

Article

New Deterministic Mathematical Model for Estimating the Useful Energy Output of a Medium-Sized Solar Domestic Hot Water System

Mirosław Zukowski ^{1,*}  and Walery Jezierski ²

¹ Department of HVAC Engineering, Faculty of Civil Engineering and Environmental Sciences, Białystok University of Technology, Wiejska 45E Street, 15-351 Białystok, Poland

² Department of Energy Efficient Construction and Geodesy, Faculty of Civil Engineering and Environmental Sciences, Białystok University of Technology, Wiejska 45E Street, 15-351 Białystok, Poland; w.jezierski@pb.edu.pl

* Correspondence: m.zukowski@pb.edu.pl

Abstract: According to the authors of this paper, the mathematical point of view allows us to see what sometimes cannot be seen from the designer's point of view. The aim of this study was to estimate the influence of the most important parameters (volume of heat storage tanks, daily consumption of domestic hot water, optical efficiency, heat loss coefficient, and total area of a solar collector) on the thermal power output of solar domestic hot water (SDHW) system in European climatic conditions. Three deterministic mathematical models of these relationships for Madrid, Budapest, and Helsinki were created. The database for the development of these models was carried out using computer simulations made in the TRNSYS software environment. The SDHW system located at the Białystok University of Technology (Poland) was the source of the measurement results used to validate the simulation model. The mathematical optimization procedure showed that the maximum annual useful energy output that can be obtained from 1 m² of gross collector area is 1303 kWh in the case of Madrid, 918.5 kWh for Budapest, and 768 kWh for Helsinki weather conditions.

Keywords: thermal solar collectors; useful energy output; energy simulations; solar domestic hot water system; deterministic mathematical model



Citation: Zukowski, M.; Jezierski, W. New Deterministic Mathematical Model for Estimating the Useful Energy Output of a Medium-Sized Solar Domestic Hot Water System. *Energies* **2021**, *14*, 2753. <https://doi.org/10.3390/en14102753>

Academic Editor: Carlo Renno

Received: 7 April 2021

Accepted: 10 May 2021

Published: 11 May 2021

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

The effects of climate change are becoming more and more visible around the world. It manifests itself mainly in extreme weather events, loss of biodiversity and acidification of waters. The continuous increase in the global mean temperature directly affects the global rise in sea level. The global action plan to combat far-reaching climate change was set out in the Paris Agreement and signed by 195 countries in December 2015 [1]. The next step towards achieving climate neutrality was the decision of the European Commission (EC) taken in December 2020 [2]. EC proposed a 55% reduction in CO₂ emissions in 2030 compared to 1990 levels.

One way to achieve this difficult goal is the widespread use of the conversion of solar radiation into heat and electricity. The most effective devices implementing this process are thermal solar collectors, which support the heating of domestic hot water (DHW). It is difficult to achieve optimal performance of these systems at the design stage and later in the operating conditions. This is due to the large number of components in this system combined with the time-varying effects of weather conditions. Therefore, there are many methods of designing solar domestic hot water (SDHW) systems. Generally, we can divide them into two groups [3]. The first type of methods are based on analytical formulas describing the heat exchange phenomena occurring in these systems. An example might be one of the most frequently used method, i.e., the f-Chart [4,5]. Based on the value of

solar radiation intensity, ambient temperature, and DHW demand, the amount of energy that can be provided by the designed system is calculated. The calculation algorithm of this method is based on the function of two variables whose values are influenced by the design and operational parameters. The error of the f-Chart method largely depends on the accuracy of the data describing a given system and ranges from 5% to 10% [6].

The second way to determine the performance of SDHW systems are computer energy simulations. They allow for very accurate modelling of the equipment characteristics, the system of pipes connecting them, and weather conditions. A large number of parameters describing the entire system requires the use of specialized software. TRNSYS, Design-Build (based on a computing engine EnergyPlus), GetSolar, T*SOL, RETScreenL, and Polysun are the most popular computer programs often used in scientific analysis.

The authors of this paper focused the most attention on the application of the TRNSYS (Transient System Simulation Program) software package to simulate the operation of the SDHW system during the literature review. This computer program was used in this study.

Essabbani et al. [7] simulated the operation of a SDHW system consisting of 20 m² of flat plate solar collectors (FPC), a 300-L storage tank, and an electric heater. The testing facility was located in Oujda (Morocco) and the period of analysis was from the December 2011 to March 2012. The influence of total solar radiation, ambient temperature, and daily DHW consumption on the system performance was analyzed experimentally. Additionally, a numerical simulation of this installation was applied using the TRNSYS software. As a result of this research, it was found that the average share of solar energy in the total energy demand was 45% in the winter season. In March, the solar fraction was higher than in other months. However, the maximum required auxiliary energy was 740 MJ in December.

Ayompe et al. [8] developed and validated a model of a solar water heating system with forced circulation used in typical European residential homes. The tested rig consisted of two flat plate collectors (a total area of 4 m²), an evacuated tube collector (ETC) with 30 heat pipe tubes, two 300 L hot water storage tanks, a control unit, and a pump station. A quasi-steady state simulation of this SDHW system were made using the TRNSYS software. This model was validated based on the results of measurements performed on a bench located in Dublin (Ireland). As it turned out, the mean absolute error in calculations of the heat collected by the collectors was 14.1% in the case of FPC circuit and 16.8% in the case of ETC loop.

The optimization of SDHW systems intended for single-family housing was performed by Hobbi and Siddiqui [9]. In this analysis, a forced indirect circulation system with a heat exchanger separating the solar circuit from the DHW circuit was considered. A south-facing flat collector with a slope equal to Montreal's latitude (45.5 degrees) was assumed. The share of energy obtained from solar collectors in the total energy demand for water heating was used as the optimization parameter. Computer simulations using TRNSYS software were used in this study. It was found that the designed system can satisfy 83% to 97% (in summer) and 30% to 62% (in winter) of the demand for DHW. It should be noted that the authors of this article did not use any special optimization methods.

The model of the SDHW system consisting of two FPC (5 m² of the absorber area), a heat storage tank (volume of 0.4 m³), and a coil heat exchanger was developed by Mongibello et al. [10]. The operation of this system was simulated using the code written in MatLab environment and using TRNSYS software. The input data was the results of the measurements made at the ENEA Portici Research Center. The results of the calculations from both solvers were compared and it turned out that they are characterized by the good accordance.

Lima et al. [11] simulated a thermosiphon water heating system equipped with FPC, a storage tank, and an auxiliary heater in the climate conditions of Sao Paulo (Brazil). The hourly meteorological data collected by the Institute of Astronomy and Geosciences of the University of de Sao Paulo were used in the calculations. The main conclusion of this analysis was the following: the optimal configuration of this kind of thermosiphon system depends on the investor's concerns and local electricity prices.

The model of the SDHW system equipped with a heat pipe ETC and cooperating with an additional gas heater was made by Mehmood et al. [12] in the TRNSYS software environment. An experimental setup installed at National University of Sciences and Technology, Islamabad (Pakistan) was developed to validate the simulation model. A comparison of the results of calculations and experimental tests showed that the discrepancy between them ranges from 5 to 9%. Based on the computational analysis, it was found that the modelled system can replace conventional gas water heaters, providing 23% to 56% fossil fuel savings annually.

The performance of the ETC and FPC collectors were compared by Mohasseb and Kasaeian [13] in two different climatic conditions: cold weather-Tabriz (Iran) and hot-Bandar-Abbas (Iran). The operation of two SDHW systems with different collectors was simulated using the TRNSYS software. Based on the results of the calculations, it was found that the climatic conditions and inlet temperature had a greater influence on the operation of FPC than ETC. Besides, the annual useful energy from ETC is 30% higher than from FPC in cold climate, and 15% in hot weather conditions.

Mathematical equations that give relationships between the characteristic parameters of the SDHW system were not developed in the analyses described above. Additionally, optimization algorithms were not used to solve the issues related to the selection of the optimal variant.

The main disadvantages of computer simulations is the relatively long time needed for creating a model of the system and the large number of input data. They are dedicated to scientific analysis rather than being a tool used by designers. Therefore, it was decided to apply a deterministic mathematical modelling in order to simplify the method of the energy yield estimation from thermal solar collectors. The article presents a new approach for determining the annual useful energy output based on a multivariable function of volume of heat storage tanks, daily consumption of DHW, and the basic parameters of a solar collector. Climatic conditions are not included as one of the variables because they consist of many factors and cannot be described by one coherent indicator. In optimization analysis, this type of parameter is called uncontrollable and heterogeneous. That is why the numerical experiment was performed separately for three European weather zones. This is a drawback of the method presented here, but on the other hand, the development of a model for a specific climate zone allows to achieve high accuracy of the modelling results.

