation period, $T = \{1, 2, \dots, 24\}$ , $t \in T$ .
$I$	Set of indices of the reservoirs/plants, $I = \{1, 2, 3, 4, 5, 6\}$ , $i \in I$ .
$J$	Set of indices of the perf. curves $J = \{1\text{-high lvl.}, 2\text{-middle lvl.}, 3\text{-low lvl.}\}$ , $j \in J$ .

$U_i$	Set of upstream reservoirs of plant $i$ .
$B$	Set of indices of the blocks of the piecewise linearization of the unit performance curve $B = \{1,2,3\}$ , $b \in B$ .
$M$	Conversion factor equal to 3600 [m <sup>3</sup> /m <sup>3</sup> h].
$x(i, t)$	Water content of the reservoir $i$ in time step $t$ [m <sup>3</sup> ].
$x_{avg}(i, t)$	Average water content of the reservoir $i$ in time step $t$ [m <sup>3</sup> ].
$X_{MIN}(i)$	Minimum content of the reservoir $i$ [m <sup>3</sup> ].
$X_1(i)$	First discrete level of the content of the reservoir $i$ [m <sup>3</sup> ].
$X_2(i)$	Second discrete level of the content of the reservoir $i$ [m <sup>3</sup> ].
$X_{MAX}(i)$	Maximum content of the reservoir $i$ [m <sup>3</sup> ].
$X(i, 0)$	Initial water content of the reservoir $i$ [m <sup>3</sup> ].
$X(i, 24)$	Final water content of the reservoir $i$ [m <sup>3</sup> ].
$W(i, t)$	Forecasted natural water inflow of the reservoir $i$ in time step $t$ [m <sup>3</sup> /s].
$\pi(t)$	Forecasted price of electricity in time step $t$ [€/MWh].
$\pi_{MAX}$	Price level that affects binary variable $F_\pi(t)$ in time step $t$ [€/MWh].
$q(i, t)$	Water discharge of plant $i$ in time step $t$ [m <sup>3</sup> /s].
$q(b, i, t)$	Water discharge of block $b$ of plant $i$ in time step $t$ [m <sup>3</sup> /s].
$Q_{MIN}(i)$	Minimum water discharge of plant $i$ [m <sup>3</sup> /s].
$Q_{MAX}(i, b)$	Maximum water discharge of block $b$ of plant $i$ [m <sup>3</sup> /s].
$Q_{MAX}(i)$	Maximum water discharge of plant $i$ [m <sup>3</sup> /s].
$B_{MIN}(i)$	Biological minimum of plant $i$ [m <sup>3</sup> /s].
$\hat{Q}_{MAX}(6)$	Maximum water intake of plant 6 (PSHPP) in pump regime [m <sup>3</sup> /s].
$\hat{q}(6, t)$	Water intake of plant 6 (PSHPP) in pump regime in time step $t$ [m <sup>3</sup> /s].
$s(i, t)$	Spillage of the reservoir $i$ in time step $t$ [m <sup>3</sup> /s].
$\tau_{ji}$	Time delay in water flow between reservoir $j$ and $i$ [h].
$E$	Large enough number for setting constraints. In this case 100,000.
$E_S$	Large enough number for setting constraints. In this case $12 \times 10^9$ .
$E_\pi$	Large enough number for setting constraints. In this case 200.
$D_1(i, t)$	0/1 variable used for discretization of performance curves.
$D_2(i, t)$	0/1 variable used for discretization of performance curves.
$V(i, t)$	0/1 variable which is equal to 1 if plant $i$ is on-line in time step $t$ .
$L(6, t)$	0/1 variable which is equal to 1 if plant 6 is in pump regime in time step $t$ .
$W(i, t, b)$	0/1 variable which is equal to 1 if water discharged by plant $i$ has exceeded block $b$ in time step $t$ .
$F(5, t)$	0/1 variable defined by Equation (19).
$F(6, t)$	0/1 variable defined by Equation (20).
$F_S(t)$	0/1 variable defined by Equation (21).
$F_\pi(t)$	0/1 variable defined by Equation (22).
$F(t)$	0/1 variable defined by Equation (23).
$P(i, t)$	Power output of plant $i$ in time step $t$ [MW].
$P_{01}(i)$	Minimum power output of plant $i$ for performance curve 1 (lower level of water content) [MW].

$P_{02}(i)$	Minimum power output of plant $i$ for the performance curve 2 (intermediate level of water content) [MW].
$P_{03}(i)$	Minimum power output of plant $i$ for the performance curve 3 (higher level of water content) [MW].
$P_{MIN}(i)$	Minimum power output of plant $i$ [MW].
$P_{MAX}(i)$	Capacity of plant $i$ [MW].
$\rho_j(b,5)$	Slope of the block $b$ of the performance curve $j$ of plant 5 [MWs/m <sup>3</sup> ].
$Lo(6,t)$	Load of PSHPP 6 when working in pump regime in time step $t$ [MW].
$L_{MAX}(6)$	Maximum load of PSHPP 6 [MW].
$L_{MIN}(6)$	Minimum load of PSHPP 6 [MW].
$profit_{HC}$	Profit of HC during optimization time step [€].
$profit_{HC,max}$	Maximum possible HC profit or HC profit without constraints on the PC profit.
$profit_{PC}$	Profit of PC during optimization time step [€].
$profit_{PC,max}$	Maximum possible PC profit or PC profit without constraints on the HC profit.
$profit_{1,2,3,4,5}$	Profit share of HC total profit gained by HPP 1 to 5 in Case A [€].
$profit_{PSHPP}$	Profit share of HC total profit gained by PSHPP 6 in Case A [€].
$\vartheta$	Arbitrarily chosen step in the optimization procedure (1, 10, 100, etc.).

## 1. Introduction

Water is scarce resource with uncertain availability, especially in comparison with fossil fuels. Hence finding an optimal production schedule for hydropower plants is usually a complex task and it is necessary to carefully balance the timing of water use. In a traditional (cost minimization) environment hydropower plant dispatching is optimized using hydro-thermo coordination processes [1–3]. On the contrary, in a deregulated environment, which is pursued in many countries, the main goal is to find the hydropower plant dispatch with the highest possible profit [4,5].

Choosing the right model and finding an optimal water use schedule depends on planning horizon. In this case a day-ahead period is considered. In short term planning most of parameters are usually considered known and resulting models are deterministic [6,7]. It is also possible to use stochastic models in short-term scheduling [8]. In long-term models most variables are stochastic [9,10]. In this paper a deterministic day-ahead model is presented and discussed.

Usually short-term hydropower plant scheduling considers the relationships between the head of the associated reservoir(s), the discharged water and the generated power [11]. A model describing these relationships is introduced in [12]. The same method is used in this paper with some modifications. This is a nonlinear and nonconcave 3-D relationship, the so-called Hill chart [13]. Model complexity grows with number of reservoirs and associated hydropower plants. Therefore a cascade hydro system is carefully modeled, considering time delays between reservoirs, which is the case in this paper.

Modeling of pump storage hydropower plants (PSHPP) requires extra effort [14,15]. These types of hydropower plants need a lower and upper reservoir. The PSHPP energy conversion cycle is inefficient therefore their main purpose is producing financial benefit. PSHPP participation in ancillary services

markets is not addressed in this paper. PSHPP shifts energy from hours with lower electricity prices (which can even be negative [16]) to hours with higher electricity prices. In Croatia, like in most other countries, most of attractive locations for hydropower plants (HPP and PSHPP) are already utilized. Therefore more intensive usage of water resources requires building of new PSHPPs as a supplement to the existing hydropower system [17]. Hence there is opportunity for a new investor to build that new PSHPP and make certain profit.

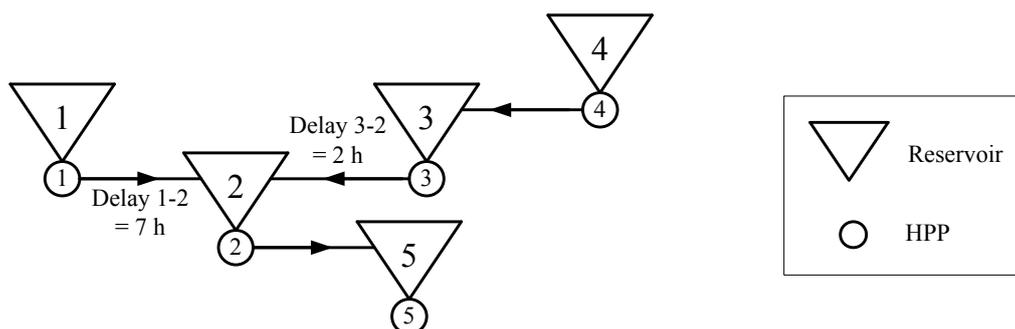
Interaction of two independent owners on the same basin (using the same water resources) is described in this paper. This interaction can be compared with symbiosis in Nature, as an interaction of two entities when both benefit from it. For this kind of interaction it is necessary to build a model of (financial) cooperation which encourages both entities to effectively coexist. In this work a term “water trading” between different hydro companies on the same river flow is introduced. In the reviewed literature analysis of this phenomenon is not significantly discussed. Introduced water trading method brings not only financial benefit with higher flexibility of new system but also more efficient exploitation of existing water resources. This is achieved by preventing spillage of the meeting reservoir when it is possible and profitable.

In this paper a real basin (river Cetina in Croatia) is modeled for the case study. It consists of five reservoirs and five hydropower plants. The rest of paper is organized in following way: in the section entitled “Base model” the characteristics of Cetina basin, assumptions and input parameters in optimization model are introduced. In the “Problem description” section circumstances which require improvement of the base model are described. The coexistence of two companies which brings benefits to both of them is also described. In the section “Model of PC–HC symbiosis” symbiosis of PC and HC companies is modeled. In the section “Search for an optimal solution” an algorithm for finding an optimal solution is introduced. In the “Case study” section results are presented. In the “Conclusion” section the main conclusions of this paper are stated.

## 2. Base Model

### 2.1. Model Overview

The described model is deterministic. Optimization horizon is “day-ahead”, which is divided into 24 hourly steps. For problem definition 0/1 mixed-integer linear programming is used. Base model (Figure 1) will be modified after the problem definition.



In the base model all HPPs (1 to 5 in Figure 1) are owned by one company called Hydro Company (HC). HC has built all the reservoirs and associated HPPs. HC also pays concessions for water usage. These concessions are paid to the national authority.

2.2. Water Balance

The water balance of the hydro reservoirs is formulated as:

$$x(i,t) = x(i,t-1) + M \cdot \left[ W(i,t) + \sum_{j \in U_i} \{q(j,t - \tau_{ji}) + s(j,t - \tau_{ji})\} - q(i,t) - s(i,t) \right] \tag{1}$$

$$\forall i \in I, \forall t \in T$$

2.3. Input Data

The data for the base model of the Cetina basin is taken from [18]. These data are listed in Tables 1 and 2.

**Table 1.** Input parameters.

	HPP						Reservoir				
	1	2	3	4	6		1	2	3	4	6
$P_{MAX}$ [MW]	41.6	40.8	237	5	150	$X_{MIN}(i)$ [ $e^6 m^3$ ]	300	1	0.6	200	4
$P_{MIN}$ [MW]	0	0	0	0	0	$X_{MAX}(i)$ [ $e^6 m^3$ ]	800	2.6	1.6	560	13
$Q_{MAX}$ [ $m^3/s$ ]	120	220	70	50	150	$X(i,0)$ [ $e^6 m^3$ ]	500	2	1.1	250	5
$B_{MIN}$ [ $m^3/s$ ]	0	0	0	0	0	$X(i,24)$ [ $e^6 m^3$ ]	500	2	1.1	250	5

**Table 2.** Parameters for reservoir 5 and HPP 5.

	[MW]	[ $10^6 m^3$ ]	[ $m^3/s$ ]	[MWs/ $m^3$ ]
$P_{MAX}$	486	$X_{MAX}$	4.4	$Q_{MIN}(5)$ 75 $\forall j \in J$
$P_{01}$	115	$X_{MIN}$	1.6	$Q_{MAX}(5,1)$ 75 $\rho_j(1,5)$ 1.8
$P_{02}$	125	$x(5,0)$	2	$Q_{MAX}(5,2)$ 50 $\rho_j(2,5)$ 2
$P_{03}$	135	$x(5,24)$	2	$Q_{MAX}(5,3)$ 20 $\rho_j(3,5)$ 5.8

2.4. Formulation of Performance Curves

For HPP 5, the set of curves representing the relationship between the head, the power output and the water discharge is described with three curves [12], according to four discrete levels of the stored water in the reservoirs (Figure 2). In this paper, these curves have been modeled through a piecewise linear formulation.