2. Materials and Methods

2.1. Model of SDHW System Developed in the TRNSYS Software Environment

The development of a simplified method of determining the energy yield from thermal collectors consisted of three main stages. First, the SDHW model of the system was created using the TRNSYS software. The second step was to validate this model based on a measurement database. Deterministic mathematical modelling was the third stage of this study. The selected TRNSYS simulation results created a database for this modelling. The selection of appropriate variants was based on the symmetrical three-level plan.

The modelled system consisted of a circuit with a battery of solar collectors (glycol solution as a heat transfer medium (HTM)) and a DHW circuit (water as a HTM). Both loops were separated by a plate compact heat exchanger. The other elements were heat storage tanks, separate pumps for each loop, a control and regulation system, and an auxiliary water heater. The schematic diagram of the model is shown in Figure 1. The list of the main modules creating this model is presented in Table 1. The model was built on the basis of an example solution presented in [14].

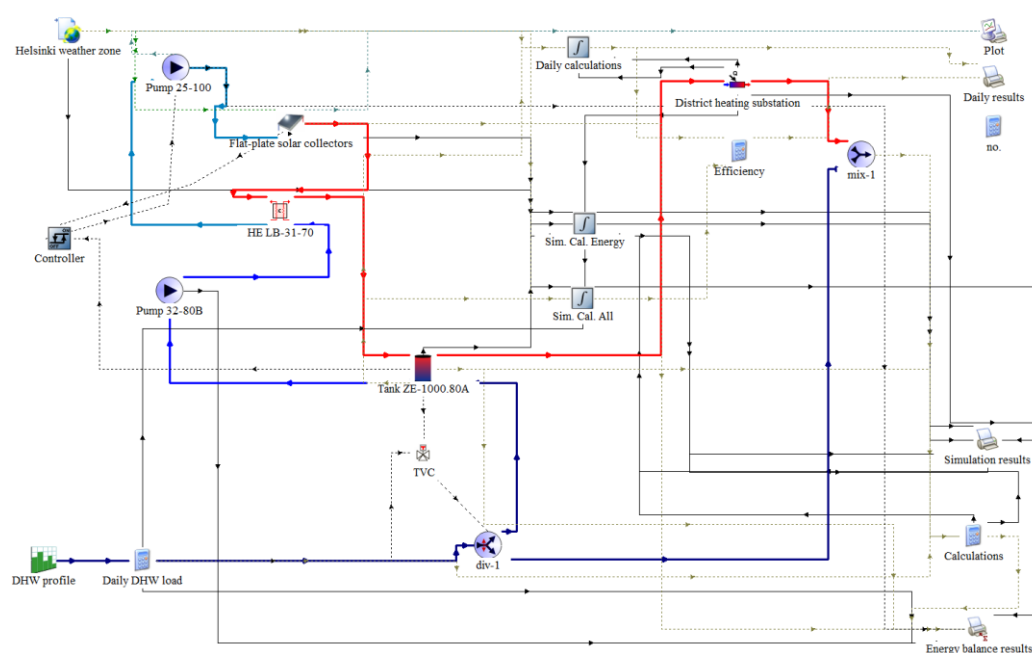


Figure 1. Scheme of the model of SDHW system developed in the TRNSYS software environment.

Table 1. List of the main modules of the SDHW system model.

Type Number	Module Name
1	Solar Collector; Quadratic Efficiency, 2nd Order Incidence Angle Modifiers
11	Controlled flow diverter
14	Time Dependent Forcing Function
15	Weather Data Processor; Combines data reading, radiation processing and sky temperature calculations
24	Quantity Integrator
91	Heat Exchanger with Constant Effectiveness
114	Single Speed Pump
115	Controller for Tempering Valves-Heating Mode
138	Auxiliary Fluid Heater
158	Cylindrical Storage Tank
165	ON/OFF Differential Controller

An important parameter that must be considered when creating this type of model is the consumption profile of DHW for occupants in the housing sector. An example of hourly prediction of DHW consumption was presented in [15]. This research work was based on data from the historical time series of DHW heat use in hotels in Norway. In the review of existing studies on hot water demand [16], we can find structured information on the DHW consumption patterns specified in technical standards for different types of buildings. The authors also noted the important impact of the mains water temperature on the energy consumption for DHW heating. A Time Dependent Forcing Function (TDFF) that allowed to employ the DHW consumption profile was used in this simulation. The repeated pattern of this function is established by a set of time-dependent data points shown for this case in Table 2. In order to reduce the time step during the calculations, linear interpolation is used.

Table 2. DHW profile assumed in the model.

Hour	0–7	8	9	10	11	12	13–17	18	19	20	21	22	23–24
Fraction	0	0.2	0.2	0	0.1	0.1	0	0	0.1	0.1	0.1	0.1	0

Three different locations are considered in the current analysis. These are Madrid (Spain), Budapest (Hungary), and Helsinki (Finland) for which different mean annual values of the main water temperatures were assumed: 17 °C, 15 °C, and 12 °C, respectively. On the other hand, the temperature of the hot water collected in the storage tank was constant for each variant and amounted to 55 °C.

Presented above the model of SDHW system, due to its characteristics, is the most dedicated to typical multi-family apartment buildings. That is why the title of the article uses the term “medium size”.

2.2. Validation of the SDHW System Model

As is well known, even a precisely developed numerical model may be inaccurate. Therefore, it was decided to compare the results of computer simulations with the measurement results. The source of the measurement database was the SDHW system located at the Bialystok University of Technology (Poland). The screenshot of the automatic operating parameters recording system is shown in Figure 2. A more detailed description of this measuring stand can be found in [17].

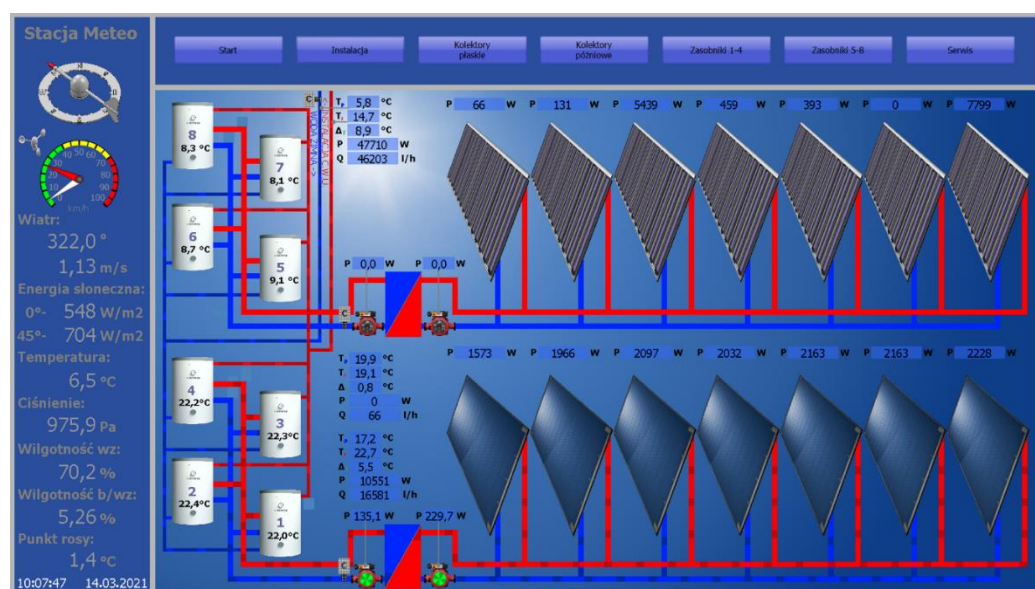


Figure 2. The screenshot of the scheme of the SDHW system and working online data acquisition system.

This active solar heating system consists of:

- Seven rows of flat plate collectors, five connected in series in each row, with a total gross area of 71.9 m²;
- Seven rows of evacuated tube collectors, three connected in series in each row, with an active total area of 74.45 m²;
- Eight heat storage tanks with a volume of 1 m³ each (four for each type of collectors).

The set of FPC was selected for comparative analysis. These solar panels have the following characteristics:

- Coefficient of collector efficiency—0.784 (-);
- Heat loss coefficient—3.64 W/(m²K);
- Temperature dependence heat loss coefficient—0.00185 W/(m²K²);
- Maximum flow rate—0.000033 m³/s;
- Gross area of single panel—2.054324 m².

In order to estimate the accuracy of the model, the measurement results of an annual useful energy output Q_{SC} (in kWh/m²) from 5 years (2016–2020), were selected. The

measurement of this value was made using a heat meter placed on the solar loop next to the heat exchanger. Figure 3 shows the amount of energy supplied by the solar collectors during the year as a function of the intensity of solar radiation Q_{SR} (in kWh/m²) incident perpendicular to the plane of the collectors (blue rhombus).