Performance curves are activated according to two binary variables  $D_1$  and  $D_2$  through the following Equations:

$$x_{avg}(5,t) = \frac{x(5,t) + x(5,t-1)}{2}, \forall t \in T \tag{2}$$

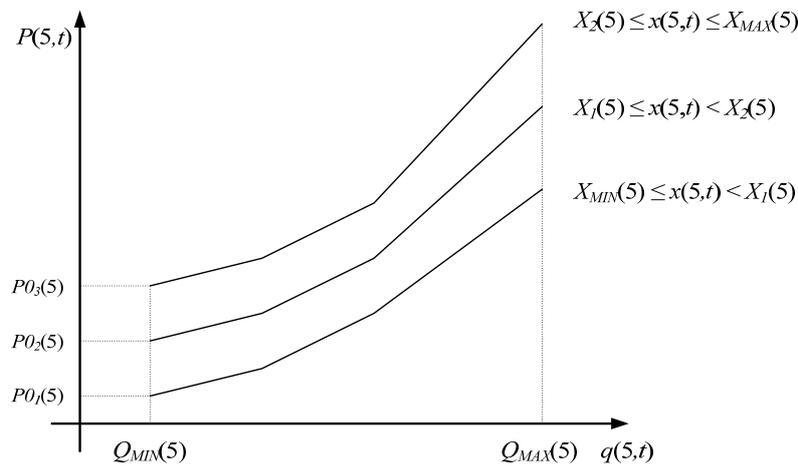
$$x_{avg}(5,t) \geq X_1(5) \cdot (D_1(5,t) - D_2(5,t)) + X_2(5) \cdot D_2(5,t) \quad \forall t \in T \tag{3}$$

$$x_{avg}(5,t) \leq X_{MAX}(5) \cdot D_2(5,t) + X_1(5) \cdot (1 - D_1(5,t)) + X_2(5) \cdot (D_1(5,t) - D_2(5,t)) \quad \forall t \in T$$

$$\begin{aligned} x(5,t) &\geq X_{MIN}(5) \quad \forall t \in T \\ x(5,t) &\leq X_{MAX}(5) \quad \forall t \in T \end{aligned} \tag{4}$$

$$D_1(5,t) \geq D_2(5,t) \quad \forall t \in T \tag{5}$$

Figure 2. HPP 5 performance curves.



Formulation of the performance curves for HPP 5 follows:

$$\begin{aligned} P(5,t) - P_{01}(5) \cdot V(5,t) - \sum_{b \in B} q(b,5,t) \cdot \rho_1(b,5) - P_{MAX}(5) \cdot [D_1(5,t) + D_2(5,t)] &\leq 0 \quad \forall t \in T \\ P(5,t) - P_{01}(5) \cdot V(5,t) - \sum_{b \in B} q(b,5,t) \cdot \rho_1(b,5) + P_{MAX}(5) \cdot [D_1(5,t) + D_2(5,t)] &\geq 0 \quad \forall t \in T \end{aligned} \tag{6}$$

$$\begin{aligned} P(5,t) - P_{02}(5) \cdot V(5,t) - \sum_{b \in B} q(b,5,t) \cdot \rho_2(b,5) - P_{MAX}(5) \cdot [1 - D_1(5,t) + D_2(5,t)] &\leq 0 \quad \forall t \in T \\ P(5,t) - P_{02}(5) \cdot V(5,t) - \sum_{b \in B} q(b,5,t) \cdot \rho_2(b,5) + P_{MAX}(5) \cdot [1 - D_1(5,t) + D_2(5,t)] &\geq 0 \quad \forall t \in T \end{aligned} \tag{7}$$

$$\begin{aligned} P(5,t) - P_{03}(5) \cdot V(5,t) - \sum_{b \in B} q(b,5,t) \cdot \rho_3(b,5) - P_{MAX}(5) \cdot [2 - D_1(5,t) - D_2(5,t)] &\leq 0 \quad \forall t \in T \\ P(5,t) - P_{03}(5) \cdot V(5,t) - \sum_{b \in B} q(b,5,t) \cdot \rho_3(b,5) + P_{MAX}(5) \cdot [2 - D_1(5,t) - D_2(5,t)] &\geq 0 \quad \forall t \in T \end{aligned} \tag{8}$$

$$q(5,t) = \sum_{b \in B} q(b,5,t) + Q_{MIN}(5) \cdot V(5,t) \quad \forall t \in T \tag{9}$$

$$\begin{aligned} q(b,5,'1') &\leq Q_{MAX}(5,'1') \cdot V(5,t) \quad \forall t \in T \\ q(b,5,'1') &\geq Q_{MAX}(5,'1') \cdot W(5,t,'1') \quad \forall t \in T \\ q(b,5,t) &\leq Q_{MAX}(5,b) \cdot W(5,t,b-1) \quad \forall b \in B \setminus \{1\}, \forall t \in T \\ q(b,5,t) &\geq Q_{MAX}(5,b) \cdot W(5,t,b) \quad \forall b \in B \setminus \{1\}, \forall t \in T \end{aligned} \tag{10}$$

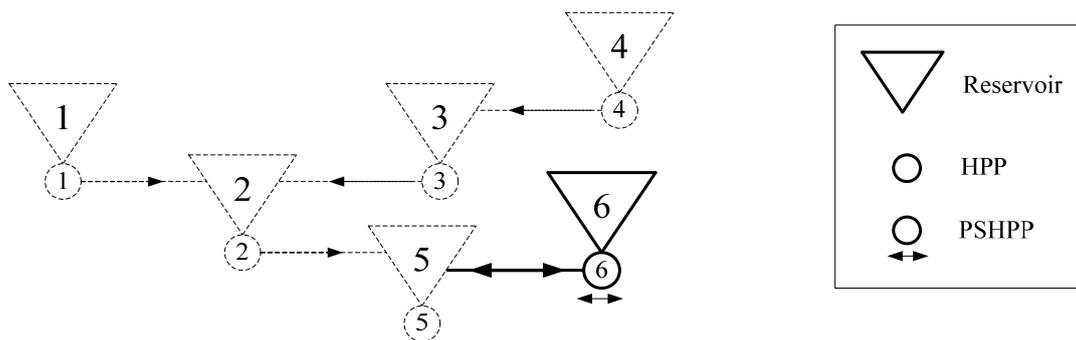
Instead of using label  $x$  as in [12] label  $x_{avg}$  was used in Equations (2) and (3). Performance curves are activated using an average value of stored water in certain time step  $t$  instead of using final reservoir water content of the time step  $t$ . This approach also allows achieving discrete levels of stored water  $X_1(5)$  and  $X_2(5)$ . Unlike HPP 5, the other HPPs are approximated with linear relationship of output power and discharge (head effects are neglected). These relationships are shown in the following Equation:

$$\begin{aligned}
 P(i,t) &\leq P_{MAX}(i) \cdot V(i,t) \quad \forall i \in I \setminus \{5\} \quad \forall t \in T \\
 P(i,t) &\geq P_{MIN}(i) \cdot V(i,t) \quad \forall i \in I \setminus \{5\} \quad \forall t \in T
 \end{aligned}
 \tag{11}$$

### 3. Problem Description

The previously presented model is now upgraded with new PSHPP (labeled 6). The new PSHPP uses existing reservoir 5 as its lower reservoir. This same reservoir 5 is used by the HC (Figure 3).

**Figure 3.** New PSHPP 6 on Cetina basin.



Introduction of the new PSHPP in addition to the base model, requires changes in reservoir 5 water balance and also defines a new water balance for reservoir 6. Maximum pump load is modeled as 200 MW, minimum load 0 MW with pump efficiency of 0.8 (state of the art PSHPP efficiency).

PSHPP 6 working regime is defined with:

$$\begin{aligned}
 L(6,t) + V(6,t) &\leq 1 \quad \forall t \in T \\
 L_o(6,t) &\leq L_{MAX}(6) \cdot L(6,t) \quad \forall t \in T \\
 L_o(6,t) &\geq L_{MIN}(6) \cdot L(6,t) \quad \forall t \in T
 \end{aligned}
 \tag{12}$$

Maximal HC profit in the base model is labeled  $O_{HC0}$  and is defined by Equation (13):

$$O_{HC0} = \max \sum_{t \in T} \left( \sum_{i \in I \setminus \{6\}} P(i,t) \cdot \pi(t) \right)
 \tag{13}$$

In Case A (Cases A and B are described in detail in the “Case study” section below) HC is gaining flexibility with the introduction of the new PSHPP 6. Maximal HC profit in Case A is labeled  $O_{HC+PC}$  and is defined by the following equation:

$$O_{HC+PC} = \max \sum_{t \in T} \left( \left( \sum_{i \in I} P(i,t) - L(6,t) \right) \cdot \pi(t) \right)
 \tag{14}$$

In Case B it is assumed that the independent company has built a new PSHPP 6, under certain conditions of coexistence with HC. This company, called Pump Company (PC) does not need to build a lower reservoir but has to compensate HC for using water from reservoir 5. Constraints for this cooperation are such that PC operates with certain profit while at a same time HC has an equal or higher profit compared to  $O_{HC0}$ . The question that then arises is under which conditions is this kind of symbiosis possible?

3.1. First Condition: Compensation for Using Water from the Reservoir 5

In the long run, PC does not reduce the available amount of water in reservoir 5. PC introduces uncertainties in scheduling of the HC. Additionally PSHPP 6 pump water intake reduces the head of reservoir 5 which has a negative effect on the output power of HPP 5. Therefore PC has to compensate HC for using water from the reservoir 5. The absolute value of this fee depends on the electricity price and the water content of reservoir 5. This fee is defined with factors  $C_1$ ,  $C_2$  and  $C_3$ .

3.2. Second Condition: Preventing Spillage of Reservoir 5 by Forcing PC in Pump Regime

It is in the interest of both companies to avoid spillage of reservoir 5, in other words to avoid the loss of resources. If, for some reason, preventing of spillage of reservoir 5 cannot be accomplished without PC’s intervention, PSHPP 6 will work at maximum pump capacity but only if the electricity price is below the price level  $\pi_{MAX}$ . In this case PC will pay a fixed fee  $R$  [€/MWh] (value of  $R$  and  $\pi_{MAX}$  are based on a long term agreement between HC and PC) for electricity used for pumping. At the same time HC pays the remaining price for electricity used for pumping [ $\pi(t) - R$ ]. In this way the total pumping cost is divided between HC and PC. HC takes market price risk in this case. The necessary condition is that PC maximal pump load does not result in spillage of its upper reservoir (reservoir 6). The value of fee  $R$  has to be low enough to motivate PC for this condition but also not too low so as to demotivate HC.

3.3. Third Condition: Preventing Spillage of Reservoir 5 by PC Refraining from Production

PC needs to refrain from production in hours when it would cause spillage of reservoir 5 (HC loses resources). In a market environment PC expects payment for this favor. For the sake of simplicity it is assumed that this payment is included in value of fixed fee  $R$  (in the HC-PC agreement) defined in second condition.

4. Model of PC-HC Symbiosis

4.1. First Condition: Compensation for Using Water from Reservoir 5

For modeling this condition binary variables  $D_1(5,t)$  and  $D_2(5,t)$  are used in a similar manner as in Equations (3–8). PC pumping cost is defined with Equations (15) and (16):

$$\begin{aligned}
 C_{PC}(t) &\geq Lo(6,t) \cdot \pi(t) \cdot C_j + L_{MAX}(6) \cdot F(t) \cdot (R - \pi(t) \cdot C_j) - E_j(t) \quad \forall t \in T, \forall j \in J \\
 C_{PC}(t) &\leq Lo(6,t) \cdot \pi(t) \cdot C_j + L_{MAX}(6) \cdot F(t) \cdot (R - \pi(t) \cdot C_j) + E_j(t) \quad \forall t \in T, \forall j \in J \\
 E_1(t) &= E \cdot (2 - D_1(5,t) - D_2(5,t)) \quad \forall t \in T \\
 E_2(t) &= E \cdot (1 - D_1(5,t) + D_2(5,t)) \quad \forall t \in T \\
 E_3(t) &= E \cdot (D_1(5,t) + D_2(5,t)) \quad \forall t \in T \\
 D_1(5,t) &\geq D_2(5,t) \quad \forall t \in T
 \end{aligned}
 \tag{15}$$

$$\begin{aligned}
 C_j &\geq 1 \quad \forall j \in J \\
 C_{j+1} &\geq C_j \quad \forall j \in J \setminus \{1\}
 \end{aligned}
 \tag{16}$$

$C_j$  depends on water content of reservoir 5. PC pumping cost,  $C_{PC}(t)$ , consists of the fee for used electricity,  $C_{PC_e}(t)$  and compensation to HC for used water,  $C_{PC_w}(t)$ .

$$\begin{aligned} C_{PC}(t) &= C_{PC_w}(t) + C_{PC_e}(t) \quad \forall t \in T \\ C_{PC_e}(t) &= Lo(6,t) \cdot \pi(t) \quad \forall t \in T \end{aligned} \quad (17)$$

$$C_{PC_w}(t) = C_{PC}(t) - Lo(6,t) \cdot \pi(t) \quad \forall t \in T \quad (18)$$

#### 4.2. Second Condition: Preventing Spillage of Reservoir 5 by forcing PC in Pump Regime

This condition is modeled by Equations (19–24):

$$\begin{aligned} E_S \cdot F(5,t) &\geq x(5,t-1) + W(5,t) + q(2,t) + s(2,t) - Q_{MAX}(5) - X_{MAX}(5) \quad \forall t \in T \\ E_S \cdot (1 - F(5,t)) &> -x(5,t-1) - W(5,t) - q(2,t) - s(2,t) + Q_{MAX}(5) + X_{MAX}(5) \quad \forall t \in T \end{aligned} \quad (19)$$

$$\begin{aligned} E_S \cdot F(6,t) &\geq x(6,t-1) + \hat{Q}_{MAX}(6) - X_{MAX}(6) \quad \forall t \in T \\ E_S \cdot (1 - F(6,t)) &> -x(6,t-1) - \hat{Q}_{MAX}(6) + X_{MAX}(6) \quad \forall t \in T \end{aligned} \quad (20)$$

$$\begin{aligned} F_S(t) &\leq F(5,t) \quad \forall t \in T \\ F_S(t) &\geq F(5,t) - F(6,t) \quad \forall t \in T \\ F_S(t) &\leq 1 - F(6,t) \quad \forall t \in T \end{aligned} \quad (21)$$