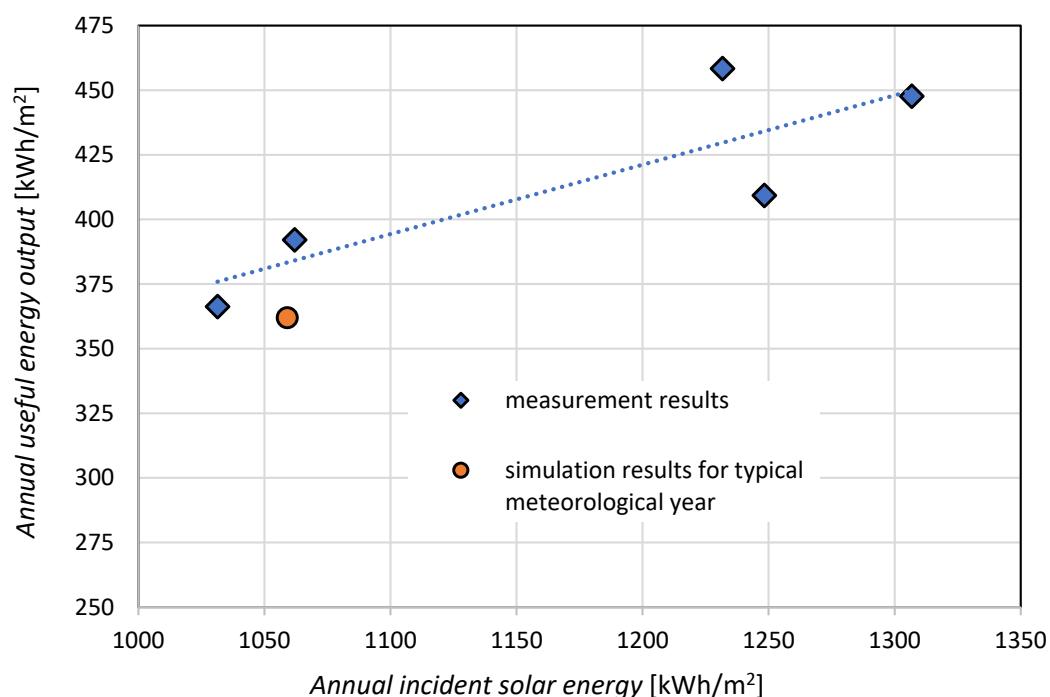


Figure 3. Dependence of annual useful energy output on the intensity of solar radiation for the FPC collector system.

The results of these measurements can be approximated (dashed line in Figure 3) by the following equation: $Q_{SC} = 0.2685 \cdot Q_{SR} + 98.946$.

The error resulting from comparing the result of the computer simulation (circle in Figure 3) for a typical meteorological year (TMY) with the value calculated from the above equation for TMY conditions is 5.87%. Therefore, it should be considered that the developed model of SDHW system reflects the trend of the real complex physical process with relatively good accuracy.

2.3. Mathematical Model for Determining the Solar Collector Useful Energy Output

Mathematical modelling was used as the research method. It allows to describe the operation of the tested object, determine the necessary output data, and determine the optimal values of the input parameters using mathematical relationships. The use of mathematical modelling allows you to resign from physical modelling, reduce the number of samples, and the labor intensity compared to the physical experiment [18]. The main component in such procedure is the mathematical model. Utilitarianism, efficiency, and practical usefulness of this type of model can be ensured by developing short algorithms. In order to obtain the desired information, the most important factors/parameters describing the object under study or phenomenon should be used.

The main goal of this research was to determine the influence of the most important parameters on thermal performance of SDHW systems in European climatic conditions. Thus, it was assumed that the annual useful energy output Q_{SC} of the solar collectors is an objective function Y , and the value of Y depends on the following parameters:

- Volume of heat storage tanks V_{ST} [m³] (variable X_1),
- Solar collector total area F_{SC} [m²] (variable X_2),
- Intercept (maximum) efficiency of solar collector η_o [-] (variable X_3),

- Heat loss coefficient a_1 [W/(m²K)] (variable X_4),
- Daily consumption of DHW m [kg/day] (variable X_5).

The useful energy output makes a clear physical sense and is strongly explicit. The selected factors (X_1 – X_5) are measurable, controllable, independent, and consistent, i.e., they meet the basic requirements of mathematical modelling [19]. It should be noted that climatic conditions cannot be considered as a parameter in such mathematical models. This factor is presented by a set of various meteorological indicators, which are difficult to combine with one coherent parameter. In optimization, this is the so-called heterogeneous and uncontrollable parameter. As mentioned before, it was decided to choose three different locations: (group I) Madrid (Spain)—($Q_{I,SC} = Y_{I,i}$), (group II) Budapest (Hungary)—($Q_{II,SC} = Y_{II,i}$), and (group III) Helsinki (Finland)—($Q_{III,SC} = Y_{III,i}$), the characteristics of which are shown in Table 3. The parameters listed in this table were calculated using the TRNSYS software using the data for a typical meteorological year. Additionally, three charts that show the variability of the basic meteorological parameters during the year for three selected locations are created. Figure 4 shows the average daily air temperature, Figure 5 shows wind velocity, and the distribution of total horizontal solar radiation is presented in Figure 6.

Table 3. List of the main modules of the SDHW system model.

Group Number	Town	Average Annual Outside Temperature [°C]	Average Annual Wind Velocity [m/s]	Solar Beam Radiation on Horizontal Surface [kWh/m ²]	Solar Diffuse Radiation on Horizontal Surface [kWh/m ²]
I	Madrid	13.9	2.6	1006	654
II	Budapest	10.4	3.0	529	668
III	Helsinki	4.5	3.6	463	483

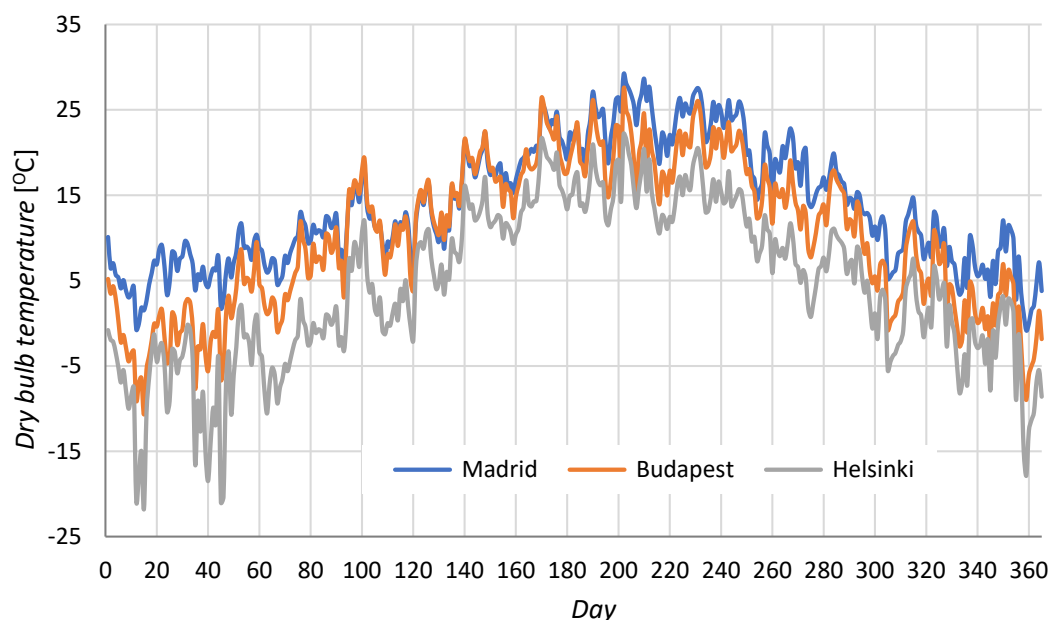


Figure 4. Average daily dry bulb temperature.