Binary variable  $F_\pi(t)$  is defined by Equation (22):

$$\begin{aligned} F_\pi(t) \cdot E_\pi &\geq \pi(t) - \pi_{MAX} \quad \forall t \in T \\ (1 - F_\pi(t)) \cdot E_\pi &> \pi_{MAX} - \pi(t) \quad \forall t \in T \end{aligned} \quad (22)$$

$$\begin{aligned} F(t) &\leq F_S(t) \quad \forall t \in T \\ F(t) &\geq F_S(t) - F_\pi(t) \quad \forall t \in T \\ F(t) &\leq 1 - F_\pi(t) \quad \forall t \in T \end{aligned} \quad (23)$$

$$\begin{aligned} F(t) &\leq L(6,t) \quad \forall t \in T \\ Lo(6,t) &\geq L_{MAX}(6) \cdot F(t) \quad \forall t \in T \\ Lo(6,t) &\leq L_{MAX}(6) \quad \forall t \in T \end{aligned} \quad (24)$$

Equation (19) introduces binary variable  $F(5,t)$  that is equal to 1 if it is impossible to avoid reservoir 5 spillage without PC intervention in time step  $t$  (otherwise 0).

Equation (20) introduces binary variable  $F(6,t)$  that is equal to 1 if PC maximal pump load results in spillage of reservoir 6 in time step  $t$  (otherwise 0).

Equation (21) introduces binary variable  $F_S(t)$  that is equal to 1 in time step  $t$  only if  $F(5,t)$  is equal to 1 and  $F(6,t)$  is equal to 0 in time step  $t$  (otherwise 0).

Equation (22) introduces binary variable  $F_\pi(t)$  that is equal to 1 in time step  $t$  if the electricity price is above price level  $\pi_{MAX}$  in time step  $t$  (otherwise 0).

PC pump should be on maximum level only if  $F_S(t)$  is equal to 1 and  $F_\pi(t) = 0$  in time step  $t$ . This condition is modeled by Equations (23) and (24) introducing binary variable  $F(t)$ . This variable is already in use in Equation (15).

4.3. Third Condition: Preventing Spillage of Reservoir 5 by PC Refraining from Production

This condition is modeled by Equation (25):

$$E \cdot (1 - V(6, t)) \geq s(5, t) \quad \forall t \in T \tag{25}$$

5. Search for an Optimal Solution

It is necessary to define the optimality criterion. The problem includes a simultaneous search for two optima, one from PC’s and the other from HC’s perspective. Criteria for finding an optimal solution in the possible solution space can be to find a solution that maximizes sum:  $profit_{HC} + profit_{PC}$ . This multiobjective problem of finding a coexistence production plan of two independent companies both holding hydro powerplants in the same river flow is handled through use of the model of PC-HC symbiosis described in Section 4 and the multi-objective procedure presented in Figure 4. This procedure will create a solution space of  $k$  possible solutions as shown in Figure 5.

Figure 4. Proposed procedure for finding an optimal solution.

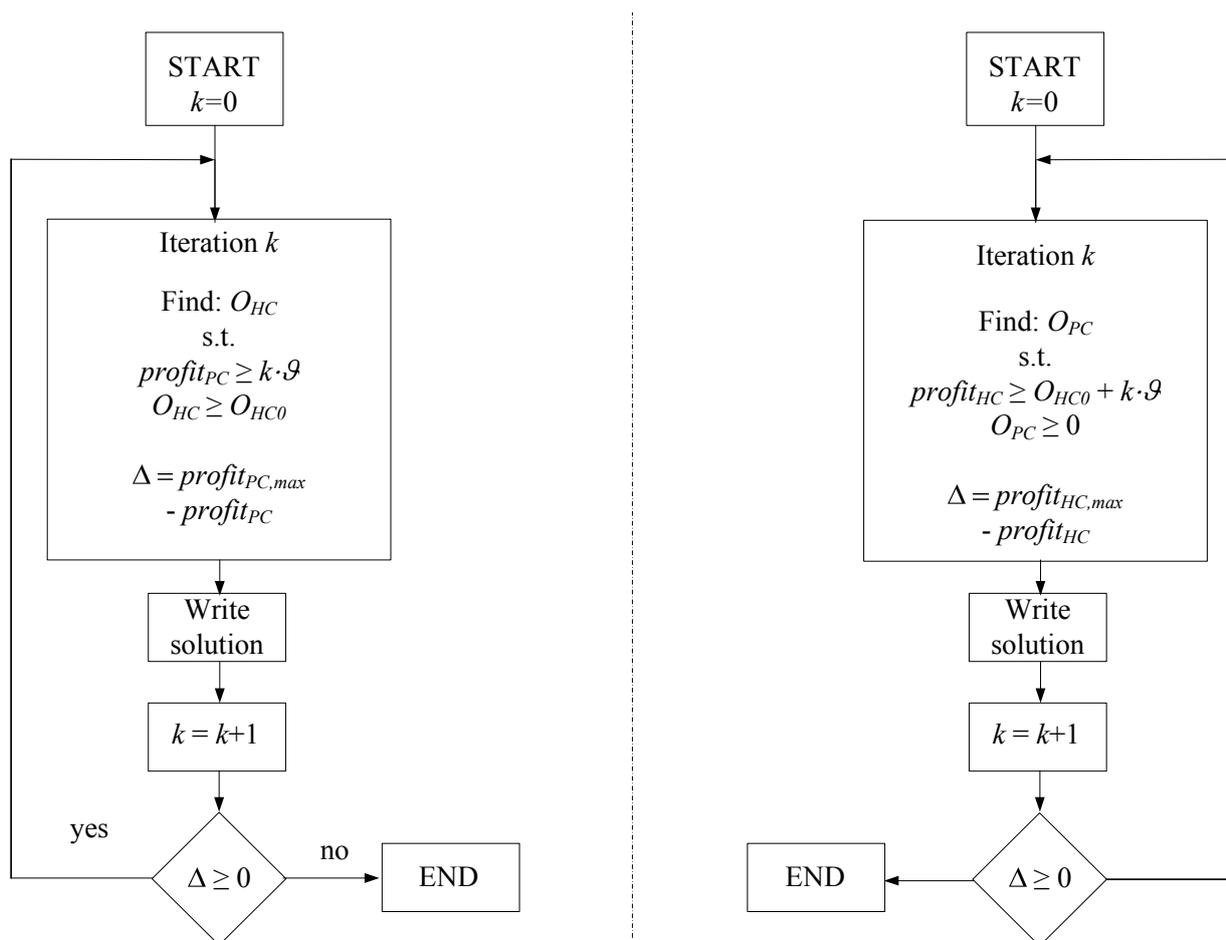
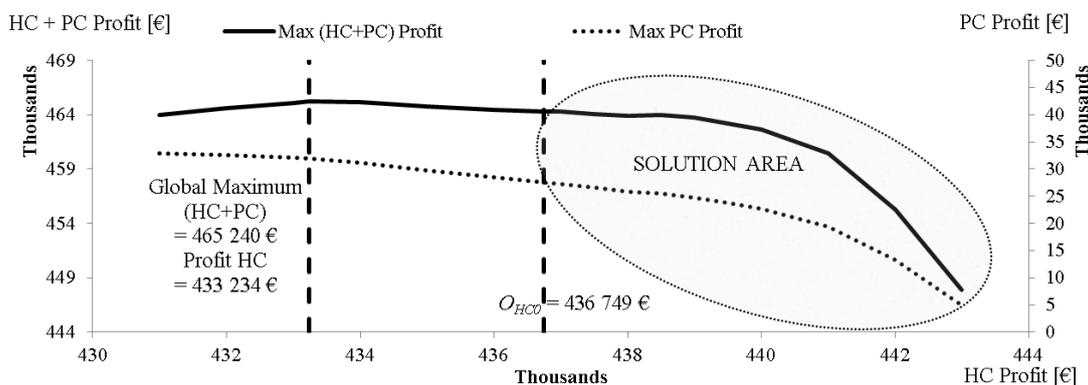


Figure 5. Set of possible solutions.



This work does not address individual strategies for PC or HC. The intention was to prove that their symbiosis is possible. HC conditions for symbiosis are defined in Equations (18) and (26):

$$\begin{aligned}
 O_{HC} &= \max \sum_{t \in T} \left( \sum_{i \in I \setminus \{6\}} P(i, t) \cdot \pi(t) + C_{PC_w}(t) \right) \\
 s.t. & \\
 O_{HC} &\geq O_{HCO} \\
 profit_{PC} &\geq 0
 \end{aligned}
 \tag{26}$$

In the same manner PC conditions for symbiosis are defined in Equation (27):

$$\begin{aligned}
 O_{PC} &= \max \sum_{t \in T} (P(6, t) \cdot \pi(t) - C_{PC}(t)) \\
 s.t. & \\
 O_{PC} &\geq 0 \\
 profit_{HC} &\geq O_{HCO}
 \end{aligned}
 \tag{27}$$

In this model, the constraints are handled through the use of constraint programming. Relations between variables are stated in form of equality and inequality constraints which are defined under General Algebraic Modeling System (GAMS) using indexed assignments. The syntax in GAMS for performing indexed assignments is extremely powerful [19]. This operation offers what may be thought of as simultaneous or parallel assignment and provides a concise way of specifying large amounts of data.

### 6. Case Study

Hourly prices are taken from the EEX pool on 20 January 2012. Initial and final water contents are arbitrary. Water flows with the time delay are taken into account. Hourly water inflows are considered known and constant over the whole optimization period. HPP production costs and start-up costs are neglected. Forecasted water inflows are shown in Table 3.

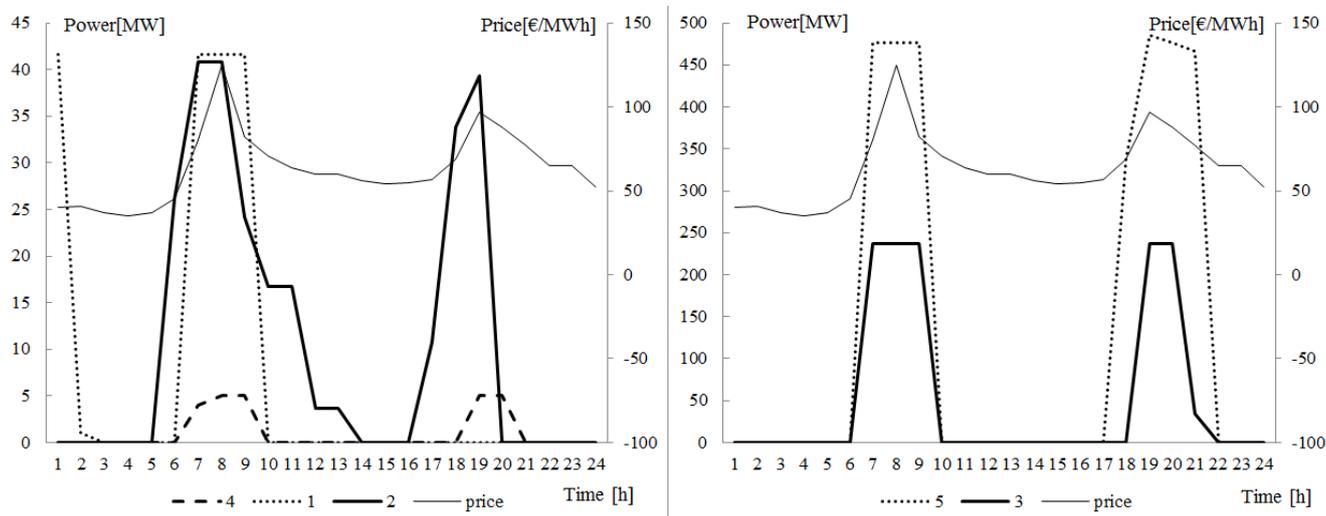
Table 3. Natural water inflow of the reservoir *i*.

Reservoir	1	2	3	4	5	6
$W(i, t)$ [m <sup>3</sup> /s]	20	20	5	15	5	0

6.1. Base Model

Characteristics of Base model are described in “Base model” chapter. Equation (13) is relevant in Base model. Optimal HC dispatch in base model is shown on Figure 6 with  $O_{HC0} = 436,749 \text{ €}$ .

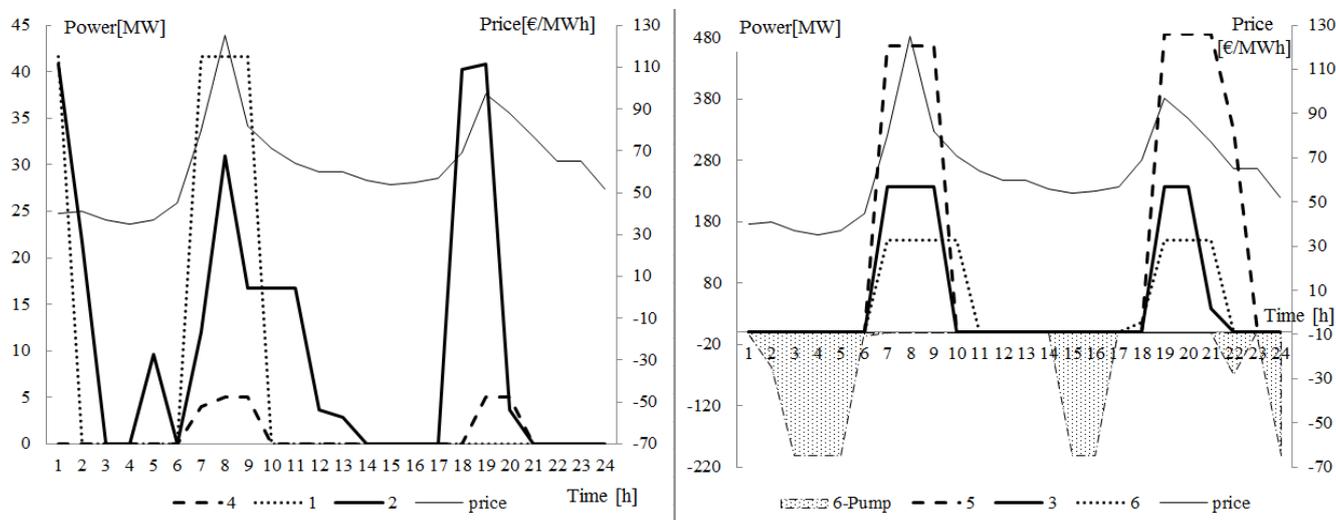
Figure 6. Optimal HC dispatch in base model.