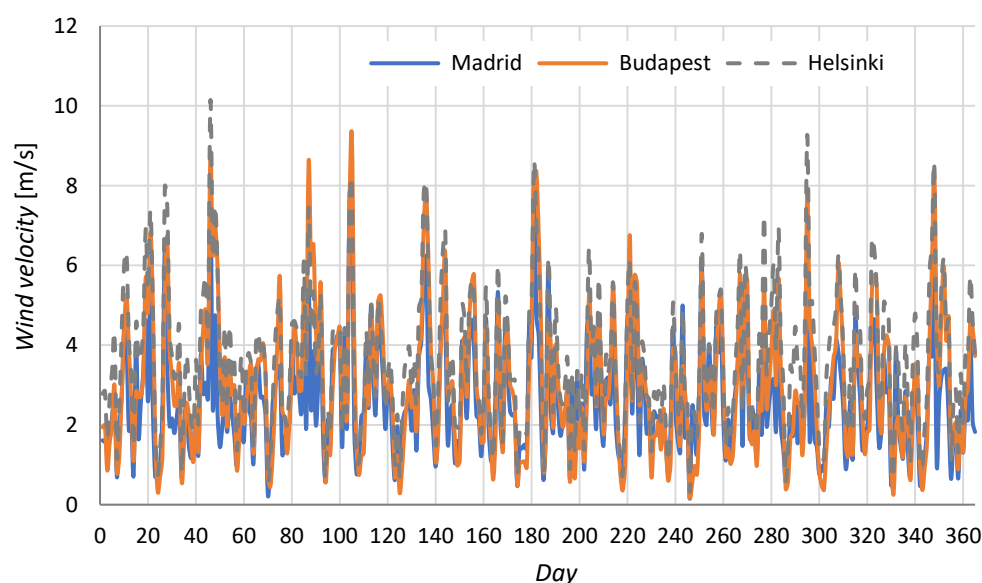


Figure 5. Average daily wind velocity.

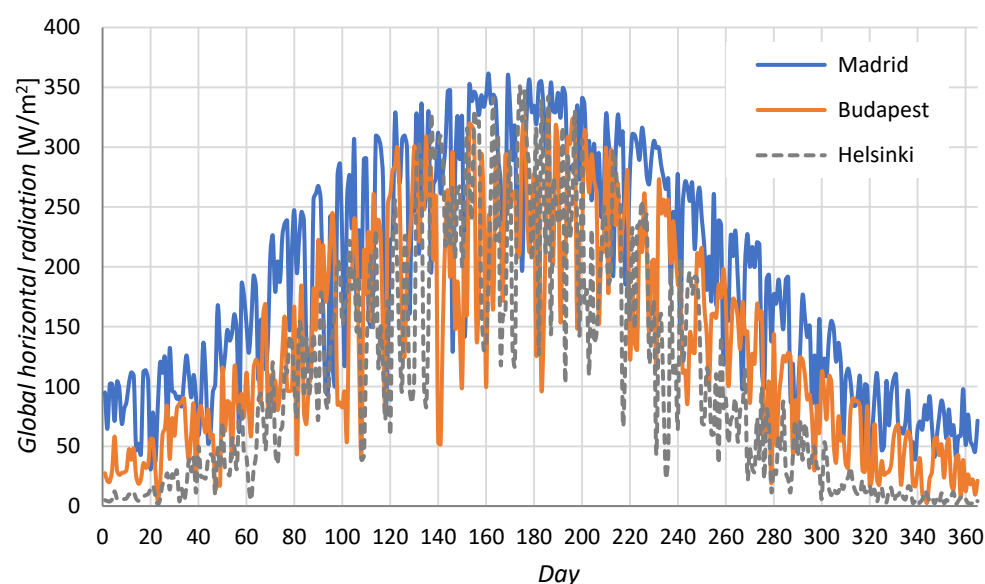


Figure 6. Average daily global solar radiation incident on a horizontal plane.

As can be seen from the characteristics presented in Table 3, the best conditions for converting solar energy into the heat exist in Madrid. This location is characterized by a high average annual temperature of the outside air, relatively low windiness, and a high value of solar radiation with a higher share of beam fraction (61%). Budapest is a city with moderate weather conditions. In this case, the total intensity of solar radiation is relatively high, but a predominance of the diffusion fraction (56%) is observed. The worst conditions in terms of installation of solar collectors are in Helsinki. There is a low outside air temperature, the lowest insolation value, and a relatively high average wind velocity.

Each of these locations differs in latitude and the share of direct and diffused solar radiation. Therefore, the optimal solar panel inclination angle β has been determined for each city separately. The results of the multivariate calculations are presented in the form of three graphs in Figure 7 for Madrid, Figure 8 for Budapest, and Figure 9 for Helsinki. The maximum annual useful energy output allowed to determine the optimal value of β , which is 36 degrees for Madrid, 39.5 degrees for Budapest, and 47.5 degrees for Helsinki. In the case of Madrid, we can observe an interesting phenomenon. It consists in the fact that the

optimal inclination of the collector is in a wide range from 35 to 44 degrees with only small deviations. The opposite is Budapest, where we have a very clearly marked maximum.

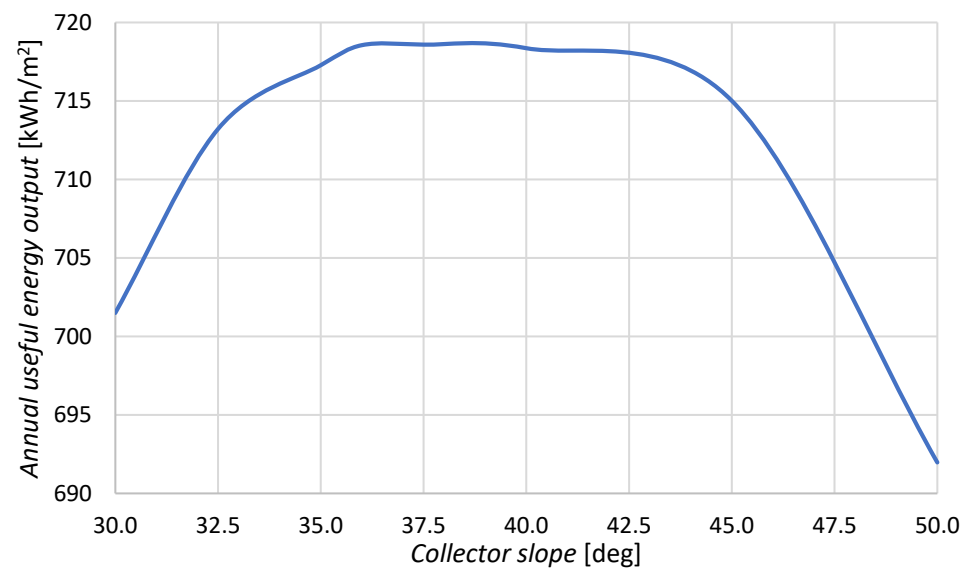


Figure 7. Dependence of the inclination angle of the solar collector on annual useful energy output for Madrid.

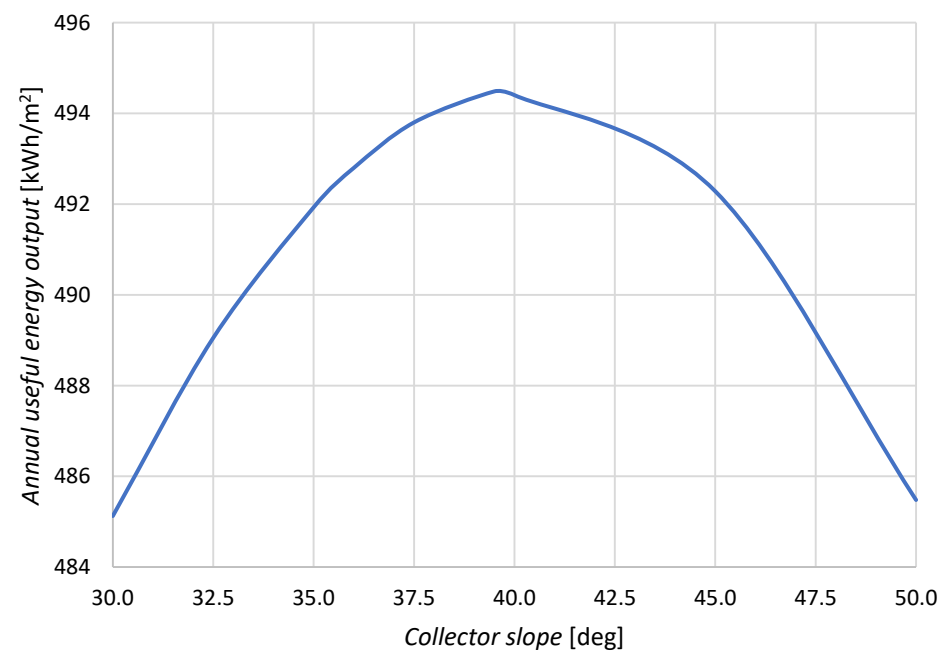


Figure 8. Dependence of the inclination angle of the solar collector on an annual useful energy output for Budapest.