6.1.1. Case A

PSHPP 6 is introduced in addition to the base model in the way described in the “Problem description” section. Case A assumes that new PSHPP 6 is owned by HC. In this way HC operation flexibility is improved, as it now operates one additional plant. Therefore it is expected that HC can attain greater profit than in the base model. For Case A Equation (14) is relevant. Optimal HC dispatch in Case A is shown in Figure 7. In this case  $O_{HC+PC} = 465,189 \text{ €}$  ( $profit_{1,2,3,4,5} = 432,180 \text{ €}$ ,  $profit_{PPSHPP6} = 33,009 \text{ €}$ ). This point was also shown in Figure 5 and named “Global Maximum”.

Figure 7. Optimal HC dispatch in Case A.



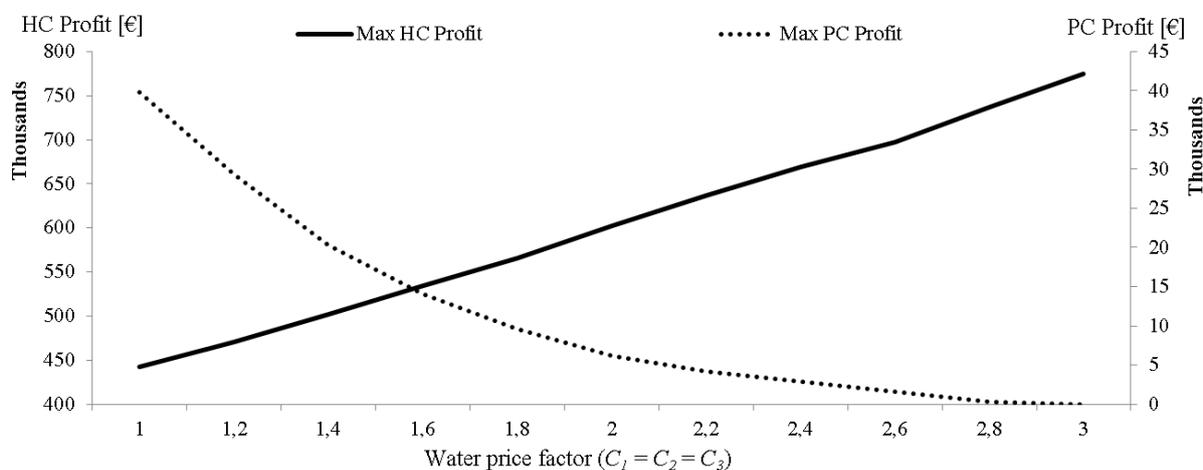
### 6.1.2. Case B

Again PSHPP 6 is introduced in addition to the base model in the way described in the “Problem description” section. However, Case B assumes that new PSHPP 6 is independently owned by PC. Equations (26) and (27) are now relevant. Maximum HC profit in Case B is  $profit_{HC,max} = 442,230$  € (without constraints on the PC profit) and maximum PC profit is  $profit_{PC,max} = 39,800$  € (without constraints on the HC profit). Based on Equations (26) and (27)  $O_{HC} = 442,192$  € and  $O_{PC} = 27,124$  €.

### 6.2. Water Price Factors Impacts on PC/HC Profit

Figure 8 shows the correlation between  $C_j$  and PC\HC profit. For simplicity it is assumed that  $C_1 = C_2 = C_3$ .

**Figure 8.** Water price factors impacts on PC\HC maximal profit.

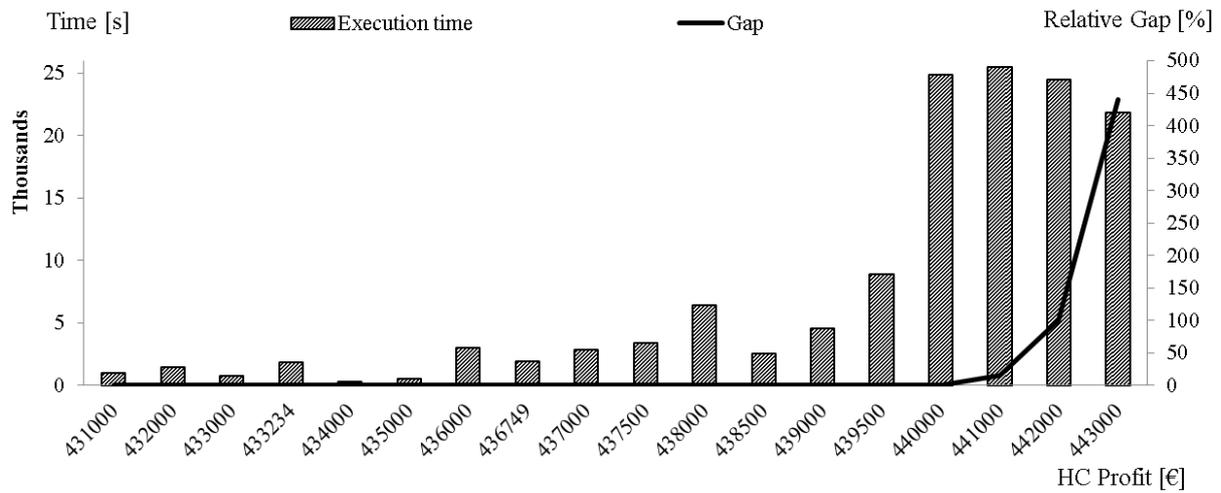


Pump efficiency (0.8) multiplied by the ratio between highest and lowest electricity price (124.99 €/MWh/35.16 €/MWh) is equal to 2.84. That is the value of water price factors for which maximal profit of PC (without constraints on the HC profit) becomes equal to 0 (Figure 8).

### 6.3. Set of Possible Solutions

The set of possible solutions is determined by the constraints in Equations (26) and (27). The model is executed for several definite intervals of free variable  $profit_{HC}$  (setting upper and lower bound of  $profit_{HC}$  each time) and  $O_{PC}$  is recorded (see Figure 5). Water price factors are set to:  $C_1 = 1.01$ ;  $C_2 = 1.02$  and  $C_3 = 1.03$  (to model the significance of the water content in reservoir 5). Relative solution gap and time for each execution are shown on Figure 9. Beside the “Global Maximum” (as per **Case A**) there is another local maximum around point  $profit_{HC} = 438,500$  which shows that the solution space is not convex. Also, it is important to emphasize that in this specific case the “Global Maximum” is not in the possible solution space.