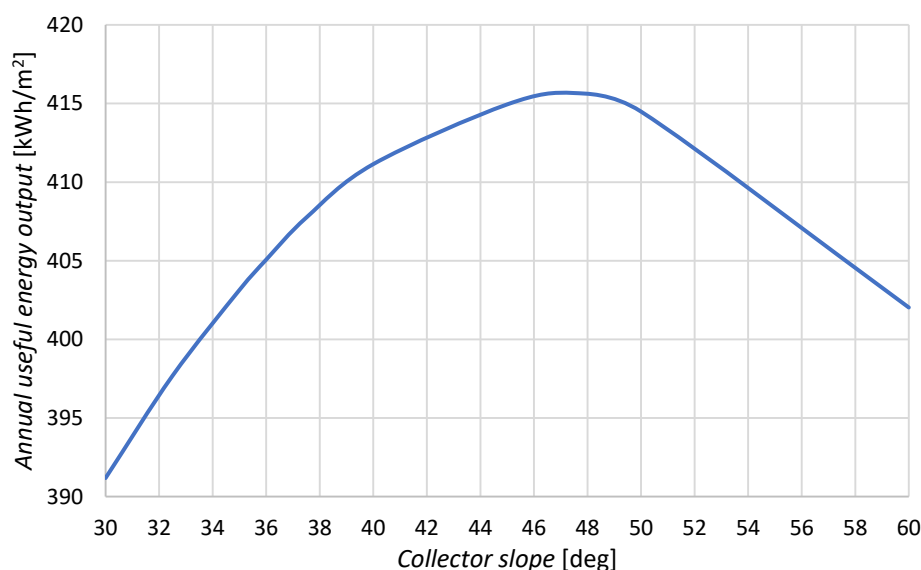


Figure 9. Dependence of the inclination angle of the solar collector on an annual useful energy output for Helsinki.

Thus, due to the heterogeneous nature of the weather factors, three new mathematical models for each of the groups of climatic conditions were developed. It was assumed that the relationship $Y = f(X_1, X_2, X_3, X_4, X_5)$ can be described as a second-degree polynomial equation in the following form:

$$Y = a_0 + a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4 + a_5X_5 + a_{12}X_1X_2 + a_{13}X_1X_3 + a_{14}X_1X_4 + a_{15}X_1X_5 + a_{23}X_2X_3 + a_{24}X_2X_4 + a_{25}X_2X_5 + a_{34}X_3X_4 + a_{35}X_3X_5 + a_{45}X_4X_5 + a_{11}X_1X_1 + a_{22}X_2X_2 + a_{33}X_3X_3 + a_{44}X_4X_4 + a_{55}X_5X_5 \quad (1)$$

The ranges of variability for each of the factors were assumed at three levels:

- Low-marked as (−1),
- Medium-marked as (0),
- High-marked as (1).

Table 4 presents the natural values of the factors $\dot{X}_1 \div \dot{X}_5$ and the corresponding normalized values $(X_1) \div (X_5)$, which are defined by the following formula [19]:

$$X_i = \frac{\dot{X}_i - 0.5 \cdot (\dot{X}_{i,max} + \dot{X}_{i,min})}{0.5 \cdot (\dot{X}_{i,max} - \dot{X}_{i,min})} \quad (2)$$

where: $\dot{X}_i$, $\dot{X}_{i,max}$, $\dot{X}_{i,min}$ —current, maximum and minimum natural values of the i -th factor, respectively.

Table 4. Natural and standardized values of the factors.

Factor Level $\dot{X}_i$	$V_{ST} [\text{m}^3]$ (X_1)	$F_{SC} [\text{m}^2]$ (X_2)	$\eta_o [-]$ (X_3)	$a_1 [\text{W}/(\text{m}^2\text{K})]$ (X_4)	$m [\text{kg}/\text{d}]$ (X_5)
Low (−1)	2.0	30.0	0.58	2.0	1200
Medium (0)	7.0	100.0	0.78	6.0	4800
High (+1)	12.0	170.0	0.98	10.0	8400
Range of variability ΔX_i	5.0	70.0	0.20	4.0	3600

The range of collector surfaces, volumes of accumulation tanks, and consumption of DHW have been selected to represent the SDHW medium-sized system.

In order to create a database for the description of Y relationship (Equation (1)), a series of computer simulations were performed in TRNSYS environment software. Based on a symmetrical three-level plan [18], only 26 series of calculations were made. The $Q_{SC,i}$ (Y_i) estimation results for three different locations are shown in Table 5.

Table 5. Planning matrix and calculation results of annual useful energy output [kWh/m²].

No	V_{ST}	F_{SC}	η_o	a_1	m	$Q_{I,SC}$	$Q_{II,SC}$	$Q_{III,SC}$
	X_1	X_2	X_3	X_4	X_5	$Y_{I,i}$	$Y_{II,i}$	$Y_{III,i}$
1	2	30	0.58	2	8400	721	504	425
	−1	−1	−1	−1	+1			
2	12	30	0.58	2	1200	570	407	339
	+1	−1	−1	−1	−1			
3	2	170	0.58	2	1200	118	108	98
	−1	+1	−1	−1	−1			
4	12	170	0.58	2	8400	500	349	293
	+1	+1	−1	−1	+1			
5	2	30	0.98	2	1200	641	545	474
	−1	−1	+1	−1	−1			
6	12	30	0.98	2	8400	1289	901	758
	+1	−1	+1	−1	+1			
7	2	170	0.98	2	8400	505	438	373
	−1	+1	+1	−1	+1			
8	12	170	0.98	2	1200	145	141	133
	+1	+1	+1	−1	−1			
9	2	30	0.58	10	1200	286	192	161
	−1	−1	−1	+1	−1			
10	12	30	0.58	10	8400	570	391	331
	+1	−1	−1	+1	+1			
11	2	170	0.58	10	8400	133	99	85
	−1	+1	−1	+1	+1			
12	12	170	0.58	10	1200	75	52	45
	+1	+1	−1	+1	−1			
13	2	30	0.98	10	8400	799	575	490
	−1	−1	+1	+1	+1			
14	12	30	0.98	10	1200	508	355	300
	+1	−1	+1	+1	−1			
15	2	170	0.98	10	1200	97	75	64
	−1	+1	+1	+1	−1			
16	12	170	0.98	10	8400	294	203	171
	+1	+1	+1	+1	+1			
17	2	100	0.78	6	4800	376	272	229
	−1	0	0	0	0			
18	12	100	0.78	6	4800	450	310	260
	+1	0	0	0	0			
19	7	30	0.78	6	4800	791	544	455
	0	−1	0	0	0			
20	7	170	0.78	6	4800	296	207	173
	0	+1	0	0	0			
21	7	100	0.58	6	4800	330	227	191
	0	0	−1	0	0			

Table 5. Cont.

No	V_{ST}	F_{SC}	η_o	a_1	m	$Q_{I,SC}$	$Q_{II,SC}$	$Q_{III,SC}$
	X_1	X_2	X_3	X_4	X_5	$Y_{I,i}$	$Y_{II,i}$	$Y_{III,i}$
22	7 0	100 0	0.98 +1	6 0	4800 0	544	384	322
23	7 0	100 0	0.78 0	2 −1	4800 0	661	505	425
24	7 0	100 0	0.78 0	10 +1	4800 0	309	213	178
25	7 0	100 0	0.78 0	6 0	1200 −1	193	159	139
26	7 0	100 0	0.78 0	6 0	8400 +1	509	354	297

The results of these calculations $Q_{I,SC}$, $Q_{II,SC}$, $Q_{III,SC}$ (Table 5) were used to approximate the functions of the examined objects. The least squares method was chosen for this approximation. This procedure was performed using matrix calculus $\mathbf{A} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}$. In this vector notation: $\mathbf{A}$ denotes the column vector of the coefficients a_i , $\mathbf{X}$ -experiment plan matrix, $\mathbf{Y}$ -column vector of the calculation results, and superscript T means matrix transpose. In this way, the three mathematical models were developed:

Madrid

$$\hat{Y}_I = 444.07 + 40.28X_1 - 222.80X_2 + 84.39X_3 - 115.50X_4 + 149.27X_5 - 20.56X_1X_2 - 16.43X_1X_3 - 24.18X_1X_4 + 21.19X_1X_5 - 54.68X_2X_3 + 24.31X_2X_4 - 23.56X_2X_5 - 2.31X_3X_4 + 38.81X_3X_5 - 44.43X_4X_5 - 31.07X_1X_1 + 99.42X_2X_2 - 7.08X_3X_3 + 40.92X_4X_4 - 93.07X_5X_5, \quad (3)$$

Budapest

$$\hat{Y}_{II} = 313.51 + 16.72X_1 - 152.33X_2 + 71.56X_3 - 96.83X_4 + 98.89X_5 - 13.3X_1X_2 - 20.56X_1X_3 - 8.94X_1X_4 + 12.06X_1X_5 - 39.56X_2X_3 + 14.81X_2X_4 - 9.94X_2X_5 - 11.43X_3X_4 + 26.06X_3X_5 - 24.81X_4X_5 - 22.51X_1X_1 + 61.98X_2X_2 - 8.02X_3X_3 + 45.48X_4X_4 - 51.01X_5X_5, \quad (4)$$

Helsinki

$$\hat{Y}_{III} = 262.68 + 12.83X_1 - 127.66X_2 + 62.02X_3 - 82.94X_4 + 81.67X_5 - 9.75X_1X_2 - 17.37X_1X_3 - 6.63X_1X_4 + 10.10X_1X_5 - 34.12X_2X_3 + 11.38X_2X_4 - 9.25X_2X_5 - 11.25X_3X_4 + 20.63X_3X_5 - 18.62X_4X_5 - 18.18X_1X_1 + 51.31X_2X_2 - 6.19X_3X_3 + 38.81X_4X_4 - 44.68X_5X_5. \quad (5)$$

A characteristic feature of deterministic models is their unambiguous agreement between the external influence and the reaction to that influence. This principle was applied when the adequacy of the model was checked. The relevance of the Equations (3)–(5) were assessed by comparing the variance of the mean value S_y^2 and the residual variance S_r^2 :

$$S_y^2 = \frac{\sum (Y_i - \bar{Y})^2}{N - 1}, \quad (6)$$

$$S_r^2 = \frac{\sum (\hat{Y}_i - \bar{Y})^2}{N - N_b}, \quad (7)$$

where N is the number of calculations ($N = 26$) and N_b is the number of coefficients in the regression equation ($N_b = 21$).

The Fisher criterion [18,20], commonly known as the F -test, was used to exam the above models:

$$F = \frac{S_y^2(N-1)}{S_r^2(N-N_b)} \quad (8)$$

The regression equation approximates the calculation results correctly if the Fisher criterion F is much higher than the tabular value F_t . For Equations (3)–(5), the value of F is 152.47, 30.98, and 32.38 for Madrid, Budapest, and Helsinki, respectively. However, the tabular value of F_t [19] is smaller and equal to 4.52. Therefore, it can be concluded that the developed models are adequate because F_t is many times lower than F . A particularly good fit of the regression predictions to the calculation results is confirmed by the coefficient of determinations R^2 , which are in the range from 0.9935 to 0.9942. Additionally, the significance of the coefficients of the regression equations was also checked using the t -criterion [19], with a positive result. The description of this procedure was omitted due to its complexity. Therefore, it was found that the models (Equations (3)–(5)) should be considered useful for further analysis.

In conclusion, it should be mentioned that the data set for the development of the mathematical model and its subsequent testing was created using the TRNSYS software. The model was validated on the basis of the measurement data collected by the corresponding author of this paper.

3. Results and Discussion

The analysis of the influence of the examined factors $X_1 \div X_5$ on the annual useful energy output Q_{SC} of the collector was performed on mathematical models described by equations Equations (3)–(5). The center of the multivariate-space is formed by the average level of coefficients, i.e., in this case:

- Volume of heat storage tanks $V_{ST} = 7 \text{ m}^3$ ($X_1 = 0$),
- Solar collector total area $F_{SC} = 100 \text{ m}^2$ ($X_2 = 0$),
- Intercept (maximum) efficiency of solar collector $\eta_o = 0.78$ ($X_3 = 0$),
- Heat loss coefficient $a_1 = 6 \text{ W/(m}^2\text{K)}$ ($X_4 = 0$),
- Daily consumption of DHW $m = 4800 \text{ kg/day}$ ($X_5 = 0$).

The annual useful energy output of the collector for selected groups of weather conditions are as follows: for Madrid (group I) $Q_{I.SC} = 444 \text{ kWh/m}^2$, for Budapest (group II) $Q_{II.SC} = 313 \text{ kWh/m}^2$, and for Helsinki (III group) $Q_{III.SC} = 263 \text{ kWh/m}^2$. As can be seen from the above results, the Q_{SC} value for the same solar collector fluctuates significantly, i.e., compared to Madrid it decreases by 29.5% for Budapest and as much as 41.0% for Helsinki. It is caused by changes in various climatic parameters, especially by total tilted surface solar radiation, which decrease by 27.6% for Budapest and 38.1% for Helsinki. However, when comparing the decrease in solar radiation on a horizontal plane, these values are slightly different: 27.9% for Budapest and 43.0% for Helsinki. This proves that the use of simple mathematical equations where Q_{SC} is only a function of the radiation intensity can cause some errors in designing SDHW systems.

The influence of each factor on the Q_{SC} was then estimated from a mathematical point of view. In each of the considered models, the level of parameters impact varies. When changing the selected factors from the lower to the higher level, as could be expected, the energy output increases with the increase of the $V_{ST}(X_1)$, $\eta_o(X_3)$, $m(X_5)$ and decreases with increasing factors $F_{SC}(X_2)$ and $a_1(X_4)$. The effects of changes in $\Delta Q_{i.SC}$ (natural and percentage) for each model are given in Table 5. As can be seen, weather conditions do not change the nature of the influence of the factors under consideration, but strongly change the degree of this influence. The most interesting and useful information for designers is the data on the interaction of factors in the obtained models. When analyzing the signs and values of the coefficients with double interactions certain regularities were noticed. They apply to all locations because the nature of the influence of the factors does not change in them.

Analysis of the results from Table 6 shows that increasing the capacity of the storage tank from 2 m³ to 12 m³ causes a change in Q_{SC} : the highest increase by 21.6% is achieved in hot weather conditions (Madrid) and the smallest by 11.1% in more severe climatic conditions (Helsinki).

Table 6. Effects of changing $X_1 \div X_5$ factors from lower to higher level on annual useful energy output [kWh/m²/%].

Group Number	V_{ST}	F_{SC}	η_o	a_1	m
		X_2	X_3	X_4	X_5
I	+80.56 (21.6%)	−445.60 (−58.2%)	+168.78 (+47.9%)	−231.00 (−38.5%)	+298.54 (+148%)
II	+33.44 (+12.2%)	−304.66 (−57.7%)	+143.12 (+61.2%)	−193.66 (−42.5%)	+197.78 (+120.9%)
III	+25.66 (+11.1%)	−255.32 (−57.8%)	+124.12 (+63.8%)	−165.88 (−43.1%)	+163.34 (+119.8%)

Furthermore, it can be stated that increasing the collector area from 30 to 170 m² strongly reduces the collector's energy output and shows the highest negative effect among the factors under consideration. Despite the fact that the natural values of this decrease strongly fluctuate in various weather conditions, the percentage decrease in all locations is around −58%.

While change the optical efficiency of the collector η_o from 0.58 to 0.98 significantly increases the collector's energy production, which ranges from 48% for Madrid to 64% for Helsinki. The increase in heat loss of the collector, characterized by the a_1 coefficient, significantly reduces the Q_{SC} value. The average percentage change in this value is about 41% and slightly differs for all meteorological conditions.

The daily DHW consumption has the highest (positive) impact on the collector's energy yield. Q_{SC} increases from 148% (Madrid) to 120% (Helsinki) with a change of m from 1200 kg/day to 8400 kg/day.

The interactions between the five parameters tested in this study are discussed below. Regarding the $V_{ST}(X_1)$, it can be observed that the positive effect of this factor decreases with the increase of the factors $F_{SC}(X_2)$, $\eta_o(X_3)$, $a_1(X_4)$, but increases with the increase of $m(X_5)$. For the two factors $V_{ST}(X_1)$ and $m(X_5)$ there is a synergism phenomenon, because when increasing them together, it affects more strongly than it would be when increasing them separately. The large negative effect of $F_{SC}(X_2)$ increases with the increase of the factors $V_{ST}(X_1)$, $\eta_o(X_3)$, $m(X_5)$, but it decreases with the increase of $a_1(X_4)$.

For the factor $\eta_o(X_3)$, its positive effect decreases with the increase of the factors $V_{ST}(X_1)$, $F_{SC}(X_2)$, $a_1(X_4)$, but increases with the increase of $m(X_5)$. For the two factors $\eta_o(X_3)$ and $m(X_5)$ there is also a synergy, as when each of them increase together the influence is stronger than it would be with a separate increase. For the factor $m(X_5)$, the high positive impact will be the stronger, with the greater factors $V_{ST}(X_1)$ and $\eta_o(X_3)$. For these two pairs of factors $V_{ST}(X_1) - m(X_5)$ and $\eta_o(X_3) - m(X_5)$ there is synergism, as each of them influences more strongly when increased together than with their separate increased. The positive influence of factor $m(X_5)$ will be smaller if the factors $F_{SC}(X_2)$ and $a_1(X_4)$ increase.