**Figure 9.** Relative solution gap and execution time.



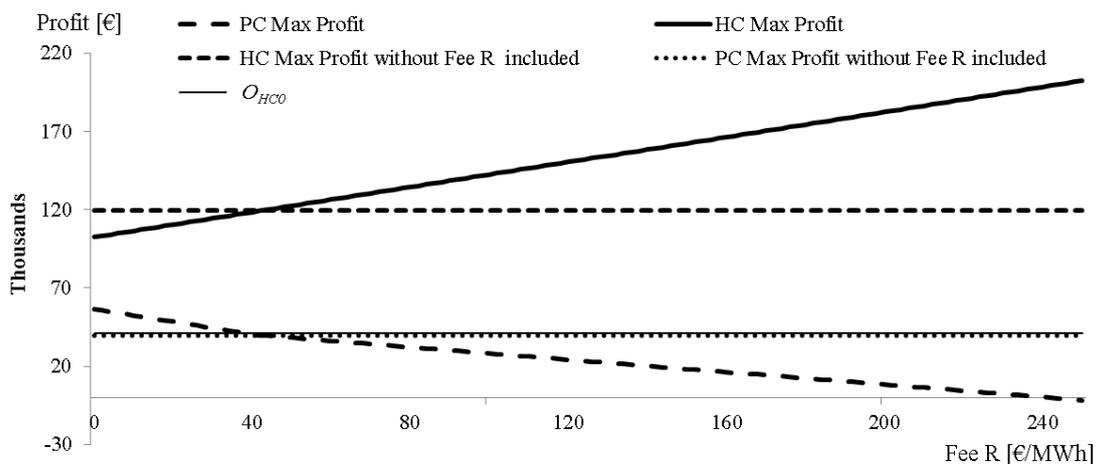
Relative solution gap in Figure 9 represents the relative difference between current primal and dual solution of the simplex search method. The presented results were obtained on Lenovo E520, 2.1 GHz based processor with 4 GB RAM using CPLEX under GAMS.

6.4. Fee R Impacts on PC/HC Profit

Two scenarios of very high inflows in reservoir 5 are analyzed. Initial and final content of reservoir 5 are set to its maximum. For the sake of simplicity and computational time savings inflows for all other reservoirs are set to 0. For simplicity in both scenarios all water price factors are set to 1 and  $\pi_{MAX}$  is set to a very high value (500 €/MWh).

Scenario 1. Inflow = 400 m<sup>3</sup>/s in hours 1 and 2. Inflows in other hours are all equal to 0.

**Figure 10.** Fee R impacts on PC/HC profit in Scenario 1.

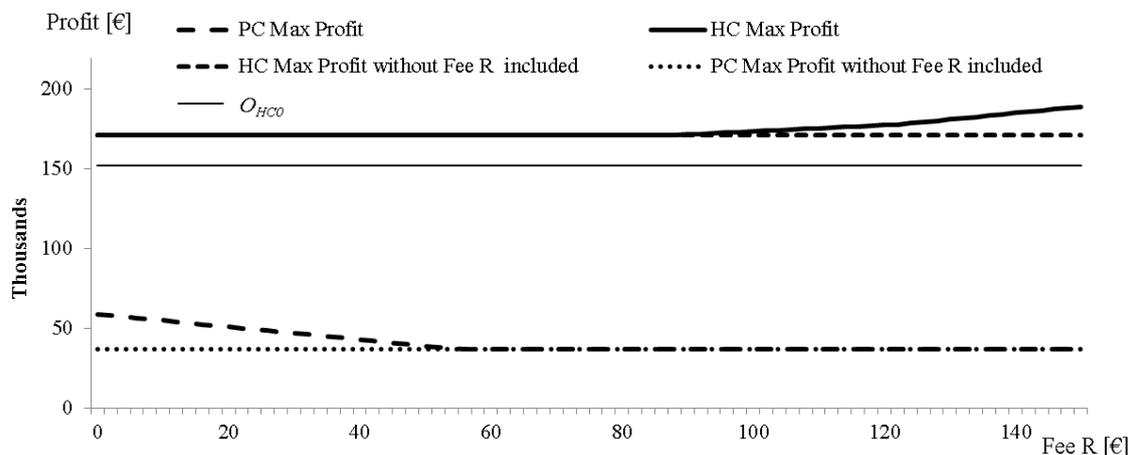


Naturally, PC prefers lower fee R values while HC prefers higher fee R values (Figure 10). Since high inflows are present in the first two hours, there is no time for actions that will prevent  $F(t)$  binary variable activation [ $F(t)$  becomes equal to 1]. Activation of  $F(t)$  forces PC in pump regime as it described in the “Model of PC-HC symbiosis” section. Therefore, for low fee R values maximum

profit of HC is lower than in model without fee  $R$  included. From the same reason PC is forced to operate with losses for high fee  $R$  values.

Scenario 2. Inflow = 400 m<sup>3</sup>/s in hours 11 and 12. Inflows in other hours are all equal to 0.

**Figure 11.** Fee  $R$  impacts on PC/HC profit in Scenario 2.



In contrast to Scenario 1, in Scenario 2 HC has equal maximum profit as in the model without fee  $R$  included for low fee values of  $R$ . Additionally PC has equal maximum profit as in model without fee  $R$  included for high fee  $R$  values (Figure 11). In this scenario both PC and HC can prepare themselves during first 10 hours for high inflows and thus are able to prevent unwilling activation of  $F(t)$  binary variable (when it would cause financial losses). In both scenarios HC's maximum profit is significantly higher than  $O_{HCO}$  for all fee  $R$  values.

## 7. Conclusions

The relationship between two independent hydropower companies that use water from the same basin is described in this paper. In this case study they shared one reservoir. One of them is the owner of that reservoir and pays a concession fee for water usage. The other one pays a compensation for water use from the same reservoir to the first company. A model of symbiosis between existing and a new company that owns the pumped storage hydropower plant was proposed and a model of financial interaction was introduced. It involves water usage compensation and stimulations to avoid spillage—the loss of resources—in case it is possible and profitable. The water payment model is based on the reservoir discretization method that is used for defining hydropower plant performance curves. There are two conditions for this kind of symbiosis. The first condition is that the new company's operating strategy does not have a negative impact on the incumbent company's profit. The second condition, just as important as the first, is that the new company generates a profit. The conducted optimization results show the existence of a set of solutions that satisfy both conditions. It is shown that in the proposed symbiosis model the incumbent company can “borrow” some space in the new company's upper reservoir to avoid loss of water. Individual company strategies can be further derived based on the proposed model. The results motivate future research.

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