Next, the procedure of optimizing the value of energy produced by the collectors was carried out. As could be expected, the optimal values of the SDHW system parameters occur on the boundaries of the factor space. The maximum of the examined functions was achieved with the extreme values of the factors $X_2 = -1$, $X_3 = +1$, $X_4 = -1$, $X_5 = +1$. Only factor X_1 changed the optimal value in different locations.

Thus, the maximum annual useful energy output (Y_I) is 1303 kWh/m² for Madrid. $Y_{I, \max}$ is achieved with the following parameters: the storage tank capacity $V_{ST} = 12$ m³, the collector area $F_{SC} = 30$ m², the collector's optical efficiency $\eta_o = 0.98$, the linear heat loss coefficient $a_1 = 2$ W/m²·K, and the daily DHW consumption $m = 8400$ kg/day. In the case

of Budapest weather zone, the function Y_{II} has a maximum of 918.5 kWh/m² for storage tank capacity $V_{ST} = 10.5$ m³, and for the function Y_{III} (Helsinki weather conditions) the maximum of 768 kWh/m² is obtained for $V_{ST} = 10$ m³.

It should be noted that the above analysis makes perfect sense from the point of view of mathematical relations. However, in design practice, sometimes a slightly different approach to such issues is needed. It often happens that in practical conditions we cannot obtain the maximum or optimal value of an objective function $Y_{i, \max}$ estimated as a result of solving the optimization problem. In the case of SDHW systems, this may be due to the inability to install the appropriate number of collectors due to the insufficient roof area. Sometimes it also happens that it is not possible to design an appropriate volume of hot water storage tanks due to the lack of space in the district heating substations.

Using mathematical models, Equations (3)–(5), we can check whether the designed system is far from the optimal parameters in a given situation. To visualize this issue, three charts have been drawn up. The first one shows the effect of changing the amount of accumulated hot water on the useful energy output of the collectors (Figure 10). The second one shows the change of DHW consumption effects on the same parameter characterizing the collectors (Figure 11). The impact of the collector surface on their performance is presented in Figure 12. The calculations were made for the parameters located in the center of the multivariate-space that is formed by the average level of coefficients, i.e., equal to 0.

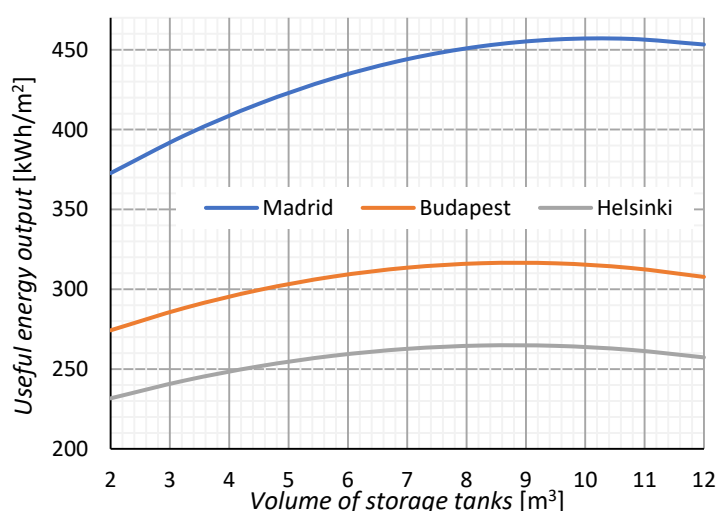


Figure 10. Dependence of the volume of heat storage tanks on an annual useful energy output.

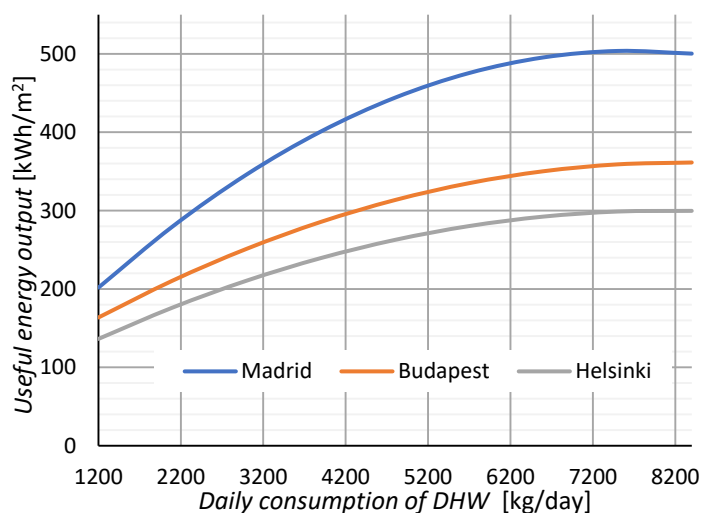


Figure 11. Dependence of the daily consumption of DHW on an annual useful energy output.

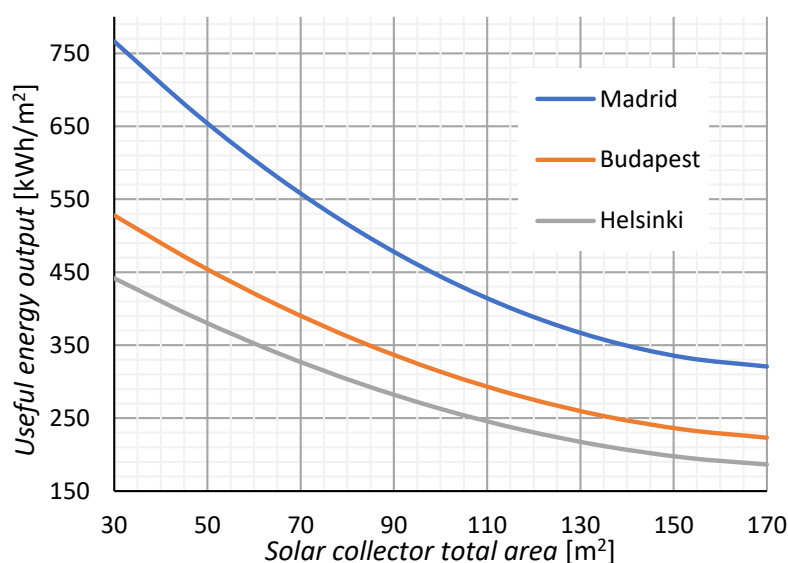


Figure 12. The influence of the collector surface area on an annual useful energy output.

The trend of the influence of the storage tanks capacity on the collector performance (Figure 10) clearly shows that an increase in tank volume above 8 m^3 does not make any practical sense in the case of Madrid, although the optimum is 10 m^3 . For the other two locations (Budapest, Helsinki), the maximum volume is between 8 m^3 and 9 m^3 . However, as in the previous case, reducing the storage tank capacity by 2 m^3 causes a decrease in useful energy output only by about 2.5%.

As can be seen in Figure 11, the change in the DHW consumption has a significant impact on the useful energy output of the collectors. An almost linear increase in the Q_{SC} of solar collectors in 70% of the range of DHW consumption change can be observed. In operating conditions, it often happens that the energy efficiency of the SDHW system is lower in summer, when the demand for DHW decreases due to holiday trips.

Sometimes it happens that an excessive number of collectors is designed when, for example, there is a large roof area to be used. As can be seen from the charts presented in Figure 12, the amount of energy converted into the heat by 1 m^2 of solar panels significantly decreases with increasing their surface area. So, as we can see, the oversizing of the SDHW system significantly reduces its efficiency.

Figure 13 shows the effect of DHW consumption on the outlet temperature T_{OUT} of the solar collectors. The calculations were made for Madrid, 30 m^3 of collector area and 4 m^3 of storage tank capacity in the period from 1 to 15 September. As could be expected, the highest temperature is achieved with the lowest DHW demand of 1200 kg/day . A 3.5-fold increase in daily consumption lowers T_{OUT} value by an average of 21.2%, while its 7-fold increase reduces this temperature by slightly less, i.e., by 25%. Lowering the temperature of the water flowing out of the solar collectors will be associated of course with the need to provide more energy from a supplementary heat source.

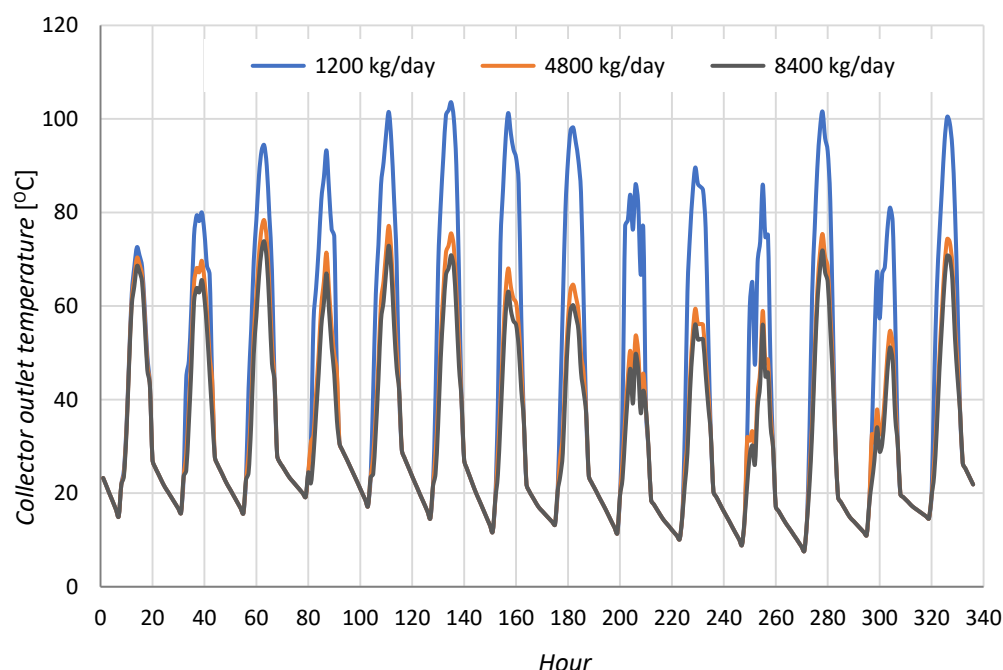


Figure 13. Outlet temperature from the solar collectors depending on the DHW consumption.

4. Conclusions

This article presents a mathematical approach to the issue of the influence of selected parameters on the thermal performance of a solar collector. Based on the results of the computational experiment for hot (Madrid), moderate (Budapest) and cold (Helsinki) climatic zones, three new deterministic mathematical models (Equations (3)–(5)) were developed. These equations can be used to determine the influence of five factors: volume of heat storage tanks, solar collector total area, maximum efficiency, and heat loss coefficient of solar collector, and daily consumption of DHW on the annual useful energy output. Based on these models, the degree and nature of the influence of the examined factors were estimated. It should be noted that the analysis presented in this article is the most appropriate for medium-sized SDHW systems.

The change of the examined factors from the lower to the upper level showed significant fluctuations in their impact both for individual factors and for the weather zones (Table 5). The useful energy output Q_{SC} of thermal solar collectors to a large extent depends on the weather conditions and, above all, the intensity of solar radiation. The Q_{SC} value is 29.5% lower for Budapest and 41% lower for Helsinki compared to the most suitable solar conversion zone in which Madrid is located.

The results of the optimization procedure according to the energy criterion showed that the maximum annual useful energy output, which is possible to supply from solar collectors, is 1303 kWh/m² for Madrid weather conditions, 918.5 kWh/m² for Budapest, and 768 kWh/m² for Helsinki conditions.

The above results can be obtained at the optimum collector inclination β for a particular location, i.e., 36 degrees for Madrid, 39.5 degrees for Budapest, and 47.5 degrees for Helsinki. These values were not calculated based on the latitude that designers usually estimate it at. The novelty is the determination of the β value using the method described in this article.

It is planned to develop similar models for selected main cities in Europe. The obtained dependencies will allow to write a simple application that can be used for a relatively precise determination of the annual useful energy output of thermal collectors. It is also planned to carry out a similar mathematical experiment using Solar Fraction as the objective function.

Author Contributions: Conceptualization, M.Z. and W.J.; methodology, M.Z. and W.J.; energy simulations, M.Z.; mathematical modelling, W.J.; validation, M.Z.; formal analysis, W.J. and M.Z.; investigation, M.Z. and W.J.; writing—original draft preparation, M.Z. and W.J.; writing—review and editing, M.Z. and W.J.; project administration, M.Z.; funding acquisition, M.Z. and W.J. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Bialystok University of Technology and financed by the Ministry of Science and Higher Education of the Republic of Poland [grant numbers WZ/WB-IIS/9/2019, WZ/WB-IIL/5/2019].

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Paris Agreement. Available online: <https://unfccc.int/process-and-meetings/the-paris-agreement/the-paris-agreement> (accessed on 15 March 2021).
2. Decision of the European Commission. Available online: https://ec.europa.eu/clima/policies/eu-climate-action/2030_ctp_en (accessed on 15 March 2021).
3. Kulkarni, G.N.; Kedare, S.B.; Bandyopadhyay, S. Determination of design space and optimization of solar water heating systems. *Sol. Energy* **2007**, *81*, 958–968. [\[CrossRef\]](#)
4. Klein, S.A.; Beckman, W.A.; Duffie, J.A. Design Procedure for Solar Heating Systems. *Sol. Energy* **1976**, *18*, 113–126. [\[CrossRef\]](#)
5. Beckman, W.A.; Klein, S.A.; Duffie, J.A. *Solar Heating Design by the F-Chart Method*; Wiley & Sons, Inc.: New York, NY, USA, 1977.
6. Haberl, J.S.; Cho, S. *Literature Review of Uncertainty of Analysis Methods (F-Chart Program)*; Report to the Texas Commission on Environmental Quality; Energy Systems Laboratory, Texas A&M University: College Station, TX, USA, 2004.
7. Essabbani, T.; Moufekkir, F.; Mezrhab, A.; Naji, H. Numerical Computation of Thermal Performance of a Simulation of a Solar Domestic Hot Water System. *Appl. Sol. Energy* **2015**, *51*, 22–33. [\[CrossRef\]](#)
8. Ayompe, L.M.; Duffy, A.; McCormack, S.J.; Conlon, M. Validated TRNSYS model for forced circulation solar water heating systems with flat plate and heat pipe evacuated tube collectors. *Appl. Therm. Eng.* **2011**, *31*, 1536–1542. [\[CrossRef\]](#)
9. Hobbi, A.; Siddiqui, K. Optimal design of a forced circulation solar water heating system for a residential unit in cold climate using TRNSYS. *Sol. Energy* **2009**, *83*, 700–714. [\[CrossRef\]](#)
10. Mongibello, L.; Bianco, N.; Caliano, M.; de Lucad, A.; Graditi, G. Transient analysis of a solar domestic hot water system using two different solvers. *Energy Procedia* **2015**, *81*, 89–99. [\[CrossRef\]](#)
11. Lima, J.B.A.; Prado, R.T.A.; Taborianski, V.M. Optimization of tank and flat-plate collector of solar water heating system for single-family households to assure economic efficiency through the TRNSYS program. *Renew. Energy* **2006**, *31*, 1581–1595. [\[CrossRef\]](#)
12. Mehmood, A.; Waqas, A.; Said, Z.; Rahman, S.M.A.; Akram, M. Performance evaluation of solar water heating system with heat pipe evacuated tubes provided with natural gas backup. *Energy Rep.* **2019**, *5*, 1432–1444. [\[CrossRef\]](#)
13. Mohasseb, S.; Kasaeian, A. Comparing the Performance of Flat Plate Collector and Evacuated Tube Collector for Building and Industrial Usage in Hot and Cold Climate in Iran with TRNSYS Software. In Proceedings of the Ninth International Conference on Engineering Computational Technology, Napoli, Italy, 2–5 September 2014.
14. TRNSYS 18. *TRaNsient SYstem Simulation Program*; Tutorials; Solar Energy Laboratory, University of Wisconsin-Madison: Madison, WI, USA, 2020; Volume 9.
15. Ivanko, D.; Sorensen, A.L.; Nord, N. Selecting the model and influencing variables for DHW heat use prediction in a hotel in Norway. *Energy Build.* **2020**, *228*, 110441. [\[CrossRef\]](#)
16. Fuentes, E.; Arce, L.; Salom, J. A review of domestic hot water consumption profiles for application in systems and buildings energy performance analysis. *Renew. Sustain. Energy Rev.* **2018**, *81*, 1530–1547. [\[CrossRef\]](#)
17. Zukowski, M.; Radzajewska, P. A New Method to Determine the Annual Energy Output of Liquid-Based Solar Collectors. *Energies* **2019**, *12*, 4586. [\[CrossRef\]](#)
18. Durakovic, B. Design of Experiments Application, Concepts, Examples: State of the Art. *Period. Eng. Nat. Sci.* **2017**, *5*, 421–439. [\[CrossRef\]](#)
19. Korzynski, M. Metodyka eksperymentu. In *Planowanie, Realizacja i Statystyczne Opracowanie Wynikow Eksperymentow Technologicznych*; WNT: Warszawa, Poland, 2006.
20. Hartmann, K.; Lezki, E.; Schär, W. *Statistische Versuchsplanung und—Auswertung in der Stoffwirtschaft*; VEB: Leipzig, Germany, 1977.